流体力学

静压强及其特性

- 流体静压强对某个表面的作用所产生的静压力必指向作用面的内法线方向
- 静止流体内任意一点处，压强的大小与作用面的方位无关。

平衡微分方程

流体平衡微分方程式

$$f_x - \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$
$$f_y - \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$
$$f_z - \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

也称欧拉平衡微分方程，无论质量力有哪些类型，流体是否可压缩、流体是否可压缩、流体有无粘性，欧拉平衡方程式都是普遍适用的。

方程也可以整理成

$$f_x dx + f_y dy + f_z dz = \frac{1}{\rho}\left(\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz\right) = \frac{1}{\rho}dp$$

如果单位质量力与某一个坐标函数 $U(x,y,z)$满足

$$f_x = -\frac{\partial U}{\partial x}, f_y = -\frac{\partial U}{\partial y}, f_z = -\frac{\partial U}{\partial z}$$

于是就可以改写成

$$dp + \rho dU = 0$$

$U(x,y,z)$是一个决定流体质量力的函数，称为力势函数，而具有这样力势函数的质量力称为有势力，不可压缩流体只有在有势的质量力的作用下才能保持平衡。

等压面

1. 等压面也是等势面
2. 通过任意一点的等压面必与该点所受质量力相垂直
3. 两种互不相混的流体处于平衡状态时，他们的分界面为等压面

静压强基本公式

$$p = p_0 + \rho g H$$

相对压力/表压：$p - p_a$

真空度：$p_v = p_a - p$

流体的相对平衡

$$\begin{aligned}
&\nabla p = \rho(g - a) \\
&\mathbf{V} = \mathbf{V}_0 + \boldsymbol{\Omega} \times \mathbf{r}_0 \\
&\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_0) + \frac{d\boldsymbol{\Omega}}{dt} \times \mathbf{r}_0
\end{aligned}$$

等加速直线运动流体的平衡

容器以等加速度 a 沿坐标轴方向运动，在新的平衡时，容器内的液体所受质量力除了重力还有一个与运动方向相反的惯性力

$$f_x = -a, f_y = 0, f_z = -g$$

所以有

$$dp + \rho adx + \rho gdz = 0$$

积分得

$$p + \rho ax + \rho gz = C$$

C由边界条件决定。当 $x = 0, z = 0$时 $p = p_0$，解得 $C = p_0$
在液面上有

$$\rho ax + \rho gz = 0$$

等角度旋转流体的平衡

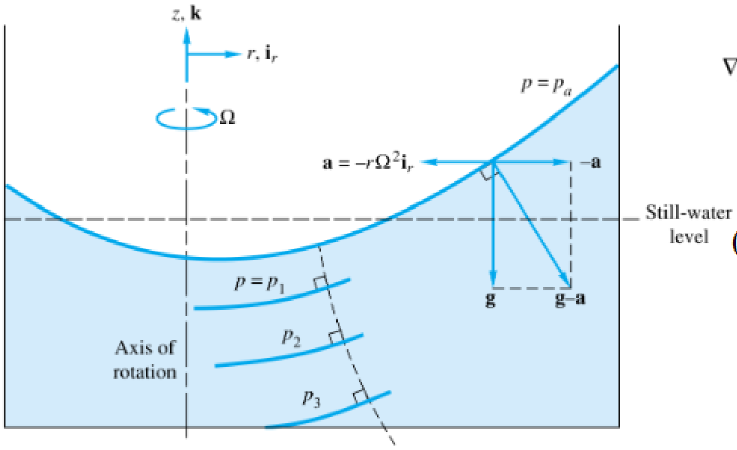
Pressure Distribution in Rigid-Body Motion

Assume that the container has been rotating long enough at constant angular velocity for the fluid to have attained rigid-body rotation.

