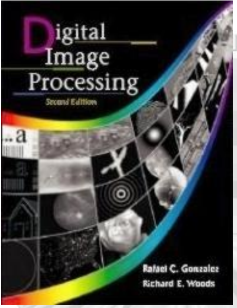

Chapter 2
Digital Image Fundamentals

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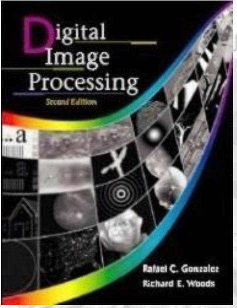
Light

- **Light** is a particular type of *electromagnetic radiation* that can be sensed by eye.
- *Visible band of EM spectrum* spans from approx. 0.43 μm(violet) to about 0.70 μm (red).
- The colors of an object are determined by the nature of the light reflected
  - A body that reflects light relatively balanced in all visible wavelengths appears white to the observer.
  - Whichever favors reflectance in a limited range of the visible spectrum exhibits some shades of color (**Chromatic light**).
- Light that is void of color is called **monochromatic (achromatic)** light.
  - The only attribute of monochromatic light is its **intensity** or amount.
  - Achromatic images known as ***gray scale image***.
- **Radiance** is the total amount of energy that flows from the light source, and it is usually measured in watts (W).
- **Luminance**, measured in lumens (lm), gives a measure of the amount of energy an observer perceives from a light source.
- **Brightness** is a subjective descriptor of light perception that is practically impossible to measure.
  - It embodies the achromatic notion of intensity



Electromagnetic Spectrum

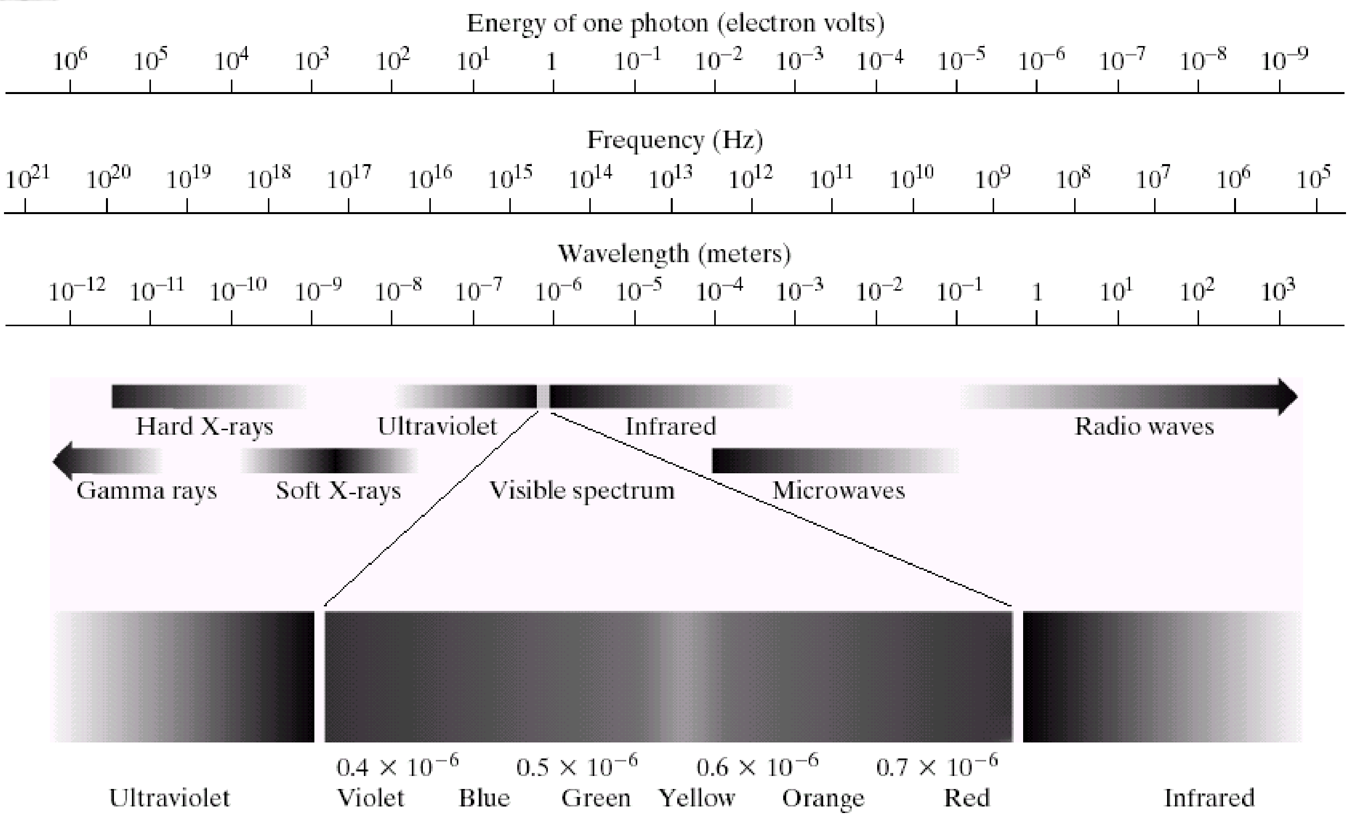
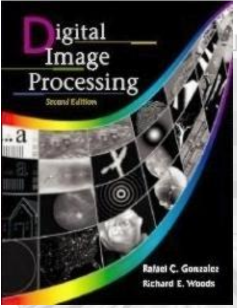
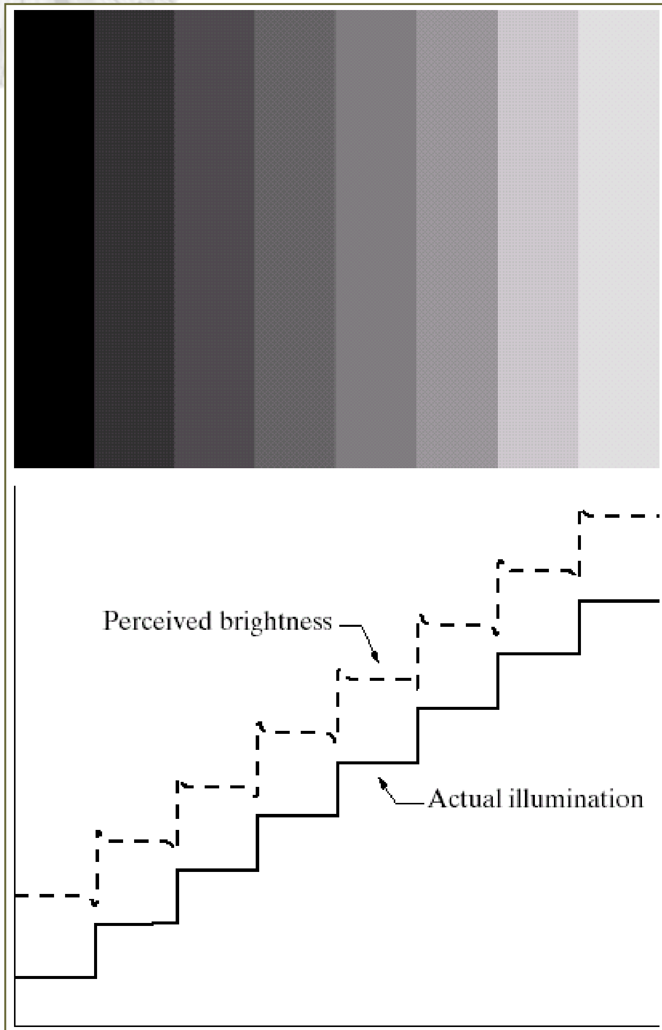


FIGURE 2.10 The electromagnetic spectrum. The visible spectrum is shown zoomed to facilitate explanation, but note that the visible spectrum is a rather narrow portion of the EM spectrum.



Brightness vs. Function of intensity



a
b

FIGURE 2.7 (a) An example showing that perceived brightness is not a simple function of intensity. The relative vertical positions between the two profiles in (b) have no special significance; they were chosen for clarity.

✓Visual system tends to undershoot or overshoot around the boundary of regions of different intensities.

Simultaneous Contrast

a b c

FIGURE 2.8 Examples of simultaneous contrast. All the inner squares have the same intensity, but they appear progressively darker as the background becomes lighter.

