———— × ————

31.08.2022

- $L_1 = N_1$

  $L_2 = N_2$

  $N = L = L_1 \cup L_2$

  $L' = L_1 \cdot L_2$

$w \in L'$

if $x \in L_1$ & $y \in L_2$

then $w = xy$

* $L_1 = \{a, aa, b\}$

  $L_2 = \{aa, ab, abb\}$

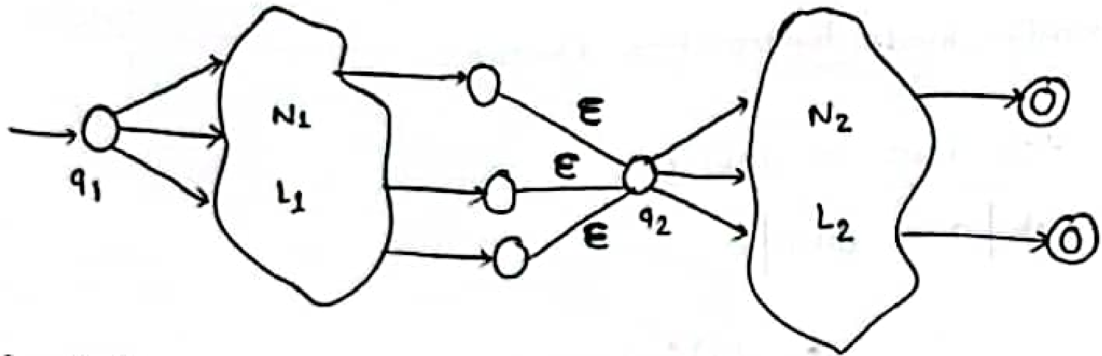
  $L' = \{aaa, aab, aabb, aaaa, aaab, aaabb, baa, bab, babb\}$

- $N_1 = \{Q_1, \Sigma, \delta_1, q_{01}, F_1\}$ $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$

N_1 accepts L_1 N_2 accepts L_2

what will be the formal definition of the machine N that accepts L_1L_2 ?

$\longrightarrow$ $N = (Q, \Sigma, \delta, q_0, F)$



$Q = Q_1 \cup Q_2$

$\Sigma = \Sigma$

$q_0 = q_1$

$F = F_2$

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & \text{if } q \in Q_1 \\ \delta_2(q,a) & \text{if } q \in Q_2 \\ \delta_1(q,a) \cup \{q_2\} & \text{if } q \in F_1 \wedge a = \varepsilon \end{cases}$$

Regular Expression:

a is a regular expression where $a \in \Sigma$

$R_1 \cup R_2$ " " " " R_1 is regular & R_2 is regular

$R_1 \cdot R_2$ " " " " .

- R_1^* ⟶ Regular expression

  ε ⟶ "

  ϕ ⟶ "

  (R_1) ⟶ "

- ① Star binds tighter. $ab^* = a(b^*) \neq (ab)^*$

  ② concatenation binds tighter than Union.

$$a \cdot b \cup c = (ab) \cup c$$

$$\Rightarrow a \cdot b \mid c = (ab) \mid c$$

$$a^* = \{a\}^*$$

$$a^+ = aa^*$$

⊙ $aa\ b \cup c\ aa\ b \cup c\ aa = ?$

⟶ $(aab) \mid (caab) \mid (caa)$

⊙ $d \cup ab^*\ cd^* = ?$

⟶ $d \cup \left(a(b^*)c(d^*)\right)$

⊙ Let, $\Sigma = \{a, b, c, d\}$

a → regular expression	$L_1 = \{a\}$
$abccd$	$L_2 = \{abccd\}$
$ab \cup cd$	$L_3 = \{ab, cd\}$
$a(b \cup c)d$	$L_4 = \{abd, acd\}$

