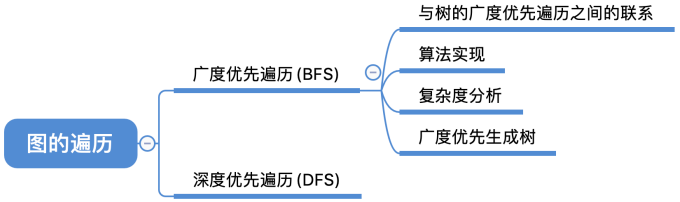
本节内容

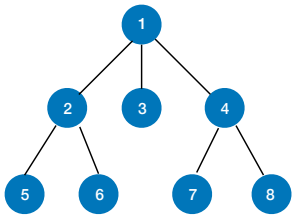
图的遍历

BFS

知识总览

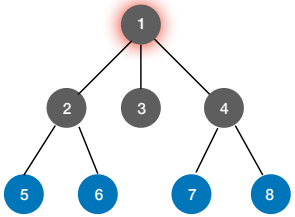


树的广度优先遍历

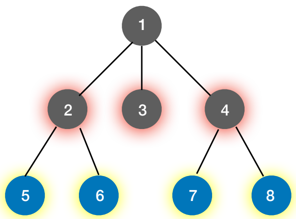


树的广度优先遍历：
通过根节点，可以找到下一层的结点2，3，4.通过234又可以找到再下一层的结点5678

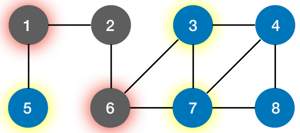
树的广度优先遍历



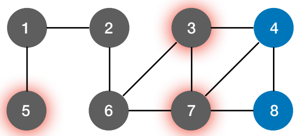
树的广度优先遍历



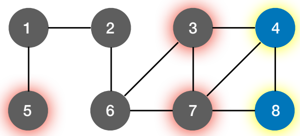
图的广度优先遍历



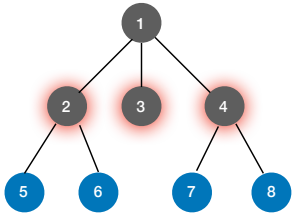
图的广度优先遍历



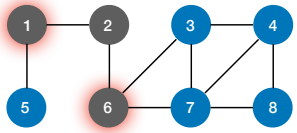
图的广度优先遍历



树 vs 图



不存在“回路”，搜索相邻的结点时，不可能搜到已经访问过的结点



搜索相邻的顶点时，有可能搜到已经访问过的顶点

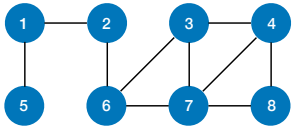
树的广度优先遍历（层序遍历）：

①若树非空，则根节点入队

②若队列非空，队头元素出队并访问，同时将该元素的孩子依次入队

③重复②直到队列为空

代码实现



广度优先遍历（Breadth-First-Search, BFS）要点：

1. 找到与一个顶点相邻的所有顶点
2. 标记哪些顶点被访问过
3. 需要一个辅助队列

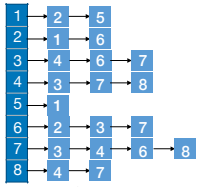
•FirstNeighbor(G,x)：求图G中顶点x的第一个邻接点，若有则返回顶点号。若x没有邻接点或图中不存在x，则返回-1。

•NextNeighbor(G,x,y)：假设图G中顶点y是顶点x的一个邻接点，返回除y之外顶点x的下一个邻接点的顶点号，若y是x的最后一个邻接点，则返回-1。

```
bool visited[MAX_VERTEX_NUM];    //访问标记数组
```

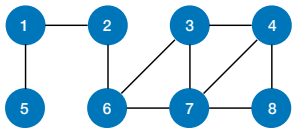
	1	2	3	4	5	6	7	8
1	0	1	0	0	1	0	0	0
2	1	0	0	0	0	1	0	0
3	0	0	0	1	0	1	1	0
4	0	0	1	0	0	0	1	1
5	1	0	0	0	0	0	0	0
6	0	1	1	0	0	0	1	0
7	0	0	1	1	0	1	0	1
8	0	0	0	1	0	0	1	0

