Markov Decision Process II

CSE 4711: Artificial Intelligence

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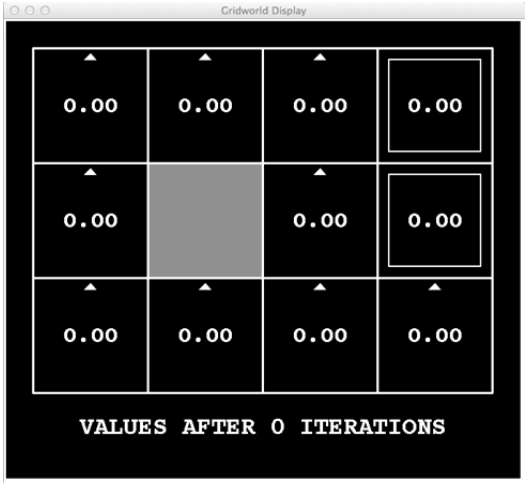
Department of Computer Science and Engineering
Islamic University of Technology

Problems with Value Iteration

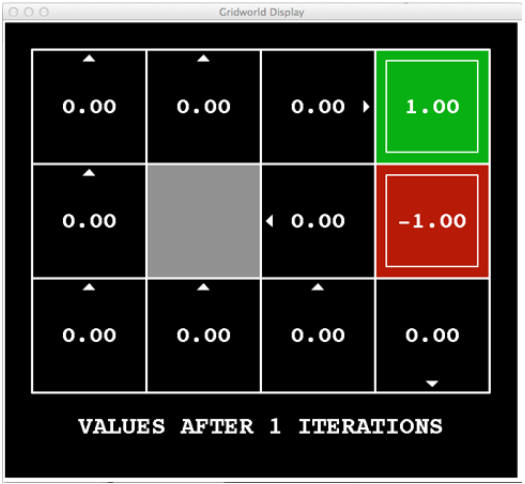
k = 0



Noise = 0.2
Discount = 0.9
Living reward = 0

Problems with Value Iteration

k = 1



Noise = 0.2
Discount = 0.9
Living reward = 0

Problems with Value Iteration

k = 2

Noise = 0.2
Discount = 0.9
Living reward = 0

Problems with Value Iteration

k = 3

Noise = 0.2
Discount = 0.9
Living reward = 0

Problems with Value Iteration

k = 4

Noise = 0.2
Discount = 0.9
Living reward = 0

Problems with Value Iteration

k = 5

Noise = 0.2
Discount = 0.9
Living reward = 0

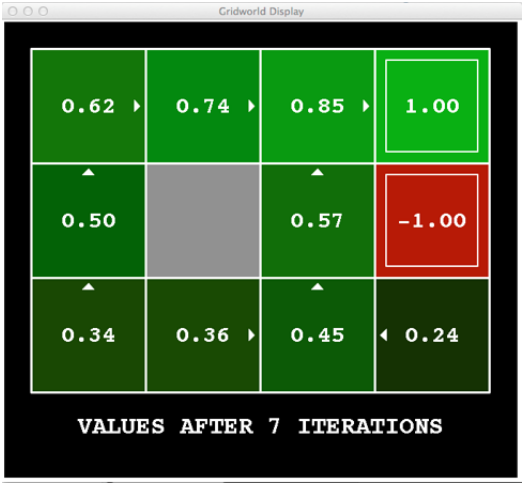
Problems with Value Iteration

k = 6

Noise = 0.2
Discount = 0.9
Living reward = 0

Problems with Value Iteration

k = 7



Noise = 0.2
Discount = 0.9
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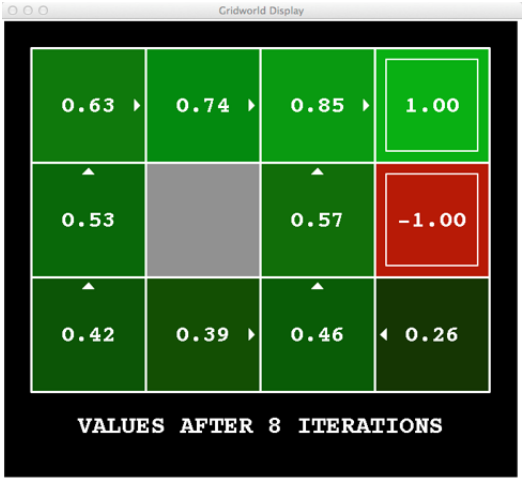
Problems with Value Iteration