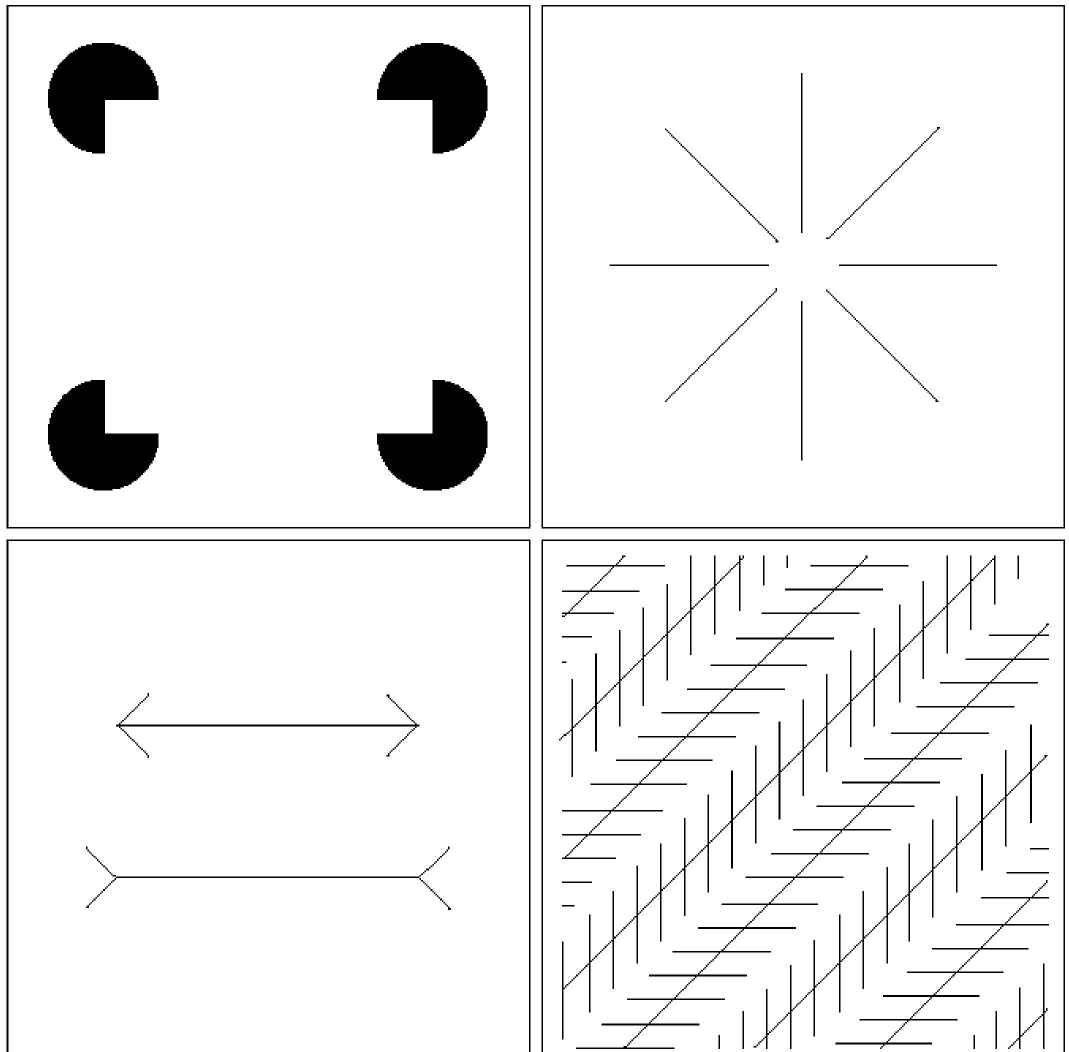
✓ All the small squares have exactly the same intensity, but they appear to the eye progressively darker as the background becomes brighter.

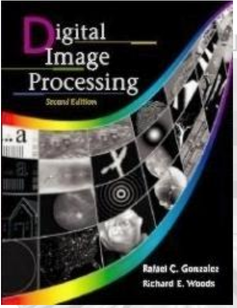
✓ Region’s perceived brightness does not depend simply on its intensity.

Optical Illusions

a b
c d

FIGURE 2.9 Some well-known optical illusions.





Signal

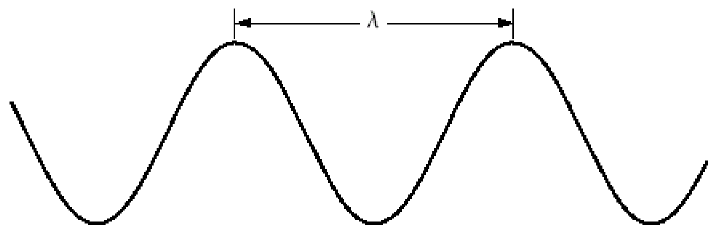


FIGURE 2.11 Graphical representation of one wavelength.

- ✓ A signal is a function that carries information.
- ✓ Usually content of the signal changes over some set of spatiotemporal dimensions.
- ✓ Time-Varying Signal: ***f(t)***

  Ex: Audio signal

Spatially-Varying Signal

- Signals can vary over space as well.
- An image can be thought of as being a function of 2 spatial dimensions:

$$f(x,y)$$

- For monochromatic images, the value of the function is the amount of light at that point.
- Medical CAT and MRI scanners produce images that are functions of 3 spatial dimensions:

$$f(x,y,z)$$

- **Spatio-temporal Signals:**

$$f(x,y,t)$$

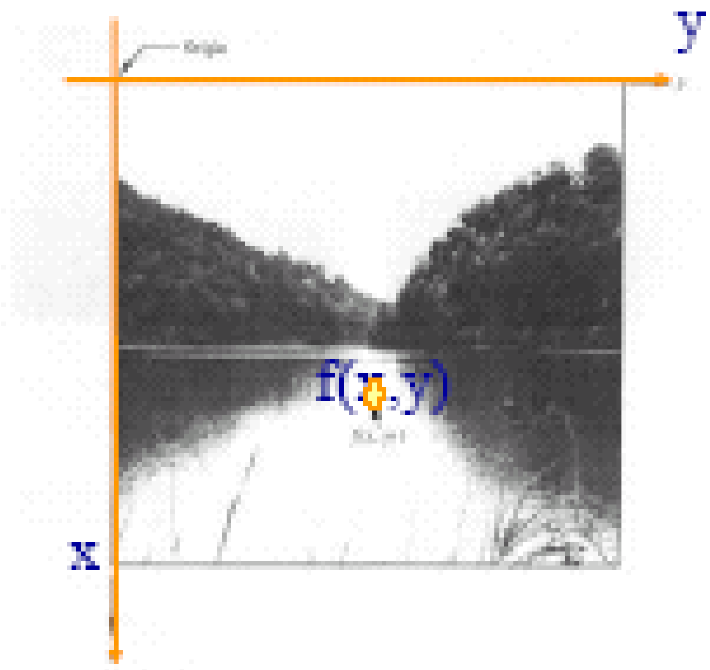
Example: a video signal

Analog and Digital Signal

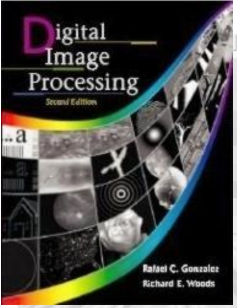
- Most naturally-occurring signals also have a real-valued range in which values occur with infinite precision.
- To store and manipulate signals by computer we need to store these numbers with finite precision.
  - These signals have a discrete range.

signal has continuous domain and range = analog
signal has discrete domain and range = digital

Digital Image Representation



- A digital image is an image ***f(x,y)*** that has been digitized both in spatial coordinates and brightness.
- The value of ***f*** at any point ***(x,y)*** is proportional to the brightness (or gray level) of the image at that point.



