Regular Expressions :-

a is a Regular Expression where $a \in \Sigma$

$R_1 \cup R_2$ is a Regular Expression where R_1 is Regular & R_2 is also Regular.

$R_1 \circ R_2$ is a " " where R_1 is Regular & R_2 is also Regular.

R_1^* is a Regular Expression where R_1 is a Regular

ϵ " " " "

$\emptyset$ " " " "

(R_1) " " " "

Rules

① Star binds tighter

$$ab^* = a(b^*)$$
$$\neq (ab)^*$$

② Concatenation binds tighter than Union

$$a \circ b \cup c = (ab) \cup c$$
$$a \circ b | c = (ab) | c$$

~~$a^* \; 2 \; aa^+$~~

$$a^+ = aa^*$$

$aa\ b \cup c\ \ aa\ b \cup c\ \ aa = ?$

$\downarrow$ khek

$(aab) \cup (caab) \cup (caa)$.

$d \cup ab^* cd^* \Rightarrow$ ~~$d \cup ac(bd)^*$~~

~~$d \cup (ab)^* (cd)^*$~~

$d \mid (a(b)^* c(d)^*)$

or

$d \cup (a(b)^* c(d)^*)$

let $\Sigma = \{a, b, c, d\}$.

a = Regular Expression

$L_1 = \{a\}$

$abccd$

$L_2 = \{abccd\}$

$ab \cup cd$

$L_3 = \{ab, cd\}$

$a(b \cup c)d$

$\rightarrow L_4 = \{abd, acd\}$

$ab^* c$

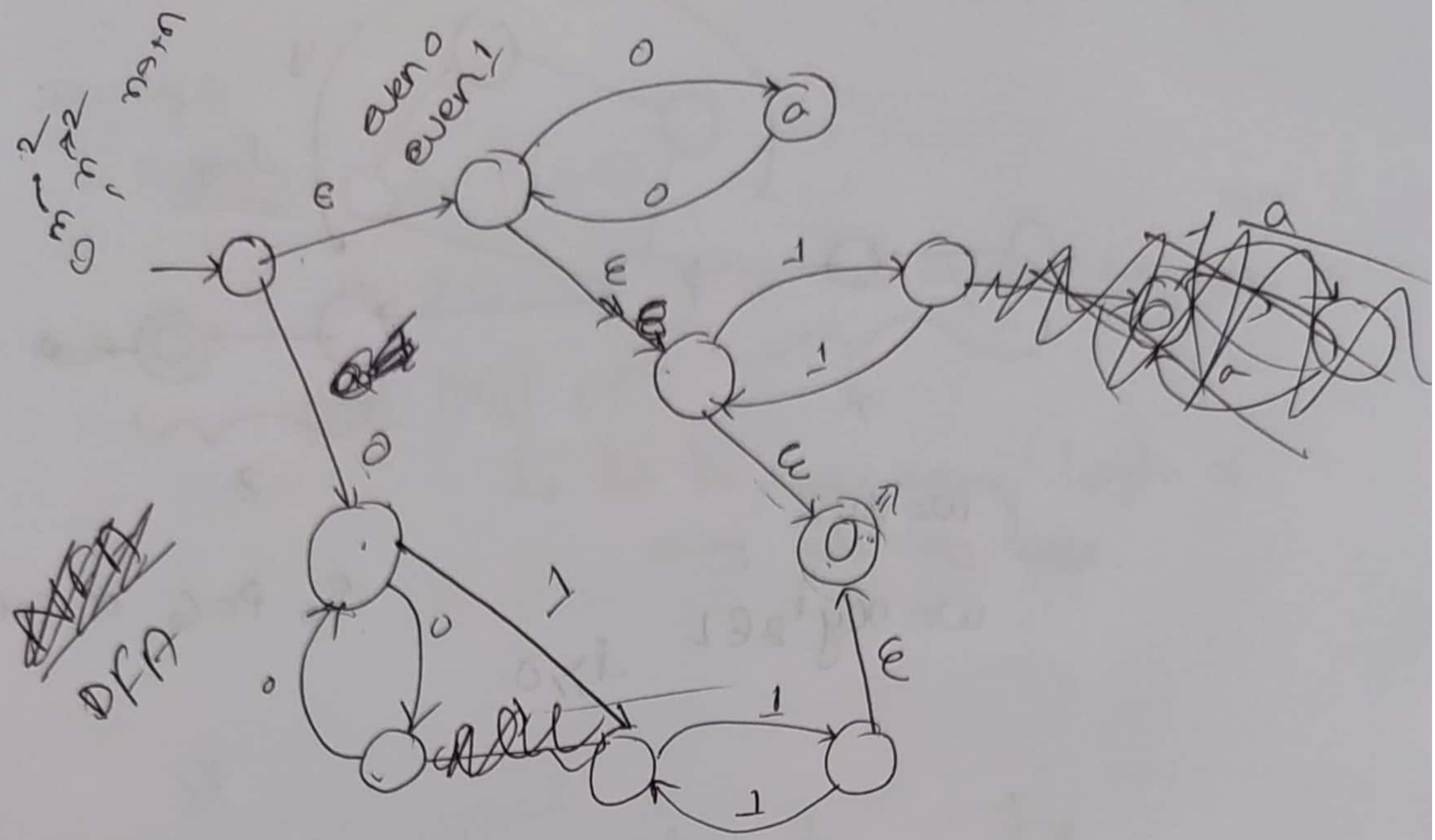
$\rightarrow L_5 = \{ac, abc, abbc, abbbc, \ldots\}$

