Coarse Graining Holographic Black Holes

Engelhardt & Wall 2018

William Arnold

HRT is great but...

HRT as fine-grained entropy

$$S_{vN} = \frac{\text{Area}[X]}{4G\hbar}$$

Limitations:

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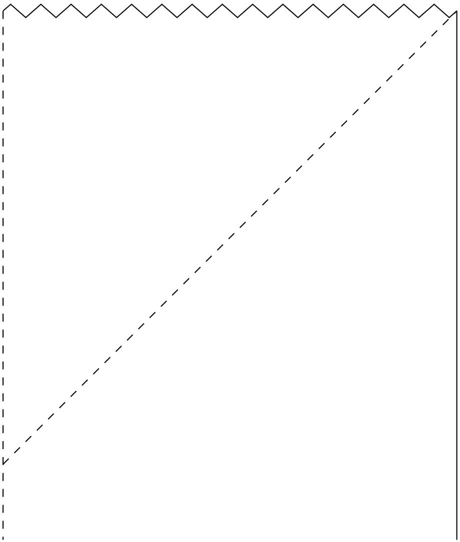
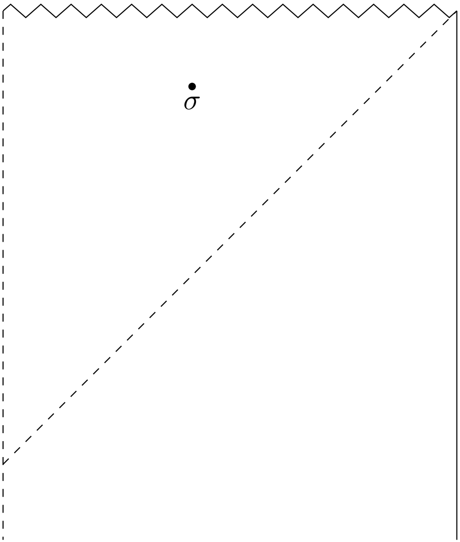
- Need to coarse-grain over thermalized degrees of freedom

Entropy for any surface?



Entropy for any surface?

- Pick a (compact) Cauchy-splitting surface σ



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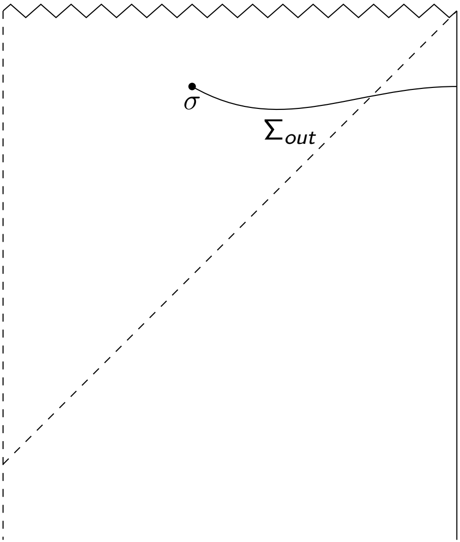

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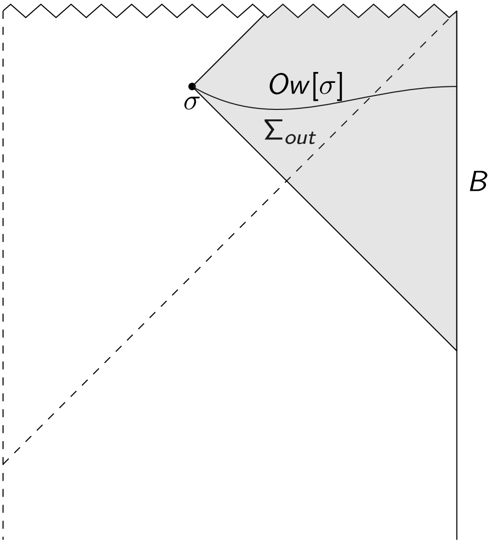

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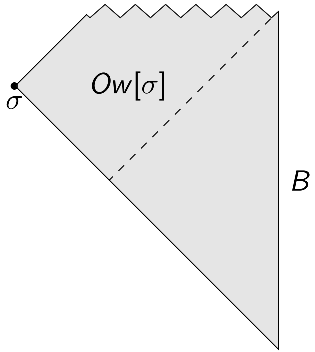

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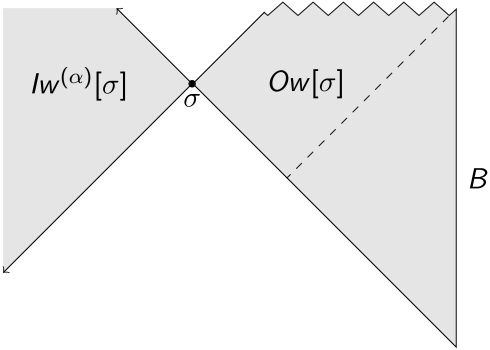

