Approximate Method for Plug Nozzle Design

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Nomenclature

x = axial coordinate
r = radial coordinate
l = length of a characteristic segment
λ = l/l_t, nondimensional length for plane nozzle
ξ = l/r_e, nondimensional length for axisymmetric nozzle
η = r/r_e, nondimensional radial coordinate
α = geometric angle with the reference axis
A = area
ϵ = A/A_t, expansion ratio
M = Mach number
μ = Mach angle
ν = $[(\gamma + 1)/(\gamma - 1)]^{1/2} \arctan\{[(\gamma - 1)/(\gamma + 1)](M^2 - 1)\}^{1/2} - \arctan(M^2 - 1)^{1/2}$, Prandtl-Meyer angle
S = surface
V = velocity
p = pressure
γ = isentropic exponent

Subscripts

t = throat
e = exit
0 = stagnation
b = base

The approximate method for the design of axisymmetrical plug nozzles described here is derived from a simple, exact technique valid in the two-dimensional case, a brief description of which is useful in understanding the limits of validity of the axisymmetrical procedure.

With reference to Fig. 1a, a sonic flow, present at the throat AB, is assumed to expand through a centered wave, originating at the plug nozzle lip. The streamline passing through point B is the required nozzle profile. A typical characteristic line is a straight, constant properties line, inclined with respect to the direction of the sonic flow, at the

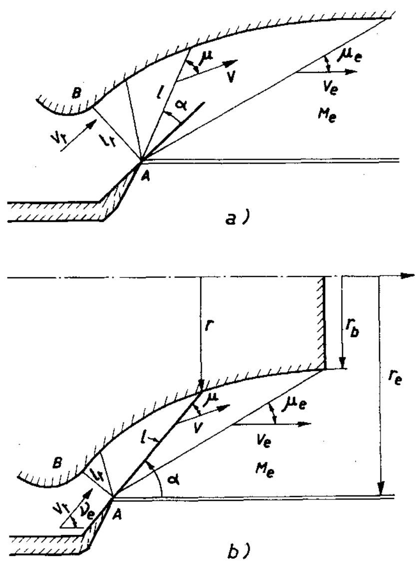

Fig. 1 a) Two-dimensional plug nozzle and b) annular plug nozzle.

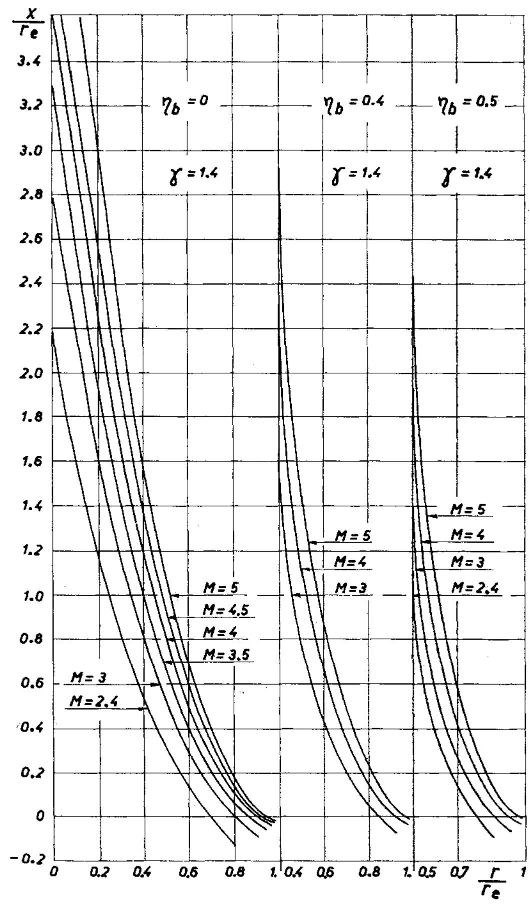

Fig. 2 Approximate plug nozzle profiles.

angle $\alpha = \mu - \nu$, where μ is the Mach angle and ν is the Prandtl-Meyer function.[1]

It crosses the nozzle boundary at a distance l from the lip, which is computed by writing the continuity equation

$$l/l_t = (A/\sin\mu)(1/A_t) \text{ or } \lambda = \epsilon M$$

where A is the passage area, normal to the velocity vector, λ is a nondimensional length, and ϵ is the expansion ratio.

The polar equations of the plug are then

$$\lambda = M\epsilon(M)$$
$$\alpha = \mu(M) - \nu(M)$$

in which the Mach number is a parameter to be varied from 1 to the desired exit value M_e

In this way, each point of the curve is computed independently, improving the accuracy of the procedure with respect to the classical technique used for drawing streamlines in a flow field.

Let us now consider an annular axisymmetrical plug nozzle, a typical example of which is shown in Fig. 1b. A report of an experimental investigation on a similar nozzle is given in Ref. 2.

The usual plug nozzle is obtained as a particular case when the radius of the base r_b becomes zero.

A proper selection of r_b permits a sort of adaptation of the nozzle to the body of the vehicle, possibly reducing its base drag.

We admit the presence of a sonic flow at the geometric throat AB, the velocity of which is inclined with respect to the nozzle axis at the Prandtl-Meyer angle corresponding to the design exhaust conditions.

The expansion fan in the vicinity of the nozzle lip follows the two-dimensional law; furthermore, the two characteristics that bind the fan are straight as in the two-dimensional case.

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We assume that the other characteristics are also straight, constant properties lines. (A detailed computation of the actual flow would show that the foregoing two assumptions, in the geometric considerations that follow, have, to a certain extent, balancing effects.)

As in the two-dimensional case, we write the continuity equation across a typical characteristic (in Ref. 3 a straight line perpendicular to the actual plug profile was instead selected).

