ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **SUMMER SEMESTER, 2020-2021**

DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

Math 4641: Numerical Methods

Programmable calculators are not allowed. Do not write anything on the question paper

Answer **all 3 (three)** questions. Marks of each question and corresponding CO and PO are written in the right margin.

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1. a) Give an analysis of the **advantages** and **drawbacks** of the Bisection method. 6 (CO3) (PO2)

   b) i. Why do we need Spline Interpolation? Demonstrate a scenario where Direct method, Newton's Divided Difference method and Lagrangian method would fail but Spline interpolation would not. 6+9 (CO1,CO2) (PO2)

   ii. Suppose you wanted to perform Spline interpolation on n data points given as $(x_0, y_0), (x_1, y_1), \dots, (x_{n-1}, y_{n-1}), (x_n, y_n)$. And suppose you wanted to fit the data using **Cubic splines** of the form $f(x) = ax^3 + bx^2 + cx + d$. How many unknowns and how many equations would you have to deal with, and how would you arrive at those equations?

   c) Determine how many significant digits are correct in each of these cases: 4 (CO3) (PO2)

   i. 3.141590

   ii. 1.0790×10^3

   iii. 0.0098

   iv. 0.00980

2. a) Many supercomputers do not have any **division** operation. The reason for this is that a single divide operation can take 20 to 25 clock cycles, which is 5 times what it takes for a single multiplication. So instead, a divide unit is developed that numerically solves a nonlinear equation in order to get the inverse of a number, and the overall time taken to solve that equation using just subtraction and multiplication becomes less than performing the divide operation directly. The method that is used to solve the equation is **Newton Raphson**. Suppose we want to divide b by a. In paper, it can be written as $\frac{b}{a} = b * x, where\ x = \frac{1}{a}$. If we can numerically find the root of the equation $x = \frac{1}{a}$, then we directly multiply b with x to get the same effect as division. Derive the Newton Raphson expression for x_{i+1} that can be used to iteratively find the inverse of a. 10 (CO1) (PO2)

   Note that the expression cannot contain any division operation itself, otherwise the whole process becomes redundant since we are having to do a division anyway.

   b) Refer to the previous problem in question 2(a) where we derived an expression for iteratively finding the inverse of a. Suppose $a = x.y$, where x and y are the 1st and 2nd of the last two digits of your student ID. For example, if you ID = 180041001, then $a = 0.1$, so we are trying to find $\frac{1}{0.1}$. Obviously, using calculator we will get 10, but that is not our target. We are trying to iteratively get the result using Newton Raphson method. Choose an initial guess for the inverse of a **(don't choose the actual result as the initial guess)** and set it to x_0. For example, you can choose $x_0 = 5$ when $a = 0.1$. Use your previously derived Newton Raphson expression and perform as many iterations as required for your case showing the absolute relative approximate error at 9 (CO2) (PO1)

every step. If the inverse cannot be found within 4 steps, then truncate the process there and show the result you obtained within the first 4 steps.

c) Explain the differences between **Interpolation** and **Extrapolation.** When would we need which? — 6 (CO3) (PO2)

3. a) Suppose you are working at a start-up computer assembly company and you are tasked with determining the minimum number of computers the shop will have to sell to make a profit during the first year in business, which is also known as the **break-even point**. With such businesses, there is always some fixed capital cost, and some other costs which vary based on the number of computers assembled. At the break-even point, the total cost of operating the business and assembling the computers should become equal to the total sales minus the discount price. Based on this, suppose you found that the following equation should give you the break-even point: $\$35000 - 875n + 40n^{1.5} = 0$, where n is the number of computers sold. Find the minimum number of computers that need to be sold to obtain a profit by solving the above nonlinear equation using **Secant method**. Perform at least 3 iterations and show the number of significant digits that are at least correct in each iteration. — 10 (CO2) (PO3)

b) Derive the Quadratic form of Newton-s Divided Difference polynomial and using its pattern, determine the Cubic form of Newton-s Divided Difference polynomial. — 8 (CO1) (PO2)

c) The Maclaurin series is a special case of Taylor series. The Maclaurin series for $sin\ (x)$ is given as $sin\ sin\ (x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$. Use the **Remainder Theorem** to find the bounds of the truncation error in the representation of $sin\ (2)$ if only the first 3 terms of the Maclaurin series are used. — 7 (CO2,CO3) (PO2)