Analysis for phase chirp by switching AOMs

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I. Introduction

In order to achieve 10-19 precision in our clock, we need to take care of every step which can induce additional error in laser system or frequency measurements. In Stephan's article[1] they give a frequency shift below $2*10^{-17}$ after switching on acousto-optic modulator (AOM) and phase lock. With their results, we're concerned about how our AOMs influence our laser's phase.

In this report we describe a phase contrast system including 2 switching AOMs using 701nm laser. The system can be unlocked or phase-locked, which provides a more practical circumstances in real optical clock experiments. By comparing the beat signal with a local oscillator we are able to get the phase difference between two paths and see the phase chirp when switching AOMs.

The following report will be divided for three parts, first is a detailed description of our experiment's setup, second is the theory of sensitivity function and how we calculate our frequency chirp using phase chirp data, third is the data analysis with USRP. In analysis parts we studied two different situations: switching one AOM and two AOMs with phase lock. Each parts we will change the control voltage of AOMs, which means change their efficiency, to find the best efficiency which bring the lowest chirp. By using the sensitivity function we can connect phase chirp with frequency's chirp, which will give us the magnitude of error introduced by AOMs.

II. Setup

Our laser system is as shown below

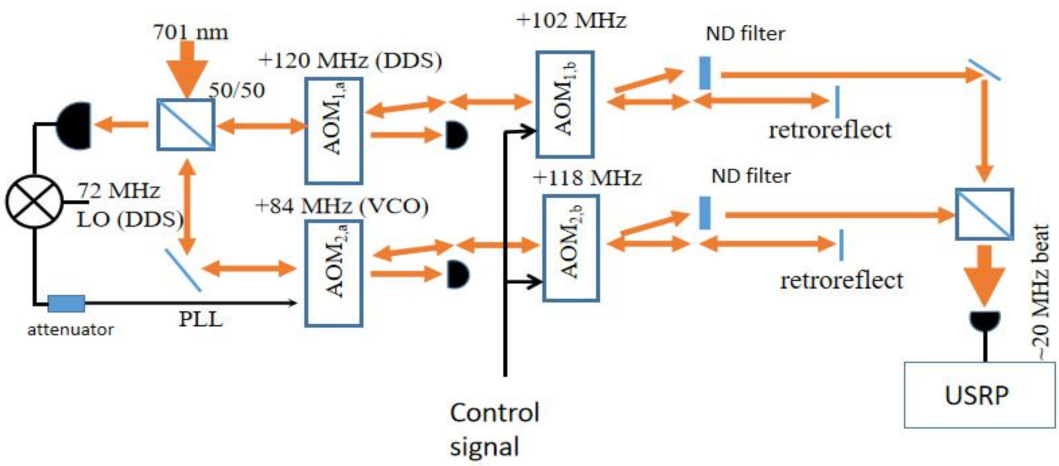


Fig.1 Switching 1 AOM Set Up

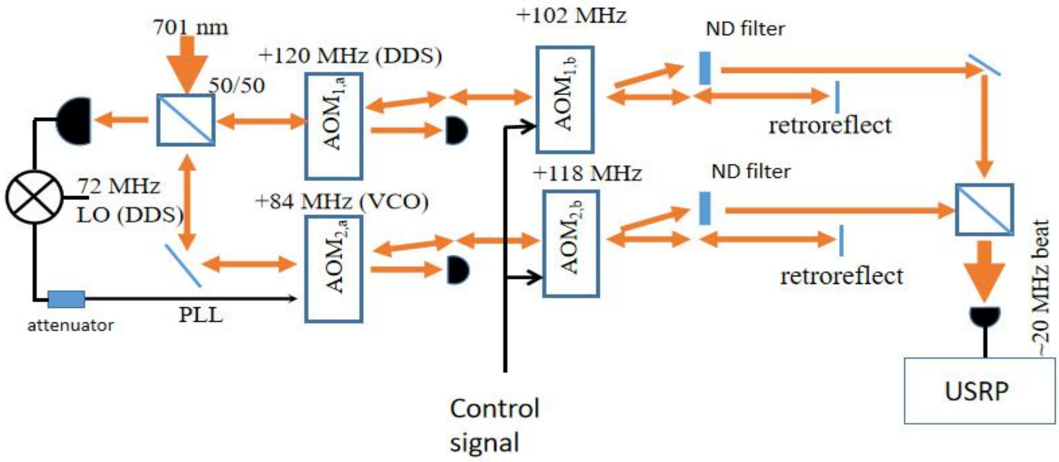


Fig .2 Switching 2 AOMs Set Up

Our 701nm laser is split for two paths. Their zero order lights at second set of AOMs are reflected back and beat at the first beam splitter(BS). This 72MHz beat signal is mixed with another 72MHz signal which comes from local oscillator(LO) and give out error signal. This error signal is used to accomplish the phase-lock. In other words, the phase difference between two paths is stable before two retroreflect mirrors, so only paths after them will be hugely affected by environment. In the analysis without phase lock we will take this part out, in which situation the whole system will be out of lock and has some drift. The detectors after first set of AOMs are used to reach power stabilization, but since it will introduce extra noise it will be turned off through the whole experiment.

After two reflect mirrors, the 1st order lights will beat at another BS and give a near 20MHz beat signal. This beat signal will go into USRP and mix with another 20MHz. Then the difference of these two signal is the phase difference we want. And the changes of this signal will tell us how much our phase changes. The control signal is using to switch on and off AOMs and help us to analysis different situations.

