CSED433 Computational Logic – HW 1

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Due: Tuesday 24th September, 2024

1 Matched Parentheses

1.1 Problem 1

Theorem 1. *If* s lparen*,* s mparen*.*

Proof. by Rule Induction,

Case $\dfrac{}{\epsilon \text{ lparen}}$ *Leps* where $s = \epsilon$:
s mparen — by the rule *Meps*

Case $\dfrac{s_1 \text{ lparen} \quad s_2 \text{ lparen}}{(s_1)s_2 \text{ lparen}}$ *Lseq* where $s = (s_1)s_2$:
s_1 mparen — by induction hypothesis on s_1 lparen
s_2 mparen — by induction hypothesis on s_2 lparen
(s_1) mparen — by the rule *Mpar* on s_1
$(s_1)s_2$ mparen — by the rule *Mseq* on (s_1) and s_2

□

1.2 Problem 2

Lemma 1. *If* s tparen *and* s' tparen*, then* ss' tparen*.*

Proof. by Rule Induction,

Case $\dfrac{}{\epsilon \text{ tparen}}$ *Teps* where $s' = \epsilon$:
$ss' = s\epsilon = s$
s tparen — by assumption $s = \epsilon$

Case $\dfrac{s_1 \text{ tparen} \quad s_2 \text{ tparen}}{s_1(s_2) \text{ tparen}}$ *Tseq* where $s' = s_1(s_2)$:
$ss' = ss_1(s_2)$ — by assumption $s' = s_1(s_2)$
"s tparen implies ss_1 tparen" — by induction hypothesis on s tparen
ss_1 tparen — by assumption s tparen
$ss_1(s_2)$ tparen — by the rule *Tseq* on ss_1 tparen and s_2 tparen

□

1.3 Problem 3

Theorem 2. *If* s mparen*, then* s tparen*.*

Proof. by Rule Induction,

Case $\dfrac{}{\epsilon \text{ mparen}}$ *Meps* where $s = \epsilon$:
s tparen — by the rule *Teps*

Case $\dfrac{s' \text{ mparen}}{(s') \text{ mparen}}$ *Mpar* where $s = s'$:
s' tparen — by induction hypothesis on s' mparen
(s') tparen — by the rule *Tseq* on ϵ tparen and s' tparen

Case $\dfrac{s_1 \text{ mparen} \quad s_2 \text{ mparen}}{s_1 s_2 \text{ mparen}}$ *Mseq* where $s = s_1 s_2$:
s_1 tparen — by induction hypothesis on s_1 mparen
s_2 tparen — by induction hypothesis on s_2 mparen
$s_1 s_2$ tparen — by **Lemma 1.**
□

1.4 Problem 4

Theorem 3. *If* s lparen*, then* s tparen*.*

Lemma 2. *If* s tparen *and* s' tparen*, then* ss' tparen*.*

Proof of Lemma 2. by Rule Induction,

Case $\dfrac{}{\epsilon \text{ tparen}}$ *Teps* where $s' = \epsilon$:
$ss' = s\epsilon = s$
s tparen — by assumption $s = \epsilon$

Case $\dfrac{s_1 \text{ tparen} \quad s_2 \text{ tparen}}{s_1(s_2) \text{ tparen}}$ *Tseq* where $s' = s_1(s_2)$:
$ss' = ss_1(s_2)$ — by assumption $s' = s_1(s_2)$
"s tparen implies ss_1 tparen" — by induction hypothesis on s tparen
ss_1 tparen — by assumption s tparen
$ss_1(s_2)$ tparen — by the rule *Tseq* on ss_1 tparen and s_2 tparen
□

Proof of Theorem 3. by Rule Induction,

Case $\dfrac{}{\epsilon \text{ lparen}}$ *Leps* where $s = \epsilon$:
s tparen — by the rule *Teps*

Case $\dfrac{s_1 \text{ lparen} \quad s_2 \text{ lparen}}{(s_1)s_2 \text{ lparen}}$ *Lseq* where $s = (s_1)s_2$:
s_1 tparen — by induction hypothesis on s_1 lparen
s_2 tparen — by induction hypothesis on s_2 lparen
(s_1) tparen — by the rule *Tseq* on s_1 and ϵ
$(s_1)s_2$ tparen — by the **Lemma 2.** on (s_1) and s_2
□