TUTORIAL 8

COMPLEX NUMBERS

1. Simplify the following:
   1.1 j^{403}
   1.2 $(j^{891}+1)^2$
   1.3 $j^8-\frac{1}{j^6}+(-j)^3-7(1-j)^3$ $[16+15j]$

2. Solve the following equations:
   2.1 $x^2+25=0$
   2.2 $2x^2=2x-1$
   2.3 $x^2-6x+11.25=0$

3. Simplify:
   3.1 $\frac{3(\cos 20°+j\sin 20°)\times 5(\cos 10°+j\sin 10°)}{2(\cos 30°+j\sin 30°)}$ $\left[\frac{15}{2}\right]$

   3.2 $\frac{\left(1-j\sqrt{3}\right)^3.\ (1+j)^8}{(-2-j2)^4}$ … (Hint: convert to polar form) [2]

   3.3 $\frac{(1+j1.5)(1+j2)}{e^{j1.2}.\ \underline{3|30°}}$. Give your answer in polar form $\left[1.03|\underline{21°}\right]$

   3.4 $\frac{3j(4+j4)}{8-j8}$. Express your answer in polar form $\left[1.5|\underline{\pi}\right]$

4. Evaluate the following, giving your answer in rectangular form:
   4.1 $\ln \underline{2|75°}$
   4.2 $\ln 5|\underline{210°}$ $[1.609-j2.618]$
   4.3 $\ln z$ if $z=\underline{\left(2|15°\right)^2}$ $[1.386+j0.524]$
   4.4 $\frac{\ln 2|\underline{60°}}{(1+j2)^2}$ $[2.109-j5.914]$

5. Solve for x and y in the following equations:

5.1 $2(x + jy) = 6 - j4$

5.2 $(1 + j2)(-2 - j3) = x + jy$

5.3 $\frac{2+j}{1-j} = j(x + jy)$ $\left[x = \frac{3}{2}\,; y = -\frac{1}{2}\right]$

5.4 $(x - j2y) - (y - jx) = 2 + j$ $[x = 3\,; y = 1]$

6. Answer the following questions based on the given complex numbers:

6.1 Given $z_1 = 5 + j2$; $z_2 = -4 + j$ and $z_3 = -j3$, evaluate
$\ln z_2 + \frac{(z_1)^2}{z_3}$

6.2 Given $z_1 = 5 + j2$; $z_2 = -3 + j4$ and $z_3 = 4 - j4$, evaluate

6.2.1 $2z_2 - z_1 + \frac{1}{2}(z_3)$

6.2.2 $(2 - j4)z_1 + (z_3)^2$

6.3 Given $z_1 = 4 + j$; $z_2 = -3 + j3$ and $z_3 = -4 - j2$, evaluate

6.3.1 $2z_1 + (z_3)^2$ in rectangular form

6.3.2 $z_1 z_2$ in polar form where θ is in radian measure $[17.49|\underline{2.6}]$

7. Solve the equations below, based on the complex numbers listed:

7.1 Given $z_1 = 1 + j2$ and $z_2 = 4(\cos 60° + j \sin 60°)$, solve
$z_1 - z_2 = x + j2y$ correct to 2 decimal places.

7.2 Given $z_1 = 4 - j3$ and $z_2 = j$, solve
$\frac{1}{8}(x + jy) = \frac{1}{z_1 - z_2} + z_1 z_2$ $[x = 25\,; y = 33]$

7.3 Given $z_1 = 3 + j\frac{1}{2}$ and $z_2 = -1 - j2$, solve
$(z_1)^2 + x = \frac{z_1}{z_2} - jy$ $\left[x = -\frac{51}{4}\,; y = \frac{5}{2}\right]$