Graph Theory and Network Analysis

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1 Basic Definitions

A **graph** $G = (V, E)$ consists of a set of vertices V and a set of edges E. If the edges are ordered pairs, the graph is *directed*; otherwise, it's *undirected*.

2 Graph Properties

2.1 Degree

The **degree** of a vertex v in an undirected graph is the number of edges incident to v:

$$\deg(v) = |\{e \in E : v \in e\}| \tag{1}$$

2.2 Connectivity

A graph is **connected** if there exists a path between any two vertices. The **connectivity** $\kappa(G)$ is the minimum number of vertices whose removal disconnects the graph.

3 Important Theorems

3.1 Handshaking Lemma

For any graph $G = (V, E)$:

$$\sum_{v \in V} \deg(v) = 2|E| \tag{2}$$

This implies that the sum of all vertex degrees equals twice the number of edges.

3.2 Euler's Formula

For a connected planar graph with V vertices, E edges, and F faces:

$$V - E + F = 2 \tag{3}$$

4 Applications

Graph theory has numerous applications:

- Social network analysis
- Computer network topology
- Transportation systems

- Molecular structure analysis
- Web page ranking algorithms

5 Algorithmic Considerations

Many graph problems are computationally challenging. For example:

- **Hamiltonian Path**: NP-complete
- **Graph Coloring**: NP-complete for $k \geq 3$
- **Shortest Path**: Polynomial time (Dijkstra's algorithm)

6 Conclusion

Graph theory provides powerful tools for modeling and analyzing complex systems with discrete structures.