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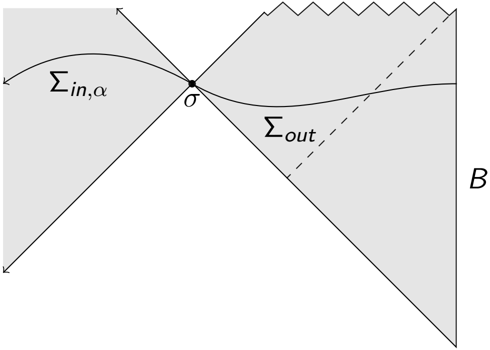

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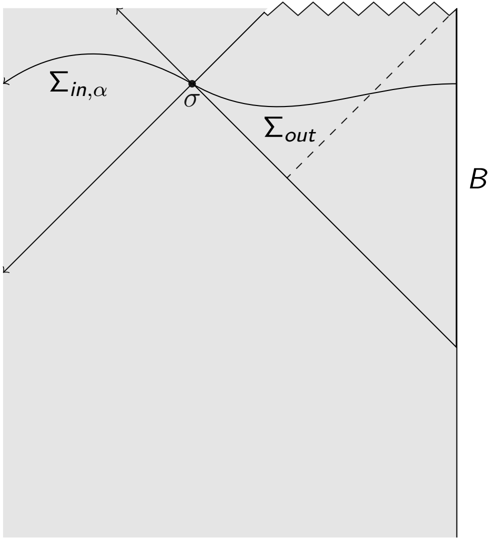

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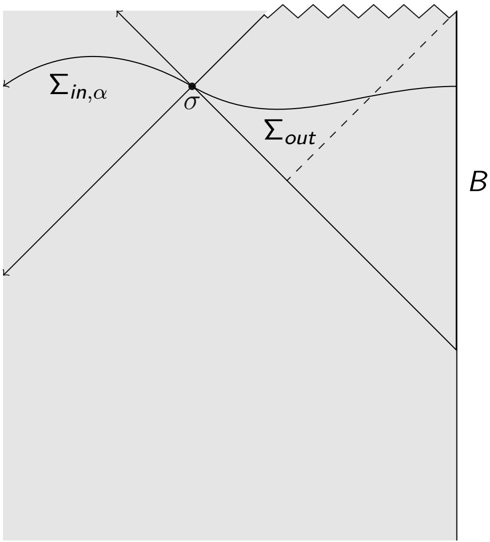

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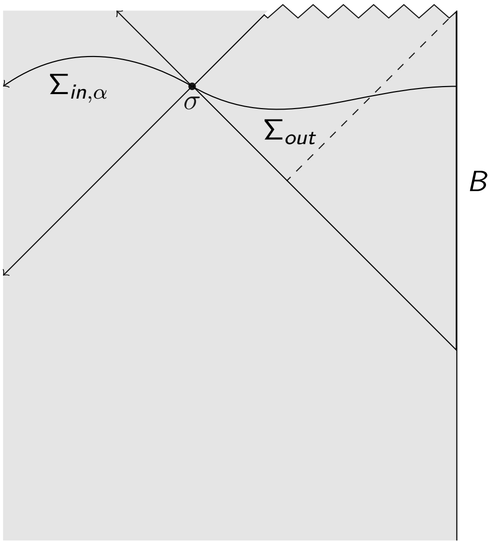
- IVP $\rightarrow$ full spacetime
- Some state $\rho_B^{(\alpha)}$ on boundary
- Compute $S_{vN}[\rho_B^{(\alpha)}]$!



Can define a new entropy for *any* surface homologous to B

$$S^{(outer)}[\sigma] = \max_{\{\alpha\}} \left[-\operatorname{tr}\left(\rho_B^{(\alpha)} \ln \rho_B^{(\alpha)} \right) \right]$$

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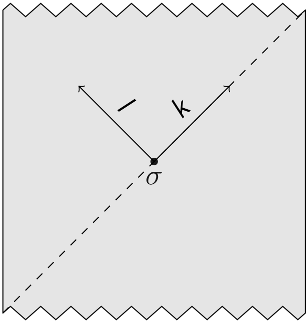
For an HRT Surface, X,

$$S^{(outer)}[X] = -\operatorname{tr}\left(\rho_B \ln \rho_B\right) = \frac{\text{Area}[X]}{4G\hbar}$$

Surfaces

What about other surfaces?



