

# CSED433 Computational Logic – HW 2

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## 1 Problem 1

**Lemma 1.**  $((A \vee B) \supset C) \supset ((A \supset C) \wedge (B \supset C)) \text{ true}$

$$\begin{array}{c}
 \frac{\frac{\frac{}{A \vee B \supset C \text{ true}} x}{\frac{C \text{ true}}{A \supset C \text{ true}} \supset I^a} \supset E \quad \frac{\frac{\frac{}{A \text{ true}} a}{A \vee B \text{ true}} \vee I_L}{\frac{C \text{ true}}{A \supset C \text{ true}} \supset I^a} \supset E \quad \frac{\frac{\frac{}{B \text{ true}} b}{A \vee B \text{ true}} \vee I_L}{\frac{C \text{ true}}{B \supset C \text{ true}} \supset I^b} \supset E \\
 \frac{\frac{C \text{ true}}{A \supset C \text{ true}} \supset I^a \quad \frac{C \text{ true}}{B \supset C \text{ true}} \supset I^b}{(A \supset C) \wedge (B \supset C) \text{ true}} \wedge I \\
 \frac{(A \supset C) \wedge (B \supset C) \text{ true}}{((A \vee B) \supset C) \supset ((A \supset C) \wedge (B \supset C)) \text{ true}} \supset I^x
 \end{array}$$

## 2 Problem 2

**Lemma 2.**  $((A \supset C) \wedge (B \supset C)) \supset ((A \vee B) \supset C) \text{ true}$

$$\begin{array}{c}
 \frac{\frac{\frac{}{(A \supset C) \wedge (B \supset C) \text{ true}} x}{A \supset C \text{ true}} \wedge E_L \quad \frac{}{A \text{ true}} a}{\frac{C \text{ true}}{A \supset C \text{ true}} \supset E} \supset E \quad \frac{\frac{\frac{}{(A \supset C) \wedge (B \supset C) \text{ true}} x}{B \supset C \text{ true}} \wedge E_R \quad \frac{}{B \text{ true}} b}{\frac{C \text{ true}}{B \supset C \text{ true}} \supset E} \supset E \\
 \frac{\frac{C \text{ true}}{A \supset C \text{ true}} \supset E \quad \frac{C \text{ true}}{B \supset C \text{ true}} \supset E}{\frac{C \text{ true}}{(A \vee B) \supset C \text{ true}} \supset I^{a,b}} \supset E \\
 \frac{\frac{C \text{ true}}{(A \vee B) \supset C \text{ true}} \supset I^{a,b}}{((A \supset C) \wedge (B \supset C)) \supset ((A \vee B) \supset C) \text{ true}} \supset I^x
 \end{array}$$

### 3 Problem 3

**Lemma 3.**  $\neg\neg\neg A \supset \neg A \text{ true}$

$$\begin{array}{c}
 \frac{\frac{\frac{}{\neg\neg\neg A \text{ true}} tna}{\neg\neg\neg A \text{ true}} \quad \frac{\frac{\frac{}{\neg A \text{ true}} na}{\neg A \text{ true}} \quad \frac{\frac{}{A \text{ true}} a}{A \text{ true}}}{\neg A \text{ true}} \neg E}{\frac{\frac{}{\neg\neg\neg A \text{ true}} tna}{\neg\neg\neg A \text{ true}} \quad \frac{\frac{\frac{}{\neg A \text{ true}} na}{\neg A \text{ true}} \quad \frac{\frac{}{A \text{ true}} a}{A \text{ true}}}{\neg A \text{ true}} \neg E} \neg I^{na} \\
 \frac{\frac{\frac{}{\neg\neg\neg A \text{ true}} tna}{\neg\neg\neg A \text{ true}} \quad \frac{\frac{}{\neg A \text{ true}} na}{\neg A \text{ true}}}{\neg\neg\neg A \text{ true}} \supset I^{tna}
 \end{array}$$

### 4 Problem 4

**Lemma 4.**  $(\neg\neg(A \vee \neg A) \supset (A \vee \neg A)) \supset (A \vee \neg A) \text{ true}$

$$\begin{array}{c}
 \frac{\frac{\frac{}{A \text{ true}} a}{A \text{ true}} \vee I_L \quad \frac{\frac{}{\neg(A \vee \neg A) \text{ true}} an}{\neg(A \vee \neg A) \text{ true}}}{\frac{}{A \vee \neg A \text{ true}} \vee I_L} \neg E \\
 \frac{\frac{\frac{}{\neg A \text{ true}} na}{\neg A \text{ true}} \neg I^a \quad \frac{\frac{}{A \vee \neg A \text{ true}} \vee I_R}{A \vee \neg A \text{ true}}}{\frac{}{A \vee \neg A \text{ true}} \vee I_R} \neg E \\
 \frac{\frac{\frac{}{\neg\neg(A \vee \neg A) \text{ true}} \neg I^{an}}{\neg\neg(A \vee \neg A) \text{ true}} \quad \frac{\frac{}{\neg\neg(A \vee \neg A) \supset (A \vee \neg A) \text{ true}} f}{\neg\neg(A \vee \neg A) \supset (A \vee \neg A) \text{ true}}}{\frac{}{(\neg\neg(A \vee \neg A) \supset (A \vee \neg A)) \supset (A \vee \neg A) \text{ true}} \supset I^f} \supset E
 \end{array}$$