Reinforcement Learning II

CSE 4711: Artificial Intelligence

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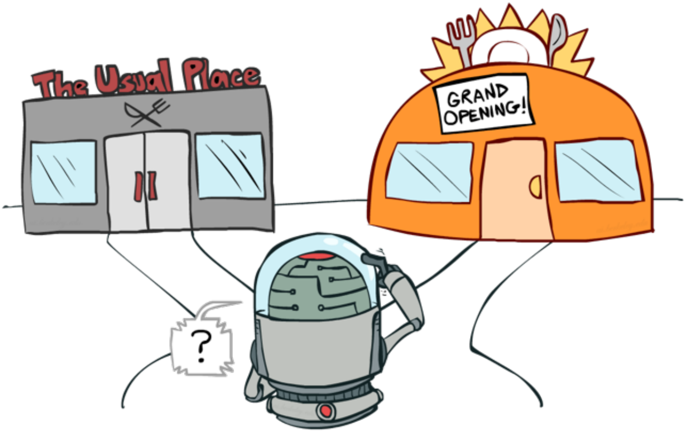
Islamic University of Technology

Exploration vs. Exploitation



How to Explore?

- Several schemes for forcing exploration
  - Simplest: random actions (ϵ-greedy)
    - Every time step, flip a coin
    - With (small) probability ϵ, act randomly
    - With (large) probability $1 - \epsilon$, act on current policy

Videos: q-bridge, q-epsilon

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    - Every time step, flip a coin
    - With (small) probability ϵ, act randomly
    - With (large) probability $1 - \epsilon$, act on current policy
  - Problems with random actions?
    - You do eventually explore the space, but keep thrashing around once learning is done
    - One solution: lower ϵ over time
    - Another solution: exploration functions

Videos: q-bridge, q-epsilon

Exploration Functions

- When to explore?
  - Random actions: explore a fixed amount
  - Better idea: explore areas whose badness is not (yet) established, eventually stop exploring

Video: q-explorer-crawler

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  - Note: this propagates the "bonus" back to states that lead to unknown states as well!

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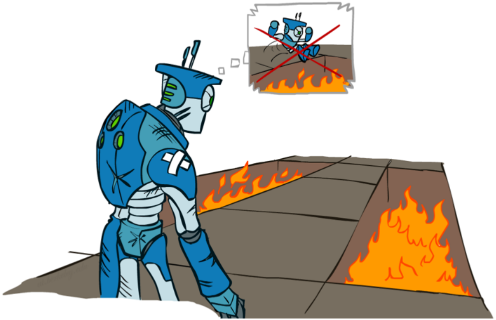
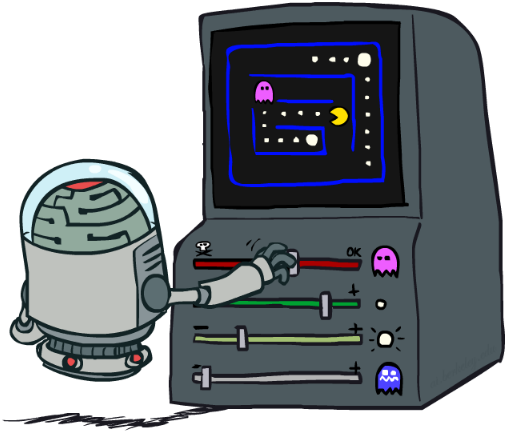
Regret

- Even if you learn the optimal policy, you still make mistakes along the way
- Regret is a measure of your total mistake cost: the difference between your (expected) rewards, including youthful suboptimality, and optimal (expected) rewards
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret



Approximate Q-Learning



Generalizing Across States

- Basic Q-Learning keeps a table of all q-values

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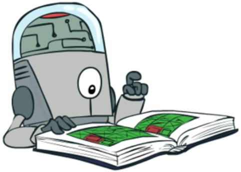

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Generalizing Across States

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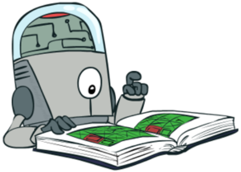
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory



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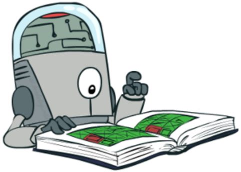

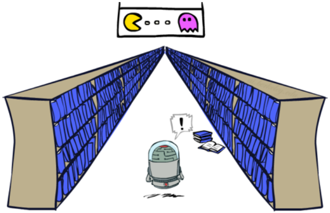

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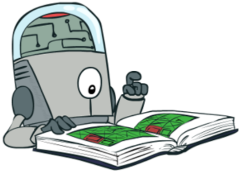
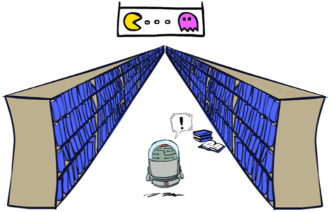
Generalizing Across States

- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - This is a fundamental idea in machine learning, and we'll see it over and over again





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Example: Pacman

Let's say we discover
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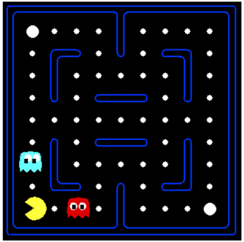

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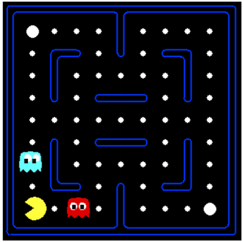
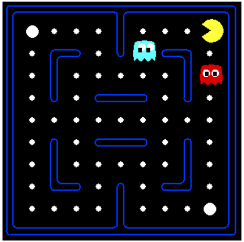


In naïve q-learning, we know nothing about this state:



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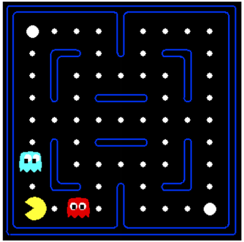

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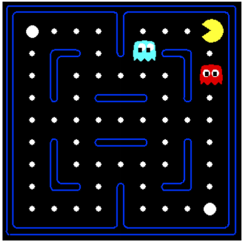
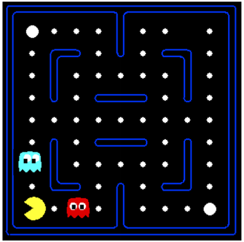
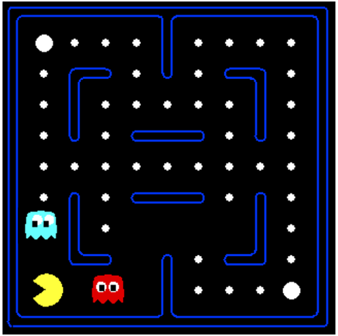


Or even this one!



Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - $1/(\text{dist to dot})^2$
    - Is Pacman in a tunnel? (0/1)
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Linear Value Functions

- Using a feature representation, we can write a Q-function for any state using a few weights:

$$\hat{Q}(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \cdots + w_n f_n(s,a)$$

- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but actually be very different in value!

Approximate Q-Learning

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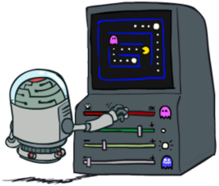

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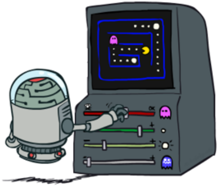
- Q-learning with linear Q-functions:



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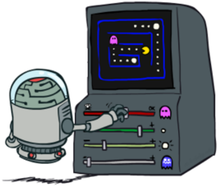

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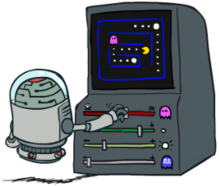
  Transition = (s, a, r, s')

  Difference = $\left[r + \gamma \max_{a'} Q(s', a')\right] - \hat{Q}(s, a)$



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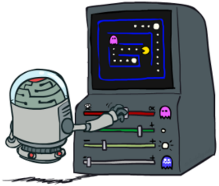
  Exact Q's $\quad Q(s,a) \leftarrow Q(s,a) + \alpha[\text{difference}]$



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  sample − $Q(s)$

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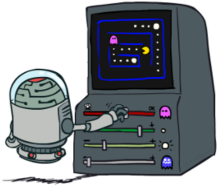
$W = W +$



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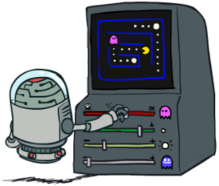
- Intuitive interpretation:
  - Adjust weights of active features
  - E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features



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- Intuitive interpretation:
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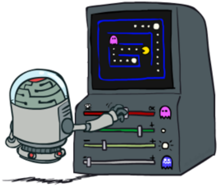
- Formal justification: online least squares



Example: Q-Pacman

$$\hat{Q}(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$$

Video: q-approx-pacman

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$f_{DOT}(s, \text{NORTH}) = 0.5$