邻接矩阵



邻接表

代码实现



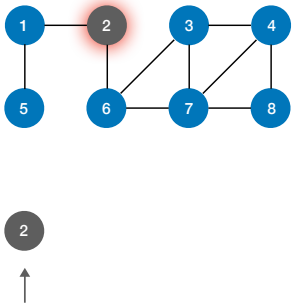
初始都为false

```
bool visited[MAX_VERTEX_NUM];    //访问标记数组

//广度优先遍历
void BFS(Graph G,int v){    //从顶点v出发，广度优先遍历图G
    visit(v);                   //访问初始顶点v
    visited[v]=TRUE;            //对v做已访问标记
    Enqueue(Q,v);               //顶点v入队列Q
    while(!isEmpty(Q)){
        DeQueue(Q,v);           //顶点v出队列
        for(w=FirstNeighbor(G,v);w>=0;w=NextNeighbor(G,v,w))
            //检测v所有邻接点
            if(!visited[w]){    //w为v的尚未访问的邻接顶点
                visit(w);       //访问顶点w
                visited[w]=TRUE;//对w做已访问标记
                EnQueue(Q,w);   //顶点w入队列
            }//if
    }//while
}
```

	1	2	3	4	5	6	7	8
visited	false	false	false	false	false	false	false	false

代码实现



初始都为false

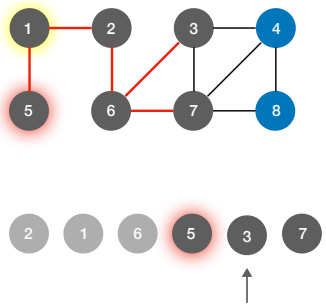
```
bool visited[MAX_VERTEX_NUM];    //访问标记数组

//广度优先遍历
void BFS(Graph G,int v){    //从顶点v出发，广度优先遍历图G
    visit(v);                   //访问初始顶点v
    visited[v]=TRUE;            //对v做已访问标记
    Enqueue(Q,v);               //顶点v入队列Q
    while(!isEmpty(Q)){
        DeQueue(Q,v);           //顶点v出队列
        for(w=FirstNeighbor(G,v);w>=0;w=NextNeighbor(G,v,w))
            //检测v所有邻接点
            if(!visited[w]){    //w为v的尚未访问的邻接顶点
                visit(w);       //访问顶点w
                visited[w]=TRUE;//对w做已访问标记
                EnQueue(Q,w);   //顶点w入队列
            }//if
    }//while
}
```

	1	2	3	4	5	6	7	8
visited	false	true	false	false	false	false	false	false

代码实现

初始都为false



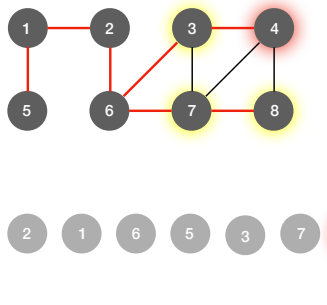

```
bool visited[MAX_VERTEX_NUM];   //访问标记数组

//广度优先遍历
void BFS(Graph G,int v){    //从顶点v出发，广度优先遍历图G
    visit(v);                   //访问初始顶点v
    visited[v]=TRUE;            //对v做已访问标记
    Enqueue(Q,v);               //顶点v入队列Q
    while(!isEmpty(Q)){
        DeQueue(Q,v);           //顶点v出队列
        for(w=FirstNeighbor(G,v);w>=0;w=NextNeighbor(G,v,w))
            //检测v所有邻接点
            if(!visited[w]){    //w为v的尚未访问的邻接顶点
                visit(w);       //访问顶点w
                visited[w]=TRUE;//对w做已访问标记
                EnQueue(Q,w);   //顶点w入队列
            }//if
    }//while
}
```

	1	2	3	4	5	6	7	8
visited	true	true	true	false	true	true	true	false

代码实现

初始都为false



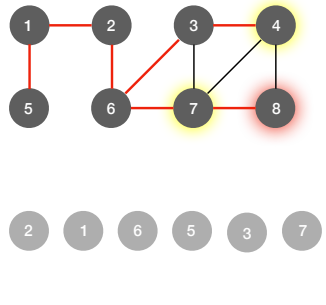

```
bool visited[MAX_VERTEX_NUM];   //访问标记数组

//广度优先遍历
void BFS(Graph G,int v){    //从顶点v出发，广度优先遍历图G
    visit(v);                   //访问初始顶点v
    visited[v]=TRUE;            //对v做已访问标记
    Enqueue(Q,v);               //顶点v入队列Q
    while(!isEmpty(Q)){
        DeQueue(Q,v);           //顶点v出队列
        for(w=FirstNeighbor(G,v);w>=0;w=NextNeighbor(G,v,w))
            //检测v所有邻接点
            if(!visited[w]){    //w为v的尚未访问的邻接顶点
                visit(w);       //访问顶点w
                visited[w]=TRUE;//对w做已访问标记
                EnQueue(Q,w);   //顶点w入队列
            }//if
    }//while
}
```

