Solutions to Problems in Chapter 14 of
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14.1. The arrival rate is $\lambda = 1/1.25 = 0.8$ and the processing rate is $\omega = 0.9$. Therefore, the utilization factor is $\rho = 0.8/0.9 = 0.889 < 1$.

14.2. No. As a machine ages, you might expect the U_i's to get smaller and the D_i's to get larger.

14.3. No. For example, U_i could be small because the previous repair might have not been completely successful, which might result in only a small D_i.

14.4. The number of parts produced before the first jam has a geometric distribution with parameter p (see. Sec. 6.2.3) and mean $(1 - p) / p$.

14.5. If the machine operator is present at all times, then a breakdown when the machine is idling might be discovered immediately. If, however, there are long periods of time during which the machine is starved, then the operator might be away doing other tasks.

14.6. Let $\overline{U}$ = sample mean of the U_i's (in hours)

$\overline{D}$ = sample mean of the D_i's (in hours)

$\overline{P}$ = average processing time for parts (in hours)

$\overline{N}$ = average number of parts produced per 8-hour shift

$\hat{\mu}_B$ = estimate of unknown mean busy time (in hours)

Then the proportion of time that the machine is busy is estimated by both sides of the following equation:

$$\frac{\hat{\mu}_B}{\overline{U} + \overline{D}} = \frac{\overline{N}\,\overline{P}}{8}$$

which gives

$$\hat{\mu}_B = \frac{\overline{N}\,\overline{P}(\overline{U} + \overline{D})}{8}$$

14.7. Solving the equations

$$E(U)+E(D)=4$$

and

$$\frac{E(U)}{E(U)+E(D)}=0.9$$

gives $E(U)=3.6$ and $E(D)=0.4$.

14.8. (*a*) Generate busy times b_1 and b_2 for components 1 and 2, respectively. Then the system operates until the minimum of the times required for components 1 and 2 to accumulate b_1 and b_2 units of busy (processing) time, respectively. Suppose for definiteness that component 1 fails first and is repaired. (Assume that component 2 accumulated a_2 units of busy time during the time required for component 1 to fail, where $a_2 < b_2$.) Generate another busy time, say, b_1' for component 1. Then the system operates until the minimum of the time required for component 1 to accumulate b_1' additional units of busy time and the time required for component 2 to accumulate $b_2 - a_2$ additional units of busy time, etc.

(*b*) Generate uptimes u_1 and u_2 for components 1 and 2, and let $u = \min\{u_1, u_2\}$. Then the system operates until time *u*. Assume for definiteness that component 1 fails first and let d_1 be its corresponding downtime. If $u + d_1 < u_2$, then the system goes back up at time $u + d_1$. Let u_1' be the second uptime for component 1. Then the system operates until the minimum of time $u + d_1 + u_1'$ and time u_2, etc.

If $u_2 < u + d_1$, then component 2 fails while component 1 is still being repaired. (This may not be very realistic.) Let d_2 be the downtime for component 2. Then the system goes back up *no sooner* than the maximum of time $u_1 + d_1$ and time $u_2 + d_2$, etc. (The first component to be repaired could fail again before the other component is repaired.)

14.9. If we multiply the mean times for component A by 250/46.5, we get the following table:

Component	Mean busy time	Mean repair time
A	250	8.065
B	250	6.000

Therefore, for every 250 hours of system busy time, there will be 14.065 = 8.065 + 6.000 hours of repair time. Hence,

$$e = \frac{E(B)}{E(B) + E(R)} = \frac{250}{250 + 14.065} = 0.947$$

14.10. Let B_{A} and B_{B} be the busy times of components A and B before their first failure, and let B be the busy time of the system before the first failure. Clearly, $B = \min\{B_{\mathrm{A}}, B_{\mathrm{B}}\}$. Then

$$\begin{aligned} P(B > x) &= P(B_{\mathrm{A}} > x, B_{\mathrm{B}} > x) \\ &= P(B_{\mathrm{A}} > x)P(B_{\mathrm{B}} > x) \quad \text{(by independence)} \\ &= e^{-x/46.5}e^{-x/250} \\ &= e^{-(x/46.5+x/250)} \\ &= e^{-x/39.207} \end{aligned}$$

Therefore, B is exponentially distributed with a mean of 39.207 hours.