Coordinate Conversion used in this book

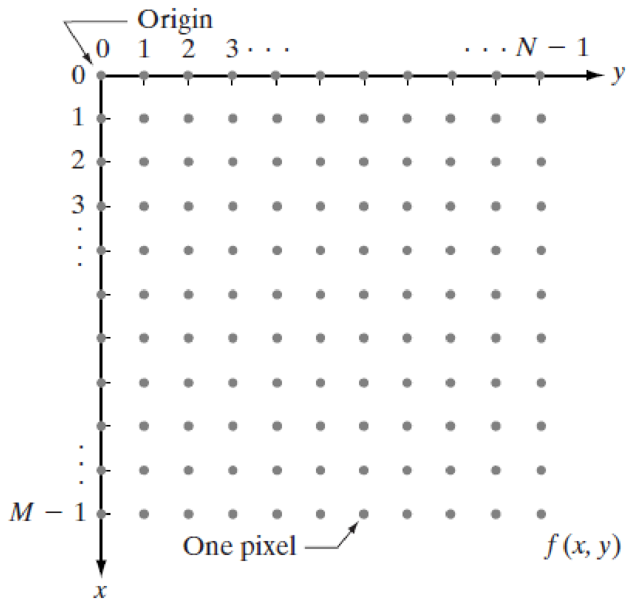


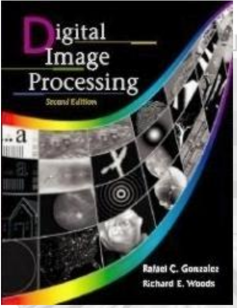
FIGURE: Coordinate convention to represent Digital Image

$$f(x, y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0, N-1) \\ f(1,0) & f(1,1) & \cdots & f(1, N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1, N-1) \end{bmatrix}.$$

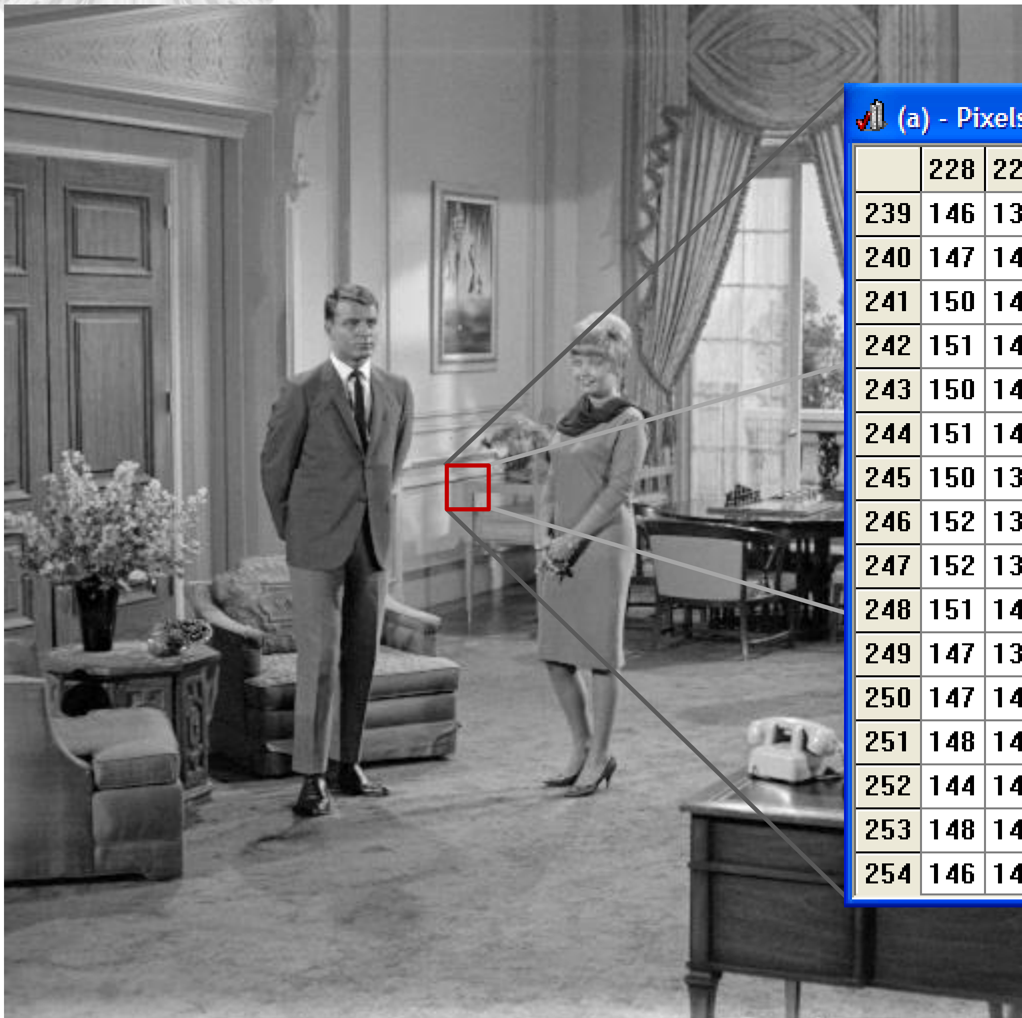
FIGURE: Matrix Representation

$$\mathbf{A} = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \vdots & & \vdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}.$$

FIGURE: Elementwise Representation

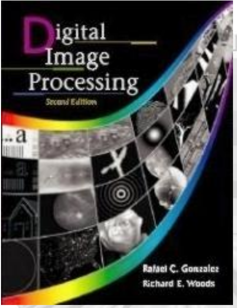


Digital Image Representation

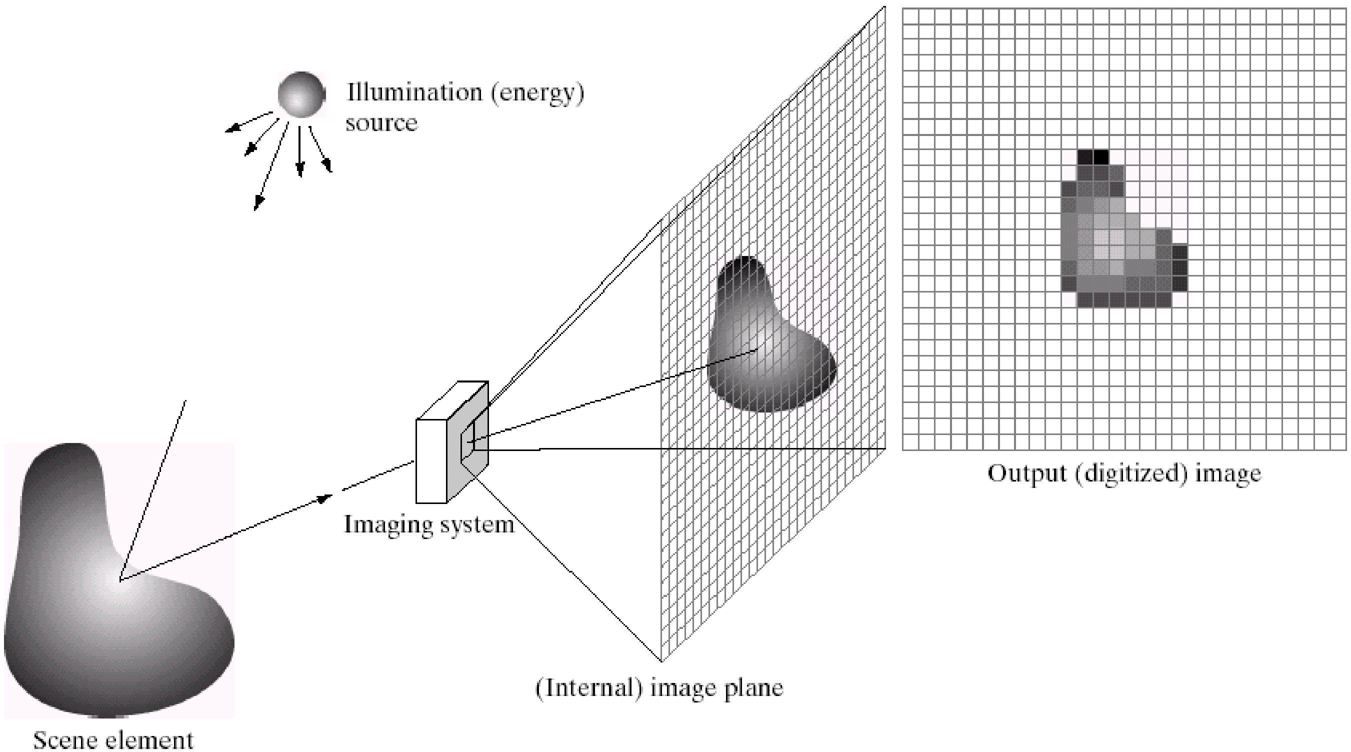


(a) - Pixels Value

	228	229	230	231	232	233	234	235	236	237	238	239	240	241
239	146	137	126	126	134	130	135	133	133	135	137	144	141	141
240	147	141	131	129	135	137	139	141	135	134	135	138	134	137
241	150	141	135	138	138	138	141	141	138	135	134	131	133	135
242	151	144	134	128	141	139	135	138	134	133	133	135	135	129
243	150	142	133	131	143	139	124	146	146	141	138	139	138	135
244	151	146	133	130	134	131	96	155	159	160	169	172	174	174
245	150	138	131	134	134	130	100	157	161	164	174	174	180	174
246	152	139	129	133	137	135	95	154	164	169	174	178	173	173
247	152	139	134	129	135	129	91	148	167	176	174	178	178	172
248	151	143	134	133	135	130	95	157	168	177	176	176	176	180
249	147	138	137	135	131	128	95	160	176	174	177	174	173	173
250	147	141	137	134	129	128	96	159	177	178	172	180	176	177
251	148	142	135	135	134	129	94	142	182	177	168	168	163	156
252	144	143	138	133	134	131	90	112	143	138	133	129	128	129
253	148	144	142	137	141	135	98	85	104	122	133	134	137	133
254	146	142	142	141	138	135	96	86	96	128	135	139	146	142