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difference $= -501$

$w_{DOT} \leftarrow 4.0 + \alpha\,[-501]\,0.5$

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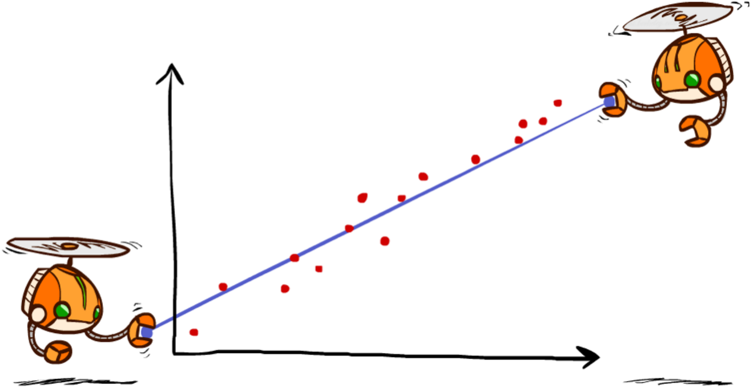
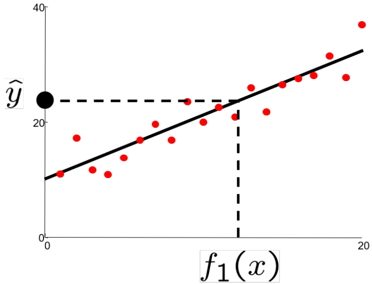
Q-Learning and Least Squares



Linear Approximation: Regression



Prediction:

$$\hat{y} = w_0 + w_1 f_1(x)$$

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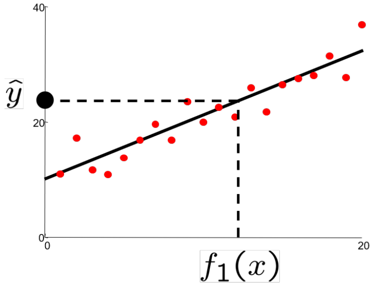

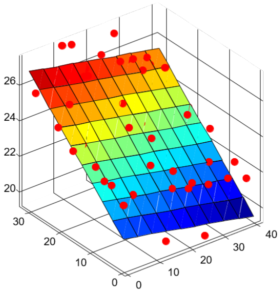

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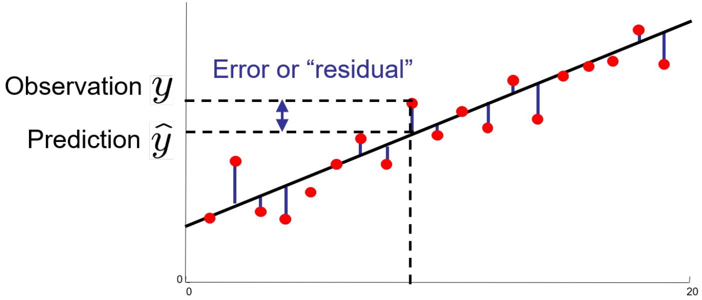
$$\hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

Optimization: Least Squares



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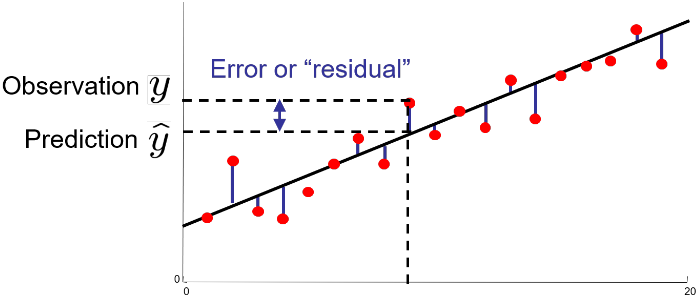

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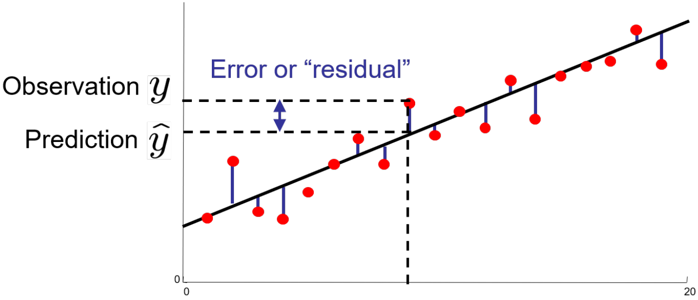

total error = $\sum_i (y_i - \widehat{y}_i)^2 = \sum_i \left(y_i - \sum_k w_k f_k(x_i) \right)^2$

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Imagine we had only one point x, with features $f(x)$, target value y, and weights w:

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$$w_m \leftarrow w_m + \underline{\alpha}\left(\underline{y} - \underbrace{\sum_k (w_k f_k(x))}_{\text{exp}}\right) \underline{f_m(x)}$$

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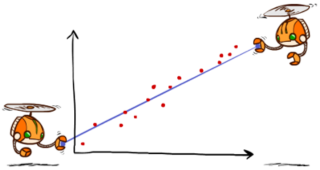