There is one significant thing we should keep in mind is that our DDS control boards which control AOMs cannot give a completely accurate frequency since they are digital devices. So our last beat signal is not exactly 20MHz but an unknown number near it. Here we have two ways to find the exact frequency. One is set the signal generator to be 20MHz, read the frequency of beat signal on oscilloscope at this time then set signal generator to be higher or lower than this frequency. The other is take the data sheet of the DDS boards and find their accuracy. In this situation our board's accuracy is 1GHz /2^{32}. Once we get this accuracy we are able to find a frequency which is it's integer multiple and nearest the frequency we set. That frequency is the exact frequency DDS board gives us. After clarifying all the frequency of four AOMs we can determine the final beat frequency. In our experiment it's 19.999999995MHz.

The reason why our AOMs' frequency is set like that is because our DDS borads' center frequency is near 110MHz, so if we want to control two AOMs by the same borad it's batter to set their frequencies on two sides of 110MHz. And the difference between AOMs should be set exactly equal to the beat signal we want. Note that zero order lights both go through AOMs twice so their frequency difference is two times of the frequency difference of first two AOMs.

When operating the experiment we will give second set of AOMs control signal with different attenuators and then see how the last beat signal changes. Notes here that when adding different attenuators, we need to add additional ND filters in the optical path and attenuators behind 72MHz lock signal to maintain the power of beat signal and the power of lock signal. Otherwise we cannot tell whether the phase change we get results from switching AOMs or changing beat signal's power. With that phase chirp we can calculate our frequency chirp by using sensitivity function, which will be introduced in the next part.

III. Theory

In this part we will have a detailed description about how to calculate the frequency change using the phase change data we have collected. Firstly I will introduce the sensitivity function which we mainly use in our calculation.

1. Sensitivity function

Generally speaking, the influence of a phase change to frequency detected, or in other words, to the excitation probability depends on when it happens in the pulse and what type of pulse it happens in. When a small frequency change $\delta\upsilon$ happens, excitation probability P will change by [1,2]

$$\delta p = \frac{1}{2}\int_0^T 2\pi\ \delta\nu(t)\cdot g(t)\,\mathrm{d}t \tag{1}$$

$$= -\frac{1}{2}\int_0^T \phi(t)\cdot\frac{\mathrm{d}}{\mathrm{d}t}g(t)\,\mathrm{d}t, \tag{2}$$

Where $\phi(t)$ is the phase signal and $g(t)$ is the sensitivity function. This function has different form in different kind of pulses(Rabi, Ramsey and Hyper Ramsey). Since $\delta\upsilon$ has very small fluctuation compared with laser frequency ν, we can treat it as a constant number and take it out from Eq.(1). After this treatment we will find out that the change of excitation probability δp only depends on the total pulse time T and the form on function $g(t)$. That's the relationship between frequency change and the change of excitation probability. Getting this relationship, we can use Eq.(2) to calculate δp with our phase chirp data. Once we get δp we can easily link it to frequency by Eq.(1).

After showing the basic ideas about frequency change calculation, I will introducing the specific form of sensitivity function in different kind of pulses, including its differential form and its integral result.

