B.Sc. Engg. CSE 5th Semester 167 15 March 2018 (Afternoon)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **WINTER SEMESTER, 2017-2018**

DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

CSE 4549: Simulation and Modeling

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 4 (four) questions. Answer any 3 (three) of them.

Figures in the right margin indicate marks.

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1. Consider a university cafeteria where no waiting staff service or table service is available. At present there exits two food-serving counters named *Main Counter* (serves main meal) and *Dessert Counter* (serves dessert item). Students enjoy their meal service from *Main Counter*, and/or *Dessert Counter* by placing their individual service request at respective counters one by one. If a student arrives and finds the *Main Counter* idle, he/she is served immediately with his requested service, else he/she waits in a *FIFO* Queue. The students have the exponential inter arrival times with mean 2.1 minutes and the service time at *Main Counter* is also exponentially distributed with mean service time 2.0 minutes.

   At the completion of service by the *Main Counter*, a student either departs with *p* probability or requests next service to *Dessert Counter* with *(1-p)* probability. The *Dessert Counter* requires exponential service time with mean 2.3 minutes. When the *Dessert Counter* remains busy at the time of service request, arriving students again wait in a FIFO queue associated with it. Upon service completion from *Dessert Counter*, students then return for additional service at the *Main Counter* again. For both counters, as any (served) student departs, if the queue is empty then the counter becomes idle, else a student from top of the queue is served immediately.

   Initially the system is empty and idle, and the simulation is to run for exactly 8 hours. The purpose of the simulation is to improve the system in terms of followings: average delay in each queue, the time average number in each queue, and the utilization of the each counter.

   a) What are the state variables and output variables for this simulation model? 4
   b) Identify the set of events for this simulation model. 5
   c) Write down the state equations for this simulation model. 7
   d) Write down the state space for this simulation model. 4
   e) Write down the output equations for this simulation model. 5

2. For the scenario given in Question 1 answer the followings:
   a) Draw a sample path of the system for a few initial minutes showing the change of the state variable(s) over time. 13
   b) Draw the separate flow charts of the events routines (i.e. the event handler functions) for any two of the events of the system. 12

3. a) Describe the characteristic properties of *Discrete Event Systems (DES)*. 5
   b) List the steps of *Simulation Development Life Cycle*. 6
   c) Describe what you think would be the most effective way to study each of the following systems in terms of given possibilities below and discuss why. 6
      - Possible study approaches:
        i. Experiment with the physical model of the system
        ii. Experiment with the mathematical model of the system through simulation

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- Given systems:
  i. Earth's thermodynamic in a particular geographic area
  ii. Small section of an existing inventory system
  iii. Traffic system in a metropolitan area
  iv. Water supply system in a commercial building
  v. Digital communication system in a battlefield

d) For each of the systems in Question 3.c, suppose that it has been decided to make a study via a simulation model. Discuss whether the simulation should be deterministic or stochastic, time-varying or time-invariant, and continuous state or discrete state. Justify your answers considering appropriate assumptions. 8

4. Consider a single server queuing system in which customers arrive according to Poisson process with rate λ1. Upon arriving, they either enter into service if the server is free or they join the queue. However, it is assumed that, each customer will only wait a random amount of time, having distribution *F*, in queue before leaving the system. Service time of a customer is exponential with rate λ2.

Suppose that each time the server completes a service, the next customer to be served is the one who has the earliest queue departure time. That is, if two customers are waiting and one would depart the queue if his/her service has not yet begun by time ***t1*** and the other if his/her service had not yet begun by time ***t2***, then the former would enter service if ***t1*** < ***t2***, and the later otherwise.