k = 8



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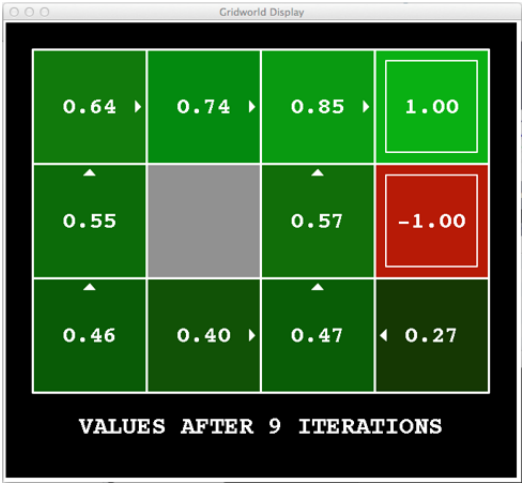
Problems with Value Iteration

k = 9



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Living reward = 0

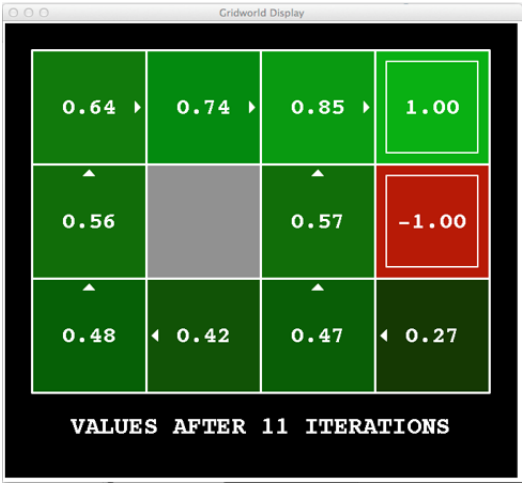
Problems with Value Iteration

k = 10

Noise = 0.2
Discount = 0.9
Living reward = 0

Problems with Value Iteration

k = 11



Noise = 0.2
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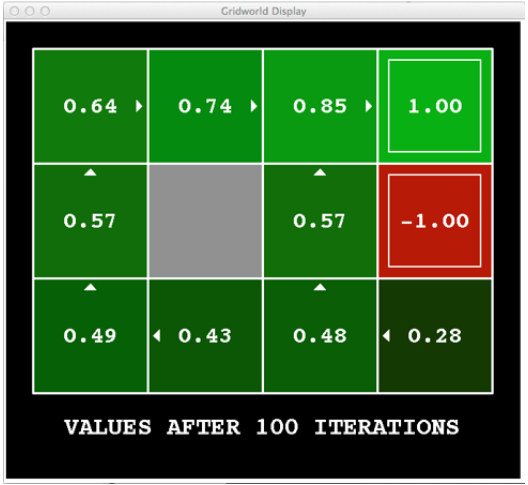
Problems with Value Iteration

k = 12

Noise = 0.2
Discount = 0.9
Living reward = 0

Problems with Value Iteration

k = 100



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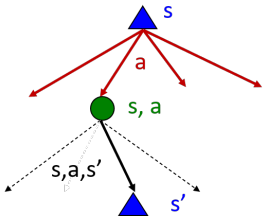
- Problem 1: It's slow - $O(S^2A)$ per iteration



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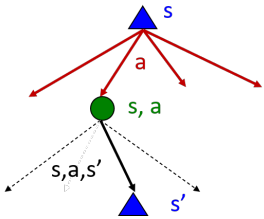

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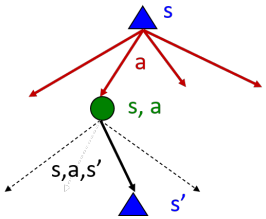
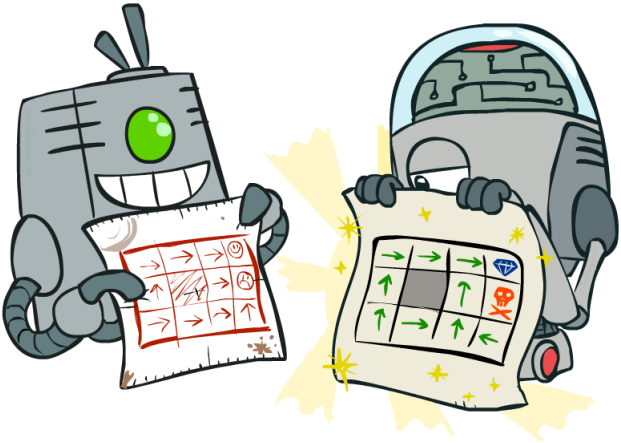
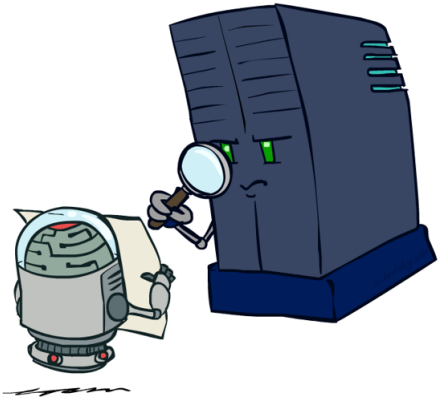
- Problem 2: The "max" at each state rarely changes
- Problem 3: The policy often converges long before the values



