B.Sc. Eng. (CSE)/ 5th Sem. 03 March, 2020 (Morning)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF MECHANICAL AND PRODUCTION ENGINEERING

Mid Semester Examination — Winter Semester: 2019-2020

COURSE NO.: Math-4541 — TIME : 1½ Hours

COURSE TITLE: Multivariable Calculus and Complex Variables — FULL MARKS: 75

There are **4 (Four)** questions. Answer any **3 (Three)** of them. Programmable calculators are not allowed. Do not write anything on this question paper. The figures in the right margin indicate full marks. The Symbols have their usual meaning.

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1. a) (i) Let z_1, z_2, z_3 represent vertices of an equilateral triangle. Prove that 12

$$z_1^2+z_2^2+z_3^2=z_1z_2+z_2z_3+z_3z_1$$

(ii) An airplane travels 150 miles southeast, 100 miles due west, 225 miles 30^0 north of east, and then 200 miles northeast. Determine by the concept of polar form of a complex number (a) analytically and (b) graphically how far and in what direction it is from its starting point.

b) (i) Find an equation using the complex number system for (a) a circle of radius 4 with center at (2, 1), (b) an ellipse with major axis of length 10 and foci at (3, 0) and (3, 0). 13

(ii) State De Moivre's Theorem and using this theorem prove that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

2. a) Find each of the indicated roots and locate them graphically 12

$$(i)\ (-1+i)^{1/3}\ (ii)\left(-2\sqrt{3}-2i\right)^{1/4}$$

b) Solve the equation: 13

$$(i) z^2+(2i-3)z+5-i=0\ (ii)\ z^5=1$$

3. a) Consider the transformation $w=\ln z$. 12

Show that

(i) circles with center at the origin in the z plane are mapped into lines parallel to the v axis in the w plane.

(ii) lines or rays emanating from the origin in the z plane are mapped into lines parallel to the u axis in the w plane.

(iii) the z plane is mapped into a strip of width 2π in the w plane. Illustrate the results graphically.

b) (i) Suppose the principal branch of $\sin^{-1}z$ to be that one for which $\sin^{-1}0=0$. 13

Prove that $\sin^{-1}z=\frac{1}{i}\ln\left(iz+\sqrt{1-z^2}\right)$

(ii) Prove that $f(z)=z^2$ is uniformly continuous in the region $|z|<1$

4. a) (i) Write necessary and sufficient conditions of $f(z)=u(x,y)+v(x,y)i$ be analytic in a region R. 12

(ii) Prove that $u=3x^2y+2x^2-y^3-2y^2$ is harmonic and Find v such that $f(z)=u+iv$ is analytic.

b) Locate and name all the singularities of $f(z)=\dfrac{z^8+z^4+2}{(z-1)^3(3z+2)^2}$ 13

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B.Sc. Eng. (CSE)/ 5th Sem. 07 March, 2019 (Afternoon)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF MECHANICAL AND CHEMICAL ENGINEERING

Mid Semester Examination | Winter Semester: 2018-2019

COURSE NO. : Math-4541 | TIME : 1½ Hours

COURSE TITLE: Multivariable Calculus and Complex Variables | FULL MARKS: 75

There are **4 (Four)** questions. Answer any **3 (Three)** of them. Programmable calculators are not allowed. Do not write anything on this question paper. The figures in the right margin indicate full marks. The Symbols have their usual meaning.

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1. a) (i) Explain how the complex function e^z and the real function e^x are different. How are they similar? **13**
(ii) Show that the Cauchy-Riemann equations hold for the functions u, v given by $u(x,y) = x^3 - 3xy^2$, $v(x,y) = 3x^2y - y^3$. Show that u, v are the real and imaginary parts of a holomorphic function $f : \mathbb{C} \rightarrow \mathbb{C}$.

b) (i) Define a harmonic function and conjugate harmonic function. **12**
(ii) Show that $u(x, y) = xy^3 - x^3y$ is a harmonic function, and find a conjugate harmonic function $v(x, y)$.