2. Rabi Pusle

In a Rabi π-pulse with total pulse time T, the sensitivity function $g(t)$ has the form[2]:

$$\begin{aligned} g(t) &= \sin^2\vartheta\ \cos\vartheta \\ &\quad\times[(1-\cos\Omega_2)\sin\Omega_1\ +\ (1-\cos\Omega_1)\sin\Omega_2] \\ \text{with}\ \ \vartheta &= \frac{\pi}{2}-\arctan(2T\Delta) \\ \text{and}\ \ \Omega_1 &= \pi\sqrt{1+(2T\Delta)^2}\times\frac{t}{T} \\ \Omega_2 &= \pi\sqrt{1+(2T\Delta)^2}\times\frac{T-t}{T} \end{aligned} \tag{3}$$

Where Δ is the detuning between clock laser and resonance frequency. For a full 80ms pulse $\mathrm{g(t)}$ looks like the following:

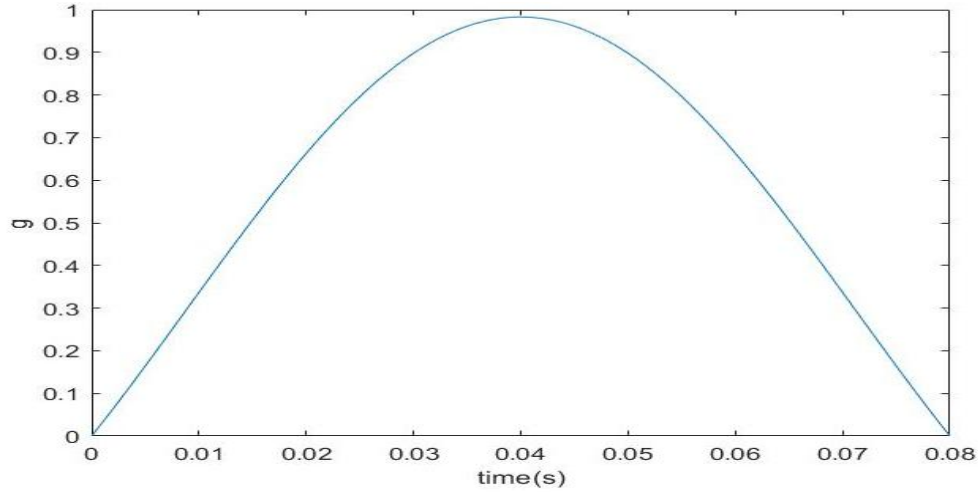


Fig .3 Sensitivity function for Rabi pulse

And its time difference $\mathrm{dg/dt}$ which we used in calculation is:

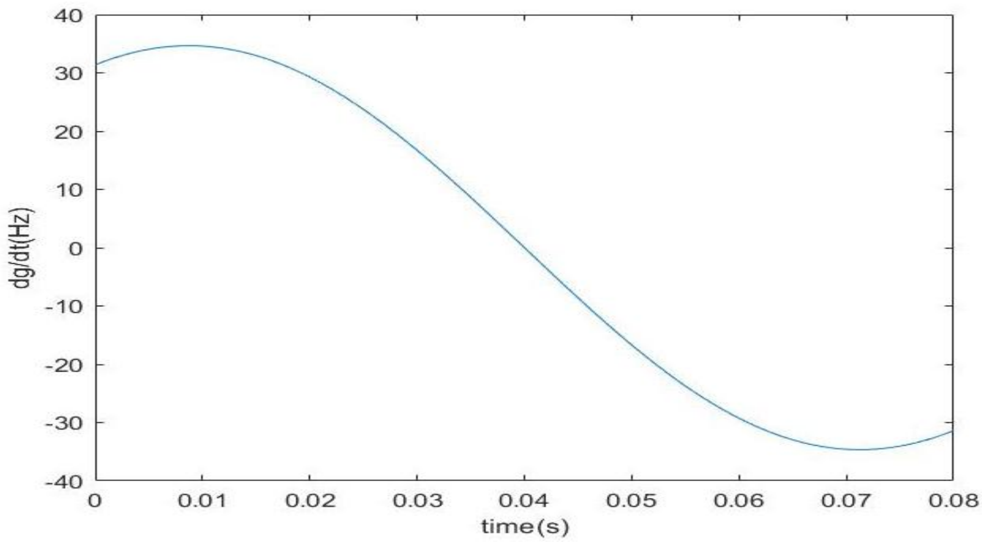


Fig .4 Time derivative of Sensitivity function for Rabi pulse

According to Fig.4, dg/dt begins at a high frequency, which means this pulse is more susceptible at the beginning of the pulse. So whether switching AOMs will introduce additional phase chirp draws our attention.