$a(b \cup \epsilon)c$.

$\rightarrow L_6 = \{abc, ac$

$\phi^* = \{\varepsilon\}$.

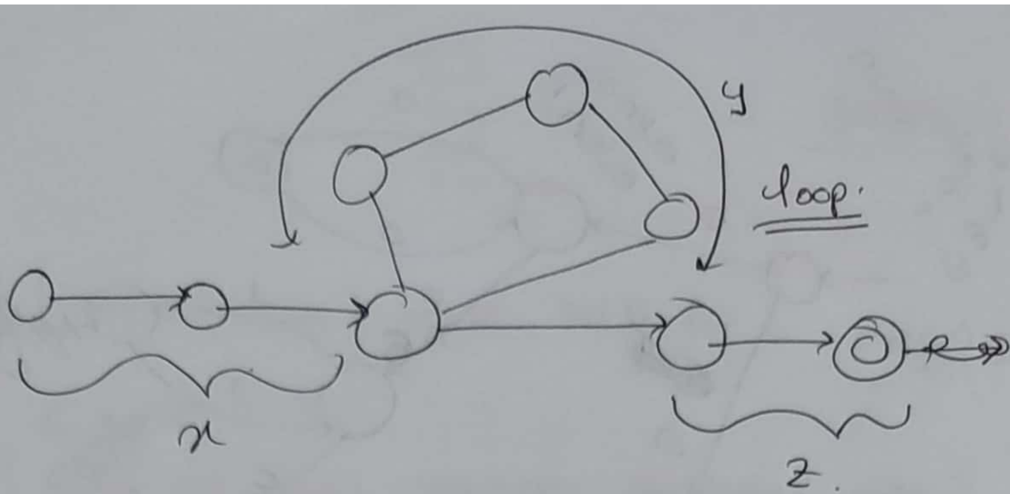
$\phi = \{ \}$



Pumping Lemma

↳ To show that a language is not regular.

↳ some loop in final state allows us to generate longer strings.



$w = xyz$

$w = xy^i z \in L$ $i \geq 0$.

$\therefore$ $P = 6$ or more.

if L is a Regular language, & w is sufficiently long enough string

i.e. $|w| \geq P$ ← the pumping length

length of string

Depends on the languge any specific FSM

⇓ that means.

long enough that FSM will have cycle

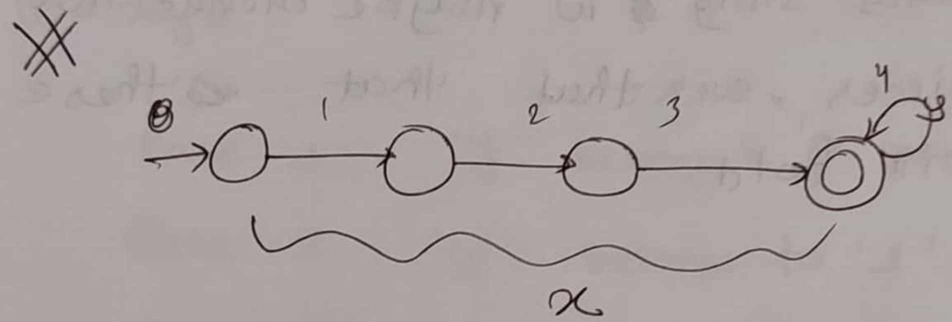
$w = xyz$

$= xy^i z \in L$ for $(i \geq 0)$

if $|y| > 0 \rightarrow$ cycle has atleast one edge in it. (Path)

~~$|xz|$~~ $|xy| \leq P$

↳ hit the cycle before length of string gets very large.