2. a) (i) Find the zeros of the following functions $1 + e^z$ and $1 + i - e^z$ **8**
(ii) State whether the following series converge or diverge. Justify your answers.

$$\sum_{n=1}^{\infty} (1+i)^n \text{ and } \sum_{n=1}^{\infty} \frac{1+in(-1)^n}{n^2}$$ **8**

b) Find the radii of convergence of the following power series: **9**

$$(i) \sum_{n=1}^{\infty} \frac{2^n z^n}{n} \quad (ii) \sum_{n=1}^{\infty} n! z^n \quad (iii) \sum_{n=1}^{\infty} n^p z^n \, (p \in N)$$

3. a) State and verify Cauchy Integral Theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points $1+i$, $-1+i$ and $-1-i$. **13**

b) (i) State the Cauchy's integral formula. **12**
(ii) Evaluate, using Cauchy's integral formula,

$$\int_c \frac{z^2 - 2z}{(z+1)^2(z^2+4)} dz,$$ where c is the circle $|z| = 10$.

4. a) Transform the rectangular region ABCD in z-plane bounded by $x = 1$, $x = 3$; $y = 0$ and $y = 3$. Under the transformation $w = z + (2 + i)$. **13**

b) Consider the transformation $w = ze^{i\pi/4}$ and determine the region R′ in w-plane corresponding to the triangular region R bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$ z-plane. **12**

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B.Sc. Eng. (CSE)/ 5th Sem. 13 March, 2018 (Afternoon)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF MECHANICAL AND CHEMICAL ENGINEERING

TERM : Mid Semester Examination Winter Semester: 2017-2018
COURSE NO. : Math-4541 TIME : 1½ Hours
COURSE TITLE: Multivariable Calculus and Complex Variables FULL MARKS: 75

There are 4 (Four) questions. Answer any 3 (Three) of them. Programmable calculators are not allowed. Do not write anything on this question paper. The figures in the right margin indicate full marks. The Symbols have their usual meaning.

1. a) Show that the function $f(z) = u + iv$, where 13

$$f(z)=\begin{cases} \dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2}; & z\neq 0 \\ 0 & ;\ z=0 \end{cases}$$

satisfies the Cauchy-Riemann equations at $z = 0$. Is the function analytic at $z = 0$? Justify your answer.

b) Define a harmonic function and conjugate harmonic function. Are the following function harmonic? If your answer is yes, find a corresponding analytic function $f(z) = u(x,y) + iv(x,y)$. 12

$$u=\frac{x}{x^2+y^2}$$

2. a) (i) Find the following functions in the form of $u + iv$ 8

$$e^{2+3\pi i} \text{ and } \cosh(-1+2i)$$

(ii) Find all solutions and graph in the complex plane

$$e^{z=1} \text{ and } \sinh z=0$$ 8

b) Evaluate $\int_C (z-z^2)dz$ where C is the upper half of the circle $|z-2| = 3$ and z is the complex variable. What is the value of the integral if C is the lower half of the above given circle? 9

3. a) State and verify Cauchy Integral Theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points $1+i,\ -1+i$ and $-1-i$. 13

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b) Evaluate, using Cauchy's integral formula, 12

(i) $\int_c \frac{3z^2+z}{z^2-1}dz$, If c is circle $|z-1| = 1$.

(ii) $\int_c \frac{z^2-2z}{(z+1)^2(z^2+4)}dz$, where c is the circle $|z| = 10$.

4. a) (i) $z_n = \frac{n\pi}{4+2ni}$ is a sequence. Is it bounded? Convergent? Find its limit points. 5

(ii) What is radius of convergence? Write its role in complex series. Find the center and radius of converges of the following series: $\sum_{n=0}^{\infty}\frac{(2n)!}{4^n(n!)^2}(z-2i)^n$ 8

b) Find the Taylor series with center z_o and its radius of convergence.

(i) $\frac{1}{1-z}, z_o = i,\ (ii) \sin 2z^2, z_o = 0$ 12

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B.Sc. Eng. (CSE)/ 5th Sem. 29 May 2019

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF MECHANICAL AND CHEMICAL ENGINEERING

Semester Final Examination — Winter Semester: 2018-2019
Course No.: Math-4541 — Time: 3 Hours
Course Title: Multivariable Calculus and Complex Variables — Full Marks: 150

There are **8 (Eight)** questions. Answer any **6 (Six)** of them. Programmable calculators are not allowed. Do not write anything on this question paper. The figures in the right margin indicate full marks. The Symbols have their usual meaning.