ab^*c $\quad L_5 = \{ac, abc, abbc, abbbc, \ldots\ldots\}$

$a(b \cup \varepsilon)c$ $\quad L_6 = \{abc, ac\}$

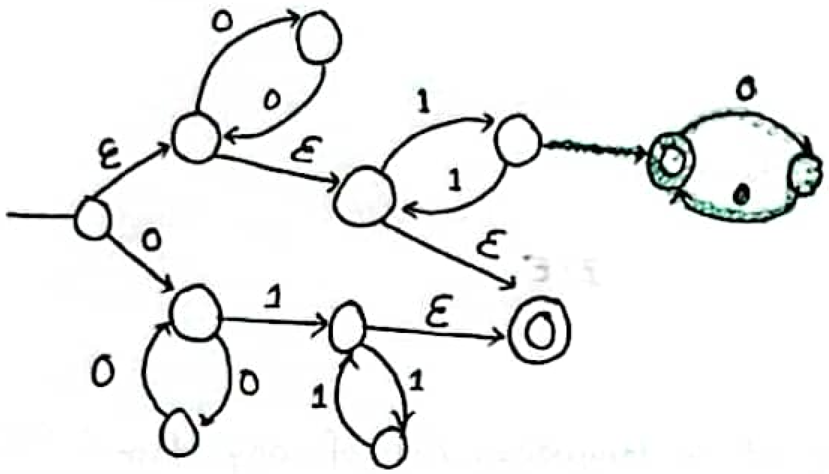
ϕ^* $\quad L_7 = \{\varepsilon\}$

ϕ $\quad L_8 = \{\}$

———x———

05.09.2022

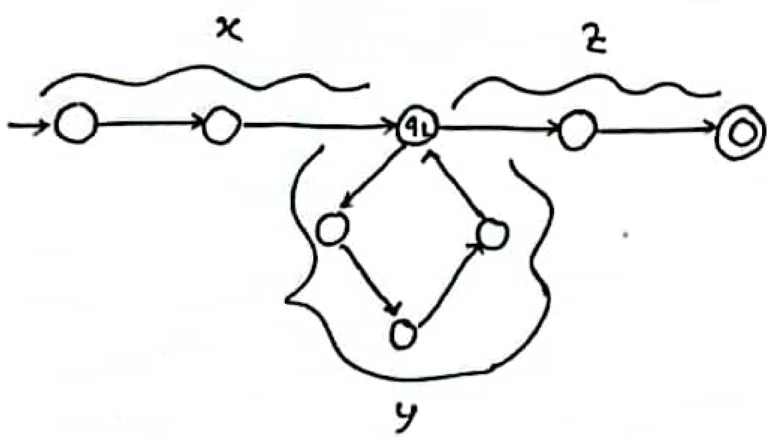
• $L = \{0^m 1^n \mid m+n = \text{even}\}$ build NFA for L over $\Sigma = \{0,1\}$



m	n	m+n
o	o	e
o	e	o
e	o	o
e	e	e

Pumping lemma

↳ whether a language is regular or not



$\omega = xyz$

$= xy^i z \; ; i \geq 0$

$\in L$

if L is regular language and ω is a sufficienty long enough string ie $|\omega| \geq P$ ($P \rightarrow$ the pumping length)

depends on the language, not specific FSM

$\Downarrow$

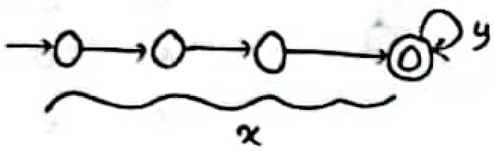
long enough that FSM will have cycle

$\omega = xyz$

$= xy^iz \in L$ for $i \geq 0$

if $|y| > 0 \longrightarrow$ cycle has at least one edge in it

$|xy| \leq P$



$z = \epsilon$

$|xy| \leq$

- pumping length is a properdy of a language, not of any fsm

* If L is a regular language, then L has a pumping length P such that any string w maybe divided into three pieces $\omega = xyz$ such that these three condition holds —

(i) $xy^iz \in L$

(ii) $|y| \geq 0$

(iii) $|xy| \leq P$

- $L = \{ 0^n 1^n \mid n \geq 0 \}$, prove that L is not a regular langage.

$\longrightarrow$

* Assume L is regular
* L have pumping length 'p'

$$w = 0^p 1^p$$

$w = xyz$

case 01 : 'y' belongs to '0' part of the string

case 02 : 'y' " " '1' " " "

case 03 : 'y' " " both '0' & '1' " " "

$p = 7$: $w = 0000000\,1111111$

y

y

y

case 01 : $i = 2$ for xy^iz

$$0000000000\,1111111$$

$$= 0^{10} 1^7 \notin L$$

case 02 :

$$0000000\,111\,111\,111$$

$$= 0^7 1^9 \notin L$$

- Show that $L = \{ ww \mid w \in \{0,1\}^* \}$ is not regular.

* Show that, $L = \{ww \mid w \in (0,1)^*\}$ is not regular.

