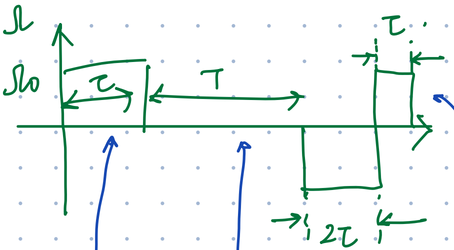
Derivation of HyperRamsey Excitation Probability



$$\begin{pmatrix} c(\tau) & is(\tau) \\ is(\tau) & c(\tau) \end{pmatrix} \quad \begin{pmatrix} e^{i\Delta T/2} & 0 \\ 0 & e^{-i\Delta T/2} \end{pmatrix} \quad \begin{pmatrix} c(2\tau) & -is(2\tau) \\ -is(2\tau) & c(2\tau) \end{pmatrix} \quad \begin{pmatrix} c(\tau) & is(\tau) \\ is(\tau) & c(\tau) \end{pmatrix}$$

$$U_1(t;\phi) = \begin{pmatrix} c(t) & ie^{i\phi}s(t) \\ ie^{-i\phi}s(t) & c(t) \end{pmatrix}$$

$$U_0(t) = \begin{pmatrix} e^{i\Delta T/2} & 0 \\ 0 & e^{-i\Delta T/2} \end{pmatrix}$$

where $c(t) \equiv \cos\left(\frac{|\tilde{\Omega}|t}{2}\right)$

$s(t) \equiv \sin\left(\frac{|\tilde{\Omega}|t}{2}\right)$.

U_1 is propagator during pulse for duration t

U_0 is propagator during dark time

$|e\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|g\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

assuming $\Delta \ll |\tilde{\Omega}| = \sqrt{|\Omega|^2 + \Delta^2}$.

Note that τ is the $\pi/2$ time

(i.e. $|\tilde{\Omega}|\tau \approx |\Omega|\tau = \pi/2$)

Hence

$$c(\tau) = s(\tau) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}.$$
$$c(2\tau) = \cos(\pi/2) = 0.$$
$$s(2\tau) = \sin(\pi/2) = 1.$$

Hence full propagator is:

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}\begin{pmatrix} e^{i\Delta T/2} & 0 \\ 0 & e^{-i\Delta T/2} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

$$P_e = |\langle e|U|g\rangle|^2$$

$$= \frac{1}{4}\left|\begin{pmatrix} 1 & i \end{pmatrix}\begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}\begin{pmatrix} e^{i\Delta T/2} & 0 \\ 0 & e^{-i\Delta T/2} \end{pmatrix}\begin{pmatrix} i \\ 1 \end{pmatrix}\right|^2$$

$$= \frac{1}{4}\left|\begin{pmatrix} 1 & -i \end{pmatrix}\begin{pmatrix} ie^{i\Delta T/2} \\ e^{-i\Delta T/2} \end{pmatrix}\right|^2.$$

$$= \frac{1}{4}\left|ie^{i\Delta T/2} - ie^{-i\Delta T/2}\right|^2.$$

$$= \frac{1}{4}\, 4\sin^2\left(\frac{\Delta T}{2}\right).$$

$$= \underline{\sin^2\left(\frac{\Delta T}{2}\right)} \quad (\text{for } \Delta \ll |\tilde{\Omega}|).$$