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1. a) (i) What is a graph of a function of two variables? How is it interpreted geometrically? Describe level curves. **12**
(ii) Explain, why $z^2 = x + 3y$ is not a function of x and y.
Consider a function $f(x,y)=\sqrt{9-x^2-y^2}$
(iii) Find the domain and range of the function.
(iv) Sketch a contour map of this surface using level curves corresponding to $k = 0, 1, 2, 3, \ldots.8$.

b) (i) If $\lim_{(x,y)\to(2,3)} f(x,y)=4$, can you conclude anything about $f(2, 3)$? Explain. **13**
(ii) Discuss the continuity of $f(x,y,z)=\dfrac{1}{x^2+y^2-z^2}$

2. a) (i) Sketch the graph of a function $z = f(x, y)$ whose derivatives f_x and f_y are always positive. **12**
(ii) Explain the geometrical interpretation of partial derivatives.
(iii) Find the slopes of the surface $f(x,y)=1-(x-1)^2-(y-2)^2$ at the point $(1, 2, 1)$ in the x-direction and in the y-direction.

b) (i) What is meant by a linear approximation of $z = f(x, y)$ at the point $P(x_0, y_0)$? **13**
A function is given $z = f(x,y)=x^2+3xy-y^2$,
(ii) Find the differential dz.
(iii) If x changes from 2 to 2.05 and y changes from 3 to 2.96, compare the values of Δz and dz.

3. a) (i) Consider a point (x_0, y_0, z_0) on a surface given by $F(x, y, z) = 0$. What is the relationship between $\nabla F(x_o, y_0, z_0)=0$ and any tangent vector v at (x_0, y_0, z_0)? How do you represent this relationship mathematically? **13**
(ii) Find the tangent plane to the elliptic paraboloid $z = 2x^2+y^2$ at the point $(1, 1, 3)$.

b) (i) What is the meaning of the gradient of a function f at a point (x, y)? **12**
(ii) Find the directional derivative of the function $f(x,y)=x^2y^3-4y$ at the Point $(2, -1)$ in the direction of the vector $\vec{v}=2\hat{i}+5\hat{j}$

4. a) (i) For a function of two variables, describe (a) relative minimum, (b) relative maximum, (c) critical point, and (d) saddle point **12**

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(ii) A rectangular box without a lid is to be made from 12 m of cardboard. Find the maximum volume of such a box.

b) (i) Explain what is meant by constrained optimization problems. 13
(ii) In your own words, describe the Method of Lagrange Multipliers for solving constrained optimization problems.
(iii) Under what condition does the Second Partials Test fail?

5. a) Find the maximum value of the function $f(x,y,z)=x+2y+3z$ on the curve of intersection of the plane $x-y+z=1$ and the cylinder $x^2+y^2=1$. 13

b) (i) What does it mean for $f(z)$ to be continuous at z_0 or on a domain D? 12
(ii) Show that the function $e^x(\cos y + i \sin y)$ is an analytic function, find its derivative.

6. a) (i) What are the Cauchy-Riemann Equations? 13
(ii) What does it mean for a function $f(z)$ to be analytic?
(iii) Show that the function $f(z) = u + iv$,

$$\text{where } f(z)=\begin{cases}\dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2}; & z\neq 0\\ 0 & ; z=0\end{cases}$$

satisfies the Cauchy-Riemann equations at $z = 0$. Is the function analytic at $z = 0$? Justify your answer.

b) (i) What is the relationship between harmonic and analytic functions? 12
(ii) Prove that $u=x^2-y^2-2xy-2x+3y$ is harmonic. Find a function v such that $f(z) = u + iv$ is analytic. Also, express $f(z)$ in terms of z.

