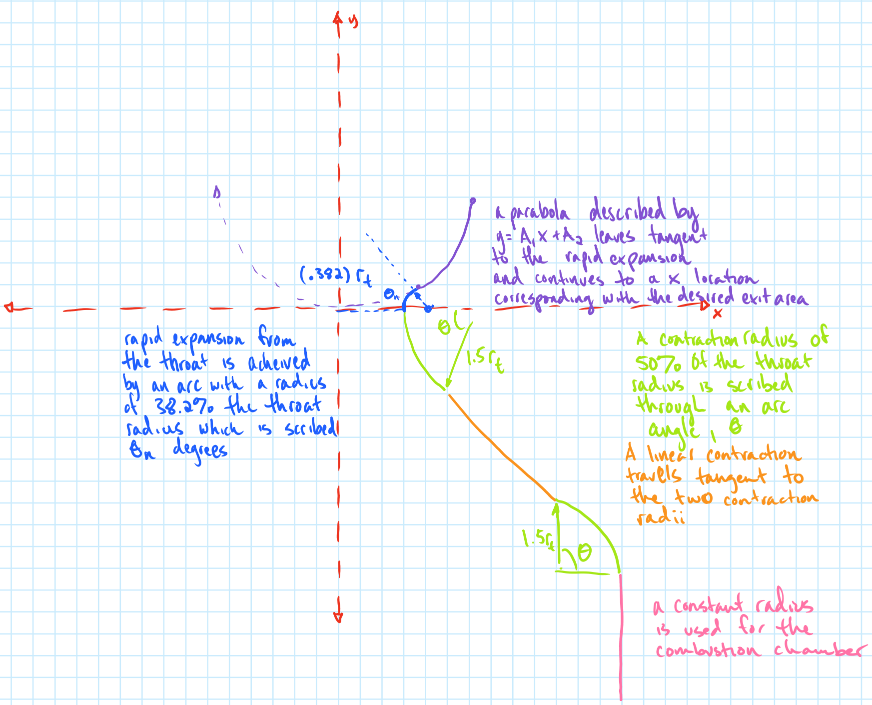
Cooling Channel Geometry

Sunday, January 24, 2016 7:08 PM

The cooling channel of the rocket can be broken up into several piecewise portions each with their own representative equations for any given geometry.

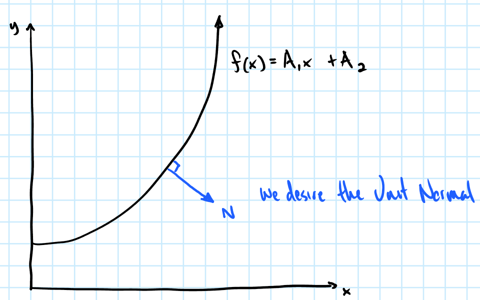
The geometry of the rocket itself is as follows:



This collection of paths should be continuous (tangent at any piecewise joining point) and should be given thickness before rotating about the Y axis in order to create the nozzle.

It is desired to create a constant cross sectional area for the cooling channels in this thickness. The cross sectional area should be measured normal to the curve at any given point and is a measure of the surface area of the conical frustrum created when bisecting the rotated 3D shape along a given normal vector.

Calculating a constant cross section for the Parabolic Expansion:



The unit Normal Vector is given by:

$$\left(\frac{t}{N}, \frac{-p}{N}\right) \quad \text{where } y = \frac{1}{2p}t^2, \quad N = \sqrt{t^2 + p^2}, \quad x = t$$

So, $A_1 = \frac{1}{2p} \longrightarrow p = \frac{1}{2A_1}$

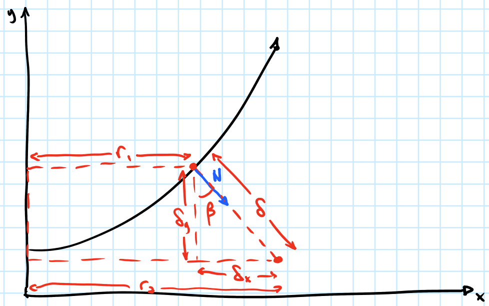
And,

$$\vec{N}_x = \frac{t}{\sqrt{t^2 + \frac{1}{4A_1^2}}} \qquad \vec{N}_y = \frac{-1}{2A_1\sqrt{t^2 + \frac{1}{4A_1^2}}}$$

And,

$$\vec{N}_x = \frac{t}{\sqrt{t^2 + \frac{1}{4A_1^2}}} \qquad \vec{N}_y = \frac{-1}{2A_1\sqrt{t^2 + \frac{1}{4A_1^2}}}$$

It is then necessary to calculate the formula for a conical frustrum, with geometry along the normal which has been calculated.