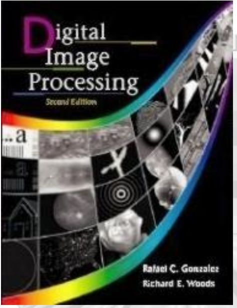


Digital Image Acquisition Process

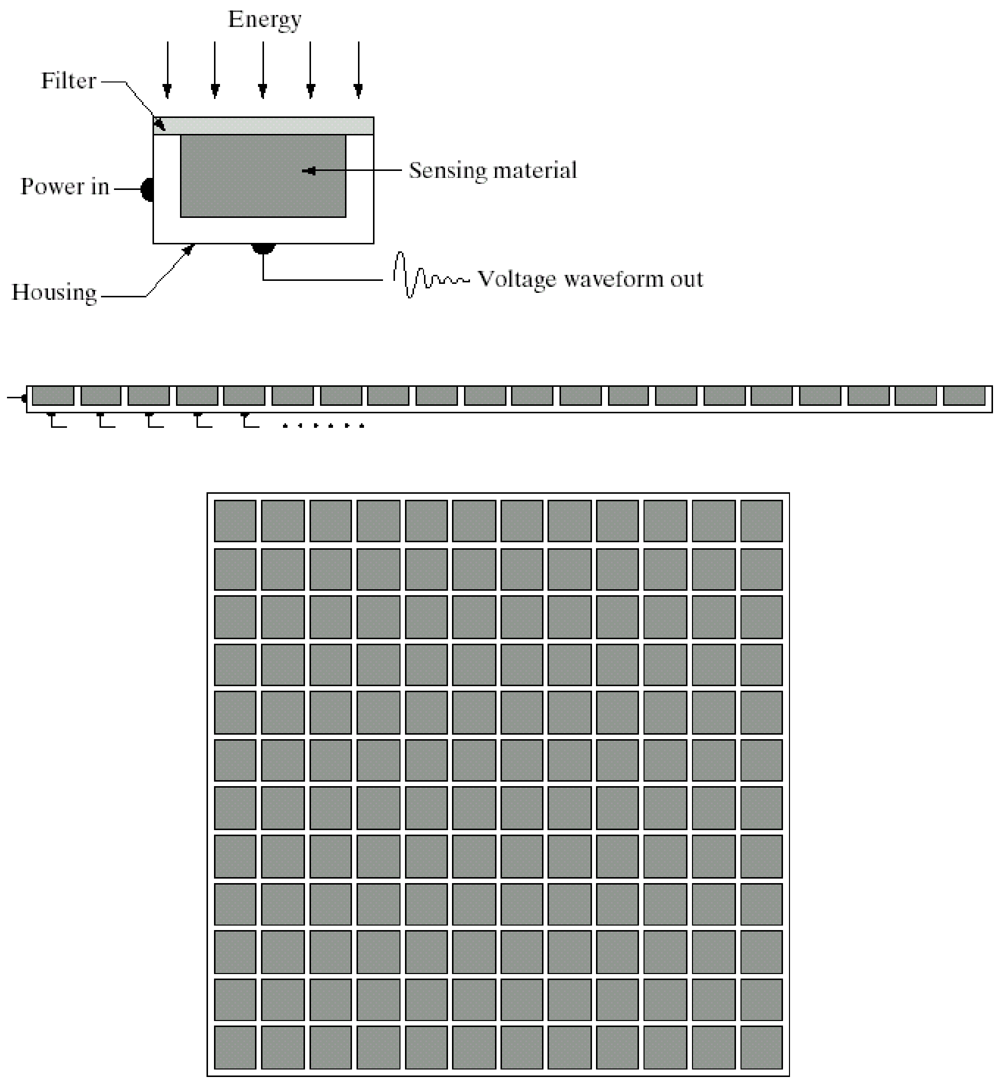


a b c d e

FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.



2D Imaging Sensor



a
b
c

FIGURE 2.12
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

Image Acquisition

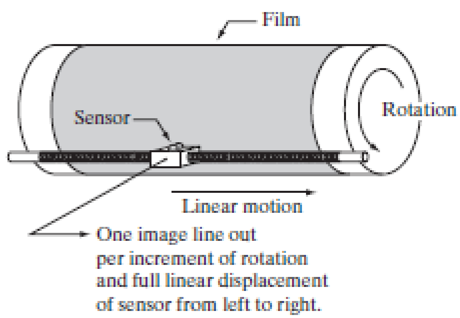
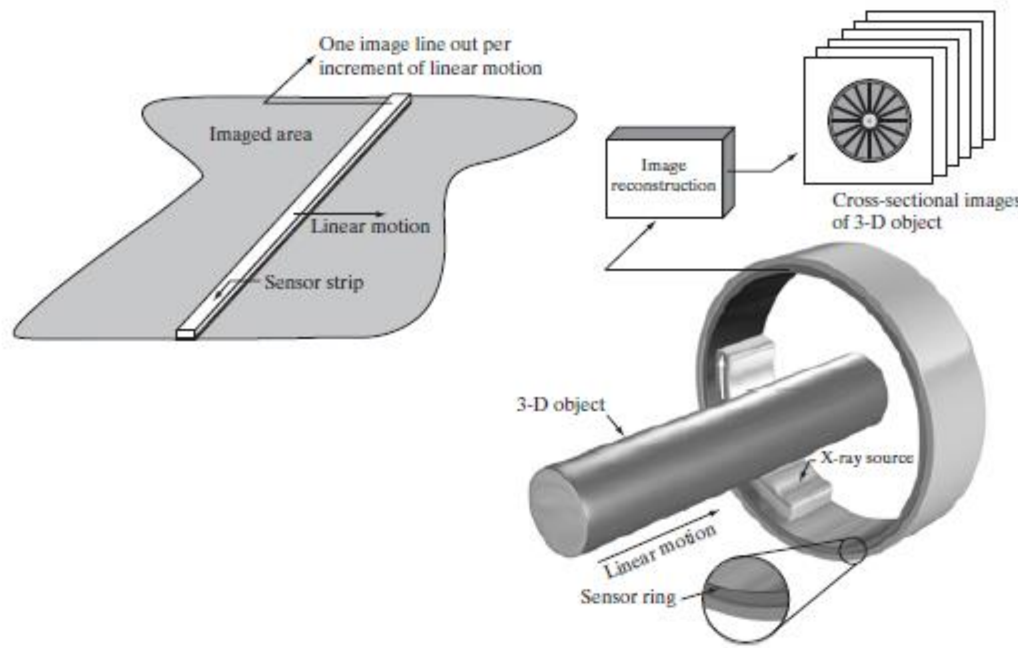


FIGURE 2.13 Combining a single sensor with motion to generate a 2-D image.



a b

FIGURE 2.14 (a) Image acquisition using a linear sensor strip. (b) Image acquisition using a circular sensor strip.