	1	2	3	4	5	6	7	8
visited	true	true	true	true	true	true	true	true

代码实现

初始都为false

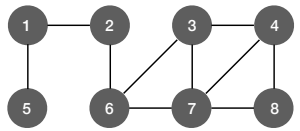



```
bool visited[MAX_VERTEX_NUM];   //访问标记数组

//广度优先遍历
void BFS(Graph G,int v){    //从顶点v出发，广度优先遍历图G
    visit(v);                   //访问初始顶点v
    visited[v]=TRUE;            //对v做已访问标记
    Enqueue(Q,v);               //顶点v入队列Q
    while(!isEmpty(Q)){
        DeQueue(Q,v);           //顶点v出队列
        for(w=FirstNeighbor(G,v);w>=0;w=NextNeighbor(G,v,w))
            //检测v所有邻接点
            if(!visited[w]){    //w为v的尚未访问的邻接顶点
                visit(w);       //访问顶点w
                visited[w]=TRUE;//对w做已访问标记
                EnQueue(Q,w);   //顶点w入队列
            }//if
    }//while
}
```

	1	2	3	4	5	6	7	8
visited	true	true	true	true	true	true	true	true

广度优先遍历序列

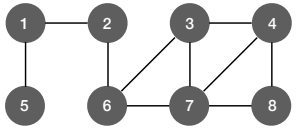


从顶点1出发得到的广度优先遍历序列：
1, 2, 5, 6, 3, 7, 4, 8

从顶点3出发得到的广度优先遍历序列：
3, 4, 6, 7, 8, 2, 1, 5

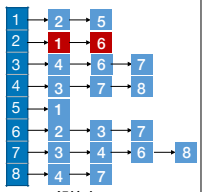
从顶点2出发得到的广度优先遍历序列

遍历序列的可变性



	1	2	3	4	5	6	7	8
1	0	1	0	0	1	0	0	0
2	1	0	0	0	0	1	0	0
3	0	0	0	1	0	1	1	0
4	0	0	1	0	0	0	1	1
5	1	0	0	0	0	0	0	0
6	0	1	1	0	0	0	1	0
7	0	0	1	1	0	1	0	1
8	0	0	0	1	0	0	1	0

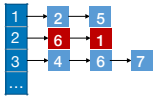
邻接矩阵



邻接表

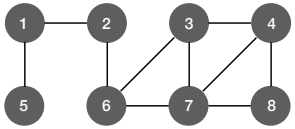
从顶点2出发得到的广度优先遍历序列

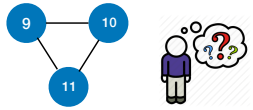
广度优先遍历序列：
2，6，1....



同一个图的邻接矩阵表示方式唯一，因此广度优先遍历序列唯一
同一个图邻接表表示方式不唯一，因此广度优先遍历序列不唯一

算法存在的问题





如果是非连通图，则无法遍历完所有结点

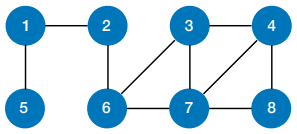
初始都为false

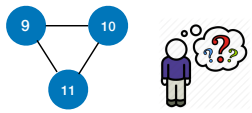
```
bool visited[MAX_VERTEX_NUM];    //访问标记数组

//广度优先遍历
void BFS(Graph G,int v){    //从顶点v出发，广度优先遍历图G
    visit(v);                    //访问初始顶点v
    visited[v]=TRUE;             //对v做已访问标记
    Enqueue(Q,v);                //顶点v入队列Q
    while(!isEmpty(Q)){
        DeQueue(Q,v);            //顶点v出队列
        for(w=FirstNeighbor(G,v);w>=0;w=NextNeighbor(G,v,w))
            //检测v所有邻接点
            if(!visited[w]){     //w为v的尚未访问的邻接顶点
                visit(w);        //访问顶点w
                visited[w]=TRUE;//对w做已访问标记
                EnQueue(Q,w);    //顶点w入队列
            }//if
    }//while
}
```

	1	2	3	4	5	6	7	8	9	10	11
visite	true	true	true	true	true	true	true	true	false	false	false

