$$S_n = \sum_{1}^{n} (7i+2)^3 \rightarrow \sum_{1}^{n} (343i^3 + 294i^2 + 84i + 8) \rightarrow$$

$$\sum_{1}^{n} 343i^3 + \sum_{1}^{n} 294i^2 + \sum_{1}^{n} 84i + \sum_{1}^{n} 8 \rightarrow 343\sum_{1}^{n} i^3 + 294\sum_{1}^{n} i^2 + 84\sum_{1}^{n} i + 8\sum_{1}^{n} 1$$

$$343 \cdot \left(\frac{n(n+1)}{2}\right)^2 + 294 \cdot \left(\frac{n(n+1)\cdot(2n+1)}{6}\right) + 84 \cdot \frac{n(n+1)}{2} + 8n$$

$$343 \cdot \frac{(n^4 + 2n^3 + n^2)}{4} + 294 \cdot \frac{(2n^3 + 3n^2 + n)}{6} + 84 \cdot \frac{(n^2 + n)}{2} + 8n$$

$$\frac{343n^4 + 686n^3 + 343n^2}{4} + \frac{588n^3 + 882n^2 + 294n}{6} + \frac{84n^2 + 84n}{2} + 8n$$

$$\frac{1029n^4 + 2058n^3 + 1029n^2 + 1176n^3 + 1764n^2 + 588n + 504n^2 + 504n + 96n}{12}$$

$$\boxed{\frac{1029n^4 + 3234n^3 + 3297n^2 + 1188n}{12}}$$

Indução: $S_n = \dfrac{1029(n-1)^4 + 3234(n-1)^3 + 3297(n-1)^2 + 1188(n-1)}{12} + (7n+2)^3$

$(7n+2)^3 \rightarrow 343n^3 + 294n^2 + 84n + 8$

$\cancel{1029n^4} - \cancel{4116n^3} + \cancel{6174n^2} - \cancel{4116n} + 1029 + \cancel{3234n^3} - \cancel{9702n^2} + \cancel{9702n}$

$-\cancel{1} + \cancel{3297n^2} - \cancel{6594n} + \cancel{1} + \cancel{1188n} - 1188 + \cancel{4116n^3} + \cancel{3528n^2} + \cancel{1008n} + 96$

$$\boxed{\dfrac{1029n^4 + 3234n^3 + 3297n^2 + 1188n}{12}}$$