CSED433 Computational Logic – HW 2

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Due: Tuesday 15$^{\text{th}}$ October, 2024

1   Problem 1

Lemma 1. $((A \lor B) \supset C) \supset ((A \supset C) \land (B \supset C))$ *true*

$$
\cfrac{
\cfrac{
\cfrac{
\cfrac{\overline{A \lor B \supset C \ true}\ x \qquad \cfrac{\overline{A\ true}\ a}{A \lor B\ true}\ \lor I_L}{C\ true}\ \supset E
}{A \supset C\ true}\ \supset I^a
\qquad
\cfrac{
\cfrac{\overline{A \lor B \supset C\ true}\ x \qquad \cfrac{\overline{B\ true}\ b}{A \lor B\ true}\ \lor I_L}{C\ true}\ \supset E
}{B \supset C\ true}\ \supset I^b
}{(A \supset B) \land (A \supset C)\ true}\ \land I
}{((A \lor B) \supset C) \supset ((A \supset C) \land (B \supset C))\ true}\ \supset I^x
$$

2   Problem 2

Lemma 2. $((A \supset C) \land (B \supset C)) \supset ((A \lor B) \supset C)$ *true*

$$
\cfrac{
\cfrac{
\cfrac{
\cfrac{\cfrac{\overline{(A \supset C) \land (B \supset C)\ true}\ x}{A \supset C\ true}\ \land E_L \qquad \cfrac{}{A\ true}\ a}{C\ true}\ \supset E
\qquad
\cfrac{\cfrac{\overline{(A \supset C) \land (B \supset C)\ true}\ x}{B \supset C\ true}\ \land E_R \qquad \cfrac{}{B\ true}\ b}{C\ true}\ \supset E
\qquad A \lor B\ true
}{C\ true}\ \lor E^{a,b}
}{(A \lor B) \supset C\ true}\ \supset I^{a,b}
}{((A \supset C) \land (B \supset C)) \supset ((A \lor B) \supset C)\ true}\ \supset I^x
$$

3   Problem 3

Lemma 3. $\neg\neg\neg A \supset \neg A \ true$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\neg\neg\neg A \ true} \ tna
    \qquad
    \cfrac{
      \cfrac{
        \cfrac{}{\neg A \ true} \ na
        \qquad
        \cfrac{}{A \ true} \ a
      }{\bot \ true} \ \neg E
    }{\neg\neg A \ true} \ \neg I^{na}
  }{\bot \ true} \ \neg E
}{
  \cfrac{\neg A \ true}{\neg\neg\neg A \supset \neg A \ true} \ \supset I^{tna}
} \ \neg I^{a}
$$

4   Problem 4

Lemma 4. $(\neg\neg(A \vee \neg A) \supset (A \vee \neg A)) \supset (A \vee \neg A) \ true$

$$
\cfrac{
  \cfrac{}{\neg\neg(A \vee \neg A) \supset (A \vee \neg A) \ true} \ f
  \qquad
  \cfrac{
    \cfrac{
      A \vee \neg A \ true
      \qquad
      \cfrac{}{A \ true} \ a
      \qquad
      \cfrac{}{\neg A \ true} \ na
    }{A \vee \neg A \ true} \ \vee E^{a,na}
    \qquad
    \cfrac{
      \cfrac{}{\neg(A \vee \neg A) \ true} \ an
    }{\neg\neg(A \vee \neg A) \ true} \ \neg E
  }{\cfrac{\bot \ true}{\neg\neg(A \vee \neg A) \ true} \ \neg I^{an}}
}{
  \cfrac{A \vee \neg A \ true}{(\neg\neg(A \vee \neg A) \supset (A \vee \neg A)) \supset (A \vee \neg A) \ true} \ \supset I^{f}
} \ \supset E
$$