Area of a conical frustrum, A_c

- $A_c = \pi(r_1 + r_2)\delta$

  $r_2 = r_1 + \delta_x$, so,

- $A_c = \pi(2r_1 + \delta_x)\delta$

  finding δ_x in terms of δ

  $$\delta_x = \delta \sin\beta$$

  finding β as a function of t

  $$\tan\beta = \frac{N_x}{N_y} = \left[\frac{t}{\cancel{\sqrt{t^2 + \frac{1}{4A_1^2}}}}\right]\left[\frac{-2A_1\cancel{\sqrt{t^2 + \frac{1}{4A_1^2}}}}{1}\right] = -2A_1 t$$

  $$\beta = \arctan(-2A_1 t)$$

  Solve for δ

  $$A_c = \pi(2r_1 + \delta\sin\beta)\delta$$

  $$0 = \delta^2\sin\beta + 2r_1\delta - \frac{A_c}{\pi}$$

which has solutions:

$$\delta = \frac{-r_1 \pm \sqrt{r_1^2 + \frac{A_c}{\pi}\sin\beta}}{\sin\beta} \qquad r_1 = t$$

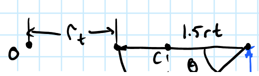
Substituting,

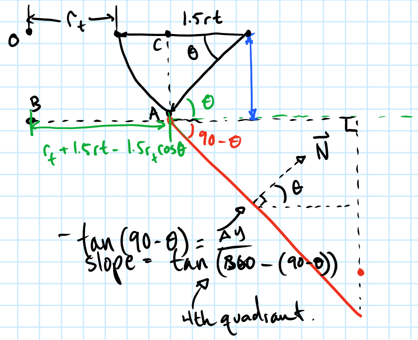
$$\delta = \frac{-t \pm \sqrt{t^2 + \frac{A_c}{\pi}\sin[\arctan(-2A_1 t)]}}{\sin(-2A_1 t)}$$

finding $\delta\vec{N}$,

$$\delta_y = (\delta)(N_y) = \frac{-t \pm \sqrt{t^2 + \frac{A_c}{\pi}\sin[\arctan(-2A_1 t)]}}{2A_1\sqrt{t^2 + \frac{1}{4A_1^2}}\,\sin[\arctan(-2A_1 t)]}$$

$$\delta_x = (\delta)(N_x) = \frac{-t\left(-t \pm \sqrt{t^2 + \frac{A_c}{\pi}\sin[\arctan(-2A_1 t)]}\right)}{\sqrt{t^2 + \frac{1}{4A_1^2}}}$$





$$\overline{BA} = r_t(1+1.5-1.5\cos\theta)$$
$$= r_t(2.5-1.5\cos\theta)$$
$$= 2.5r_t(1-0.6\cos\theta)$$

$$\overline{CA} = 1.5r_t\sin\theta$$

$$A = (\overline{BA}, \overline{AC}) = (2.5r_t(1-0.6\cos\theta), -1.5r_t\sin\theta)$$

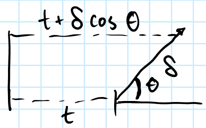
$$y = mx + b$$

$$-1.5r_t\sin\theta = (-\tan(90-\theta))[2.5r_t(1-0.6\cos\theta)] + b$$

$$b = \tan(90-\theta)[2.5r_t(1-0.6\cos\theta)] - 1.5r_t\sin\theta$$

Parametric: $x = t$, $y(x(t))$

find length of xs area



$$A = \pi(t + t + \delta\cos\theta)\delta$$
$$0 = \delta^2\cos\theta + 2t\delta - \frac{A}{\pi}$$

$$a = \cos\theta \qquad b = 2t \qquad c = -\frac{A}{\pi}$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\delta = \frac{-2t \pm \sqrt{4t^2 - 4(\cos\theta)(-\frac{A}{\pi})}}{2\cos\theta} = \frac{-t \pm \sqrt{t^2 + \frac{A}{\pi}\cos\theta}}{\cos\theta}$$

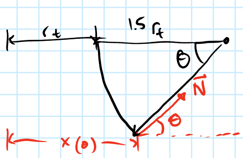
$$\vec{N}_x = \cos\theta \qquad \vec{N}_y = \sin\theta$$

$$\delta N_x = -t \pm \sqrt{t^2 + \frac{A}{\pi}\cos\theta} \qquad \delta N_y = \left[-t \pm \sqrt{t^2+\frac{A}{\pi}\cos\theta}\right]\tan\theta$$

Parametric equation of curve

$$x(t) = t + (\delta)N_x = \sqrt{t^2 + \frac{A}{\pi}\cos\theta}$$
$$y(t) = -\tan(90-\theta)t + \left(\tan(90-\theta)[2.5r_t(1-0.6\cos\theta)] - 1.5r_t\sin\theta\right) + \left[-t \pm \sqrt{t^2+\frac{A}{\pi}\cos\theta}\right]\tan\theta$$

note: positive roots will be used.