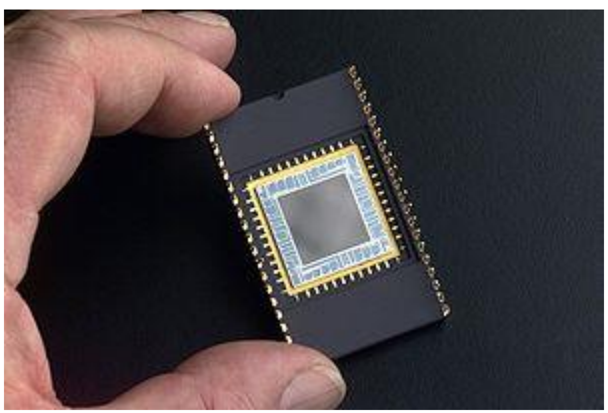
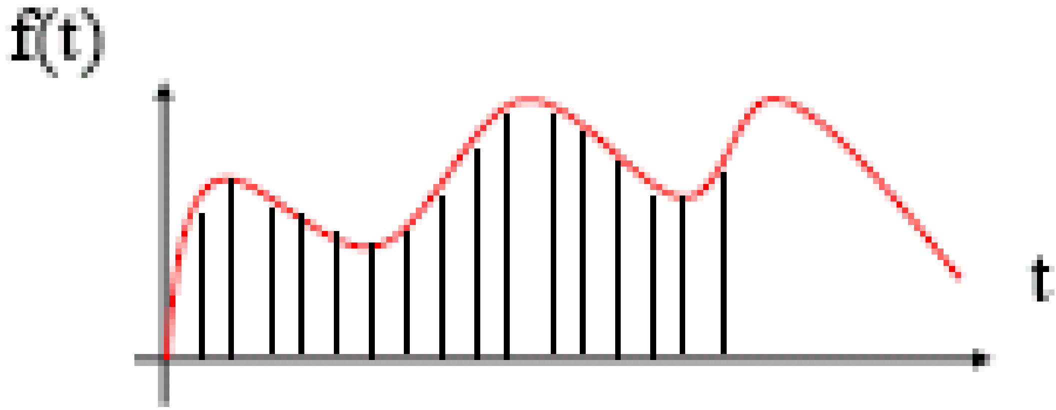


FIGURE: CCD

Sampling

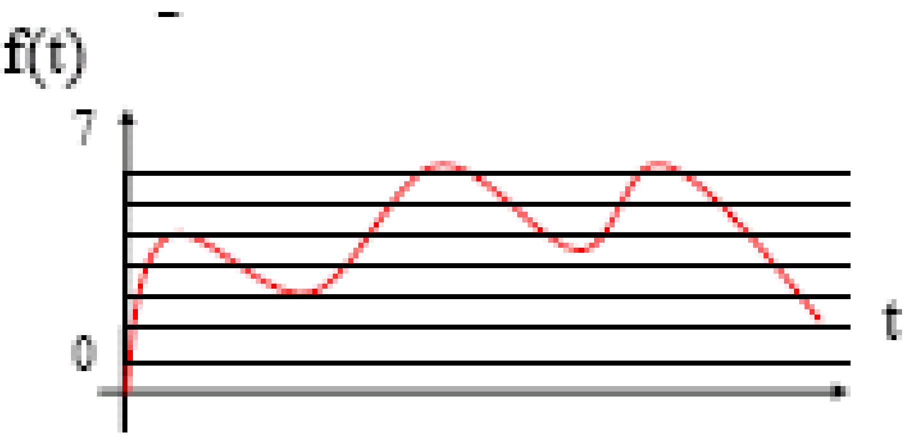
➢ sampling = the spacing of discrete values in the domain of a signal.

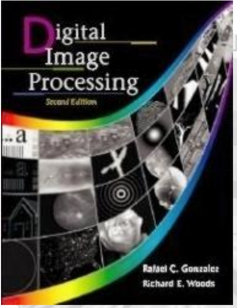
➢ sampling-rate = how many samples are taken per unit of each dimension. e.g.,samples per second, frames per second



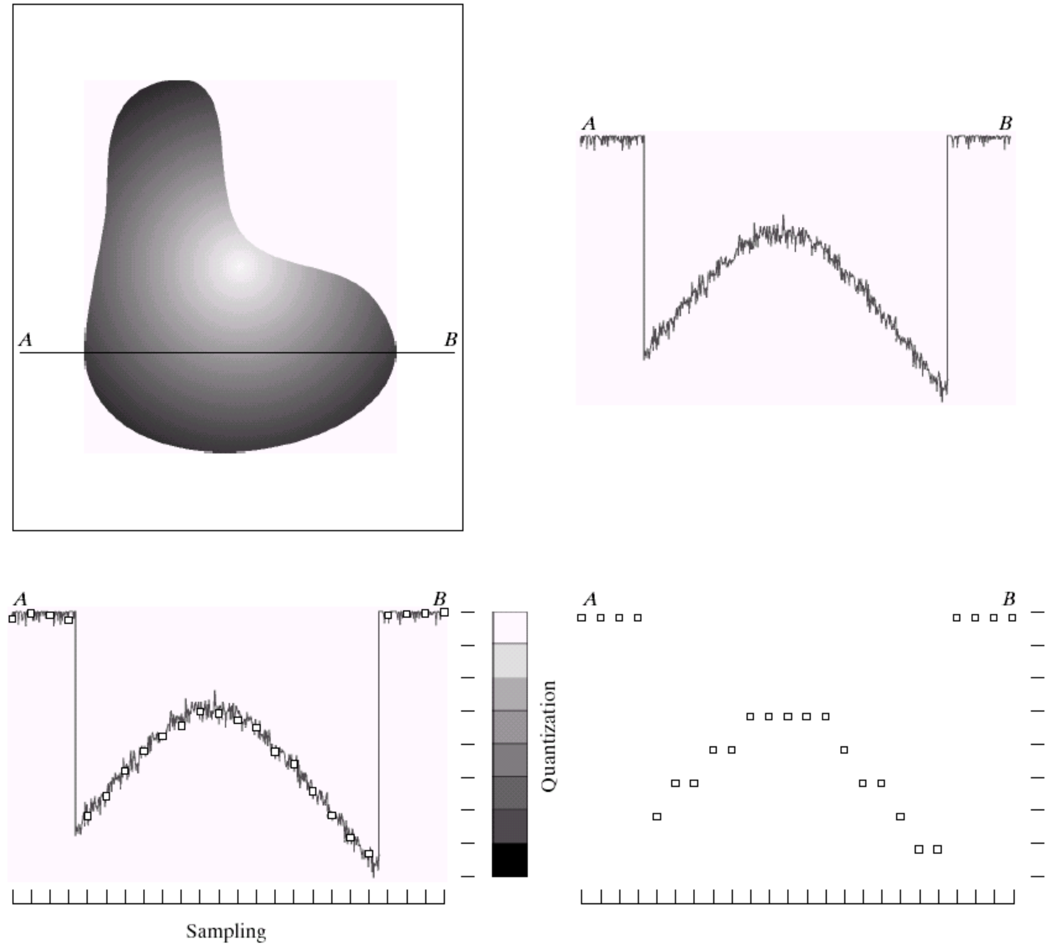
Quantization

- Quantization = spacing of discrete values in the range of a signal.
- Usually thought of as the number of bits per sample of the signal.
  e.g., 1, 8, 24 bit images, 16-bit audio.