$z = \epsilon$

$|xy| \leq P$

$\therefore P = 4.$

Pumping length is a property of a languge not of any specific FSM.

if any of these conditions are not met, then it is not a Regular language.

Def of Pumping Lemma

If the L is a regular language, then L has a pumping length P such that any string w maybe divided into three pieces, such that that these conditions will hold.

Condition 1 :- $xy^iz \in L$ $\quad i \geq 0$

Condition 2 :- $|y| > 0$

Condition 3 :- $|xy| \leq P$.

CamScanner

$L = \{0^n 1^n \mid n \geq 0\}$ L is not a Regular language

* Assume L is Regular

* L have a pumping Length P.

If pumping length available,

$w = 0^p 1^p$

$(000)^2$

$w = xyz$.

Case : 1 'y' belongs to '0' part of the string

Case : 2 'y' belongs to '1' part of the string

Case : 3 'y' belongs to '0' and '1' part of the string

assumption

$i = 2$

xy^iz.

if $P = 7$. → assumption

$w = 0000000\,1111111$

y or y or y

$\therefore xy^2z \notin L$.

case:1. ⇒ $y = 000$

$00\ \underbrace{000000}_{y^2}\ 00 1111 1$ 11

$= 0^{10}1^7 \notin L$.

Case:2 ⇒ $y = \cancel{001}$ 11

0000000 ~~111111111~~1

111111111

$0^7 1^9 \notin L$.

$L = \{ww \mid w \in \{0,1\}^*\}$

Show that it is not regular

assume string $= 0^p 1$

Language $= 0^p 1\, 0^p$ (p is the pumping length of the string).

$= \underbrace{0000}_{x}\ \underbrace{0010}_{y}\ \underbrace{00001}_{z}$ assume $p=6$.

$= 0^6 1\, 0^6 \underline{1}$.

$x = 0000 \quad y = 0010 \quad z = 000001$

$xy^iz \Rightarrow xy^2z = 0000\ 0010\ 0010\ 000001$

$= 0^6 1\, 0^3 1\, 0^6 1 \notin L.$

i) $L = \{1^{p} \mid p \text{ is prime}\}$ is it RL?

ii) $L_2 = \{ww^R \mid w^R \text{ is the reverse of } w\}$ " ?

i) $L = \{1, 111, 11111, 1111111 \dots\}$.

string $= 1^p$. assume $p = 7$.

$$\underbrace{11}_{x}\underbrace{111}_{y}\underbrace{11}_{z}$$ P(1)

$= 1^2 1^3 1^2$

~~x=0000~~ $x = 11$ $y = 111$ $z = 11$

$xy^iz \Rightarrow xy^2z = 1111111111$

$= 1^{10} \notin L$

different way

$i = p+1$

$|xy^{p+1}z| = |xyz| + |y|p$

$= p + p(y)$

$= p(1+|y|)$ shudhu 1 ashle Prime hoito 1 na, so not RL.

Assignments

Book: 121 pg.

Ex → 4.1.1

c, e, f

→ 4.1.2

g, h

$L = \{0^n 1^n \mid n < 10\}$

Regular Expression

$+ \longrightarrow$ means union $(\cup)$

i) $L_1 + L_2 = L_2 + L_1 \rightarrow$ commutative.

ii) $L_1 + (L_2 + L_3) = (L_1 + L_2) + L_3 \rightarrow$ associative.

iii) $L_1(L_2 + L_3) = L_1L_2 + L_1L_3 \rightarrow$ distributive.

concatenation

new language

$w = xy$

$x \in L_1$ and $y \in (L_2 + L_3)$

$= y \in L_2$ or L_3

$w \in (L_1L_2 + L_1L_3)$

$w \in L_1L_2$ or $w \in L_1L_3$

$x \quad y \qquad\qquad x \quad y.$

1) $\phi + R = R$

2) $\phi R = \phi$

3) $\epsilon R = R$

4) $\epsilon^* = \epsilon$

$\{\epsilon, \epsilon^2, \epsilon^3, \ldots\}$

$= \{\epsilon, \epsilon, \epsilon, \epsilon \ldots\}$

$= \{\epsilon\}.$

5) $\phi^* = \epsilon$

6) $R+R = R.$

7) $R^* R^* = R^*$

$\cancel{(RR)^*}$

$\{\epsilon, R, R^2, \ldots\} \{\epsilon, R, R^2 \ldots\}$

$= \{\epsilon, R, R^2 \ldots\}$

⑧ $RR^* = R^*R$

$\cancel{\#}(R^*)^* = R^*$ $\qquad (((R^*)^*)^*)^* = R^*$

$\{\epsilon, R, R^2, R^3 \ldots\}^*$

⑨ $\epsilon + RR^* = R^*$

$R\{\epsilon, R, R^2, R^3, \ldots\}$.

