Introduction to Quantum Mechanics

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1 The Wave Function

In quantum mechanics, the state of a particle is described by a wave function $\psi(x,t)$ that satisfies the Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi \tag{1}$$

where \hat{H} is the Hamiltonian operator and \hbar is the reduced Planck constant.

2 The Uncertainty Principle

Heisenberg's uncertainty principle states that:

$$\Delta x \Delta p \ge \frac{\hbar}{2} \tag{2}$$

This fundamental limit means we cannot simultaneously know both the position and momentum of a particle with arbitrary precision.

3 Quantum Superposition

One of the most fascinating aspects of quantum mechanics is superposition. A quantum system can exist in multiple states simultaneously until measured. For example, an electron can be in a superposition of spin-up and spin-down states:

$$|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle \tag{3}$$

where $|\alpha|^2 + |\beta|^2 = 1$.

4 The Double-Slit Experiment

The double-slit experiment demonstrates wave-particle duality. When electrons pass through two slits, they create an interference pattern on a screen, showing their wave-like nature.

The probability amplitude for an electron to reach point x on the screen is:

$$\psi(x) = \psi_1(x) + \psi_2(x) \tag{4}$$

where ψ_1 and ψ_2 are the amplitudes from each slit.

5 Conclusion

Quantum mechanics reveals a world far stranger than classical physics, where particles can be in multiple places at once and measurement fundamentally changes the system being observed.