It can be shown that components A and B will fail first with respective probabilities

$$0.843 = (1/46.5)/[(1/46.5)+(1/250.0)]$$

and

$$0.157 = (1/250.0)/[(1/46.5)+(1/250.0)]$$

Therefore, by conditional probability we get a mean repair time $E(R)$ given by

$$E(R) = 0.843(1.5) + 0.157(6.0) = 2.207 \text{ hours}$$

And the efficiency e is given by

$$e = \frac{E(B)}{E(B)+E(R)} = \frac{39.207}{39.207+2.207} = 0.947$$

Note that $E(B)$ and $E(R)$ are the same for each up-down cycle because of the memoryless property of the exponential busy times (see Prob. 4.30).

14.11. One approach is to fit distributions F_1 and F_2 to the type 1 and type 2 repair time data, respectively. When the machine fails, we generate a repair time from F_i with probability $p_i = n_i / n$ for $i = 1, 2$. Another approach is to construct an empirical distribution from all n observations. (This approach makes no assumptions about the form of the n observations, such as their histogram having a single local mode.) For both approaches we are assuming that D_i is independent of U_i.

14.12. The estimated performance measures for the two system designs are almost identical with the exception of proportion forklifts moving empty, which went from 0.27 to 0.34 for the new design. Thus, there is no reason to change from system design 3.

14.13. The mean total service times for job types 1, 2, and 3 are 0.8, 0.65, and 1, respectively. Furthermore, the probabilities of a job being of types 1, 2, and 3 are 0.3, 0.5, and 0.2, respectively. Therefore, the mean total service time of a job is

$$E(S) = 0.3(0.8) + 0.5(0.65) + 0.2(1) = 0.765$$

14.15. The expected throughput cannot exceed 120 jobs per 8-hour day because this is the arrival rate.

14.16. We have not taken into account that a machine might be blocked and, thus, not able to process available parts.

14.17. We have not taken into account the time that a forklift travels empty.

14.18. Since the machine at station 2 is busy or blocked 100 percent of the time and the size of its queue tends to grow with time, the arrival rate λ_2 to station 2 must be larger than its effective service rate ω_2' (taking into account blocking). Therefore, in the long run jobs will be added to queue 2 at a rate of $\lambda_2 - \omega_2'$.

14.19. It will look similar to Fig. 14.30 but will level out at 11.87 (94.94/8).

14.20. The departure rate at station 2 increased because another machine was added there, and all jobs leaving this station go directly to station 5. This causes the utilization factor at station 5 to increase.

14.21. In Table 14.7 what we computed was the number of forklifts required to move jobs along their routes.

14.22. The expected throughput will be unchanged, because the arrival rate is the same. The expected time in system will be less, because there is less variability in the arrival process.

14.23. The expected throughput will be unchanged. The expected time in system will be smaller, because the service times are less variable.

14.24. Let $E(e_i)$ be the expected main effect for the number of machines in station i (for $i = 1, 2, \ldots, 5$), and let $E(e_6)$ be the expected main effect for the number of forklift trucks. Then our point estimates of the expected main effects are given in the following table:

Expected main effect	Point estimate (hours)
$E(e_1)$	-0.17
$E(e_2)$	-0.01
$E(e_3)$	-0.11
$E(e_4)$	-0.02
$E(e_5)$	-0.33
$E(e_6)$	-0.15

It appears that the expected main effect of factor 5 (station 5) is the largest in magnitude, having a value of -0.33 hour. This is consistent with the fact that station 5 has the largest "Proportion machines busy" for system design 3 (Table 14.10), which corresponds to the "-" levels for our resolution IV fractional factorial design. On the other hand, stations 2 and 4 appear to have the smallest expected main effects in magnitude, as well as the smallest "Proportion machines busy" for system design 3.

The largest expected interaction effect appears to be for the 1×4 interaction, having a value of 0.056 (not shown). However, for a 2_{IV}^{6-2} fractional factorial design, two-way interactions are actually confounded with each. In particular, 0.056 is actually an unbiased estimate of the alias chain $E(e_{14}) + E(e_{56})$ [see Montgomery (2013, p. 707)], but we do not know the values of the individual expected interaction effects. Chapter 8 in Montgomery discusses what additional design points to simulate in order to break this alias chain, if desired.

14.25. No. The assembler does not operate during the third shift each day.