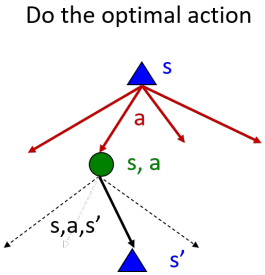
Policy Methods



Policy Evaluation



Fixed Policies



- Expectimax trees max over all actions to compute the optimal values

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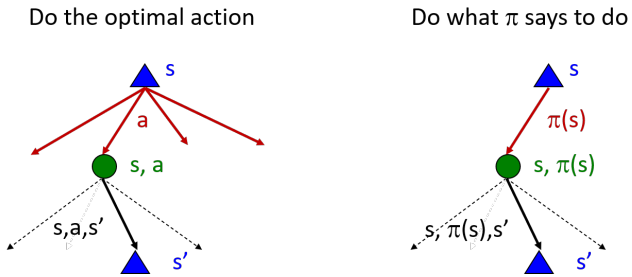
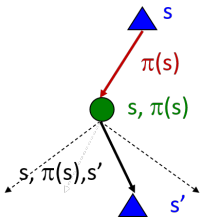


- Expectimax trees max over all actions to compute the optimal values
- If we fixed some policy $\pi(s)$, then the tree would be simpler – only one action per state
  - . . . though the tree's value would depend on which policy we fixed

Utilities for a Fixed Policy

- Compute the utility of a state s under a fixed (generally non-optimal) policy
- Define the utility of a state s, under a fixed policy π:
  $V^{\pi}(s)$ =expected total discounted rewards starting in s and following π



Utilities for a Fixed Policy

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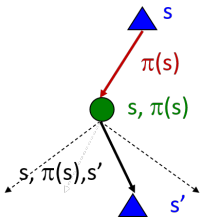
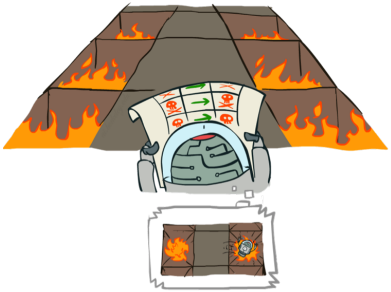
- Define the utility of a state s, under a fixed policy π: $V^{\pi}(s) =$expected total discounted rewards starting in s and following π
- Recursive relation (one-step look-ahead/Bellman equation): $V^{\pi}(s) = \sum_{s'} T(s, \pi(s), s') \left[R(s, \pi(s), s') + \gamma V^{\pi}(s')\right]$



Example: Policy Evaluation

Always Go Right



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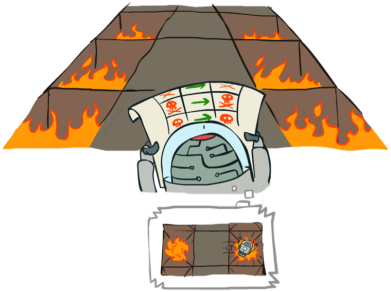
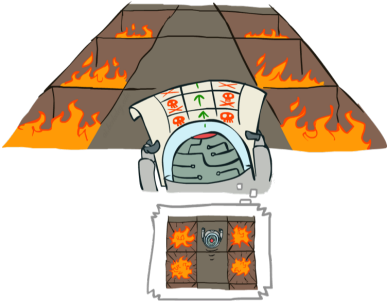


Always Go Forward



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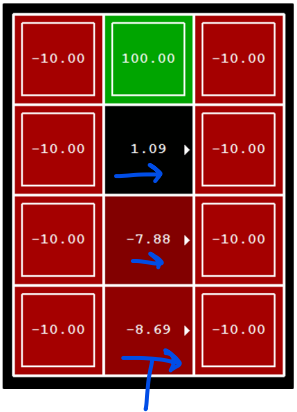