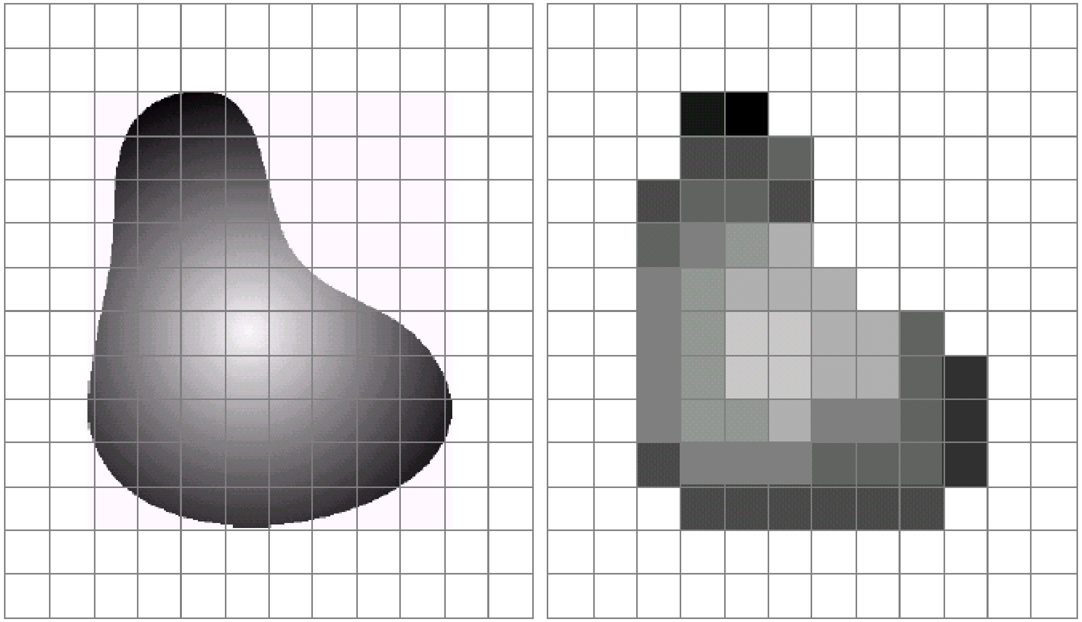
Sampling and Quantization



a b
c d

FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Example of Digital Image



a b

FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

Sampling Effect

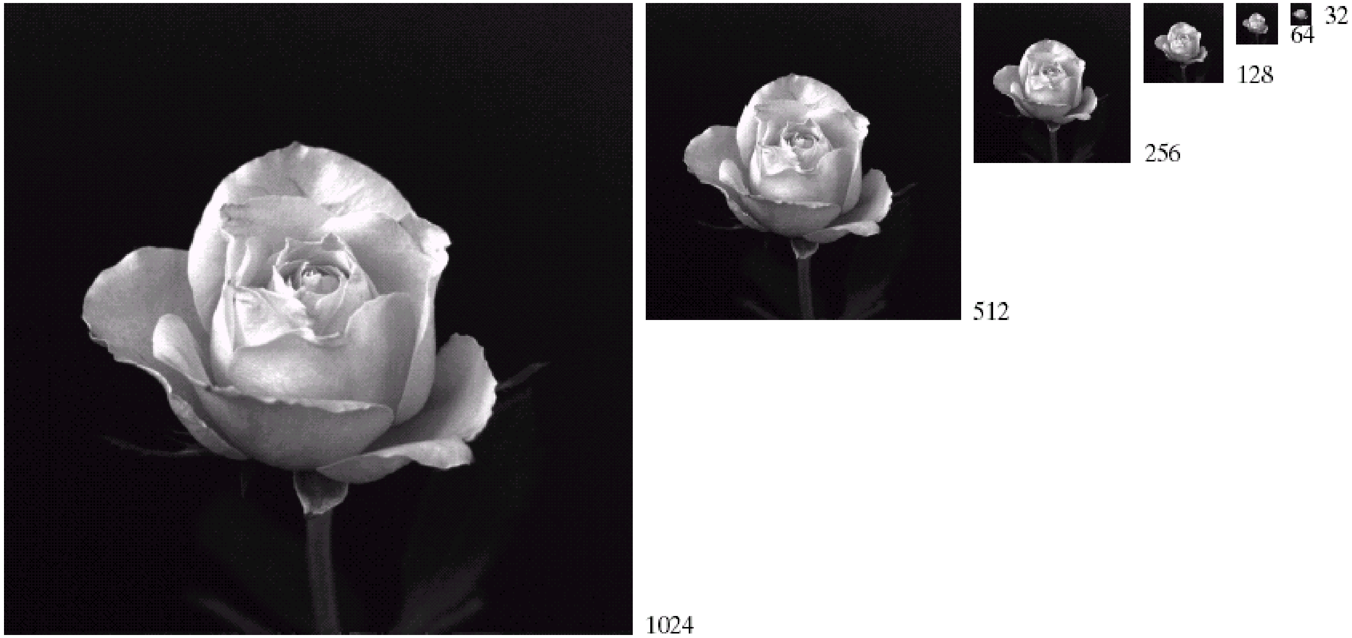
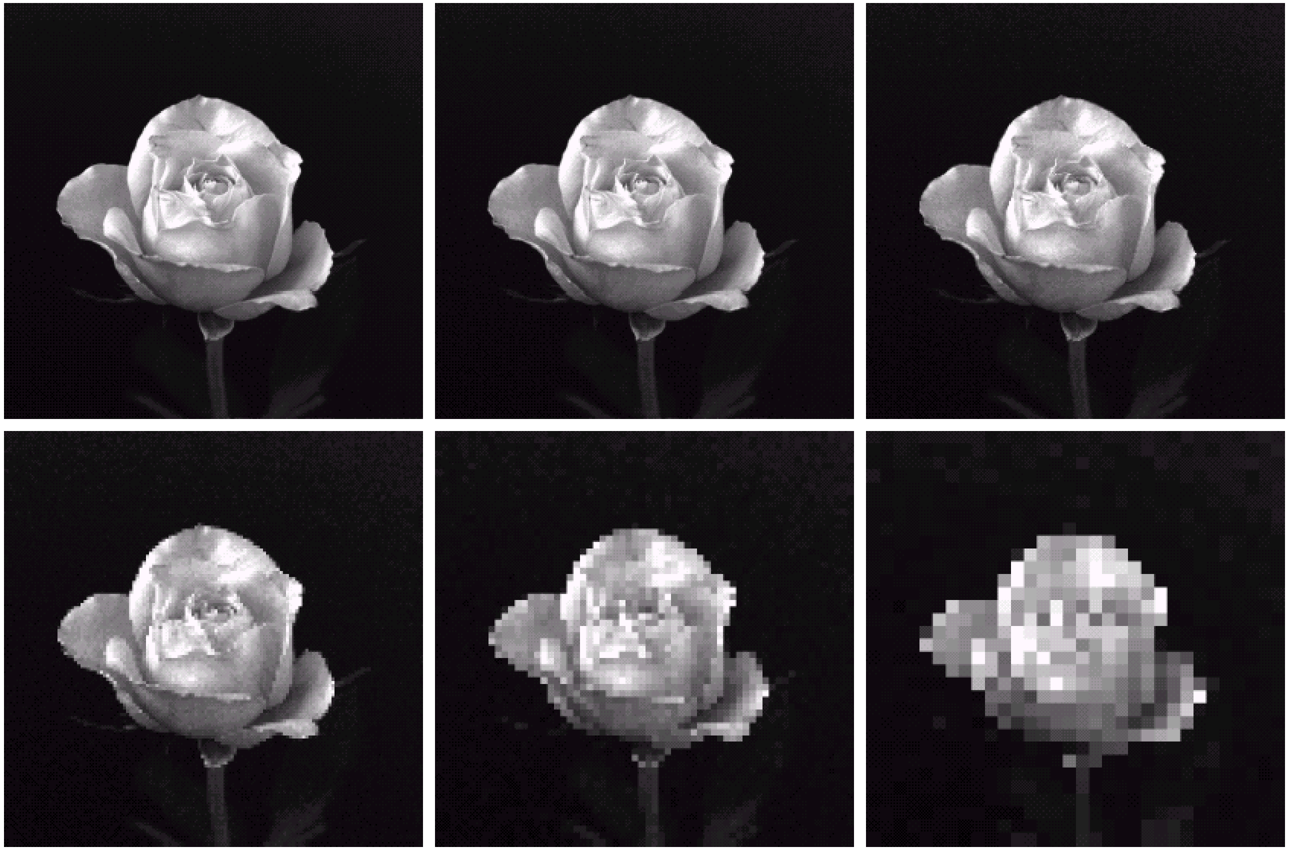


FIGURE 2.19 A 1024×1024, 8-bit image subsampled down to size 32×32 pixels. The number of allowable gray levels was kept at 256.