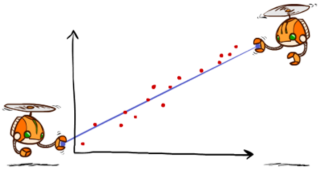
Minimizing Error

Imagine we had only one point x, with features $f(x)$, target value y, and weights w:

$$error(w) = \frac{1}{2}\left(y - \sum_k (w_k f_k(x))\right)^2$$

$$\frac{\partial error(w)}{\partial w_m} = -\left(y - \sum_k w_k f_k(x)\right) f_m(x)$$

$$w_m \leftarrow w_m + \alpha \left(y - \sum_k (w_k f_k(x))\right) f_m(x)$$

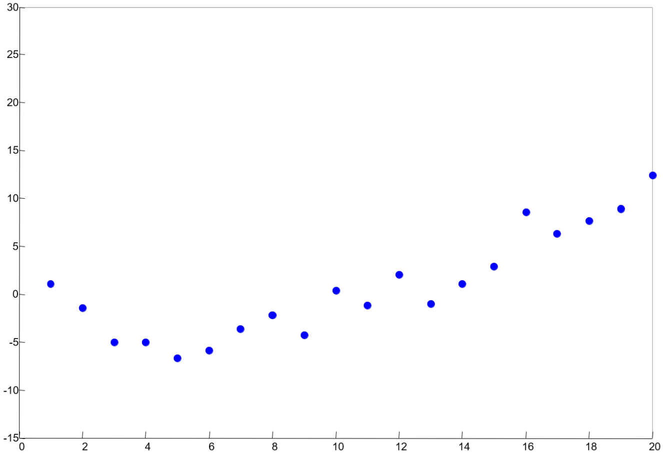


Approximate q-update:

$$w_m \leftarrow w_m + \alpha \left[r + \gamma \max_a Q(s', a') - Q(s, a) \right] f_m(s, a)$$

Q- same

Overfitting: Why Limiting Capacity Can Help



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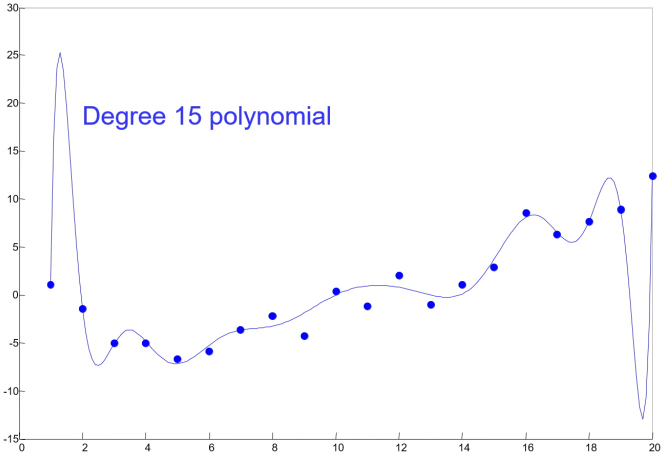

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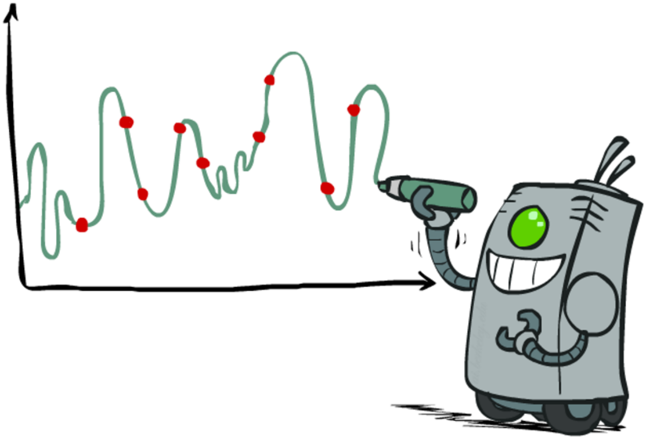
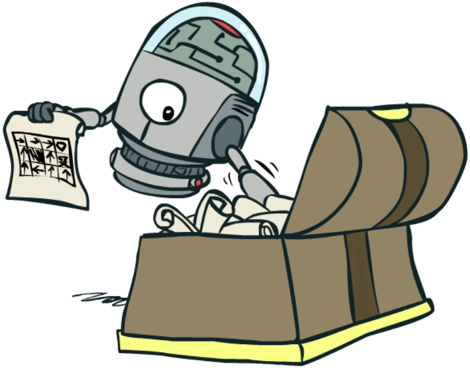


Policy Search



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- Better methods exploit lookahead structure, sample wisely, change multiple parameters...

Conclusion

- We are done with Search and Planning!
- We have seen how AI methods can solve problems in:
  - Search
  - Constraint Satisfaction Problems
  - Games
  - Markov Decision Problems
  - Reinforcement Learning
- Next? Uncertainty and Learning

Suggested Reading

- Russell & Norvig: Chapter 21