Always Go Forward

Policy Evaluation

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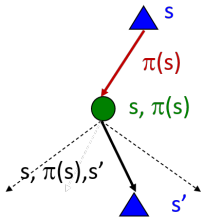

Policy Evaluation

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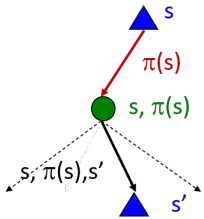
- Idea 1: Turn recursive Bellman equations into updates (like value iteration)



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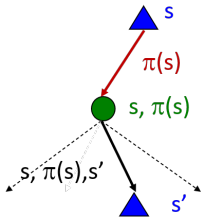

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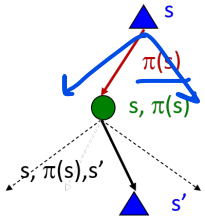

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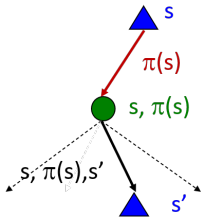

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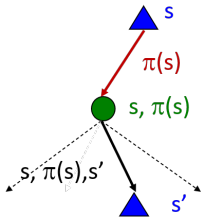
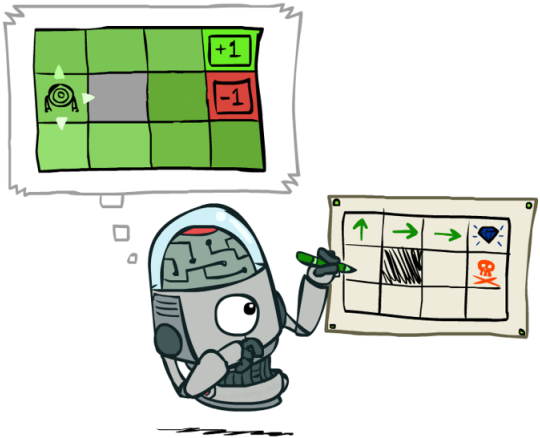


- Idea 2: Without the maxes, the Bellman equations are just a linear system
  - Solve with Matlab (or your favorite linear system solver)

Policy Extraction



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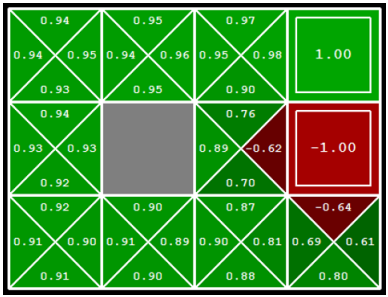
  $\pi^*(s) = \underset{a}{\operatorname{argmax}} \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma V^*(s')\right]$
- This is called **policy extraction**, since it gets the policy implied by the values

Computing Actions from Q-Values

- Let's imagine we have the optimal q-values:



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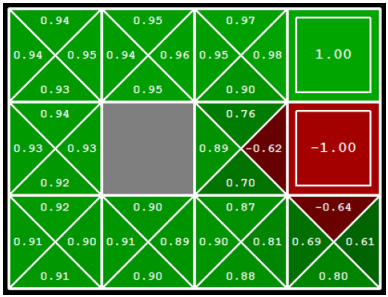

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  - Completely trivial to decide!

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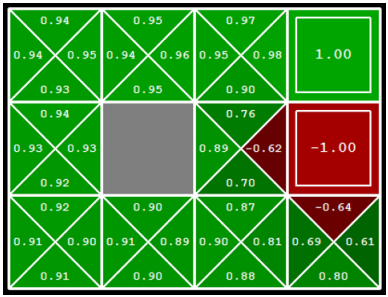

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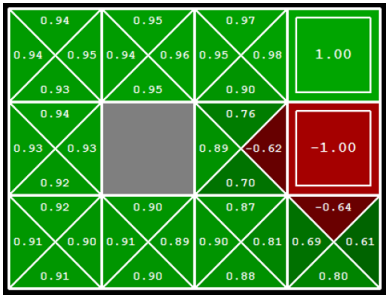
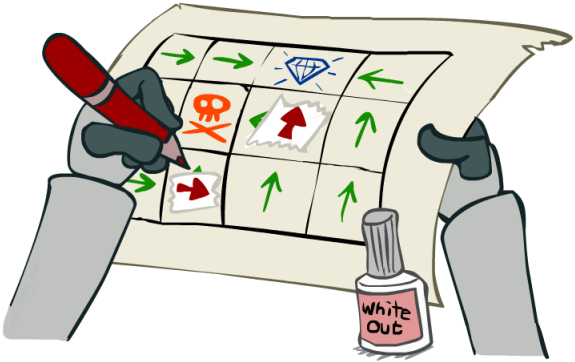
- Important: actions are easier to select from q-values than values!