Let's assume L is regular and a string $= 0^p 1$ (where p is the pumping length)

$\therefore$ The language would be $= 0^p 1 0^p 1$

$= 0000001\ 0000001$ ($P = 6$ assuming)

$= 0^6 1 0^6 1$

$\omega = xyz$

Let's assume –

$$\underbrace{0000}_{x}\underbrace{001\,0}_{y}\underbrace{000001}_{z}$$

Case 01: Assuming $i = 2$:

$xy^i z = xy^2 z$

$= 00000010\ 0010\ 0000001$

$= 0^6 1 0^3 1 0^0 1 \notin L$

So L is not regular.

Next Wednesday

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Ex 4.1.1 – c, e, f

4.1.2 – a, h

Alternative sol^n

① Assuming L is regular.

$\omega = xyz \in L$

$|xyz| = P$ $\quad\quad$ $i = P+1$

$xy^iz \in L$

$\therefore |xy^{P+1}z| = |xyz| + |y|^P$

$= P + P \cdot (|y|)$

$= P(1+|y|)$

$= P * (\text{something}) \notin L$ [For being a prime number it should be $P = 1 \cdot P$]

$\therefore$ L is not regular.

- (i) $L_1 + L_2 = L_2 + L_1$ [commutative]

  (ii) $L_1 + (L_2 + L_3) = (L_1 + L_2) + L_3$ [associative]

  (iii) $L_1 (L_2 + L_3) = L_1 L_2 + L_1 L_3$ [distributive]

Assuming

$$\omega \in L_1 (L_2 + L_3)$$

$\omega = xy$ so $x \in L_1$ and $y \in (L_2 + L_3)$

$\Rightarrow y \in L_2$ or L_3

$$\omega \in (L_1 L_2 + L_1 L_3)$$

$\omega \in L_1 L_2$ or $\omega \in L_1 L_3$

$\omega = xy$

- (i) $\phi + R = R$

  (ii) $\phi R = \phi$

  (iii) $\varepsilon R = R$

  (iv) $\varepsilon^* = \{\varepsilon^0, \varepsilon^1, \varepsilon^2, \varepsilon^3, \text{——}\} = \{\varepsilon, \varepsilon, \varepsilon, \varepsilon, \text{——}\} = \varepsilon$

  (v) $\phi^* = \varepsilon$

  (vi) $R + R = R$

  (vii) $R^* R^* = \{\varepsilon, R^1, R^2, R^3, \text{——}\} . \{\varepsilon, R^1, R^2, R^3, \text{——}\} = R^*$

  (viii) $RR^* = R^* R$

  (ix) $(R^*)^* = R^*$ $\left[(((R^*)^*)^*)^* = R \right.$

  (x) $\varepsilon + RR^* = R^*$

if

$$\varepsilon + RR^* = \varepsilon + R\{\varepsilon, R^1, R^2, R^3, \text{———}\}$$

$$= \varepsilon + \{R, R^2, R^3, \text{———}\}$$

$$= \{\varepsilon, R, R^2, R^3, \text{———}\}$$

$$= R^*$$

(xi) $R^*R + \varepsilon = R^*$

(xii) $(PQ)^*P = P(PQ)^*$

i (xiii) $(P+Q)^* = (P^*Q^*)^* = (P^* + Q^*)^*$

- Prove that, $(1+00^*1) + (1+00^*1)(0+10^*1)^*(0+10^*1) = 0^*1\,(0+10^*1)^*$

$$= (1+00^*1)\left[\varepsilon + \underbrace{(0+10^*1)^*}_{R^*}\underbrace{(0+10^*1)}_{R}\right]$$

| $\varepsilon + R^*R = R^*$

$$= (1+00^*1)(0+10^*1)^*$$

*

$$= (\varepsilon+00^*)1\,(0+10^*1)^*$$

$$= 0^*1\,(0+10^*1)^* \quad \text{(Proved)}$$

* Language to RE :