Assume that the following random variates are available:

- Inter-arrival Times (in second) are:
  Y_1= 0.4, Y_2= 0.3, Y_3= 0.4, Y_4= 1.7, Y_5= 1.7, Y_6= 0.5, and Y_7= 0.9
- Waiting Times (in second) are:
  X_1= 0.3, X_2= 0.8, X_3= 1.5, X_4= 0.6, X_5= 1.3, X_6= 0.2, and X_7= 1.1
- Service Times (in second) are:
  Z_1= 1.6, Z_2= 0.5, Z_3= 1.0, Z_4= 0.9, Z_5= 0.8, Z_6= 0.7, and Z_7= 1.1

2.1
1.5
0.9

a) Draw the sample path of the system for the above data 10
b) Mention the state(s) of the system at every event occurrence time. 5
c) Find the number of customers those left the queue 5
d) Find the average waiting time of the customer in the queue. 5

B.Sc. Engg. CSE 8th Semester 29 October, 2018

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
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Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **SUMMER SEMESTER, 2017-2018**
DURATION: 3 Hours **FULL MARKS: 150**

CSE 4831: Simulation, Modeling and Performance Evaluation

Programmable calculators are not allowed. Do not write anything on the question paper.
There are 8 (Eight) questions. Answer any 6 (Six) of them.
Figures in the right margin indicate marks.

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1. Jobs arrive at a single-CPU computer facility with interarrival times that are IID exponential random variables with mean 1 minute. Each job specifies upon its arrival the maximum amount of processing time it requires, and the maximum times for successive jobs are IID exponential random variables with mean 1.1 minutes. However, if m is the specified maximum processing time for a particular job, the actual processing time is distributed uniformly between $0.55m$ and $1.05m$. The CPU will never process a job for more than its specified maximum; a job whose required processing time exceeds its specified maximum leaves the facility without completing service. You are asked to develop a simulation program to study the computer facility until 1000 jobs have left the CPU assuming that jobs in the queue are processed in a FIFO manner.
The system is studied to compute the average and maximum delay in queue of jobs, the proportion of jobs that are delayed in queue more than 5 minutes.
   a) What are the state variable(s) and output variable(s) for the simulation model? 7
   b) Identify the set of events for the simulation model. Assume that the simulation terminates by a terminating event. 4
   c) Write down the state equation(s) and output equation(s) for the simulation model. 10
   d) Write down the state space for the simulation model. 4

2. For the scenario given in Question 1, answer the followings:
   a) Draw a sample path of the system for a few breakdowns of machines showing the change of the state variable(s) over time. 5
   b) Draw separate flow charts of the event routines (i.e., the event handler functions) for each of the events of the simulation model. 12
   c) Draw the flow chart of the function that updates the necessary statistical variables according to the output equations of the simulation model. 8

3. An instructor knows from past experience that student exam scores have mean 77 and standard deviation 15. At present the instructor is teaching two separate classes – one of size 25 and the other of size 64.
   a) Approximate the probability that the average test score in the class of size 25 lies between 72 and 82. 8
   b) Repeat part (a) for a class of size 64. 6
   c) Find out the approximate probability that the average test score in the class of size 25 is higher than that of the class size 64. 5

4. a) Let $X_1, X_2, \ldots, X_n$ be a sample from the distribution whose density function is 10

$$f(x) = \begin{cases} e^{-(x-\theta)}, & x \geq \theta \\ 0, & otherwise. \end{cases}$$

Determine the maximum likelihood estimator of θ .

b) Consider the discrete uniform distribution 8

$$P(x) = \frac{x}{b-a}, \quad x = a, a+1, \ldots, b$$

Find the joint maximum likelihood estimations (MLEs) of $a$and b, based on a random sample of size n.

c) The number of traffic accident in a city in 10 randomly chosen days is as follows: 7

4, 0, 6, 5, 2, 1, 2, 0, 4, 3

If the number of accidents in a day follows a Poisson distribution, find the proportion of the days that had 2 or fewer accidents.

5. The following data represent the time to perform transaction in a bank, measured in minutes:

0.70, 1.28, 1.46, 2.36, 0.354, 0.750, 0.912, 4.44, 0.114, 3.08, 3.24, 1.10, 1.59, 1.47, 1.17, 1.27, 9.12, 11.5, 2.42, 1.77.