$= \{R, R^2, R^3, R^4 \ldots\} \cup \{\epsilon\}$

$= \{\epsilon, R, R^2, R^3 \ldots\}$.

$= R^*$

(10) $R^*R + \epsilon = R^*$

11) $(PQ)^*P = P(QP)^*$

12) $(P+Q)^* = (P^*Q^*)^*$

$= (P^* + Q^*)^*$

Prove that,

$$(1+00^*1)+(1+00^*1)(0+10^*1)^*(0+10^*1) \text{ is}$$

equal to $0^*1(0+10^*1)^*$

$$= (1+00^*1)\left[\epsilon + (0+10^*1)^*(0+10^*1)\right]$$

R R

$$= (1+00^*1)(0+10^*1)^*$$

We knows

$$\epsilon + R^*R = R^*$$

$$= (\epsilon+00^*)1\ (0+10^*1)^*$$

$$= 0^*1(0+10^*1)^*$$

proved.

$0.(\epsilon+11)^*$

12/09/22

$((((R^*))^*))^*$ $\quad R^+ = R^+$

$R^* R^+ = R^+$

1. $L = \{ w c w^R \mid c \in \Sigma^* \}$

2. $L = \{ w c w^R \mid c, w \in \Sigma^* \}$

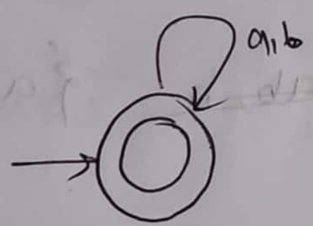
w are c er jomno eki alphabet.

w can be empty

$w = abb$
$w^R = bba$

abb ...c... bba

$\varepsilon = w \longleftarrow c \longrightarrow R\varepsilon$



evenif w & w^R empty then only c will be valid language

using pumping lemma

$x = w = \varepsilon$ $\quad y = c = abab \ldots$ $\quad z = w^R = \varepsilon$

anything Σ^*

$L = \{$ accepting string exactly length of $2\}$

$\Sigma = \{a, b\}$.

aa, ab, ba, bb.

Language to RE (Regular Expression).

$$\text{a or b} \leftarrow (a+b)(a+b) \rightarrow \text{a or b}$$

So, $RE(L) = (a+b)(a+b)$.

$L = \{$Length is greater $2\}$.

~~{aa, ab, b~~ {aaa . . . . . . }.

$(a+b)(a+b)(a+b)^+$

Exactly 2 or more

$\hookrightarrow (a+b)(a+b)(a+b)^*$

$L = \{$accepting length of string at most $2\}$.

$\Sigma = \{a, b\}$

- length of string can be 0, 1, 2.

~~$\varepsilon + (a+b)$~~ ~~$\varepsilon + ab$~~

$(\varepsilon + a + b)(\varepsilon + a + b)$

OR

$\varepsilon + (a+b) + (a+b)(a+b)$.

$L = \{$started with $ab\}$

$\Rightarrow (a)(b)(a+b)^*$

$\Rightarrow (ab)(a+b)^*$

$L =$ ends with ab.

$(a+b)^* ab$.

$L = \{$string contains $ab\}$

$(a+b)^* (aba)(a+b)^*$

CamScanner

$L = \{$ starting with 'a' ending with 'a' $\}$

$(a)(a+b)^*(a)$

$RE(L) = \{a(a+b)^* a\}$ → doesn't accept a

$\boxed{a + a(a+b)^* a}$

also accepts just a.

$L = \{$ The no. of a exactly 3 $\}$

~~$(aaa)(b)^*$~~

~~ababa~~

$RE(L) = \{(b)^* a (b)^* a (b)^* a (b)^*\}$

L { start and ends with different symbol}.

a —— b

~~(a+b)(a+b)* (~~

RE(L: ~~R(L)~~ = $\{a(a+b)^* b\} + \{b(a+b)^* a\}$.