BFS算法（Final版）





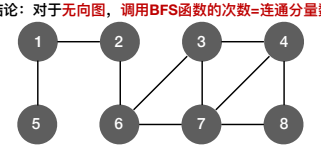
如果是非连通图，则无法遍历完所有结点

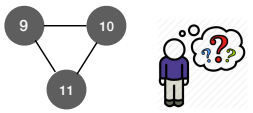
```
bool visited[MAX_VERTEX_NUM];    //访问标记数组
void BFSTraverse(Graph G){  //对图G进行广度优先遍历
    for(i=0;i<G.vexnum;++i)
        visited[i]=FALSE;        //访问标记数组初始化
    InitQueue(Q);                    //初始化辅助队列Q
    for(i=0;i<G.vexnum;++i)      //从0号顶点开始遍历
        if(!visited[i])          //对每个连通分量调用一次BFS
            BFS(G,i);            //vi未访问过，从vi开始BFS
}
//广度优先遍历
void BFS(Graph G,int v){    //从顶点v出发，广度优先遍历图G
    visit(v);                    //访问初始顶点v
    visited[v]=TRUE;             //对v做已访问标记
    Enqueue(Q,v);                //顶点v入队列Q
    while(!isEmpty(Q)){
        DeQueue(Q,v);            //顶点v出队列
        for(w=FirstNeighbor(G,v);w>=0;w=NextNeighbor(G,v,w))
            //检测v所有邻接点
            if(!visited[w]){     //w为v的尚未访问的邻接顶点
                visit(w);        //访问顶点w
                visited[w]=TRUE;//对w做已访问标记
                EnQueue(Q,w);    //顶点w入队列
            }//if
    }//while
}
```

	1	2	3	4	5	6	7	8	9	10	11
visited	false	false	false	false	false	false	false	false	false	false	false

BFS算法（Final版）

结论：对于无向图，调用BFS函数的次数=连通分量数



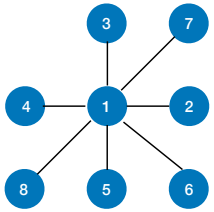


如果是非连通图，则无法遍历完所有结点

```
bool visited[MAX_VERTEX_NUM];    //访问标记数组
void BFSTraverse(Graph G){  //对图G进行广度优先遍历
    for(i=0;i<G.vexnum;++i)
        visited[i]=FALSE;        //访问标记数组初始化
    InitQueue(Q);                    //初始化辅助队列Q
    for(i=0;i<G.vexnum;++i)      //从0号顶点开始遍历
        if(!visited[i])          //对每个连通分量调用一次BFS
            BFS(G,i);            //vi未访问过，从vi开始BFS
}
//广度优先遍历
void BFS(Graph G,int v){    //从顶点v出发，广度优先遍历图G
    visit(v);                    //访问初始顶点v
    visited[v]=TRUE;             //对v做已访问标记
    Enqueue(Q,v);                //顶点v入队列Q
    while(!isEmpty(Q)){
        DeQueue(Q,v);            //顶点v出队列
        for(w=FirstNeighbor(G,v);w>=0;w=NextNeighbor(G,v,w))
            //检测v所有邻接点
            if(!visited[w]){     //w为v的尚未访问的邻接顶点
                visit(w);        //访问顶点w
                visited[w]=TRUE;//对w做已访问标记
                EnQueue(Q,w);    //顶点w入队列
            }//if
    }//while
}
```

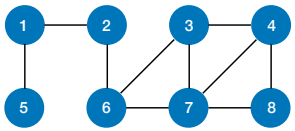
	1	2	3	4	5	6	7	8	9	10	11
visited	true	true	true	true	true	true	true	true	true	true	true

复杂度分析



空间复杂度：最坏情况，辅助队列大小为 O(|V|)

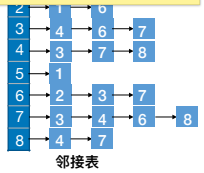
复杂度分析



	1	2	3	4	5	6	7	8
1	0	1	0	0	1	0	0	0
2	1	0	0	0	0	1	0	0
3	0	0	0	1	0	1	1	0
4	0	0	1	0	0	0	1	1
5	1	0	0	0	0	0	0	0
6	0	1	1	0	0	0	1	0
7	0	0	1	1	0	1	0	1
8	0	0	0	1	0	0	1	0

