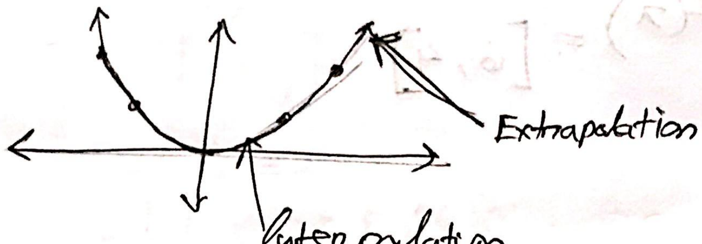
#6   **Interpolation**   MONDAY

→ Finding points within data range

Extrapolation → finding points outside data range



→ For $(n+1)$ points, ~~you~~ you can fit a polynomial of order n.

Linear Interpolation → 2 data points (Closest 2 points that bracket the required value)

$$\begin{bmatrix} 20 \\ 10 \end{bmatrix} = \begin{bmatrix} 1 & 15 \\ 1 & 20 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 & 15 \\ 1 & 20 \end{bmatrix}^{-1} \begin{bmatrix} 20 \\ 10 \end{bmatrix}$$

$$a_0 + 15a_1 = 20$$
$$a_0 + 20a_1 = 10$$

for quadratic,

$$a_0 + 15a_1 + 15^2 a_2 = 20$$
$$a_0 + 20a_1 + 20^2 a_2 = 10$$

#7

Chapter - 5.03

Newton's Divided Difference

24-Dec-21

Friday

Newton's Divided

$$f(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$$

Direct Normal Interpolation

$$f(x) = a_0 + a_1 x + a_2 x^2$$

$$f(x_1) = a_0 + a_1 x_1 \quad \text{—— (i)}$$

$$f(x_2) = a_0 + a_2 x_2 \quad \text{—— (ii)}$$

$$\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} f(x_1) \\ f(x_2) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix}^{-1} \begin{bmatrix} f(x_1) \\ f(x_2) \end{bmatrix}$$

Newton Divided:

Now, $f(x_0) = b_0$

$$f(x_1) = b_0 + b_1(x_1 - x_0)$$

$$\Rightarrow f(x_1) - f(x_0) = b_1(x_1 - x_0)$$

$$\Rightarrow b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Linear Interpolation:

$$\left[f_1(x) = f(x_0) + \left[\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right] \times (x - x_0) \right]$$

Ex - ~~$f_1(16) = f(15)$~~

#8

Larengian Interpolation

01-01-21

Direct : $f(x) = a_0 + a_1 x + a_2 x^2$

NDD : $f(x) = b_0 + b_1 (x - x_0) + b_2 (x - x_0)(x - x_1) \cancel{f(x_0)}$

$$= f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0} + \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \_\_\_}{x_2 - x_1} + \dots$$

Larengian : $f(x) = \sum_{i=0}^{n} L(x) f(x_i)$

$$= L_0 f(\;) + L_1 f(\;)$$

$$L_i = \prod_{\substack{j=0 \\ j \neq i}}^{n} \left[\frac{x - x_j}{x_i - x_j} \right]$$

for linear $\frac{x - x_0}{x_1 - x_0} f(x_1) + \frac{x - x_1}{x_0 - x_1} f(x_0)$

0	$10'' \rightarrow 500$
	$12'' \rightarrow x$
1	$15'' \rightarrow 1000$

$x = L_0 \times \cancel{500}\ 1000 + L_1 \times \cancel{1000}\ 5000$

$$= \frac{12 - 10}{15 - 10} \times 1000 + \frac{12 - 15}{10 - 15} \times \cancel{1000}\ 500$$

$$= \frac{2}{5} \times \cancel{500}\ 1000 + \frac{3}{5} \times 500$$

$= 4000 + 300$

$= 700$