7. a) (i) What is Cauchy's Integral Formula? 12
(ii) Use Cauchy's integral formula to evaluate

$$\int_c \frac{z}{(z^2-3z+2)}dz, \text{ where } c \text{ is the circle } |z-2|=\frac{1}{2}$$

b) (i) What is the ratio test? 13
(ii) Find the Taylor series and radius of convergence of the following function:

$z\sinh(z^2)$ at $z=0$

8. a) (i) What is a Laurent series expansion of $f(z)$ and where is it defined? 12
(ii) Find the Laurent series of the function

$$f(z)=\frac{z+4}{z^2\left(z^2+3z+2\right)} \text{ valid for the region } 1<|z|<2 \text{ and } |z|>2$$

b) (i) State Cauchy's Residue Theorem? 13
(ii) Using Residue theorem, evaluate

$$\frac{1}{2\pi i}\int_c \frac{e^{zt}}{z^2(z^2+2z+2)}dz, \text{ where } c \text{ is the circle } |z|=3$$

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B.Sc. Eng. (CSE)/ 5th Sem. **24 May 2018**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF MECHANICAL AND CHEMICAL ENGINEERING

Term: Semester Final Examination — Winter Semester: 2017-2018

Course No.: Math-4541 — Time: 3 Hours

Course Title: Multivariable Calculus and Complex Variables — Full Marks: 150

There are **8 (Eight)** questions. Answer any **6 (Six)** of them. Programmable calculators are not allowed. Do not write anything on this question paper. The figures in the right margin indicate full marks. The Symbols have their usual meaning.

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1. a) Define a harmonic function and conjugate harmonic function. Compute the Laplacian, where it exists, of the following functions and indicate where a function is harmonic. **12**

   (i) $x^2 + y^2$ (ii) $e^{ax}\cos\beta y$ (iii) $\ln\sqrt{(x^2+y^2)}$

   b) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function v such that $f(z) = u + iv$ is analytic. Also express $f(z)$ in terms of z. **13**

2. a) (i) Find the following functions in the form of $u + iv$ **12**

   $$e^{2+3\pi i} \text{ and } \cosh(-1+2i)$$

   (ii) Find all solutions and graph in the complex plane

   $$e^{z=1} \text{ and } \sinh z = 0$$

   b) Evaluate the following integrals along the mentioned path. **13**

   (i) $\int_0^{1+i} (x^2 - iy)dz$, along the parabola $y = x^2$

   (ii) $\int_c (12z^2 - 4iz)dz$, along the curve C joining the points (1, 1) and (2, 3).

3. a) State Cauchy integral formula and Evaluate the following integral using Cauchy integral formula: **13**

   (i) $\int_c \frac{4-3z}{z(z-1)(z-2)}dz$, where c is the circle $|z| = \frac{3}{2}$.

   (ii) $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}dz$, where c is the circle $|z| = 3$.

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b) Define the singularity of a function. Find out the zeros and discuss the nature of the singularities of **12**

$$(i)\sin\frac{1}{z}\ (ii)\ \frac{e^{\frac{1}{z}}}{z^2}\ (iii)\frac{(z-2)}{z^2}\sin\left(\frac{1}{z-1}\right)$$

4.

a) Using Residue theorem, evaluate $\dfrac{1}{2\pi i}\displaystyle\int_c \frac{e^{zt}dz}{z^2(z^2+2z+2)}$. **12**

b) Find the Laurent's series that converges for $0<|z-z_0|<R$ and determine the precise region of convergence of $\dfrac{\cos z}{(z-\pi)^2}, z_0=\pi$. **13**

5. a) (i) What is a graph of a function of two variables? How is it interpreted geometrically? Describe level curves. **13**

If $T(x, y)$ is the temperature at a point (x, y) on a thin metal plate in the xy-plane, then the level curves of T are called *isothermal curves*. All points on such a curve are at the same temperature. Suppose that a plate occupies the first quadrant and

$$T(x, y) = xy.$$

(ii) Sketch the isothermal curves on which $T = 1$, $T = 2$, and $T = 3$.
(iii) An ant, initially at (1, 4), wants to walk on the plate so that the temperature along its path remains constant. What path should the ant take and what is the temperature along that path?

b) (i) State the definition of continuity of a function of two variables. **12**

$$\text{Let, } f(x,y)=\begin{cases}-\dfrac{xy}{x^2+y^2} & ;(x,y)\neq(0,0)\\ 0 & ;(x,y)=(0,0)\end{cases}$$

(ii) Show that $f_x(x,y)$ and $f_x(x,y)$ exist at all points (x, y)
(iii) Explain why f is not continuous at (0, 0).

