PELEG BAR SAPIR

PHYSICS SIMULATIONS (LECTURE NOTES)

Introduction

Why are Simulations Used?

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Python

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Some Mathematical Background

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Harmonic Oscillator

Many systems in physics present a simple, periodic (repeating) motion. One such system is a simple mass-less spring connected to a mass m and allowed to move in a single dimension only. If we ignore the effects of gravity, the only force acting on the mass arises from the spring itself: the more we pull or push the spring, the stronger it will resist to that change. This resistant force is given by

$$F = -kx, \tag{1}$$

where k is the **spring constant**, and x is the amount by which the spring contracts or expands relative to its rest position x_0. In Figure 1 we show three cases: when the mass is at the springs rest position, $x_m = x_0$, the spring applies no force on it. When the mass is displaced by a positive amount $\Delta x > 0$, the spring applied a *negative* force on it: $F = -k\Delta x < 0$ (see note 0.1). And when the mass is displaced such that it contracts the spring, $\Delta x < 0$ and thus the force applied by the spring is positive, $F = k\Delta x > 0$.

Note 0.1 Negative and positive forces

Recall that in this context, negative force means a force in the negative x direction.

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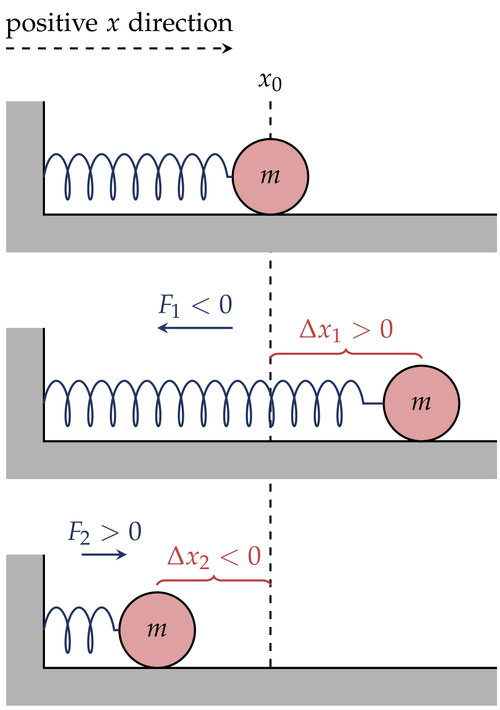

Figure 1: A simple spring-mass system with spring constant k and a mass m. The top figure shows the spring at rest - i.e. when the mass is located at position x_0 the spring applies no force on the mass (since $\Delta x = x_m - x_0 = 0$). The middle figure show the spring being at a *positive* displacement $\Delta x_1 > 0$, causing the spring to pull back with a negative force $F_1 = -k\Delta x_1$. The bottom picture shows the spring contracting by $\Delta x_2 < 0$, casing the spring to apply a positive force $F_2 = -k\Delta x_2$ on the mass.

Using Newtons second law of motion (REF?) with x being the displacement from the rest position x_0, we get the relation

$$a = -kx, \tag{2}$$

but since $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2}$, we can re-write Equation 2 as

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -kx, \tag{3}$$

or even more succinctly as

$$\ddot{x} = -kx. \tag{4}$$

Equation 4 is one of the simplest possible 2nd-order *ordinary* differential equations. Its solution is a combination of the two basic trigonometric equations:

$$x(t) = c_1 \sin(\alpha t) + c_2 \cos(\beta t), \tag{5}$$

where c_1 and c_2 are constants which we can find using the *starting conditions* (see note 0.2), and α, β are parameters of the motion. Since function arguments must be unitless, these two parameters also cause the total quantity inside the trigonometric functions to be unitless by having units corresponding to $\left[\frac{1}{\text{time}}\right]$. For example, if we measure the time in $[\mathrm{s}]$, then the units of α and β are $\left[\mathrm{s}^{-1}\right] = [\mathrm{Hz}]$.

Note 0.2 Starting conditions for solving ODEs

Recall that in order to completely solve an ordinary differential equation of order n we must have n starting conditions.

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