邻接矩阵

其实对于无向图来说，应该是O(2|E|)的时间

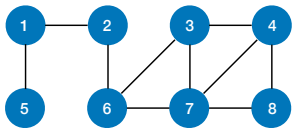


邻接表

邻接矩阵存储的图：
访问 |V| 个顶点需要O(|V|)的时间
查找每个顶点的邻接点都需要O(|V|)的时间，而总共有|V|个顶点
时间复杂度= $O(|V|^2)$

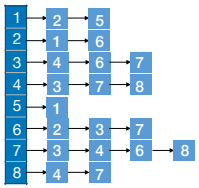
邻接表存储的图：
访问 |V| 个顶点需要O(|V|)的时间
查找各个顶点的邻接点共需要O(|E|)的时间，
时间复杂度= O(|V|+|E|)

广度优先生成树



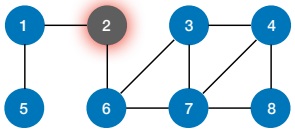
	1	2	3	4	5	6	7	8
1	0	1	0	0	1	0	0	0
2	1	0	0	0	0	1	0	0
3	0	0	0	1	0	1	1	0
4	0	0	1	0	0	0	1	1
5	1	0	0	0	0	0	0	0
6	0	1	1	0	0	0	1	0
7	0	0	1	1	0	1	0	1
8	0	0	0	1	0	0	1	0

邻接矩阵



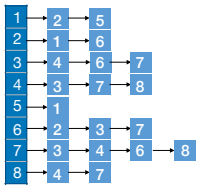
邻接表

广度优先生成树



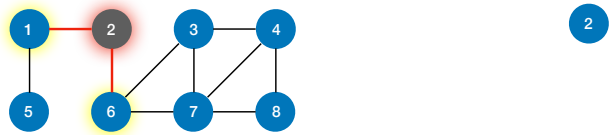
	1	2	3	4	5	6	7	8
1	0	1	0	0	1	0	0	0
2	1	0	0	0	0	1	0	0
3	0	0	0	1	0	1	1	0
4	0	0	1	0	0	0	1	1
5	1	0	0	0	0	0	0	0
6	0	1	1	0	0	0	1	0
7	0	0	1	1	0	1	0	1
8	0	0	0	1	0	0	1	0

邻接矩阵



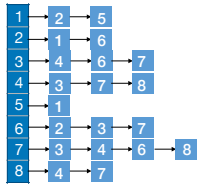
邻接表

广度优先生成树



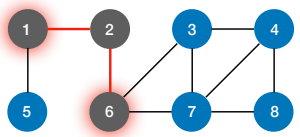
	1	2	3	4	5	6	7	8
1	0	1	0	0	1	0	0	0
2	1	0	0	0	0	1	0	0
3	0	0	0	1	0	1	1	0
4	0	0	1	0	0	0	1	1
5	1	0	0	0	0	0	0	0
6	0	1	1	0	0	0	1	0
7	0	0	1	1	0	1	0	1
8	0	0	0	1	0	0	1	0

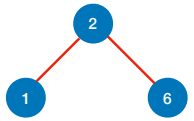
邻接矩阵



邻接表

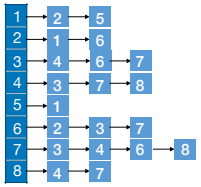
广度优先生成树





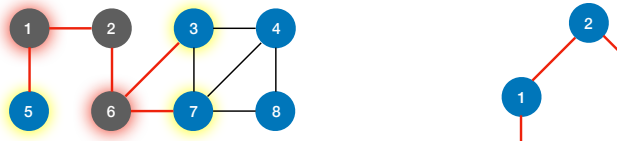
	1	2	3	4	5	6	7	8
1	0	1	0	0	1	0	0	0
2	1	0	0	0	0	1	0	0
3	0	0	0	1	0	1	1	0
4	0	0	1	0	0	0	1	1
5	1	0	0	0	0	0	0	0
6	0	1	1	0	0	0	1	0
7	0	0	1	1	0	1	0	1
8	0	0	0	1	0	0	1	0

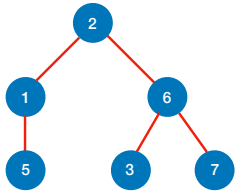
邻接矩阵



邻接表

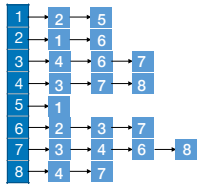
广度优先生成树





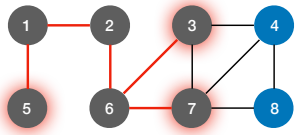
	1	2	3	4	5	6	7	8
1	0	1	0	0	1	0	0	0
2	1	0	0	0	0	1	0	0
3	0	0	0	1	0	1	1	0
4	0	0	1	0	0	0	1	1
5	1	0	0	0	0	0	0	0
6	0	1	1	0	0	0	1	0
7	0	0	1	1	0	1	0	1
8	0	0	0	1	0	0	1	0