L = { The length of string is greater than 3 or equal to }.

$(a+b)(a+b)(a+b)(a+b)^+$

$(a+b)^3 (a+b)^*$

~~a:~~

L = { length of string is less than 3 or equal }.

$\varepsilon + (a+b) + (a+b)(a+b) + (a+b)^3$

$= (\varepsilon + a + b)^3$

L = { String's 3rd symbol from Right is b.

$RE(L) = (a+b)^* \, b \, (a+b) \, (a+b)$

L = { |w| ≡ 0 mod 2 } → even.

aa, ab, ba, bb, aaaa

$\varepsilon + (a+b)(a+b) + (a+b)^4 + (a+b)^6 \ldots$

$\{ (a+b)^2 \}^*$

$L = \{$ no. of a is congruence to $\equiv 1 \bmod 3\}$.

OR

$|w|_a \equiv 1 \bmod 3$

~~3, 6, 9~~

order specified

$a\, b^* (aaa)^*$

1, 4, 7, 10 . . .

$RE(L) = b^* a b^* (b^* a b^* a b^* a b^*)^*$

2

14/09/22.

Conversion from FA to RE

ARDEN'S METHOD

If P & Q are two REs over Σ, and if P doesn't contain ϵ then equation in R given by $R = Q + RP$ has unique solⁿ, i.e., $R = QP^*$

Given Equation : $R = Q + RP$

$= Q + QP^*P$

$R = QP^*$ $\qquad = Q(\epsilon + P^*P)$ $\left[\because \epsilon + P^*P = P^*\right]$

$= QP^*$

Another way,

Given,

$R = Q + RP$

$R = Q + (Q + RP)P$

$= Q + QP + RP^2$

$= Q + QP + (Q + RP)P^2$

$= Q + QP + QP^2 + RP^3$

$= Q + QP + QP^2 + \cancel{Q} \dots QP^n + RP^{n+1}$

$= Q + QP + QP^2 + \dots QP^n + QP^*P^{n+1}$

$\boxed{R = QP^*}$

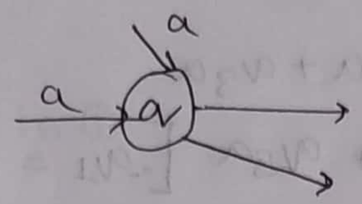
$= Q(\epsilon + P + P^2 + P^3 + \dots P^n + P^*P^{n+1})$

$= QP^*$

RE $\longrightarrow$ FA (Arden's)

Ex-1

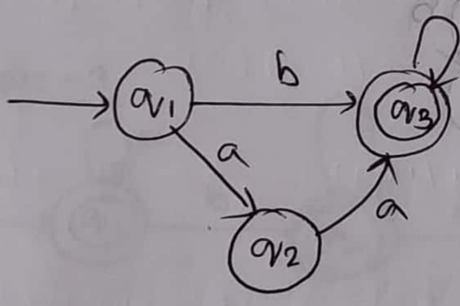
1) Write Equation for each state based on incoming edge



2) Simplify the equation using Arden's Method.

For final state

conditions:
- i) there should be no ε-transition
- ii) only one initial state.



$q_1 = \varepsilon \longrightarrow (1)$

~~$q_2 = a$~~

$q_2 = q_1 a \longrightarrow (2)$

$q_3 = q_1 b + q_2 a + q_3 a \longrightarrow (III)$

Put ① & ② in Equation ③

$$q_3 = \varepsilon . b + q_1 aa + q_3 a$$

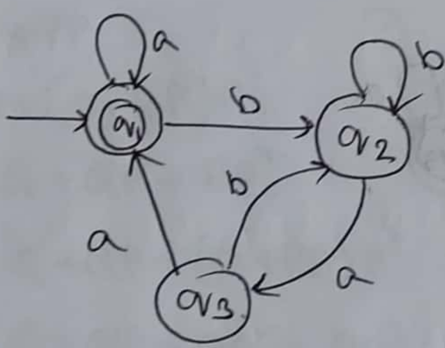
$$= b + \varepsilon . aa + q_3 a \quad [\because q_1 = \varepsilon]$$

$$\underbrace{q_3}_{R} = \underbrace{(b + aa)}_{Q} + \underbrace{q_3}_{R}\underbrace{a}_{P}$$

$$[R = Q + RP]$$

$$q_3 = (b+aa)a^* \qquad R = QP^*$$

Example.