6. a) (i) Define the total differentials of a function of two variables. **13**
(ii) When using differentials, what is meant by the terms propagated error and relative error?

(iii) Use the differential dz to approximate the change in $z=\sqrt{4-x^2-y^2}$ as (x, y) moves from the point (1, 1) to the point (1.01, 0.97). Compare this approximation with the exact change in z.

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(iv) The possible error involved in measuring each dimension of a rectangular box

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(iv) The possible error involved in measuring each dimension of a rectangular box is ± 0.1 millimeter. The dimensions of the box are x = *50* centimeters, y = *20* centimeters and z = *15* centimeters as shown in Fig.: 01. Use dv to estimate the propagated error and the relative error in the calculated volume of the box.

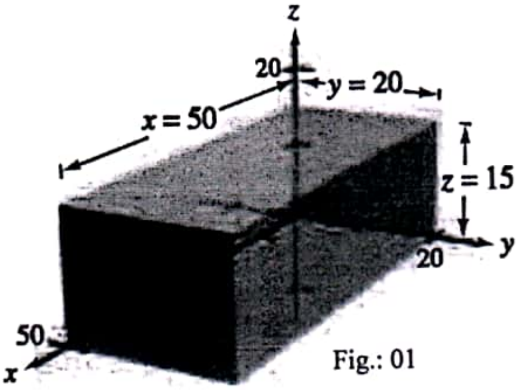

Fig.: 01

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b) (i) If $f(x,y)=0$, give the rule for finding $\frac{dy}{dx}$ implicitly.

(ii) If $f(x,y,z)=0$, give the rule for finding $\partial z/\partial x$ and $\partial z/\partial y$ implicitly.

(iii) Find $\partial z/\partial x$ and $\partial z/\partial y$, given $3x^2z - x^2y^2 + 2z^3 + 3yz - 5 = 0$

(iv) Find the directional derivative of $f(x,y)=4-x^2-\frac{1}{4}y^2$, at (1, 2) in the direction of $\bar{u}=\left(\cos\frac{\pi}{3}\right)\hat{i}+\left(\sin\frac{\pi}{3}\right)\hat{j}$.

7. a) (i) Write a paragraph describing the directional derivative of the function f in the direction $\bar{u}=\cos\theta\hat{i}+\sin\theta\hat{j}$, when $\theta=0°$ and $\theta=90°$. 12

(ii) A heat–seeking particle is located at the point (2, -3) on a metal plate whose temperature at *(x, y)* is $T(x,y)=20-4x^2-y^2$. Find the path of the particle as it continuously moves in the direction of maximum temperature increase.

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b) Consider the ellipsoid $x^2+4y^2+z^2=18$.

(i) Find an equation of the tangent plane to the ellipsoid at the point (1, 2, 1).

(ii) Find parametric equations of the line that is normal to the ellipsoid at the point (1, 2, 1).

(iii) Find the acute angle that the tangent plane at the point (1, 2, 1) makes with the *xy*-plane.

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8. a) A delivery company accepts only rectangular boxes, the sum of whose length and girth (perimeter of a cross-section) does not exceed 108 inches and shown in Fig.: 02. Find the dimensions of an acceptable box of largest volume. **12**

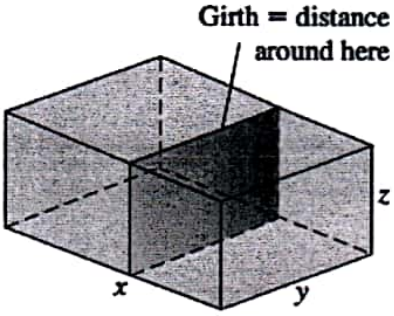

Fig.: 02

b) (i) Explain what is meant by constrained optimization problems. **13**

(ii) The operators of the Viking Princess, a luxury cruise liner are contemplating the addition of another swimming pool to the ship. The chief engineer has suggested that an area of the form of an ellipse located in the rear of the promenade deck would be suitable for this purpose. It has been determined that the shape of the ellipse may be described by the equation $x^2 + 4y^2 = 3600$ where x and y are measured in feet. Viking's operators would like to know the dimensions of the rectangular pool with the largest possible area that would meet these requirements.

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