- L = { accepting string exactly length of 2 } $\Sigma = \{a, b\}$

  = { aa, ab, ba, bb }

$RE(L) = (a+b)(a+b)$

- L = { length is greater than 2 }

  $RE(L) = (a+b)(a+b)(a+b)^+$

Exactly 2 or more : $RE(L) = (a+b)(a+b)(a+b)^*$

- $L = \{ w \mid |w| \leq 2 \}$

  $RE(L) = \varepsilon + (a+b) + (a+b)(a+b)$

  $= (\varepsilon + a + b)(\varepsilon + a + b)$

- L = { starts with ab }

  $RE(L) = ab(a+b)^*$

  Ends with ab

  $RE(L) = (a+b)^* ab$

- L = { containing aba }

  $RE(L) = (a+b)^* aba (a+b)^*$

- L = { starting with a & ending with a } ⇒ $RE(L) = a(a+b)^*a$ a+

- L = { length of a is exactly 3 }

  $RE(L) = b^*.a.b^*.a.b^*.ab^*$

- $L = \{\text{starts and ends with different symbol}\}$

$$= a(a+b)^* b \quad + \quad b(a+b)^* a$$

- $L = \{ w \mid |w| \geq 3 \}$

$$= (a+b)(a+b)(a+b)(a+b)^*$$

$|w| > 3$

$(a+b)^3 (a+b)^+$

$|w| \leq 3$

$$= (\varepsilon+a+b)(\varepsilon+a+b)(\varepsilon+a+b)$$
$$= (\varepsilon+a+b)^3$$

- $L = \{\text{string 3rd symbol from the right is } b\}$

$$= (a+b)^* \, b \, (a+b)^2$$

- $L = \{ |w| \equiv 0 \bmod 2 \}$

$$= \left((a+b)(a+b)\right)^*$$

- $L = \{ |w|_a \equiv 1 \bmod 3 \}$

$$RE(L) = b^* a (b^* a b^* a b^* a)^* b^*$$

$[\, b^* a b^* (b^* a b^* a b^* a)^* \,]$

14.09.2022

Conversion from Finite Automata to Regular Expression :

Arden's Method

If P & Q are two RE's over Σ and if P does not contain ϵ then equation in R given by $R = Q + RP$ has unique sol^n ie $R = QP^*$

$\longrightarrow$

Given eq^n :

$$R = Q + RP$$
$$= Q + QP^*P$$
$$= Q(\epsilon + P^*P) \quad [\epsilon + P^*P = P^*]$$
$$= QP^*$$

$$R = QP^* = Q(\epsilon + P + P^2 + P^3 + \dots + P^n + P^* P^{n+1})$$
$$= QP^*$$

$$R = Q + RP$$
$$= Q + (Q + RP)P$$
$$= Q + QP + RP^2 = Q + QP + (Q+RP)P^2 = Q + QP + QP^2 + RP^3$$
$$\dots\dots\dots\dots$$
$$= Q + QP + QP^2 + \dots\dots + QP^n + RP^{n+1}$$
$$= Q + QP + QP^2 + \dots\dots + QP^n + QP^* P^{n+1}$$
$$= Q(\epsilon + P + P^2 + \dots\dots + P^n + P^* P^{n+1})$$
$$= QP^*$$

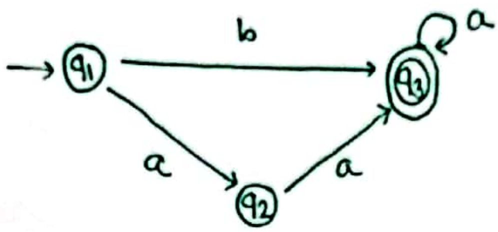
RE → FA using Arden's method

(i) write eq^n for each state based on incoming edge

(ii) simplify the equation using Arden's method for final state

1. There should be no ϵ- ~~[illegible]~~ transition
2. Only one initial state

*



$q_1 = \varepsilon$ ——— (i)

$q_2 = q_1 a$ ——— (ii)

$q_3 = q_1 b + q_2 a + q_3 a$ ——— (iii)

put (i) & (ii) into (iii) $\Rightarrow$

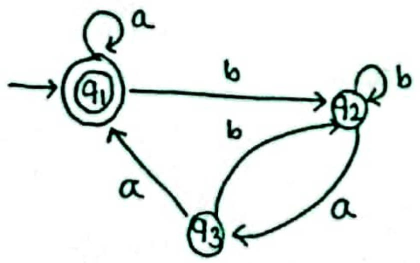
$$q_3 = \varepsilon b + q_1 aa + q_3 a$$

$$= b + \varepsilon aa + q_3 a \qquad [q_1 = \varepsilon]$$

$$\therefore \underset{R}{q_3} = \underset{Q}{\underline{(b+aa)}} + \underset{R}{q_3}\, \underset{P}{a}$$

$$\therefore q_3 = (b+aa)\,a^*$$

*



$q_1 = \varepsilon + q_1 a + q_3 a$ —— (i)

$q_2 = q_1 b + q_2 b + q_3 b$ —— (ii)

$q_3 = q_2 a$ —— (iii)

