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PHYSICS SIMULATIONS (LECTURE NOTES)

Introduction

Why are Simulations Used?

Python

A Bit About Git

Some Mathematical Background

Harmonic Oscillator

Many systems in physics present a simple, periodic (repeating) motion. One such system is a simple mass-less spring connected to a mass m and allowed to move in a single dimension only. If we ignore the effects of gravity, the only force acting on the mass arises from the spring itself: the more we pull or push the spring, the stronger it will resist to that change. This resistant force is given by

$$F = -kx, \tag{1}$$

where k is the **spring constant**, and x is the amount by which the spring contracts or expands relative to its rest position x_0. In Figure 1 we show three cases: when the mass is at the springs rest position, $x_m = x_0$, the spring applies no force on it. When the mass is displaced by a positive amount $\Delta x > 0$, the spring applied a *negative* force on it: $F = -k\Delta x < 0$ (see note 0.0.1). And when the mass is displaced such that it contracts the spring, $\Delta x < 0$ and thus the force applied by the spring is positive, $F = k\Delta x > 0$.

Note 0.0.1 Negative and positive forces

Recall that in this context, negative force means a force in the negative x direction.

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Using Newtons second law of motion (REF?) with x being the displacement from the rest position x_0, we get the relation

$$a = -kx, \tag{2}$$

but since $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2}$, we can re-write Equation 2 as

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -kx, \tag{3}$$

or even more succinctly as

$$\ddot{x} = -kx. \tag{4}$$

Equation 4 is one of the simplest possible 2nd-order *ordinary* differential equations. Its solution is a combination of the two basic trigonometric equations:

$$x(t) = c_1 \sin(at) + c_2 \cos(bt), \tag{5}$$

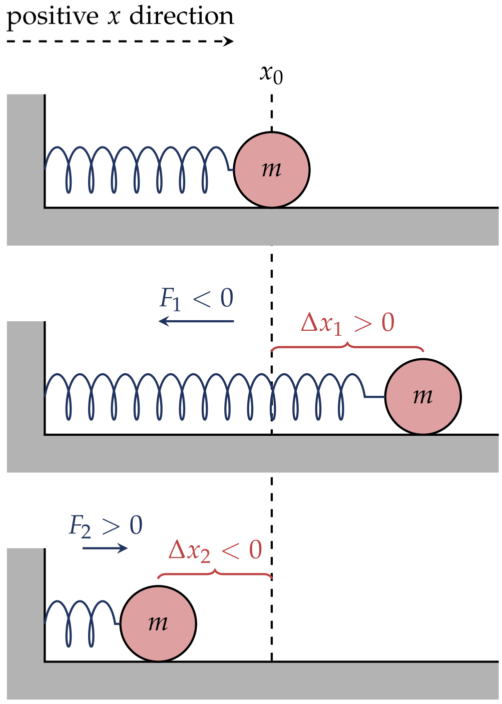

Figure 1: A simple spring-mass system with spring constant k and a mass m. The top figure shows the spring at rest - i.e. when the mass is located at position x_0 the spring applies no force on the mass (since $\Delta x = x_m - x_0 = 0$). The middle figure show the spring being at a *positive* displacement $\Delta x_1 > 0$, causing the spring to pull back with a negative force $F_1 = -k\Delta x_1$. The bottom picture shows the spring contracting by $\Delta x_2 < 0$, casing the spring to apply a positive force $F_2 = -k\Delta x_2$ on the mass.

where c_1 and c_2 are constants which we can find using the *starting conditions* (see note 0.0.2).

Note 0.0.2 Starting conditions for solving a differential equation

Recall that in order to completely solve an ordinary differential equation of order n we must have n starting conditions.

Simulating Simple Mechanics

Forward Euler Method

Backward Euler Method

Verlet Integration

Runge-Kutta Method

Thermodynamics

Waves

Molecular Dynamics