Develop an input model for these data, which includes the followings:

a) A summary statistic of the data for the functions: mean, coefficient of variation and skewness. Also, form the summary statistics comment on the possible distribution. 8

b) A graphical estimate of the distribution by drawing one or more histograms. 6

c) Estimation of the parameter(s) of the distribution. 5

d) Use the Chi-square test to test the hypothesis that the random samples have the estimated distribution. 6

6. To estimate θ, 20 independent values having mean θ have been generated. If the successive values obtained are

102, 112, 131, 107, 114, 95, 133, 145, 139, 117, 93, 111, 124, 122, 136, 141, 119, 122, 151, and 143.

a) Find a point estimate of θ. 3

b) Construct a 99% confidence interval for the estimated value. 12

c) How many additional values do you think to be generated if we want to be 99 percent certain that our final estimate of θ is correct to within ± 0.5? 10

7. a) Develop a random variate generator using the composition method for a random variable with the following distribution: 13

$$f(x) = \begin{cases} \frac{1}{2}(x-2), & 2 \leq x \leq 3 \\ \frac{1}{2}\left(2 - \frac{x}{3}\right), & 3 \leq x \leq 6 \\ 0, & \text{otherwise.} \end{cases}$$

b) Develop a random variate generator using the acceptance-rejection method for a 12

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random variable with the following distribution:

$$f(x) = \begin{cases} 1+x, & -1 \leq x \leq 0 \\ 1-x, & 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

8. a) Use the inverse transform method (or any other method) to generate random variates with the following distribution function: 15

$$f(x) = \begin{cases} \frac{x}{2}, & 0 \leq x \leq 1 \\ \frac{3}{4}, & 1 < x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

b) A machine is taken out of production if it fails, or after 5 hours, whichever comes first. By running similar machines until failure, it has been found that time to failure, X, has the uniform continuous distribution with parameters $a = 0$ and $b = 10$ hours. Thus, the time until the machine is taken out of production can be represented as $Y = \min(X, 5)$. Develop a step-by-step procedure for generating Y. 10

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DURATION: 1 Hour 30 Minutes — FULL MARKS: 75

CSE 4831: Simulation, Modeling and Performance Evaluation

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There are 4 (four) questions. Answer any 3 (three) of them.

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1. A one-pump gas station is always open and has two types of customers. A police car arrives every 30 minutes (exactly), with the first police car arriving at time 15 minutes. Regular (nonpolice) cars have exponential interarrival times with mean 5.6 minutes, with the first car arriving at time 0. Service time at the pump for all cars are exponential with mean 4.8 minutes. A car arriving to find the pump idle goes right into service, and regular cars arriving to find the pump busy join the end of a single queue. A police car arriving to find the pump busy, however, goes to the front on the line, ahead of any regular cars in the line. [If there are already other police cars at the front of the line, assume that an arriving police car gets in line ahead of them as well.]
   Initially, the system is empty and idle, and the simulation is to run until exactly 500 cars (on any type) have completed their delays in queue. The purpose of the simulation is to improve the system in terms of the followings: average delay in queue for each type of car, the time-average number of cars in queue, and the utilization of the pump.

   a) What are the state variables and output variables for the simulation model? 7

   b) Identify the set of events for the simulation model. 5

   c) Write down the state equations and output equations for the simulation model. 10

   d) Write down the state space for the simulation model. 3

2. For the scenario given in Question 1, answer the followings:

   a) Draw a sample path of the system for a few customers showing the change of the state variable(s) over time. 5

   b) Draw separate flow charts of the event routines (i.e., the event handler functions) for each of the events of the system. 12

   c) Draw the flow chart of the function that updates the necessary statistical variables according to the output equations of the simulation model. 8

3. a) Define and differentiate between Random Numbers and Pseudo-Random Numbers 7

   b) Without actually computing any Z_i's, determine whether the following to LCGs have full period: 8

      i. $Z_i = (13Z_{i-1} + 13)(mod\ 16)$
      ii. $Z_i = (Z_{i-1} + 12)(mod\ 13)$

   c) Generate 10 random numbers using the midsquare method for $Z_0 = 7367$. Discuss the disadvantages of this method. 10

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4. a) A mass is connected to a fixed point by a spring. At time $t = 0$, the mass is displaced from its rest position by an amount $u(0) = u_0 > 0$ and released. The displacement at any time $t > 0$, denoted by $y(t)$ is to be measured. Determine whether an input-output modeling or a state space modeling is appropriate for this system. Justify your answer. 8