$$\boldsymbol{\Omega} = \mathbf{k}\Omega \qquad r_0 = \mathbf{i}_r r \tag{1}$$

$$\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_0) = -r\Omega^2 \mathbf{i}_r \tag{2}$$

$$\nabla p = \mathbf{i}_r \frac{\partial p}{\partial r} + \mathbf{k}\frac{\partial p}{\partial z} = \rho(\mathbf{g} - \mathbf{a}) = \rho(-g\mathbf{k} + r\Omega^2 \mathbf{i}_r) \tag{3}$$



$$\frac{\partial p}{\partial r} = \rho r\Omega^2 \qquad \frac{\partial p}{\partial z} = -\gamma \tag{4}$$

(4) are known functions of r and z. One can proceed as follows: Integrate the first equation "partially," i.e., holding z constant, with respect to r, then

$$p = \tfrac{1}{2}\rho r^2\Omega^2 + f(z) \tag{5}$$

Now differentiate this with respect to z and compare with the second relation of (4):

$$\frac{\partial p}{\partial z} = 0 + f'(z) = -\gamma$$

$$f(z) = -\gamma z + C$$

$$p = \text{const} - \gamma z + \tfrac{1}{2}\rho r^2\Omega^2$$

中文教材（2-13），等压面：一簇对称于z轴的抛物面

$$p = p_0 - \rho g z + \frac{1}{2}\rho r^2\Omega^2$$

讨论教材56页特例1与特例2

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静止流体作用在物面上的总压力计算

静止流体在物体表面上的总压力 $\mathbf{P}$

$$\mathbf{P} = -\int_A p\mathbf{n}dA$$

A为物体与流体接触的表面面积，$\mathbf{n}$为物面单位法线向量，p为物面上的压强。

平面：

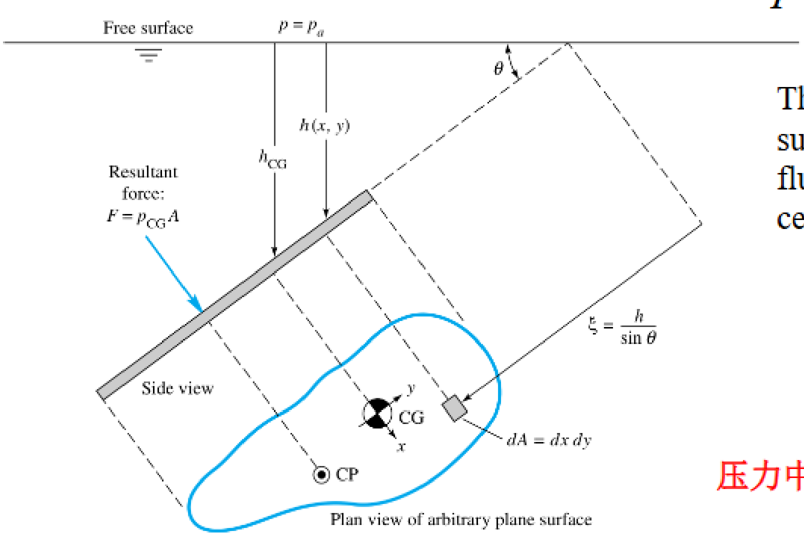
Hydrostatic Forces on Plane Surfaces

$$p = p_a + \gamma h \qquad F = \int p\, dA = \int (p_a + \gamma h)\, dA = p_a A + \gamma \int h\, dA$$

By definition, the centroidal slant distance from the surface to the plate is

$$\xi_{CG} = \frac{1}{A}\int \xi\, dA \quad \text{therefore} \quad F = p_a A + \gamma \sin\theta \int \xi\, dA = p_a A + \gamma \sin\theta\, \xi_{CG} A$$

$$F = (p_a + \gamma h_{CG})A = p_{CG} A$$



The force on one side of any plane submerged surface in a uniform fluid equals the pressure at the plate centroid times the plate area

压力中心?