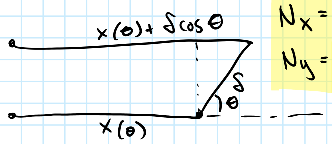


parametric eqns.

$$x(\theta) = r_t + 1.5r_t - 1.5r_t\cos\theta = 2.5r_t(1-0.6\cos\theta)$$
$$y(\theta) = -1.5r_t\sin\theta$$

$$x(\theta) = r_t + 1.5r_t - 1.5r_t\cos\theta = 2.5r_t(1-0.6\cos\theta)$$

$$y(\theta) = -1.5r_t\sin\theta$$



$$N_x = \cos\theta$$

$$N_y = \sin\theta$$

$$A = \pi\left(x(\theta) + x(\theta)\,\delta\cos\theta\right)\delta$$

$$0 = \delta^2\cos\theta + 2x(\theta)\delta - \frac{A}{\pi}$$

$$\delta = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a = \cos\theta$
$b = 2x(\theta)$
$c = -\frac{A}{\pi}$

$$\delta = \frac{-2x(\theta) \pm \sqrt{4x(\theta)^2 - 4(\cos\theta)(-\frac{A}{\pi})}}{2\cos\theta}$$

$$\delta = \frac{-x(\theta) \pm \sqrt{x(\theta)^2 + \frac{A}{\pi}\cos\theta}}{\cos\theta}$$

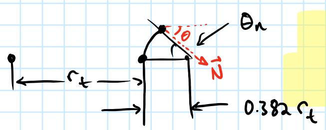
$$(\delta)(N_x) = -x(\theta) \pm \sqrt{x(\theta)^2 + \frac{A}{\pi}\cos\theta}$$

$$(\delta)(N_y) = \left(-x(\theta) \pm \sqrt{x(\theta)^2 + \frac{A}{\pi}\cos\theta}\right)\tan\theta$$

$$y(\theta) + \delta(N_y) = -1.5r_t\sin\theta + \left[-[2.5r_t(1-0.6\cos\theta)] \pm \sqrt{[2.5r_t(1-0.6\cos\theta)]^2 + \frac{A}{\pi}\cos\theta}\right]\tan\theta$$

$$x(\theta) + \delta(N_x) = 2.5r_t(1-0.6\cos\theta) + \left[-[2.5r_t(1-0.6\cos\theta)] \pm \sqrt{[2.5r_t(1-0.6\cos\theta)]^2 + \frac{A}{\pi}\cos\theta}\right]$$

6

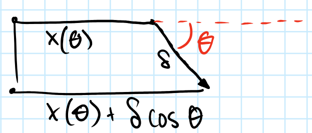


$$N_x = \cos\theta$$

$$N_y = -\sin\theta$$

Parametric equations: $x(\theta) = r_t + 0.382r_t - 0.382r_t\cos\theta = 1.382r_t - 0.382r_t\cos\theta = 1.382r_t\left(1 - \frac{0.382}{1.382}\cos\theta\right)$

$$y(\theta) = 0.382r_t\sin\theta$$



$$A = \pi\left(2x(\theta) + \delta\cos\theta\right)\delta$$

$$0 = \delta^2\cos\theta + 2x(\theta)\delta - \frac{A}{\pi}$$

this is the same as previous,

$$(\delta)(N_x) = -x(\theta) \pm \sqrt{x(\theta)^2 + \frac{A}{\pi}\cos\theta}$$

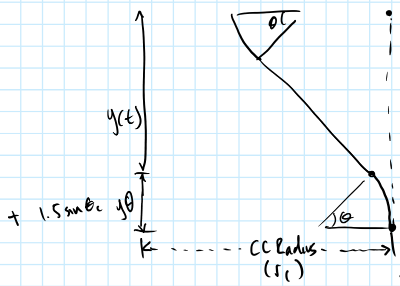
$$(\delta)(N_y) = \left(-x(\theta) \pm \sqrt{x(\theta)^2 + \frac{A}{\pi}\cos\theta}\right)(-\tan\theta)$$

from $-\sin\theta = N_y$

new $x(\theta)$ for $0.382 r_t$

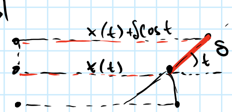
curve parametric equations

$$\boxed{\begin{bmatrix} x(\theta) + \delta(N_x) \\ y(\theta) + \delta(N_y) \end{bmatrix}}$$



$x(t) = \text{CC radius} - 1.5 r_t(1 - \cos(t))$

$y(t) = 1.5 r_t \sin(t)$



$N_x = \cos t \qquad N_y = \sin t$

$$A_c = \pi(2x(t) + \delta\cos t)\delta$$

$$0 = \delta\cos(t) + 2x(t)\delta - \frac{A_c}{\pi}$$

$$\delta = \frac{-x(t) \pm \sqrt{x(t)^2 + \frac{A_c}{\pi}\cos(t)}}{\cos(t)}$$

$$(\delta)(N_x) = -x(t) \pm \sqrt{x(t)^2 + \frac{A_c}{\pi}\cos(t)}$$

$$(\delta)(N_y) = \left(-x(t) \pm \sqrt{x(t)^2 + \frac{A_c}{\pi}\cos(t)}\right)\tan(t)$$