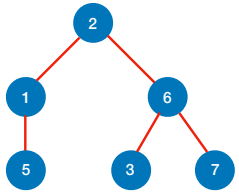
邻接矩阵



邻接表

广度优先生成树





	1	2	3	4	5	6	7	8
1	0	1	0	0	1	0	0	0
2	1	0	0	0	0	1	0	0
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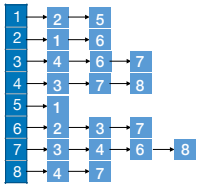
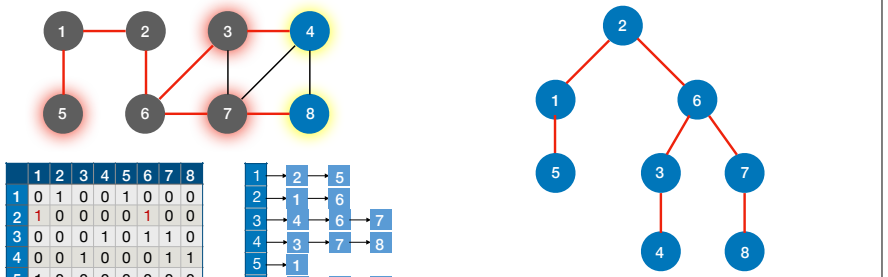
4	0	0	1	0	0	0	1	1
5	1	0	0	0	0	0	0	0
6	0	1	1	0	0	0	1	0
7	0	0	1	1	0	1	0	1
8	0	0	0	1	0	0	1	0

邻接矩阵



邻接表

广度优先生成树



	1	2	3	4	5	6	7	8
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2	1	0	0	0	0	1	0	0
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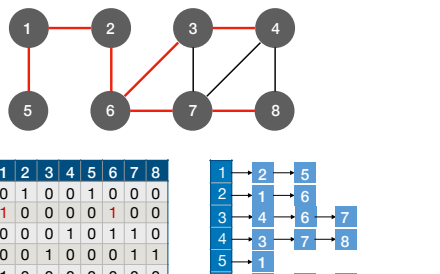
4	0	0	1	0	0	0	1	1
5	1	0	0	0	0	0	0	0
6	0	1	1	0	0	0	1	0
7	0	0	1	1	0	1	0	1
8	0	0	0	1	0	0	1	0

邻接矩阵

邻接表

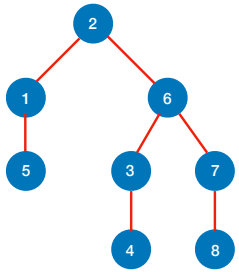
广度优先生成树



	1	2	3	4	5	6	7	8
1	0	1	0	0	1	0	0	0
2	1	0	0	0	0	1	0	0
3	0	0	0	1	0	1	1	0
4	0	0	1	0	0	0	1	1
5	1	0	0	0	0	0	0	0
6	0	1	1	0	0	0	1	0
7	0	0	1	1	0	1	0	1
8	0	0	0	1	0	0	1	0

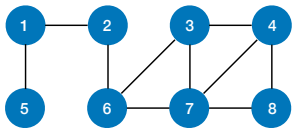
邻接矩阵

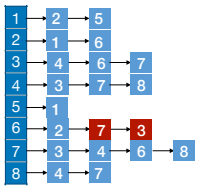
邻接表



广度优先生成树

广度优先生成树





邻接表

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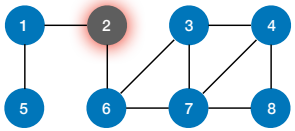
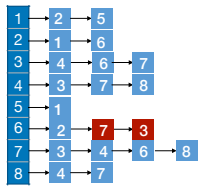
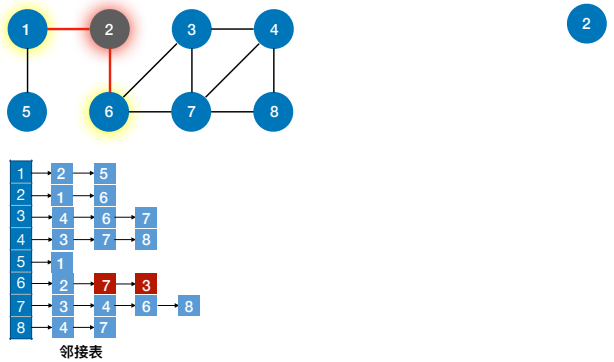