Surfaces

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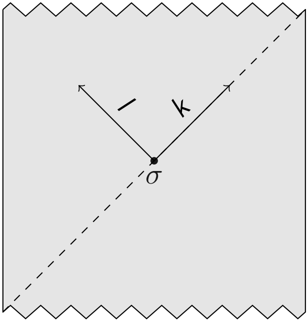
- $\theta_{(k)} = 0, \theta_{(l)} = 0$: Extremal



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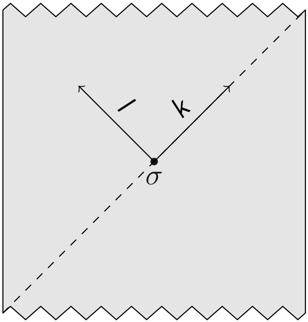
- Relax conditions: only $\theta_{(k)} = 0$



Surfaces

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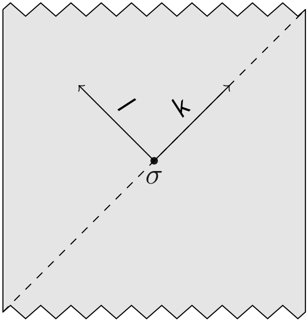
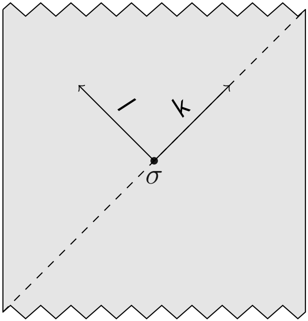
- $\theta_{(k)} = 0, \theta_{(l)} = 0$: Extremal
- Relax conditions: only $\theta_{(k)} = 0$
- Marginal surface, stationary in the k direction



Minimality

- $\theta_{(k)} = 0, \theta_{(l)} = 0$, Minimal Area: HRT



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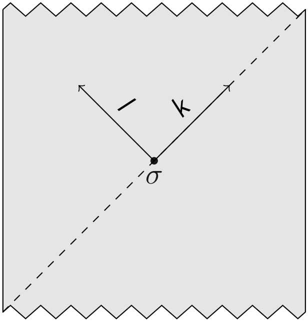

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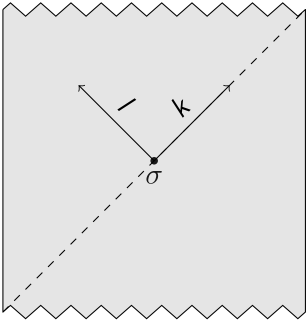

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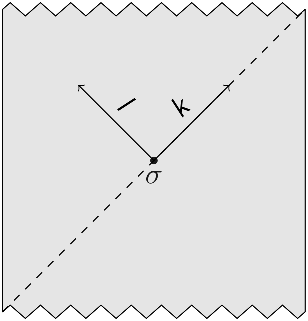
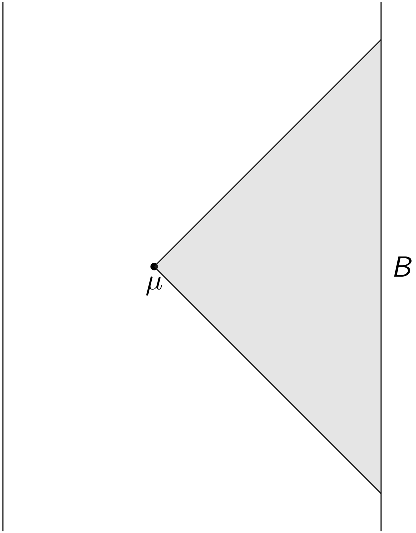
- True on HRT Surfaces



Outer Entropy & Minimar Surfaces

For choice of α, $Ow[\mu]$



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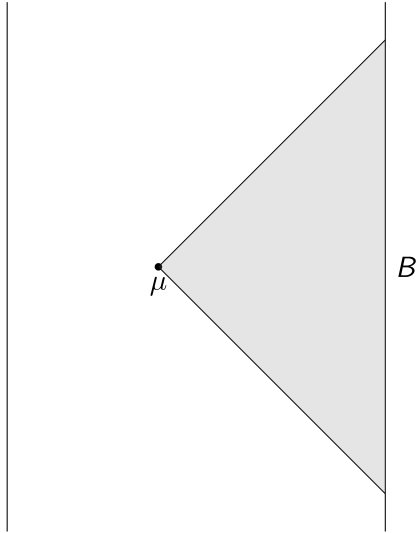
- Have outer wedge, boundary



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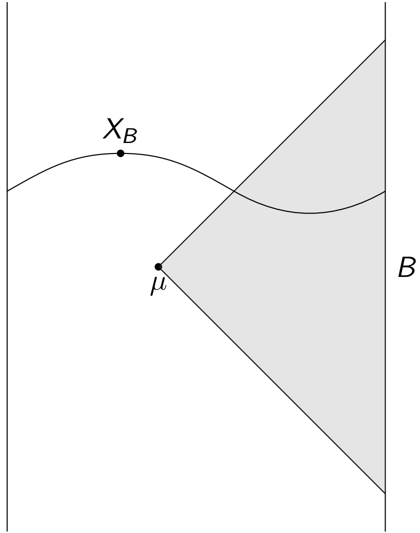