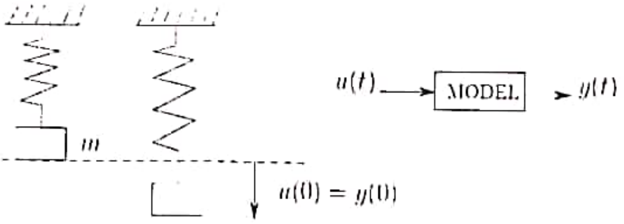

Figure 1: System for Question 4(a)

   b) Differentiate between the followings: 9

      i. Static and Dynamic Systems
      ii. Time-Varying and Time-Invariant Systems
      iii. Deterministic and Stochastic Systems

   c) Discuss the steps involved in developing the computational model of a system with an appropriate example. 8

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B.Sc. Engg. CSE 5th Semester 30 May 2018

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ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **WINTER SEMESTER, 2017-2018**
DURATION: 3 Hours **FULL MARKS: 150**

CSE 4549: Simulation and Modeling

Programmable calculators are not allowed. Do not write anything on the question paper.
There are 8 (eight) questions. Answer any 6 (six) of them.
Figures in the right margin indicate marks.

---

1. Jobs arrive at a single-CPU computer facility with interarrival times that are IID exponential random variables with mean 1.5 minutes. Each job specifies upon its arrival the maximum amount of processing time it requires, and the maximum times for successive jobs are IID exponential random variables with mean 1.1 minutes. However, if m is the specified maximum processing time for a particular job, the actual processing time is distributed uniformly between $0.77m$ and $1.04m$. The CPU will never process a job for more than its specified maximum; a job whose required processing time exceeds its specified maximum leaves the facility without completing service. You are asked to develop simulation program to study the computer facility until 5000 jobs have left the CPU assuming that jobs in the queue are ranked in increasing order of their specified maximum processing time.
The system is studied to compute the average and maximum delay in the queue of jobs, the proportion of jobs that are delayed in the queue more than 4 minutes.
   a) What are the state variable(s) and output variable(s) for this simulation model? 6
   b) Identify the set of events for this simulation model. Assume that the simulation terminates by a terminating event. 5
   c) Write down the state equations for this simulation model. 6
   d) Write down the state space for this simulation model. 3
   e) Write down the output equations for this simulation model. 5

2. For the scenario given in Question 1, answer the followings:
   a) Draw a sample path of the system for a few jobs showing the change of the state variable(s) over time. 15
   b) Draw separate flow charts of the event routines (i.e. the event handler functions) for any two of the system events. 10

3. A manufacturing system contains m machines, each subject to randomly occurring breakdowns. A machine runs for an amount of time that is an exponential random variable with mean 8 hours before breaking down. There are s (where s is fixed, positive integer) repairmen to fix broken machines, and it takes one repairmen an exponential amount of time with mean 1.5 hours to complete the repair of one machine; no more than one repairmen can be assigned to work on a broken machine even if there are other idle repairmen. If more than s machines are broken down at a given time, they form a FIFO 'repair' queue and wait for the first available repairmen. Further, a repairmen works on a broken machine until it is fixed, regardless of what else is happening in the system. Assume that it costs the system \$40 for each hour that each machine is broken down and \$20 an hour to employ each repairmen. (The repairmen are paid an hourly wage regardless of whether they are actually working.) Assume that $m = 3$, but the simulation model might accommodate a value of m as high as 20 by changing an input parameter.

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The system is studied for 500 hours for each of the employment policies $s = 1, 2,$ *and* 3 to determine which policy results in the smallest expected average cost per hour. Assume that at time 0 all the machines have just been freshly repaired.

a) Write down the goals and objectives of the simulation. 3
b) What are the state variable(s) and output variable(s) for this simulation model? 6
c) Identify the set of events for this simulation model. Assume that the simulation terminates by a terminating event. 4
d) Write down the state equations for this simulation model. 6
e) Write down the output equations for this simulation model. 6