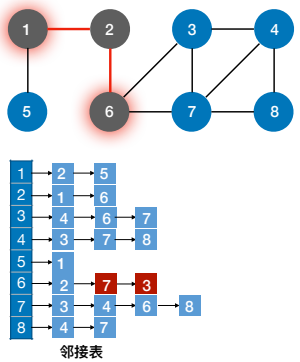


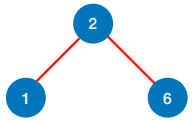
邻接表

广度优先生成树

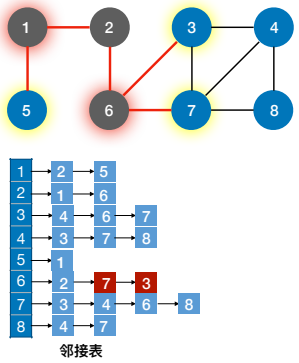


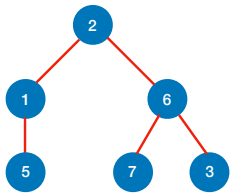
广度优先生成树



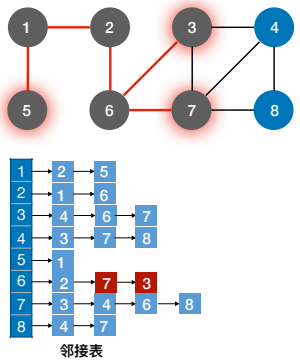


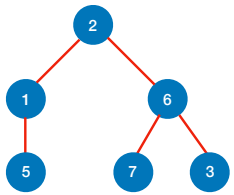
广度优先生成树



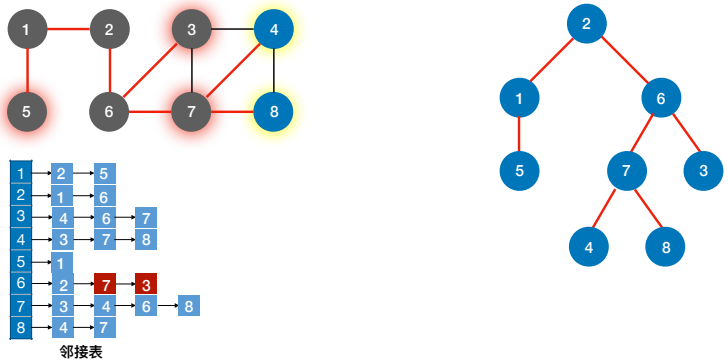


广度优先生成树

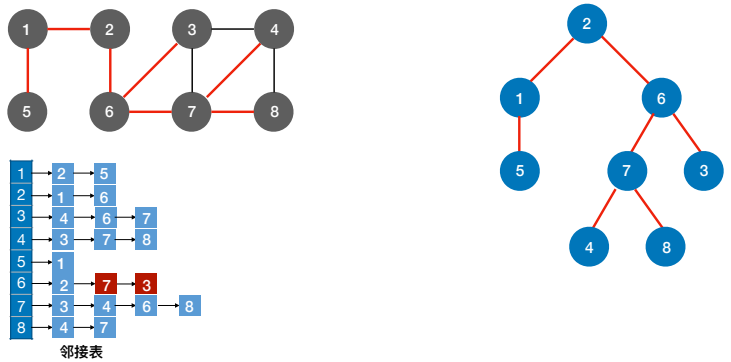




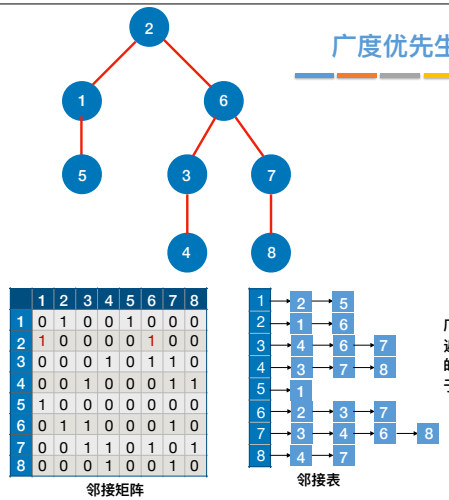
广度优先生成树



广度优先生成树



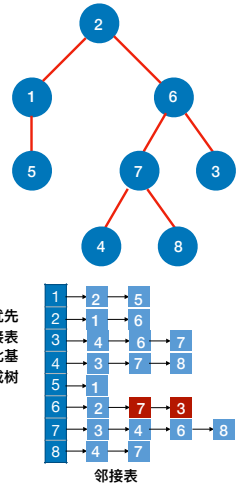
广度优先生成树



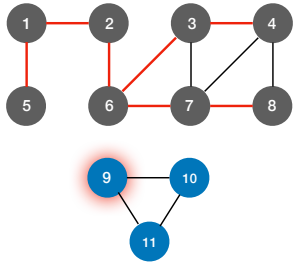
	1	2	3	4	5	6	7	8
1	0	1	0	0	1	0	0	0
2	1	0	0	0	0	1	0	0
3	0	0	0	1	0	1	1	0
4	0	0	1	0	0	0	1	1
5	1	0	0	0	0	0	0	0
6	0	1	1	0	0	0	1	0
7	0	0	1	1	0	1	0	1
8	0	0	0	1	0	0	1	0

邻接矩阵

广度优先生成树由广度优先遍历过程确定。由于邻接表的表示方式不唯一，因此基于邻接表的广度优先生成树也不唯一。

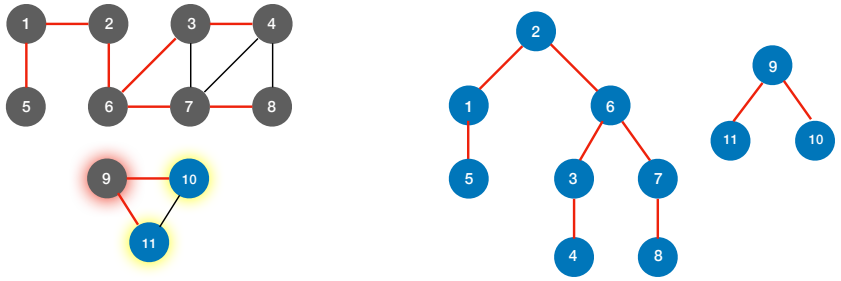


广度优先生成森林



对非连通图的广度优先遍历，可得到广度优先生成森林

广度优先生成森林

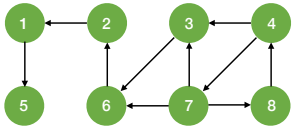


对非连通图的广度优先遍历，可得到广度优先生成森林

练习：有向图的BFS过程

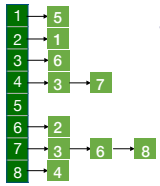
思考：

1. 从1出发，需要调用几次BFS函数?
2. 从7出发，需要调用几次BFS函数?



	1	2	3	4	5	6	7	8
1	0	0	0	0	1	0	0	0