Policy Iteration



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- This is **policy iteration**
  - Optimal
  - Can converge (much) faster under some conditions

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- Improvement: For fixed values, get a better policy using policy extraction
  - One-step look-ahead:
    $\pi_{i+1}(s) = \operatorname{argmax}_a \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma V^{\pi_i}(s')\right]$

Comparison

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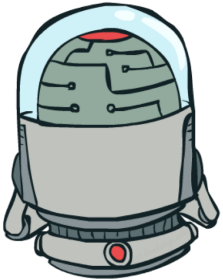
- Both are dynamic programming approaches for solving MDPs

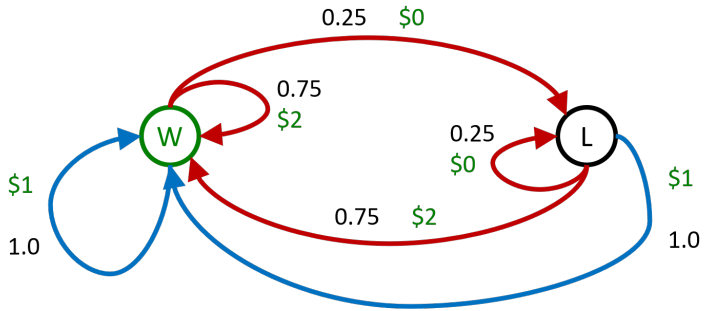
Double Bandits



Double Bandits

- Actions: *Blue*, *Red*
- States: Win, Lose

No discount
100 time steps
Both states have the same value

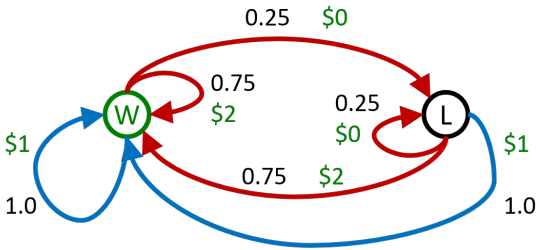


Offline Planning

- Solving MDPs is offline planning
  - You determine all quantities through computation
  - You need to know the details of the MDP
  - You do not actually play the game!

No discount
100 time steps
Both states have the same value

	Value
Play Red	150
Play Blue	100



Let's Play!

Let's Play!

$2

Let's Play!

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Let's Play!

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Let's Play!

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Let's Play!

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Let's Play!

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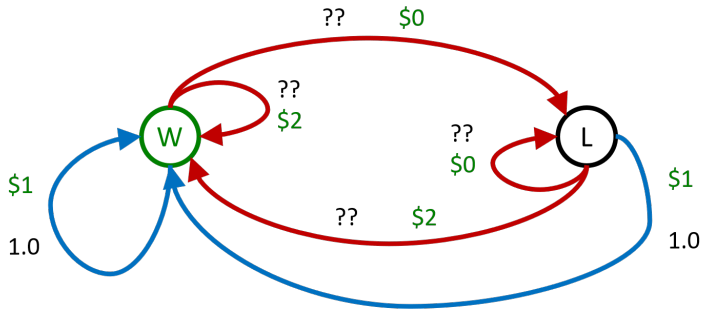
Online Planning

- Rules changed! Red's win chance is different



Let's Play Again!

Let's Play Again!

$0

Let's Play Again!

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Let's Play Again!

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Let's Play Again!

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Let's Play Again!

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Let's Play Again!

$0 $0 $0 $2 $0
$2 $0 $0 $0 $0

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  - Specifically, reinforcement learning
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- Important ideas in reinforcement learning that came up
  - Exploration: you have to try unknown actions to get information
  - Exploitation: eventually, you have to use what you know
  - Regret: even if you learn intelligently, you make mistakes
  - Sampling: because of chance, you have to try things repeatedly
  - Difficulty: learning can be much harder than solving a known MDP

Suggested Reading

- Russell & Norvig: Chapter 17.1-17.3
- Sutton and Barto: 3-4