CSED433 Computational Logic – HW 2

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Due: Tuesday 15$^{\text{th}}$ October, 2024

1 Problem 1

Lemma 1. $((A \vee B) \supset C) \supset ((A \supset C) \wedge (B \supset C))\ true$

$$
\cfrac{
  \cfrac{
    \cfrac{
      \cfrac{\overline{A \vee B \supset C\ true}\ x \qquad \cfrac{\overline{A\ true}\ a}{A \vee B\ true}\ \vee I_L}{C\ true}\ \supset E
    }{A \supset C\ true}\ \supset I^a
    \qquad
    \cfrac{
      \cfrac{\overline{A \vee B \supset C\ true}\ x \qquad \cfrac{\overline{B\ true}\ b}{A \vee B\ true}\ \vee I_L}{C\ true}\ \supset E
    }{B \supset C\ true}\ \supset I^b
  }{(A \supset B) \wedge (A \supset C)\ true}\ \wedge I
}{((A \vee B) \supset C) \supset ((A \supset C) \wedge (B \supset C))\ true}\ \supset I^x
$$

2 Problem 2

Lemma 2. $((A \supset C) \wedge (B \supset C)) \supset ((A \vee B) \supset C)\ true$

$$
\cfrac{
  \cfrac{
    \cfrac{
      \cfrac{\cfrac{\overline{(A \supset C) \wedge (B \supset C)\ true}\ x}{A \supset C\ true}\ \wedge E_L \qquad \overline{A\ true}\ a}{C\ true}\ \supset E
      \qquad
      \cfrac{\cfrac{\overline{(A \supset C) \wedge (B \supset C)\ true}\ x}{B \supset C\ true}\ \wedge E_R \qquad \overline{B\ true}\ b}{C\ true}\ \supset E
      \qquad
      A \vee B\ true
    }{C\ true}\ \vee E^{a,b}
  }{(A \vee B) \supset C\ true}\ \supset I^{a,b}
}{((A \supset C) \wedge (B \supset C)) \supset ((A \vee B) \supset C)\ true}\ \supset I^x
$$

3 Problem 3

Lemma 3. $\neg\neg\neg A \supset \neg A \; true$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\neg\neg\neg A \; true} \; tna
    \qquad
    \cfrac{
      \cfrac{
        \cfrac{}{\neg A \; true} \; na
        \qquad
        \cfrac{}{A \; true} \; a
      }{\bot \; true} \; \neg E
    }{\neg\neg A \; true} \; \neg I^{na}
  }{
    \cfrac{\bot \; true}{\neg A \; true} \; \neg I^{a}
  } \; \neg E
}{\cfrac{}{\neg\neg\neg A \supset \neg A \; true}} \; \supset I^{tna}
$$

4 Problem 4

Lemma 4. $(\neg\neg(A \vee \neg A) \supset (A \vee \neg A)) \supset (A \vee \neg A) \; true$

$$
\cfrac{
  \cfrac{
    \cfrac{
      \cfrac{
        \cfrac{
          \cfrac{\cfrac{}{A \; true} \; a}{A \vee \neg A \; true} \vee I_{L}
          \qquad
          \cfrac{}{\neg(A \vee \neg A) \; true} \; an
        }{\bot \; true} \; \neg E
      }{\neg A \; true} \; \neg I^{a}
    }{A \vee \neg A \; true} \vee I_{R}
    \qquad
    \cfrac{}{\neg(A \vee \neg A) \; true} \; an
  }{\cfrac{\bot \; true}{\neg\neg(A \vee \neg A) \; true} \; \neg I^{an}}
  \qquad
  \cfrac{}{\neg\neg(A \vee \neg A) \supset (A \vee \neg A) \; true} \; f
}{
  \cfrac{A \vee \neg A \; true}{(\neg\neg(A \vee \neg A) \supset (A \vee \neg A)) \supset (A \vee \neg A) \; true} \; \supset I^{f}
} \; \supset E
$$