Sampling Effect



a	b	c
d	e	f

FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

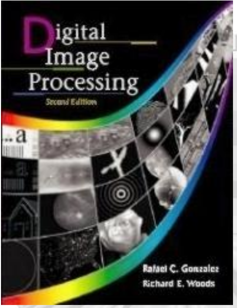
Chapter 2: Digital Image Fundamentals

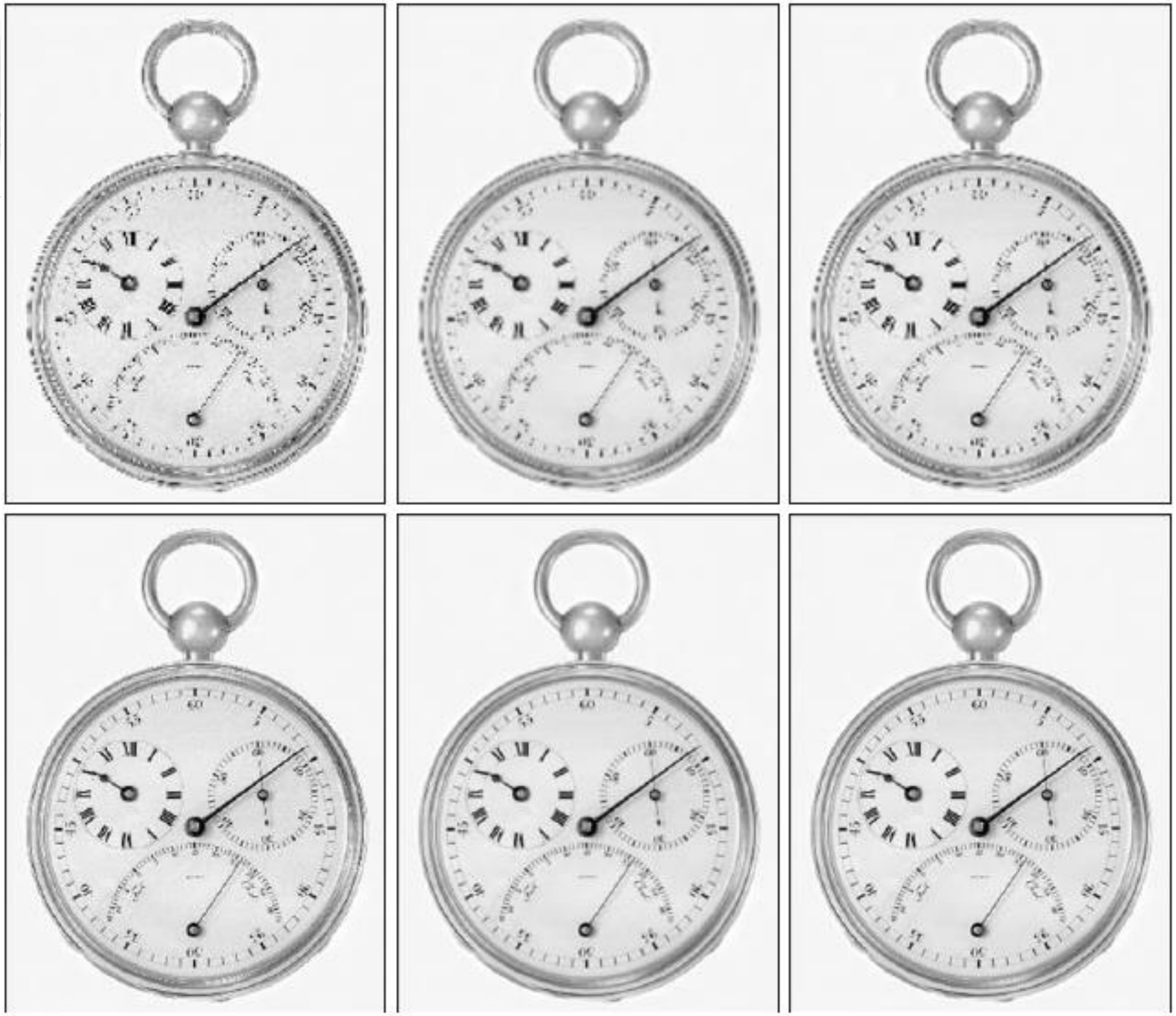
a b c
d e f

FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

Bilinear Interpolation: $v(x', y') = ax' + by' + cx'y' + d$

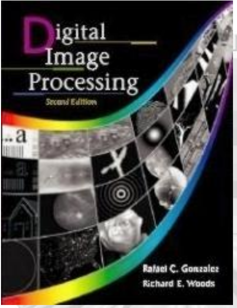
Bicubic Interpolation: $v(x, y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^i y^j$



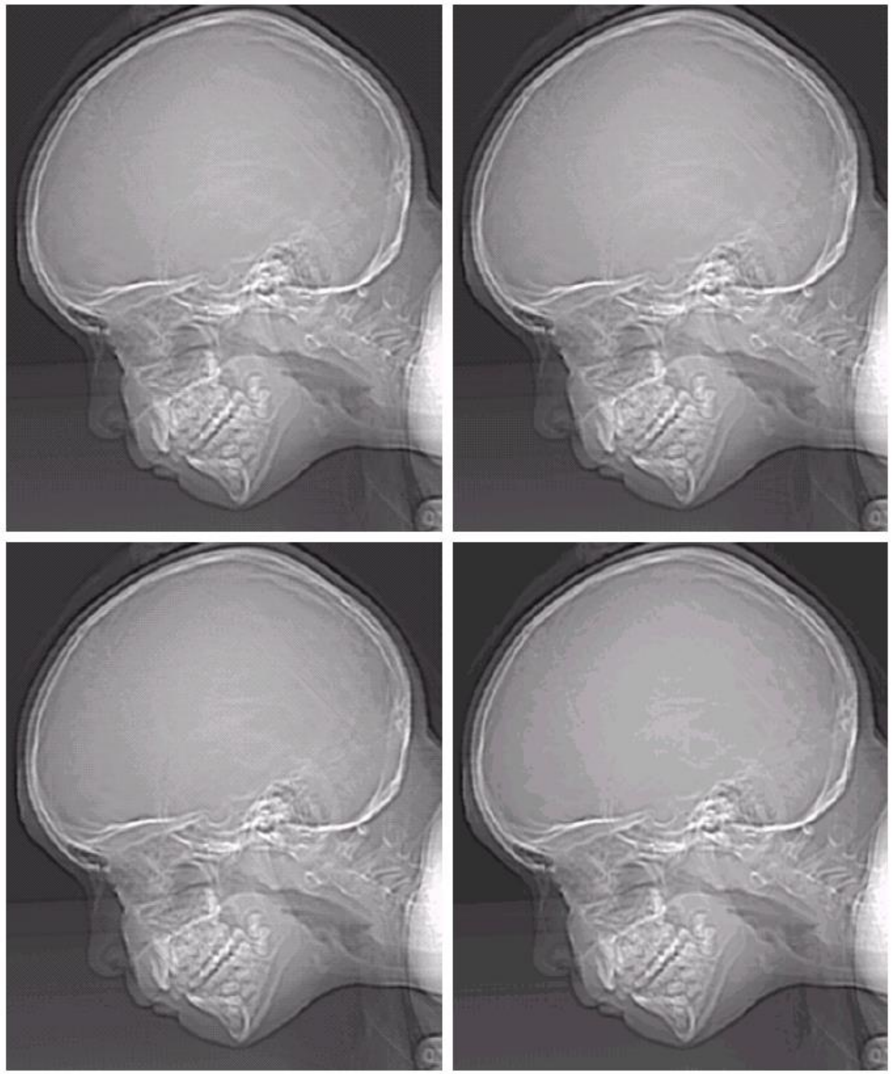


a	b	c
d	e	f

FIGURE 2.24 (a) Image reduced to 72 dpi and zoomed back to its original size (3692 × 2812 pixels) using nearest neighbor interpolation. This figure is the same as Fig. 2.20(d). (b) Image shrunk and zoomed using bilinear interpolation. (c) Same as (b) but using bicubic interpolation. (d)–(f) Same sequence, but shrinking down to 150 dpi instead of 72 dpi [Fig. 2.24(d) is the same as Fig. 2.20(c)]. Compare Figs. 2.24(e) and (f), especially the latter, with the original image in Fig. 2.20(a).