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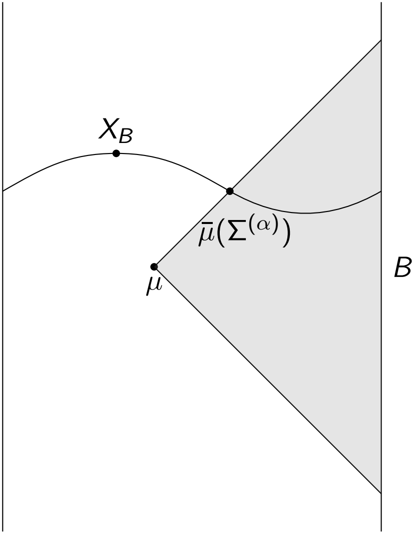

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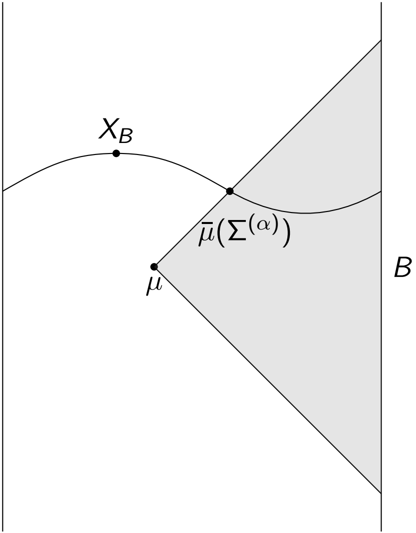

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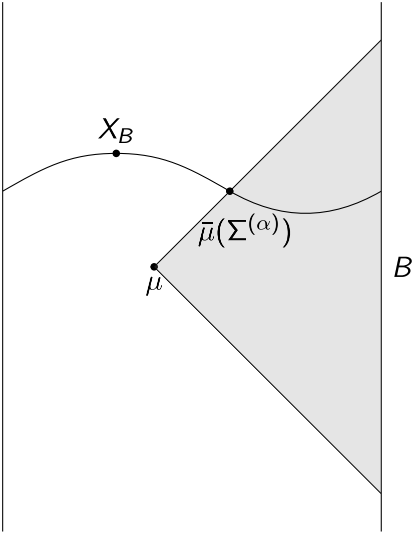
$$S[\rho_B] = \frac{\text{Area}[X_B^{(\alpha)}]}{4G\hbar} \qquad \text{(HRT)}$$



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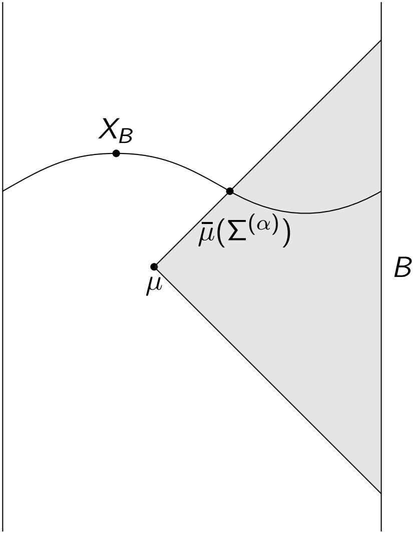
$$\begin{aligned} S[\rho_B] &= \frac{\text{Area}[X_B^{(\alpha)}]}{4G\hbar} && \text{(HRT)} \\ &\leq \frac{\text{Area}[\bar{\mu}(\Sigma^{(\alpha)})]}{4G\hbar} && \text{(maximin)} \end{aligned}$$



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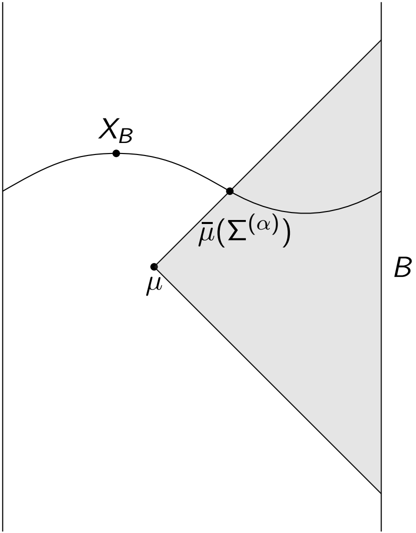
$$\leq \frac{\text{Area}[\mu]}{4G\hbar} \qquad \text{(NCC)}$$



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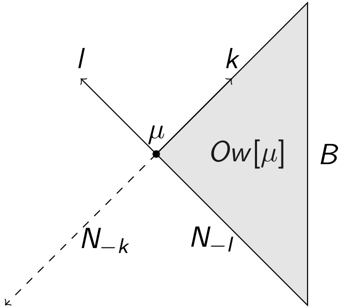
- How tight is this bound?
- Can always find α that saturates:

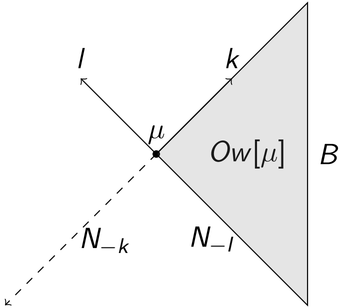
$$S^{(outer)}[\mu] = \frac{\text{Area}[\mu]}{4G\hbar}$$

Hitting the bound



Hitting the bound

- Can glue on stationary N_{-k}



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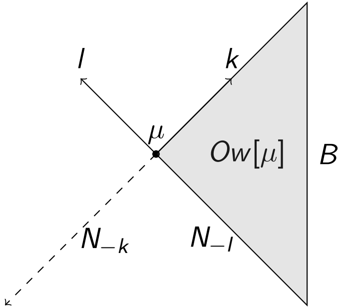

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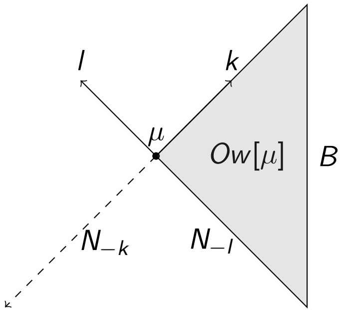

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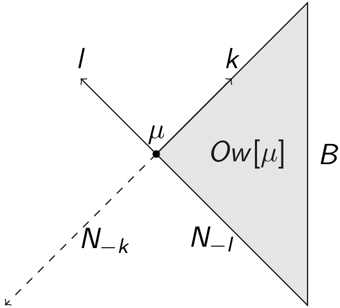

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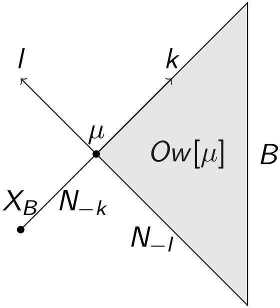

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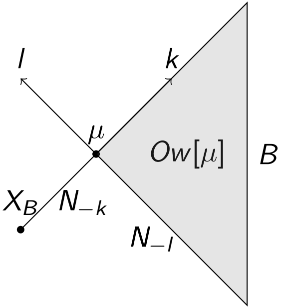
- Gives us extremal X_B



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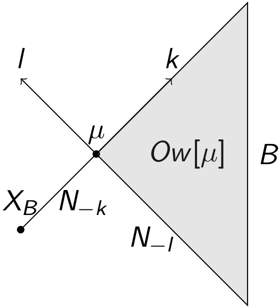
- N_{-k} stationary, so $\text{Area}[X_B] = \text{Area}[\mu]$



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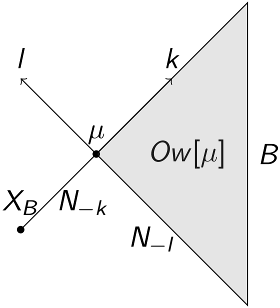
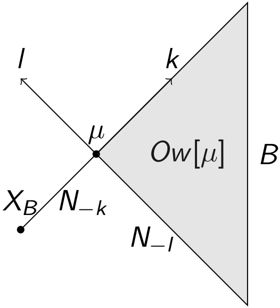
- Gives us extremal X_B
- N_{-k} stationary, so $\text{Area}[X_B] = \text{Area}[\mu]$
- Can X_B be HRT?



