

# Introduction to Topology

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## 1 Introduction

Topology is a branch of mathematics that studies properties of spaces that are preserved under continuous deformations. It provides a framework for understanding the fundamental structure of mathematical objects.

## 2 Basic Concepts

### 2.1 Topological Spaces

A **topological space** is a set  $X$  together with a collection  $\tau$  of subsets of  $X$  (called open sets) that satisfies:

1. The empty set and  $X$  itself are in  $\tau$
2. The union of any collection of sets in  $\tau$  is also in  $\tau$
3. The intersection of any finite collection of sets in  $\tau$  is also in  $\tau$

### 2.2 Continuous Functions

A function  $f : X \rightarrow Y$  between topological spaces is **continuous** if for every open set  $V \subseteq Y$ , the preimage  $f^{-1}(V)$  is open in  $X$ .

## 3 Important Theorems

### 3.1 Brouwer Fixed Point Theorem

Every continuous function from a closed ball in  $\mathbb{R}^n$  to itself has at least one fixed point.

### 3.2 Intermediate Value Theorem

If  $f : [a, b] \rightarrow \mathbb{R}$  is continuous and  $f(a) \neq f(b)$ , then for any value  $c$  between  $f(a)$  and  $f(b)$ , there exists  $x \in (a, b)$  such that  $f(x) = c$ .

## 4 Applications

Topology has applications in:

- Algebraic topology
- Differential geometry
- Functional analysis
- Data analysis and machine learning