~~put (ii) & (iii) into (i):~~

$$\cancel{q_1 = \varepsilon + q_1 a + q_2 aa}$$

From (2):

$$q_2 = q_1 b + q_2 b + q_2 ab \quad [iii]$$

$$\Rightarrow q_2 = \underset{Q}{\underline{q_1 b}} + \underset{R}{\underline{q_2}}\,\underset{P}{\underline{(b+ab)}} \qquad \left[\begin{array}{l} R = Q + RP \\ \Rightarrow R = QP^* \end{array}\right]$$

$$\therefore q_2 = (q_1 b)(b+ab)^* \quad \text{——— (iv)}$$

From (iv):

$$q_3 = q_2 a$$

$$\therefore q_3 = (q_1 b)(b+ab)^* a \quad \text{——— (v)}$$

putting (v) in (i) :

$$q_1 = \varepsilon + q_1 a + (q_1 b)(b+ab)^* aa$$

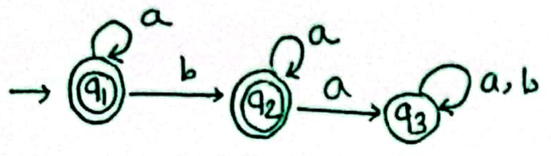
$$= \underbrace{\varepsilon}_{Q} + \underbrace{q_1}_{R} \underbrace{\left(a + b(b+ab)^* aa\right)}_{P}$$

$R = Q + RP$

$\Rightarrow R = QP^*$

$$\therefore q_1 = \varepsilon \left(a + b(b+ab)^* aa\right)^*$$

*



$q_1 = \varepsilon + q_1 a$ —— (i)

$q_2 = q_1 b + q_2 a$ —— (ii)

$q_3 = q_2 a + q_3 a + q_3 b$ —— (iii)

From (i):

$$\underbrace{q_1}_{R} = \underbrace{\varepsilon}_{Q} + \underbrace{q_1}_{R} \underbrace{a}_{P}$$

$$\therefore q_1 = \varepsilon a^*$$

$$= a^*$$

From (ii) :

$$q_2 = q_1 b + q_2 a$$

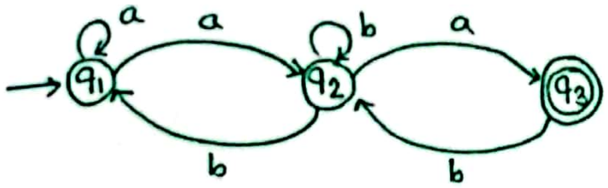
$\cancel{\therefore q_2 = \varepsilon a^* b}$

$$\therefore \underbrace{q_2}_{R} = \underbrace{a^* b}_{Q} + \underbrace{q_2}_{R} \underbrace{a}_{P}$$

$$\therefore q_2 = a^* b a^*$$

$$\therefore R = q_1 + q_2$$

$$= a^* + a^* b a^*$$



$$q_1 = \varepsilon + q_1 a + q_2 b \quad \text{—— (i)}$$

$$q_2 = q_1 a + q_2 b + q_3 b \quad \text{—— (ii)}$$

$$q_3 = q_2 a \quad \text{—— (iii)}$$

$$
\begin{aligned}
q_2 &= q_1 a + q_2 b + q_3 b \\
&= q_1 a + q_2 b + q_2 a b \qquad \text{[From iii]} \\
&= q_1 a + q_2 (b + ab) \\
&= q_1 a (b+ab)^* \quad \text{—— (iv)} \ [\, R = Q + PR \Rightarrow R = QP^* \,] \\
&\cancel{= (\varepsilon + q_1 a + q_2 b) a (b+ab)^*} \\
&\cancel{= a(b+ab)^*}
\end{aligned}
$$

$$
\begin{aligned}
q_1 &= \varepsilon + q_1 a + q_1 a (b+ab)^* b \qquad \text{[From iv]} \\
&= \varepsilon + q_1 \{a + a(b+ab)^* b\} \\
&= \varepsilon \{a + a(b+ab)^* b\}^* \\
&= \{a + a(b+ab)^* b\}^*
\end{aligned}
$$

$$
\begin{aligned}
q_2 &= q_1 a (b+ab)^* \\
&= (a + a(b+ab)^* b)^* a (b+ab)^* \quad \text{—— (v)}
\end{aligned}
$$

$$
\begin{aligned}
q_3 &= q_2 a \\
&= (a + a(b+ab)^* b)^* a (b+ab)^* a \qquad \text{[From v]}
\end{aligned}
$$