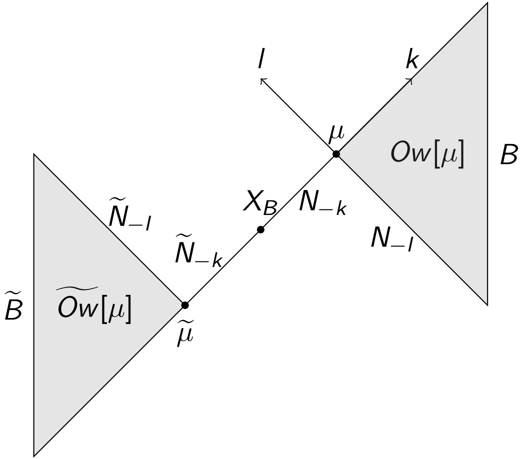
Hitting the bound

- Need to build $lw[\mu]$



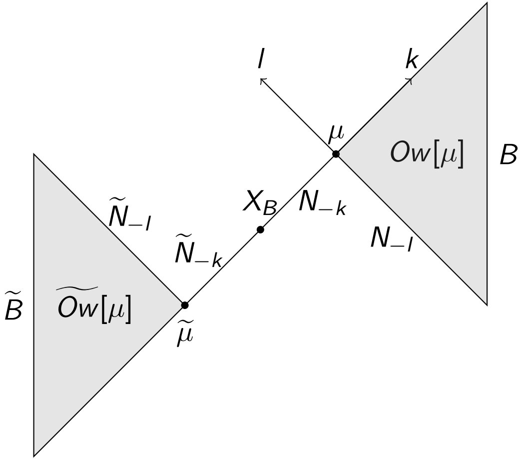
Hitting the bound

- Need to build $Iw[\mu]$
- Use CPT symmetry through X_B



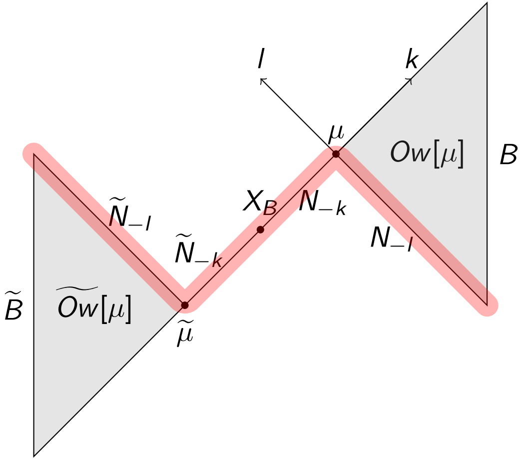
Hitting the bound

- Need to build $Iw[\mu]$
- Use CPT symmetry through X_B
- Wormhole-like geometry



Hitting the bound

- Need to build $Iw[\mu]$
- Use CPT symmetry through X_B
- Wormhole-like geometry
- Cauchy surface: $\widetilde{N}_{-l} \cup \widetilde{N}_{-k} \cup N_{-k} \cup N_{-l}$



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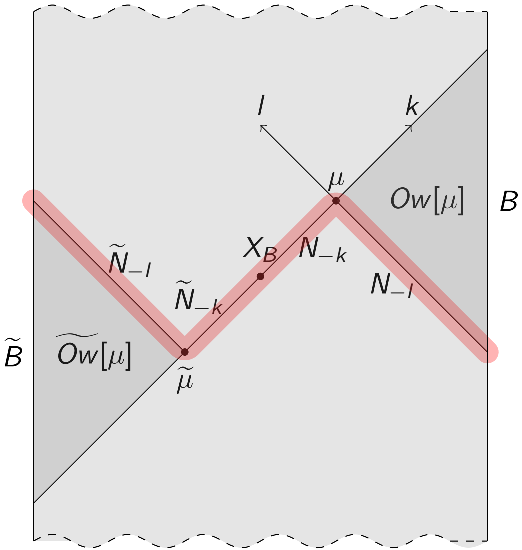
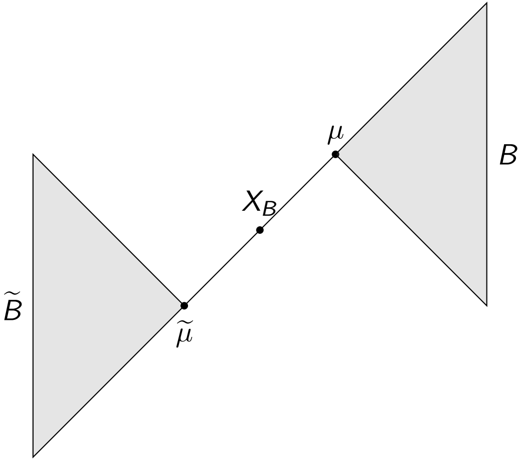
- IVP $\rightarrow$ Full spacetime



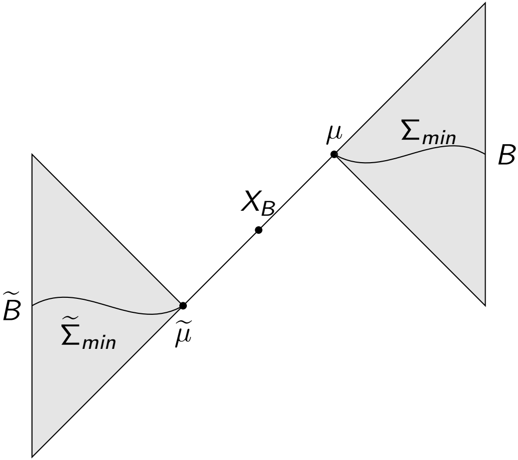
Hitting the bound

- Is it HRT?



Hitting the bound

- Is it HRT?
- Pick partial $\widetilde{\Sigma}_{min}, \Sigma_{min}$ where μ minimal area.