2	1	0	0	0	0	0	0	0
3	0	0	0	0	0	1	0	0
4	0	0	1	0	0	0	1	0
5	0	0	0	0	0	0	0	0
6	0	1	0	0	0	0	0	0
7	0	0	1	0	0	1	0	1
8	0	0	0	1	0	0	0	0

邻接矩阵



邻接表

```
bool visited[MAX_VERTEX_NUM];   //访问标记数组
void BFSTraverse(Graph G){  //对图G进行广度优先遍历
    for(i=0;i<G.vexnum;++i)
        visited[i]=FALSE;        //访问标记数组初始化
    InitQueue(Q);                    //初始化辅助队列Q
    for(i=0;i<G.vexnum;++i)      //从0号顶点开始遍历
        if(!visited[i])          //对每个连通分量调用一次BFS
            BFS(G,i);            //vi未访问过，从vi开始BFS
}
//广度优先遍历
void BFS(Graph G,int v){    //从顶点v出发，广度优先遍历图G
    visit(v);                    //访问初始顶点v
    visited[v]=TRUE;             //对v做已访问标记
    Enqueue(Q,v);                //顶点v入队列Q
    while(!isEmpty(Q)){
        DeQueue(Q,v);            //顶点v出队列
        for(w=FirstNeighbor(G,v);w>=0;w=NextNeighbor(G,v,w))
            //检测v所有邻接点
            if(!visited[w]){     //w为v的尚未访问的邻接顶点
                visit(w);        //访问顶点w
                visited[w]=TRUE;//对w做已访问标记
                EnQueue(Q,w);    //顶点w入队列
            }//if
    }//while
}
```


知识回顾与重要考点

BFS求单元最短路径问题放在6.4.2讲解