3. Ramsey Pusle

In a Ramsey $\pi/2 - \mathrm{T} - \pi/2$ pulse the sensitivity function $g(t)$ has the form[2]:

$$g(t) = \begin{cases} \sin(\pi/2 \cdot t/t_i) & \text{for } 0 \le t < t_i \\ 1 & \text{for } t_i \le t < t_i + T_d \\ \sin[\pi/2 \cdot (t - t_i - T_d)/t_i] & \text{for } t_i + T_d \le t < T_d + 2t \end{cases} \tag{4}$$

Where t_i is the pulse of two $\pi/2$ pulse and T_d is the time period between two $\pi/2$ pulse. For a full pulse $g(t)$ looks like the following:

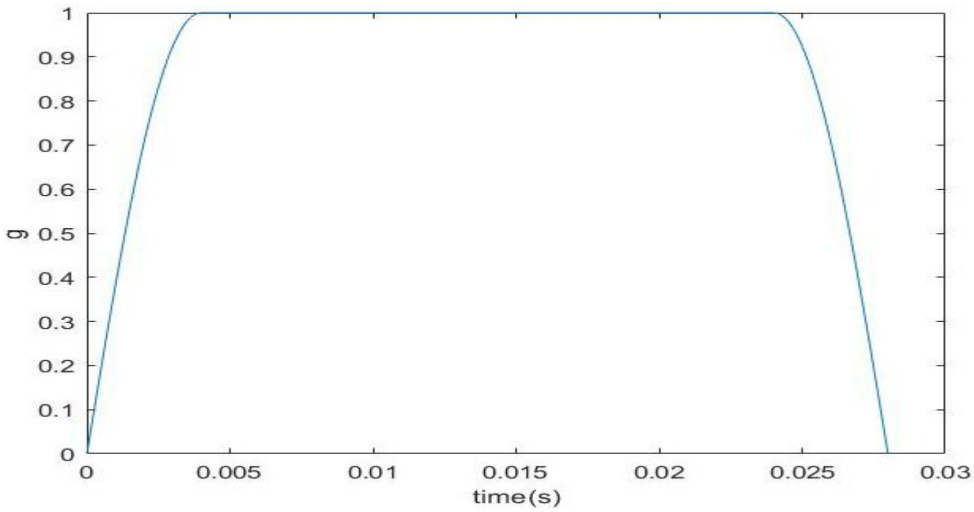


Fig .5 Sensitivity function for Ramsey pulse

And its time difference dg/dt is:

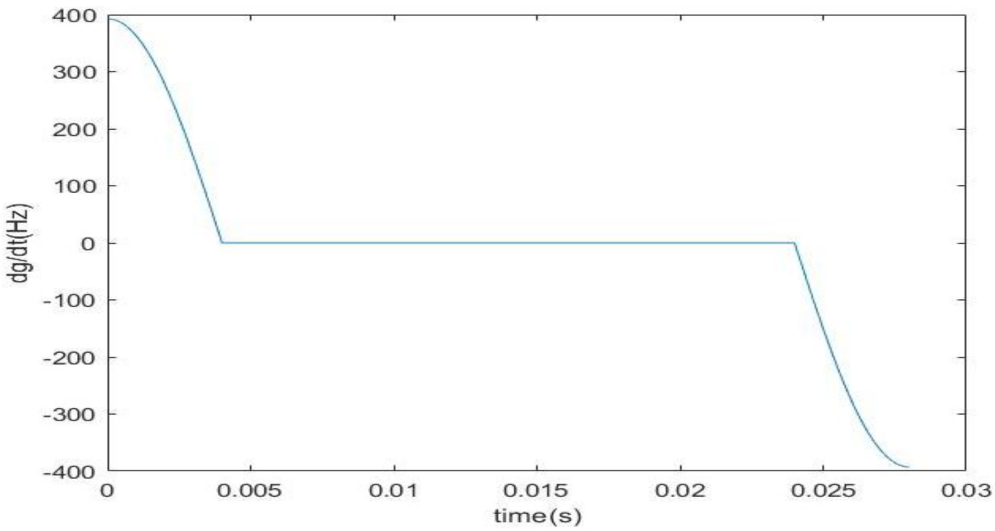


Fig .6 Time derivative of Sensitivity function for Ramsey pulse

Here $t_i = 4\text{ms}$ and $T_d = 20ms$.