Eqn:

$$q_1 = \varepsilon + q_1 a + q_3 a \longrightarrow ①$$

$$q_2 = q_1 b + q_3 b + q_2 b \longrightarrow ②$$

$$q_3 = q_2 a \longrightarrow ③$$

$$q_1 = \varepsilon + q_1 a + q_3 a$$

$$q_2 = q_1 b + q_3 b + q_2 b \quad = q_1 b(b+ab)^* \longrightarrow ④$$

$$= q_1 b + q_2 ab + q_2 b$$

$$= \underbrace{q_1 b}_{Q} + \underbrace{q_2}_{R}(\underbrace{ab + b}_{P})$$

$$[R = Q + RP]$$
$$R = QP^*$$

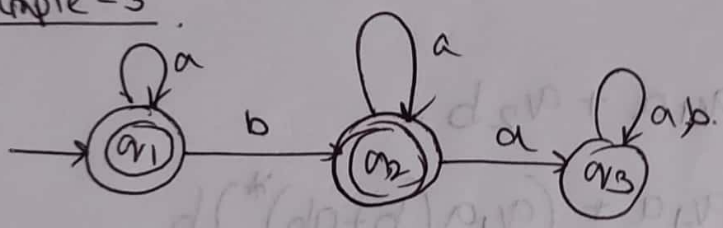
Put ~~4~~ ~~in~~ (IV) in (III)

$$q_3 = q_1 b(b+ab)^* a \longrightarrow (V)$$

Put (V) in (I)

$$q_1 = \varepsilon + q_1 a + q_3 a \longrightarrow$$
$$= \varepsilon + q_1 a + q_1 b(b+ab)^* aa$$
$$= \underbrace{\varepsilon}_{Q} + \underbrace{q_1}_{R}(\underbrace{a + b(b+ab)^* aa}_{P})$$
$$= \varepsilon \cdot (a+b(b+ab)^* aa)^*$$

Example-3



Eqn:

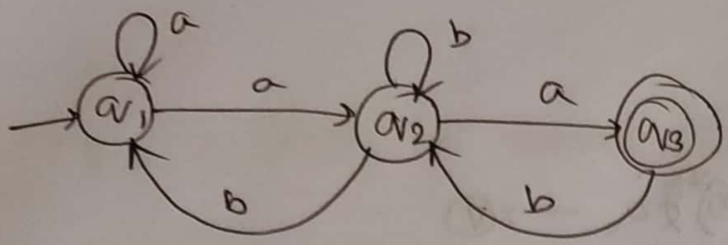
$$q_1 = \varepsilon + q_1 a$$
$$q_2 = q_1 b + q_2 a$$
$$q_3 = q_2 a + q_3 a + q_3 b.$$

~~$q_1 = q_1 a$~~

$$q_1 = QP^*$$
$$= \varepsilon a^*$$
$$= a^*$$

$$q_2 = \underbrace{a^* b}_{Q} + \underbrace{q_2}_{R} \underbrace{a}_{P}$$
$$q_2 = a^* b a^*$$

$$R = q_1 + q_2$$
$$= a^* + a^* b a^* \Rightarrow \text{Union of both.}$$



$$q_3 = \left(a + a(b+ab)^* b\right)^* a \, (b+ab)^* a.$$

$$q_1 = \varepsilon + q_1 a + q_2 b \longrightarrow (\mathrm{I})$$

$$q_2 = q_1 a + q_2 b + q_3 b \longrightarrow (\mathrm{II})$$

$$q_3 = q_2 a \longrightarrow (\mathrm{III})$$

$$q_2 = q_1 a + q_2 b + q_2 ab$$

$$q_2 = \underbrace{q_1 a}_{Q} + \underbrace{q_2}_{R} \underbrace{(b+ab)}_{P}$$

$$q_2 = q_1 a (b+ab)^* \longrightarrow (\mathrm{IV})$$

$$q_1 = \varepsilon + q_1 a + q_2 b$$

$$= \varepsilon + q_1 a + \left(q_1 a (b+ab)^*\right) b$$

$$= \varepsilon + q_1 a + \left(q_1 a (b+ab)^*\right) b.$$

$$= \underbrace{\varepsilon}_{Q} + \underbrace{q_1}_{R} \underbrace{\{a + a(b+ab)^* b\}}_{P}$$

$$= \varepsilon . \{a + a(b+ab)^* b\}^*$$