The surface crossed by the flow is

$$S = 2\pi \frac{r_e + r}{2} \frac{r_e - r}{\sin\alpha} \tag{1}$$

Since the velocity makes an angle μ with the surface, the actual passage area is

$$A = S \sin\mu = \frac{\pi(r_e^2 - r^2)}{M \sin\alpha} \tag{2}$$

The length of the characteristic from the nozzle lip to the plug surface is

$$l = \frac{r_e - r}{\sin\alpha} \tag{3}$$

whereas the exit area is

$$A_e = \pi(r_e^2 - r_b^2)$$

Equation (3), taking into account Eq. (2), may be written

$$l = \frac{r_e - [r_e^2 - (AM \sin\alpha/\pi)]^{1/2}}{\sin\alpha}$$

or, in a nondimensional form,

$$\xi = \frac{l}{r_e} = \frac{1 - \{1 - [\epsilon(1 - \eta_b^2)M \sin\alpha/\epsilon_e]\}^{1/2}}{\sin\alpha} \tag{4}$$

in which η_b represents the nondimensional base radius.

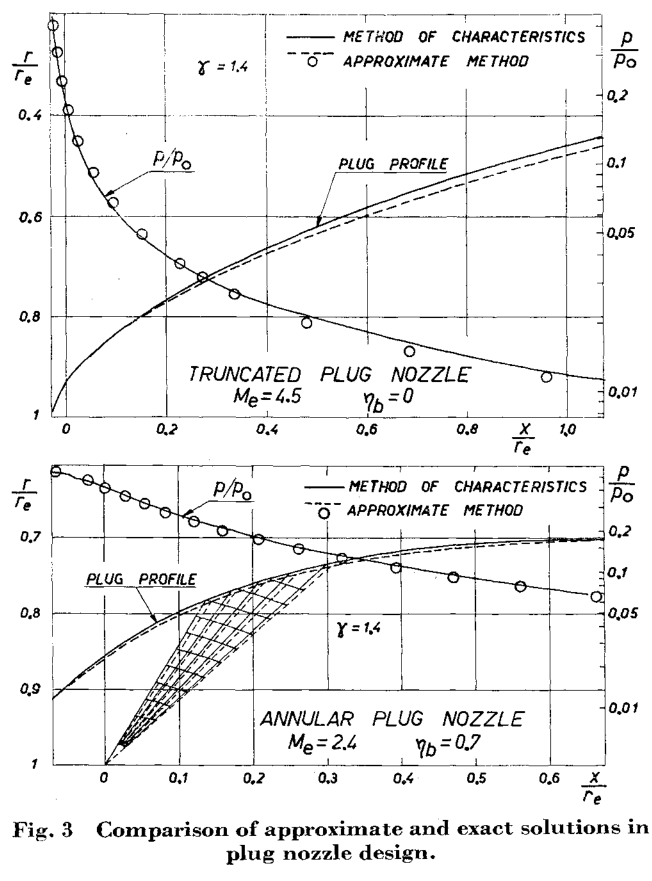

Fig. 3 Comparison of approximate and exact solutions in plug nozzle design.

The equations of the plug profile in a parametric form are then

$$\begin{aligned} \xi &= \xi(M) \\ \alpha &= \nu_e - \nu(M) - \mu(M) \end{aligned} \tag{5}$$

in which the Mach number varies from 1 to the design exit value M_e.

Figure 2 shows several plug profiles computed in this way for various Mach numbers and base radii.

In Fig. 3, shapes and pressure distributions of two isentropic plugs, computed by the method of characteristics, are compared with the results of the approximate method.

As a consequence of our assumptions, the agreement between the two design techniques is good wherever the expansion characteristics do not depart significantly from straight, constant properties lines.

This, in fact, occurs for nozzles having a large base radius, no matter what the exit Mach number, and for nozzles with a high exit Mach number in the supersonic region just downstream of the throat, no matter what the base radius.

References

[1] Ferri, A., *Elements of Aerodynamics of Supersonic Flows* (The MacMillan Co., New York, 1949), p. 27.

[2] Connors, J. F., Cubbison, R. W., and Mitchell, G. A., "Annular internal-external-expansion rocket nozzles for large booster applications," NASA TN D-1049 (1961).

[3] Greer, H., "Rapid method for plug nozzle design," ARS J. **31**, 560–561 (1961).

Plasma Radiation Shielding

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It has been recognized for some time that energetic protons constitute a serious radiation hazard in space, especially for trips lasting longer than a week or two. An important attribute of the radiation shielding problem is that very few methods are available to us for dealing with it. This note describes an approach to the problem which has not to our knowledge been suggested before. At this stage, the new approach indicates the possibility of a substantial reduction in the weight of a space radiation shield.

Three methods of shielding are currently available. First, of course, there is solid shielding. Second, pure magnetic shielding has been shown to have substantial advantages over solid shielding, but only for very large vehicle sizes.[1–3] Third, there is electrostatic shielding[4] in which the space vehicle to be protected must be kept at a positive potential of 1 or 2 × 10^8 v relative either to an outer part of the space vehicle or to "infinity." Maintaining a potential difference of this order of magnitude between two solid conductors is well beyond the limit of present-day technology using heavy ground equipment. On the other hand, the electrons present in the interplanetary plasma would rapidly discharge any positive potential of the whole space vehicle relative to infinity. The power required to maintain a potential of 2 × 10^8 v against this loss is estimated to be about 10^7 kw.

If it were possible to reduce very substantially the flow of electrons from space to the vehicle, electrostatic shielding might, after all, be feasible. Our suggestion is based on the fact that under suitable conditions electrons do not flow

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