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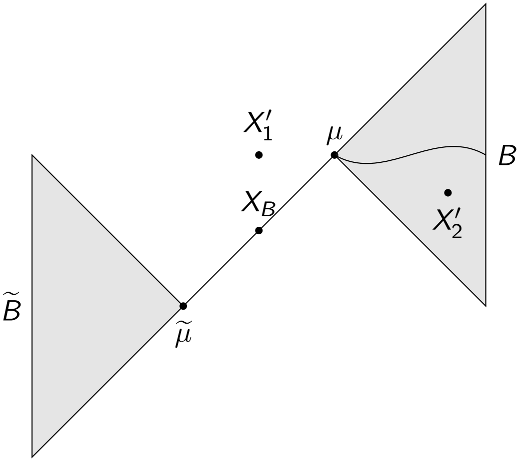

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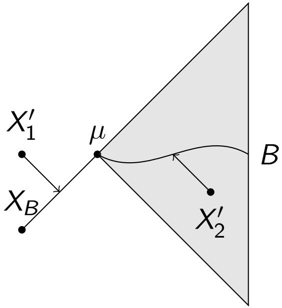

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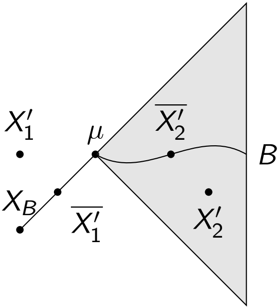

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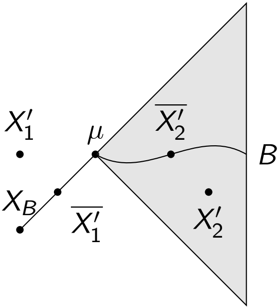

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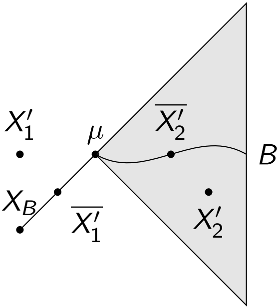
- Representatives are always smaller!



$$\text{Area}[X_{1,2}'] \geq \text{Area}[\overline{X_{1,2}'}] \geq \text{Area}[\mu] = \text{Area}[X_B]$$

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Natural generalization of HRT with coarse graining!

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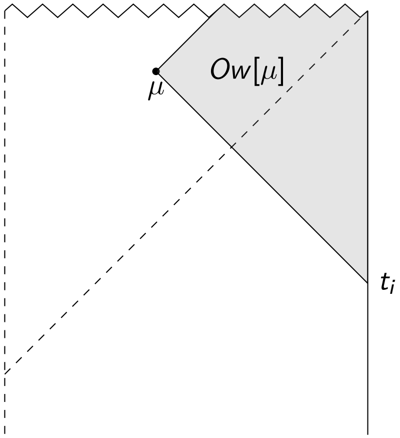

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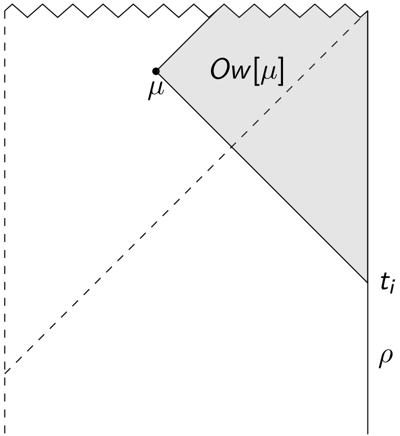

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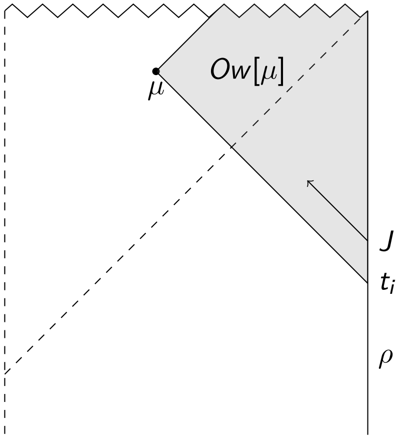

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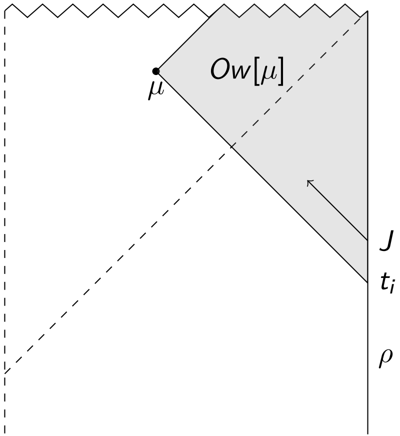

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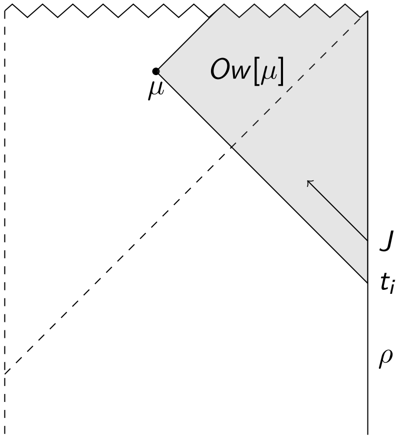

