ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID EXAMINATION **SUMMER SEMESTER, 2019-2020**
DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

CSE 4803: Graph Theory

Programmable calculators are not allowed. Answer all the questions.
Figures in the right margin indicate marks.

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1. a) Determine whether or not the following sequences represent simple graph. If the sequence represents simple graph, draw a corresponding graph. If not, justify. 9
   i. (2, 3, 3, 4, 4, 5)
   ii. (2, 3, 4, 4, 5)
   iii. Your Student ID (comma separated digits, sorted in ascending order)
   iv. (1, 3, 3, 3)
   v. (1, 2, 2, 3, 4, 4)
   vi. (1, 3, 3, 4, 5, 6, 6)

   b) One of your friends from CEE department has designed an apartment floor. Consider the drawing of an apartment with doors in Figure 1 as your friends drawing. 9

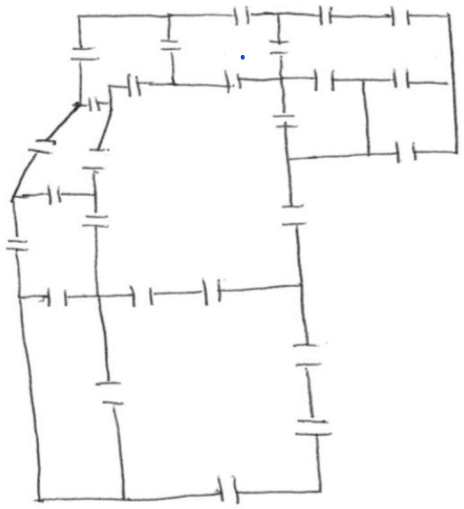


Figure 1: Floor Plan

   i. Can you find a continuous line that passes through each door exactly once? If not, At least how many doors are needed to be closed to have a continuous line that passes through each door exactly once?
   ii. If we transform this floor plan into a graph, what should the vertices and the edges represent? What does the graph look like?
   iii. Find a continuous line that passes through each door exactly once after closing the minimum numbers of doors.

   c) As a *Tom & Jerry* fan in your childhood, you used to draw *Jerry* mouse as your favorite character. One of such drawing is depicted in Figure 2(a). This drawing can be translated into a graph shown in Figure 2(b). 2.5

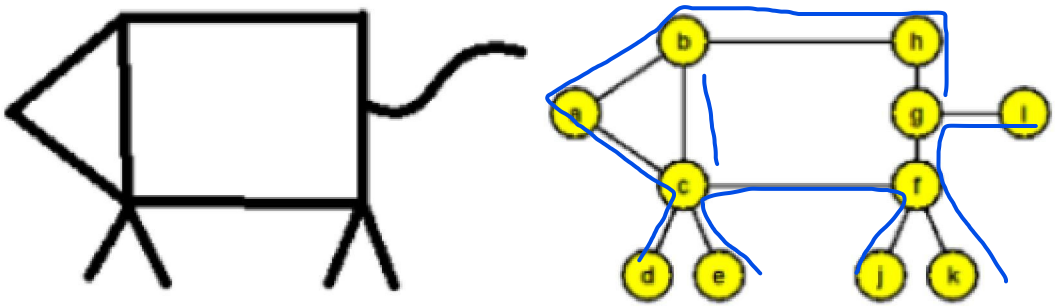


Figure 2: (a) A conceptual drawing of *Jerry* mouse, (b) Translation of the given mouse into a graph

Find the number of the minimum trails as possible covering the given graph? Draw the trails with distinguishable patterns.

[Hine: An Euler/semi Euler graph needs only one trail.]

d) Consider the graph G in Figure 3. Is G Eulerian? Is G Hamiltonian? Is G bipartite? Justify your answers. 4.5

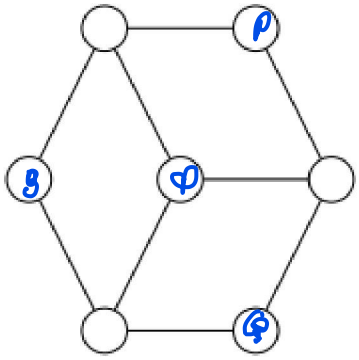


Figure 3: Simple graph G

2. a) How many isomers does Hexane (C_6H_{14}) have? Draw the structure of the carbon atoms in each isomer. 5

b) Connected acyclic graphs are known as *Tree*. 4+4

i. A finite tree T has at least one vertex v of degree 4, and at least one vertex w of degree 3. Show that T has at least 5 leaves.

ii. Let T be a tree with p vertices of degree 1 and q other vertices. What is the sum of the degrees of the vertices of degree greater than 1?

c) In Springfield Nuclear Power plant, there are 16 staff houses. An inexperienced engineer was hired to develop a network that will connect all the houses together. The engineer built a grid-like architecture for the network which is shown in Figure 4. This plan is submitted to you (an expert) for your approval. 12

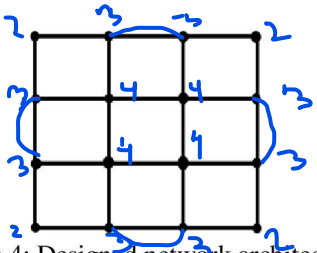
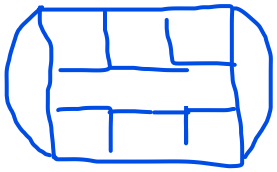


Figure 4: Designed network architecture

i. What are the Vertex and Edge connectivity of this graph?

ii. If you are asked to check all the connection starting from the top-left house and returning to it. What is the minimum unit of distance you need to cover? Given that all the connections have unit distance?

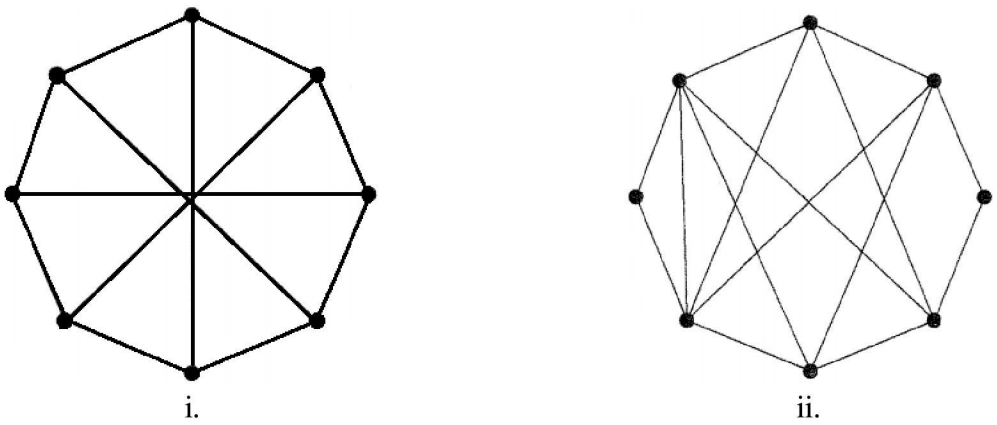
iii. Can you design a more stable architecture? If not, describe why. Otherwise, draw the network.



3. a) Draw 4 simple completely regular planar graphs with vertex degree ≥ 3. 8

b) Show that, if G is a 3-connected plane graph, then its geometric dual is a simple graph. 4

c) Determine if the following graphs in Figure 5 are planar. If yes, give a planar representation. If not justify. [Hint: drawing is not a justification] 8



i. ii.

Figure 5: Graphs for question 3(c)

d) A 5-regular planar graph has triangular regions. Find all possible number of vertices, edges and regions. 5