4. a) Mention few potential application areas of simulation. 5
b) Compare and contrast *Input-output modeling* and *State Space modeling.* 6
c) Describe the properties of *Discrete Event Systems* along with few example scenarios 7
d) Discuss the fundamental limitation of the *Midsquare* method as a random-number generator with appropriate example. 7

5. a) Consider the multiplicative congruential generator under the following circumstance: 10

$$a = 11,\ m = 16,\ X_0 = 7$$

Generate enough values to compute a cycle. What inferences can be drawn? Is maximum period/cycle achieved?

b) Without actually computing any Z_i 's, determine which of the following mixed linear congruential generator (LCGs) have the full period: 15

i. $Z_i = (13\, Z_{i-1} + 13)(mod\ 16)$
ii. $Z_i = (4951\, Z_{i-1} + 247)(mod\ 256)$

6. a) When the basketball player Wilt Chamberlain shot two free throws, each shot was equally likely either to be good (g) or bad (b). Each shot that was good was worth 1 point. Let X denote the number of points that he scored. 15

i. What is the probability mass function of random variable X?
ii. Find and sketch the cumulative distribution function of random variable X.
iii. What is the expected value of random variable X?
iv. What is the variance of random variable X?

b) Suppose that 7.3, 6.1, 3.8, 8.4, 6.9, 7.1, 5.3, 8.2, 4.9 and 5.8 are 10 observations from a distribution with unknown mean μ. Approximate the 95 percent confidence interval for μ. 10

7. a) Develop a random variate generator using the composition method for a random variable with the following distribution: 10

$$f(x) = \begin{cases} 2 - a - 2(1-a)x, & 0 \le x \le 1 \\ 0, & Otherwise. \end{cases}$$

b) Develop a random variate generator using the acceptance-rejection method for a random variable with the following distribution: 10

$$f(x) = 20x\,(1-x)^3, \quad 0 < x < 1$$

c) Generate 3 random variates using this generator. Following random numbers are available 5
0.964, 0.152, 0.759, 0.365, 0.462, 0.785, 0.218, 0.763, 0.568, and 0.631

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8. Assume that the numbers of items demanded per day from an inventory on different days are IID random variables and the data in the following Table are those demand size on 45 different days. Analyze the data performing the following steps:

0	5	4	1	3
3	2	3	6	7
1	2	1	6	1
3	3	2	2	3
1	2	5	4	5
3	2	2	6	3
4	2	3	2	2
2	4	2	1	5
1	3	6	0	8

a) Find the following summary statistics of data – coefficient of variation and skewness. Also from the summary statistics comment on the possible distribution. 8

b) Make the histogram of the data. 7

c) From the histogram determine the fitted distribution of the data. 5

d) Find the parameter value(s) of the fitted distribution. 5

Appendix A: CDF of Standard Normal Distribution

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

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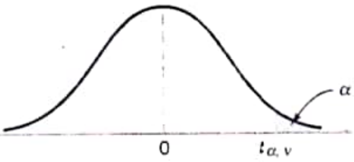
Appendix C: Chi-Square Distribution

TABLE A2 *Values of* $x^2_{\alpha,n}$

n	$\alpha = .995$	$\alpha = .99$	$\alpha = .975$	$\alpha = .95$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
1	.0000393	.000157	.000982	.00393	3.841	5.024	6.635	7.879
2	.0100	.0201	.0506	.103	5.991	7.378	9.210	10.597
3	.0717	.115	.216	.352	7.815	9.348	11.345	12.838
4	.207	.297	.484	.711	9.488	11.143	13.277	14.860
5	.412	.554	.831	1.145	11.070	12.832	13.086	16.750
6	.676	.872	1.237	1.635	12.592	14.449	16.812	18.548
7	.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582
20	7.434	8.260	9:591	10.851	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.484	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	42.557	45.772	49.588	52.336
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672

Other chi-square probabilities:
$x^2_{.9,9} = 4.2$ $\quad P\{x^2_{16} < 14.3\} = .425$ $\quad P\{x^2_{11} < 17.1875\} = .8976.$

Appendix B: Percentage Points of the t-distribution



Percentage Points $t_{\alpha,\nu}$ of the t-Distribution

ν \ α	.40	.25	.10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

ν = degrees of freedom.