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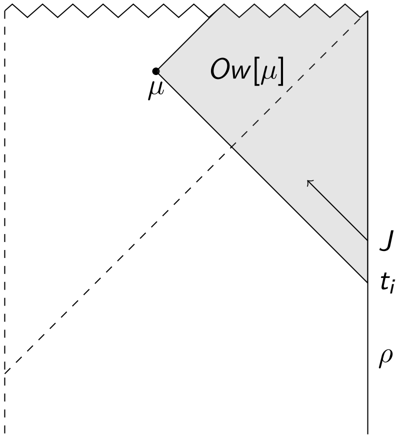

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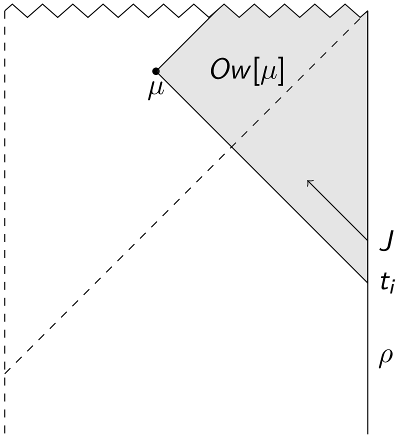
$$S^{(simple)}[t_i] = S^{(outer)}[\sigma]$$



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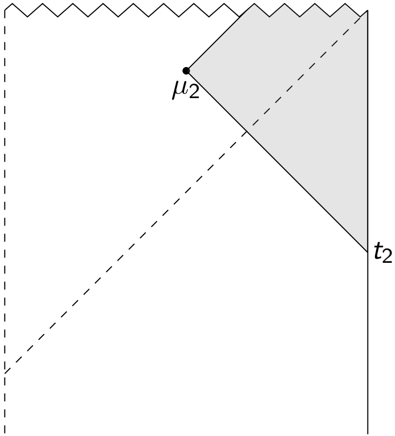

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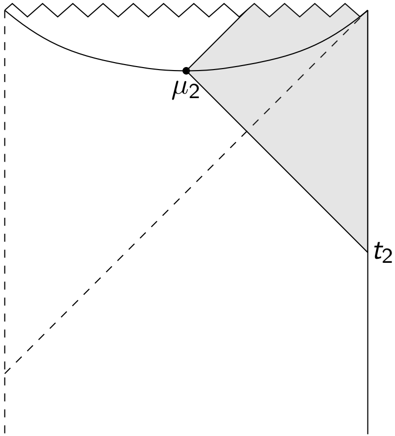

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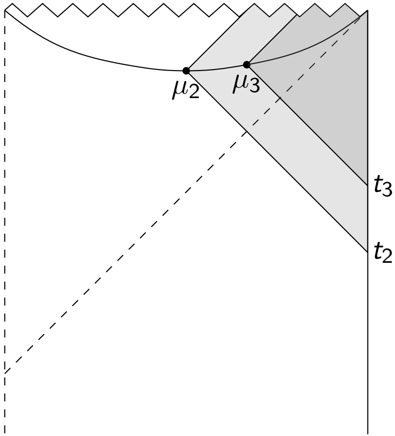

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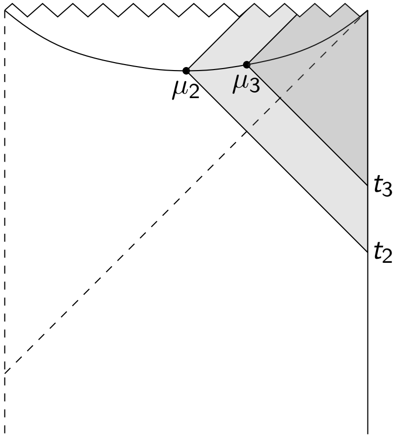

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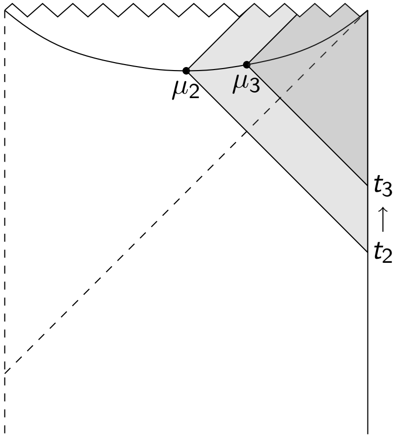

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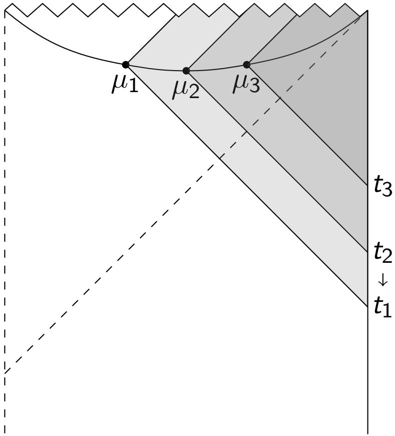
- Opposite case for moving in



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  - Raphael, Ven, Arvin did this (arxiv:1906.05299)

Thanks!