Once we know the form of the sensitivity functions of these pulses, we can get their integral and use them directly into frequency chirp calculation. For Rabi pulse its integral is

$$G = \int_0^T g(t) = 0.60386\,T \tag{5}$$

So according to Eq.(1) its relationship between excitation probability change and frequency change is:

$$\frac{dp}{dv} = \pi\, 0.60386\, p_{max} T \tag{6}$$

For Ramsey:

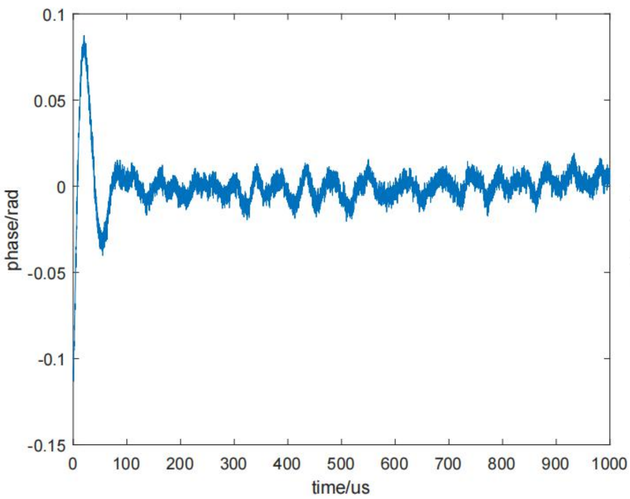
$$G = \int_0^{T+2ti} g(t) = T - \frac{4}{\pi} ti \tag{7}$$

$$\frac{dp}{dv} = \pi\, p_{max}(T - \frac{4}{\pi} ti) \tag{8}$$

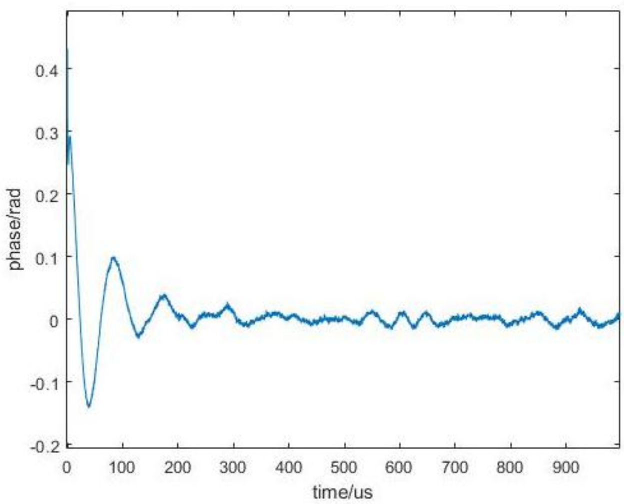
With Eq.(7) and Eq.(8), once we get the excitation probability change dp, we are able to calculate the frequency change.

IV. data analysis

In this part we will show the data collected from USRP and briefly introduce the method we treat our data. After switching on one or two AOMs, our phase difference between two paths will behave like the following:



switching 1 AOM

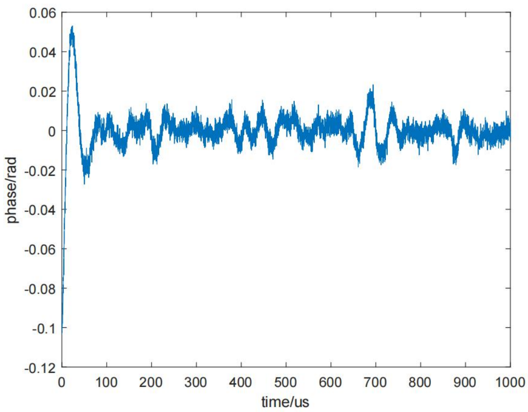


switching 2 AOMS

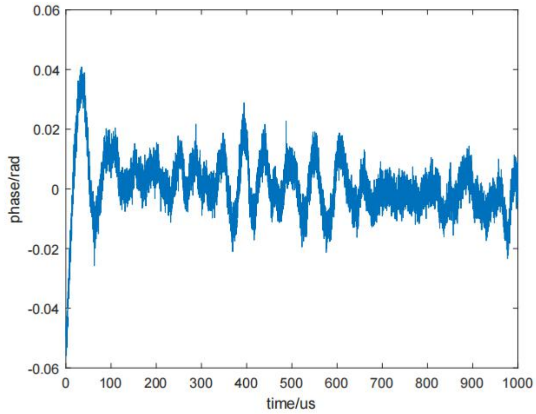
For more information about USRP, readers can refer to [3].

We can see clearly that after switching on AOMs our phase difference will have a fluctuation for about 100us long. Our ultimate goal is determining the influence of this fluctuation to our frequency. As discussed before, We added different attenuator to the RF signal to AOMs and their results look like this:

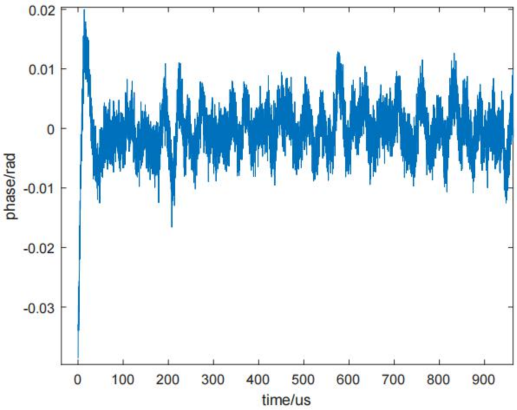
For switching 1 AOM:



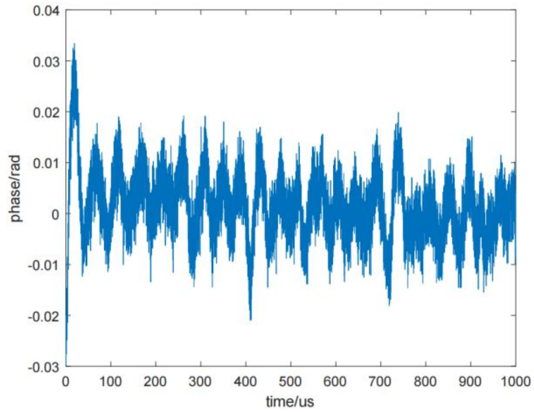
1db attenuator



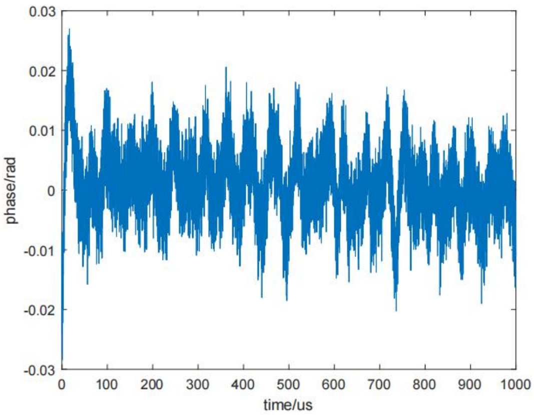
2db attenuator



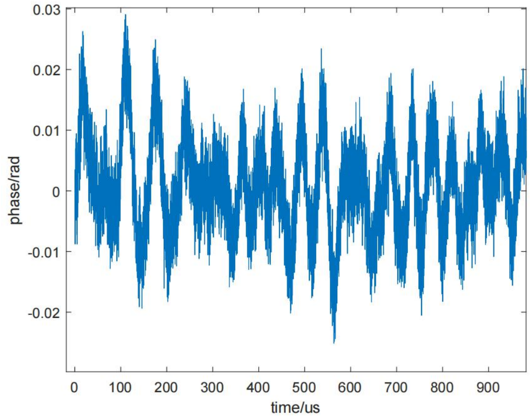
4db attenuator



6db attenuator



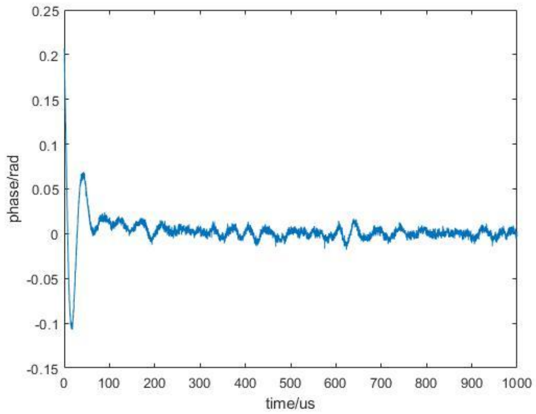
8db attenuator



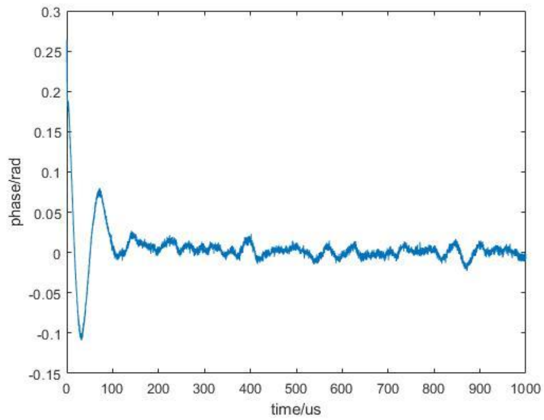
10db attenuator

With these results we are able to tell that adding attenuator, which means decreasing the power of AOM’s RF signal, will help to reduce the fluctuation. And for 2 AOMs we will get similar result.

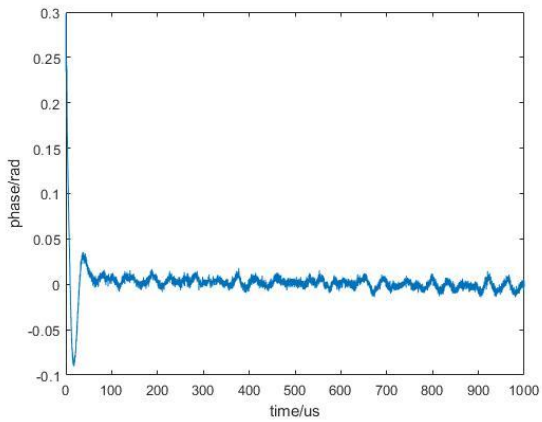
For switching 2 AOMs:



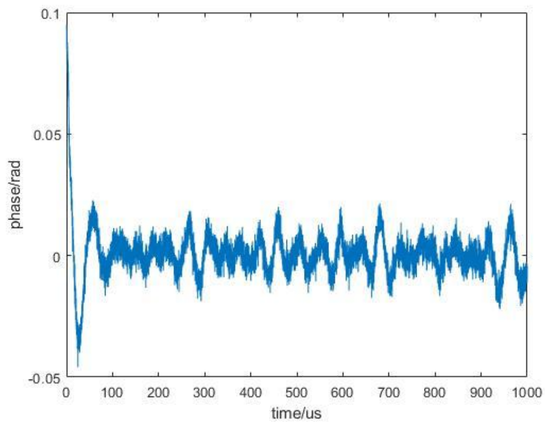
1db attenuator



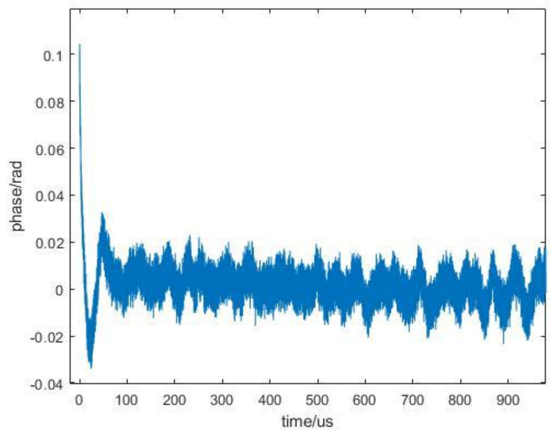
2db attenuator



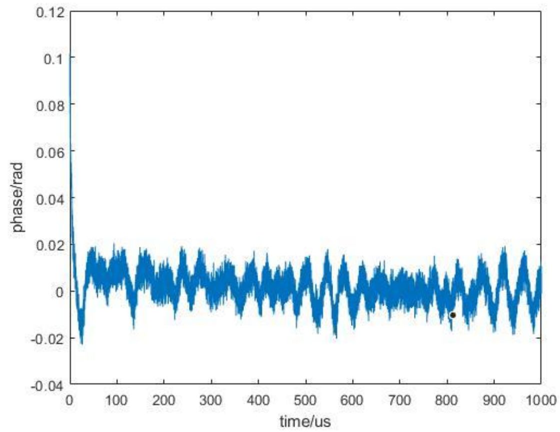
4db attenuator



6db attenuator

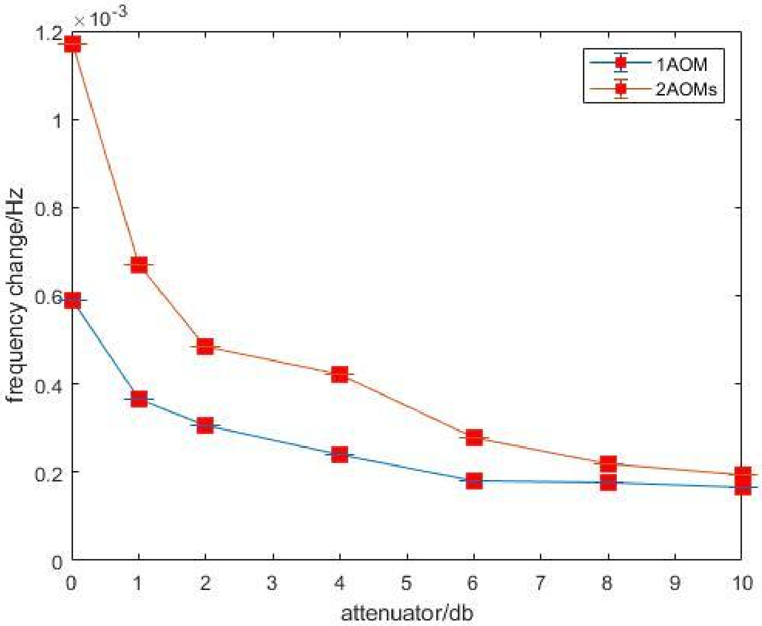


8db attenuator



10db attenuator

After getting these data we are able to use sensitivity function method we introduced before to calculate the frequency chirp. For example, if we apply this method to a 80ms Rabi pulse, we will get the result as following:



80ms Rabi Result

From this figure we can drew three conclusions, first, the influence of switching AOMs to frequency change is at the level of 10^{-3} ~ 10^{-4}Hz, which means the error level to 701nm laser(428THz) is 10^{-18} ~ 10^{-19}. Second, the influence is decreasing when adding attenuator to AOMs' power. It is obvious as the fluctuations become smaller when adding attenuator. Third, switching 2 AOMs has bigger influence than swiching 1 AOM, it's also obvious since if we have a closer look at the data, for same attenuator switching 2 AOMs has bigger fluctuation than 1 AOM at the beginning. We will have a similar result if we apply these data to a Ramsey pulse with certain pulse time, the only difference between results is the number of frequency changes and it's only comes from the ratio between two pules' time.

V. Conclusion

In this report we introduce a method to determine the influence of switching AOMs to the frequency detected. For a 80ms Rabi pulse , with phase lock on most of the path in our system, the influence of switching AOMs is at the level 10^{-18} ~ 10^{-19}. Adding attenuator to AOMs' RF signal, which means decreasing their RF signals' power, will help to decrease this influence.

VI. Future Work

The result we drew before is based on the assumption that the power of AOM will be stable at a long time period, which means the phase difference will stay at zero at a long time period. But from the data we gathered this is not the case. Our system will have a long-time drift which will also have the influence to our frequency. Determining this error is also an significant and interesting work to do.

[1] Stephan, F., Mattias, M., Uwe, Sterr., and Christian, L.(2012) *Appl. Phys. B* 107, 301-311.
[2] DIck, G.(1988) In Proceedings of 19th Annu. Precise Time and Time Interval Meeting, Reddendo Beach; 1987 Washington, DC: U.S. Naval Observatory. pp. 133-147.
[3]Sherman Jeff A. and Jordens, Robert (2016) *arXiv e-prints arXiv:1605.03505.*