Hydrostatic Forces on Plane Surfaces

$$F y_{CP} = \int y p\, dA = \int y(p_a + \gamma \xi \sin\theta)\, dA = \gamma \sin\theta \int y \xi\, dA$$

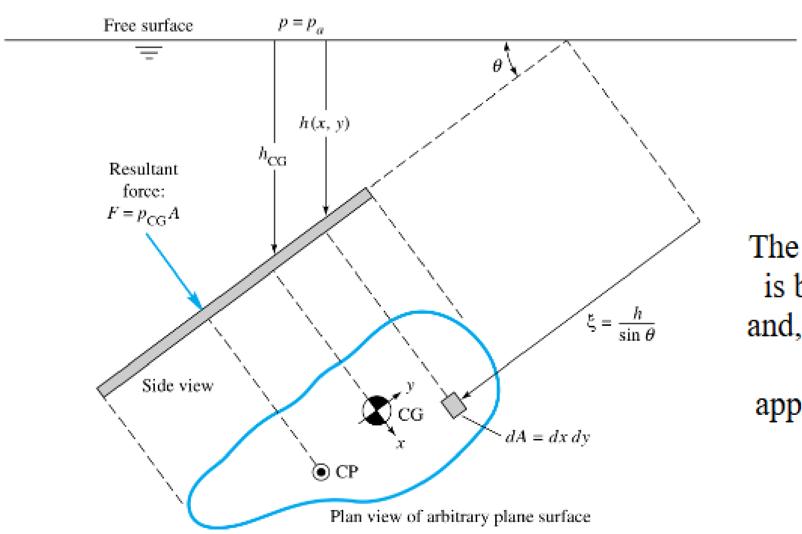
The term $\int p_a y\, dA$ vanishes by definition of centroidal axes. Introducing $\xi = \xi_{CG} - y$,

$$F y_{CP} = \gamma \sin\theta \left(\xi_{CG} \int y\, dA - \int y^2\, dA \right) = -\gamma \sin\theta\, I_{xx}$$

坐标系原点取在CG

$$y_{CP} = -\gamma \sin\theta \frac{I_{xx}}{p_{CG} A}$$

$$x_{CP} = -\gamma \sin\theta \frac{I_{xy}}{p_{CG} A}$$



The negative sign in y_{CP} shows that y_{CP} is below the centroid at a deeper level and, unlike F, depends upon angle θ. If we move the plate deeper, y_{CP} approaches the centroid because every term remains constant except p_{CG}, which increases.

曲面:

Hydrostatic Forces on Curved Surfaces

$$\mathrm{d}\boldsymbol{F} = p\mathrm{d}\boldsymbol{A} = (p_0 + \rho g h)\mathrm{d}\boldsymbol{A}$$

Its two components in (x, z) plane are

$$\mathrm{d}P_x = \sin\theta \mathrm{d}F = p\sin\theta \mathrm{d}A = p\mathrm{d}A_x = (p_0 + \rho g h)\mathrm{d}A_x$$

$$\mathrm{d}P_z = \cos\theta \mathrm{d}F = p\cos\theta \mathrm{d}A = p\mathrm{d}A_z = (p_0 + \rho g h)\mathrm{d}A_z$$

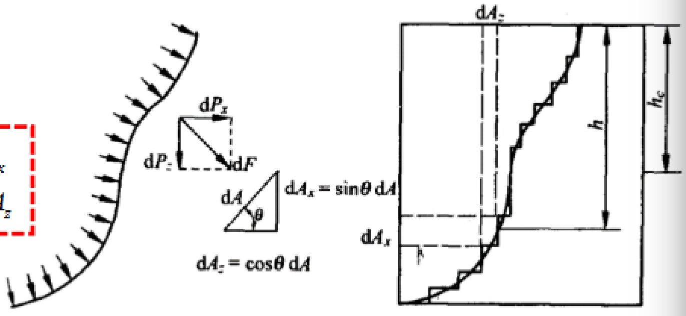


图 2-13 液体对曲面的作用

Horizontal component

$$P_x = (p_0 + \rho g h_c)A_x$$

Vertical component

$$P_z = p_0 A_z + \rho g \int h \mathrm{d}A_z = p_0 A_z + \rho g \tau$$

Pressure Volume

A_x Projection of area on vertical plane

A_z Projection of area on horizontal plane

不考虑大气压力，作用于曲面上的垂直总压力等于其压力体内的液体的重量