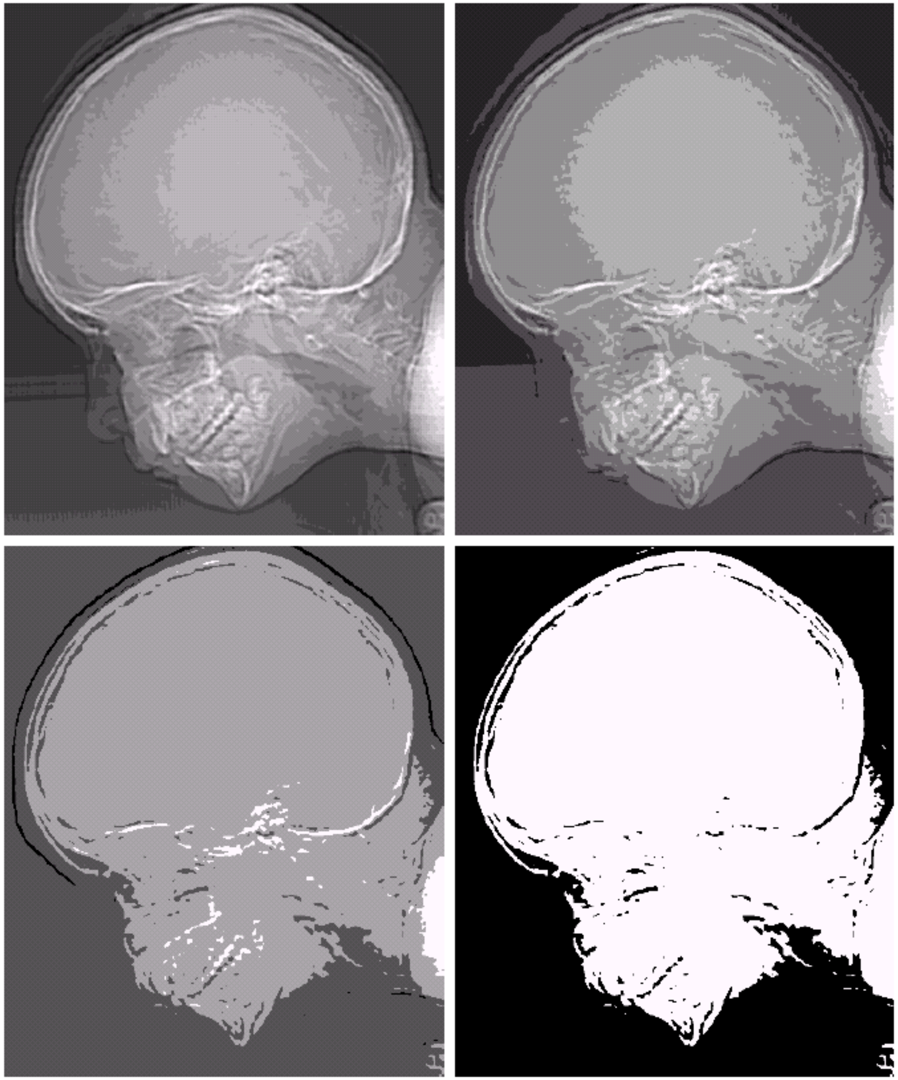
Quantization Effect



a b
c d

FIGURE 2.21 (a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

Quantization Effect



e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

- if the gray scale is not enough, the smooth area will be affected.
- False contouring can occur on the smooth area which has fine gray scales.

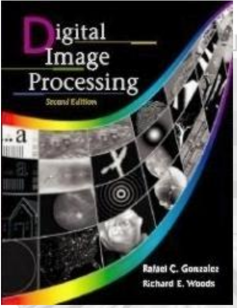


Image Formation Model

- Image refers to a 2D light-intensity function, ***f(x,y)***
- Amplitude of ***f*** at spatial coordinates ***(x,y)*** gives the intensity (brightness) of the image at that point.
- Light is a form of energy thus ***f(x,y)*** must be nonzero and finite.

$$0 < f(x,y) < \infty$$

Illumination and Reflectance

- The basic nature of ***f(x,y)*** may be characterized by 2 components:
  - ✓ the amount of source light incident on the scene being viewed
    - → Illumination, ***i(x,y)***
  - ✓ the amount of light reflected by the objects in the scene
    - → Reflectance, ***r(x,y)***

Illumination and Reflectance

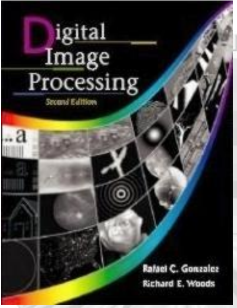
$$f(x,y) = i(x,y)r(x,y)$$

- $i(x,y)$:
  - ✓ determined by the nature of the light source
  - ✓ bounded by

$$0 < i(x,y) < \infty$$

- $r(x,y)$:
  - ✓ determined by the nature of the objects
  - ✓ bounded by

$$0 < r(x,y) < 1$$



Gray Level

- We call the intensity of a monochrome image ***f*** at coordinate ***(x,y)*** the gray level ***(l)*** of the image at that point.
  - ***l*** lies in the range $L_{min} \leq l \leq L_{max}$
  - L_{min} and L_{max} are positive and finite.
  - Gray scale = $[L_{min}, L_{max}]$
  - Common practice: Shift the interval to [***0***, ***L-1***]
  - ***0 = black** , **L-1 = white***
- The range of values spanned by the gray scale is referred to informally as the dynamic range

Resolution

- Resolution (how much you can see the detail of the image) depends on sampling and gray levels.
- The bigger the sampling rate ($M \times N$) and the gray scale level (k), the better the approximation of the digitized image from the original.
  - But the size of the image gets bigger

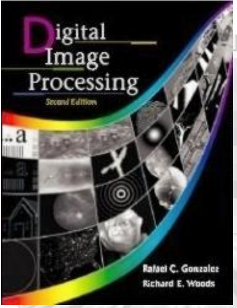
$$b = M \times N \times k$$

- Due to storage and quantizing hardware considerations, the number of intensity levels (L) typically is an integer power of 2: $L = 2^k$

Space Required for an Image

TABLE 2.1
Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912



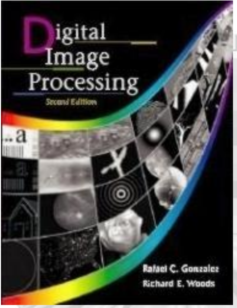
Resolution

- Spatial resolution is a measure of the smallest discernible detail in an image.
- Quantitatively ,spatial resolution can be stated in a number of ways,
  - With line pairs per unit distance, and dots(pixels) per unit distance (DPI).
- Intensity resolution similarly refers to the smallest discernible change in intensity level.
  - the number of bits used to quantize intensity as the intensity resolution.
- Lower resolution images are smaller than the original

Spatial Resolution Effects

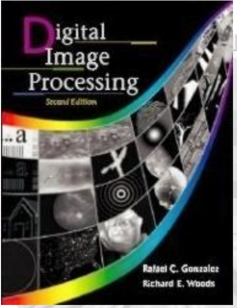
a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.



Basic Relationship among Pixels

- Neighbors of a pixel
- Connectivity
- Distance Measures
- Arithmetic/Logic Operations



Neighbors of a Pixels

- a pixel p at coordinate (x,y) has
  - $N_4(p)$: 4-neighbors of p
    (x+1, y), (x-1,y),(x,y+1), (x,y-1)

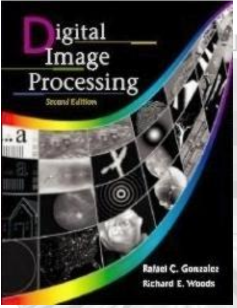
$$\begin{array}{ccc} & x & \\ x & p & x \\ & x & \end{array}$$

  - $N_D(p)$: 4-diagonal neighbors of p
    (x+1, y+1), (x+1,y-1),(x-1,y+1), (x-1,y-1)

$$\begin{array}{ccc} x & & x \\ & p & \\ x & & x \end{array}$$

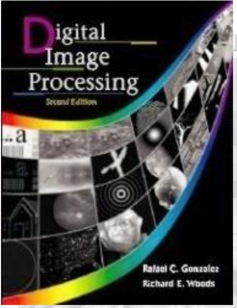
  - $N_8(p)$: 8-neighbors of p :
    a combination of $N_4(p)$ and $N_D(p)$

$$\begin{array}{ccc} x & x & x \\ x & p & x \\ x & x & x \end{array}$$



Connectivity

- Let V be the set of gray-level values used to defined connectivity
  - ✓ 4-connectivity :
    - 2 pixels p and q with values from V are 4-connected if q is in the set $N_4(p)$
- 8-connectivity :
  - ✓ 2 pixels p and q with values from V are 8-connected if q is in the set $N_8(p)$
- m-connectivity (mixed connectivity):
  - ✓ 2 pixels p and q with values from V are m-connected if
    - q is in the set $N_4(p)$ or
    - q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ is empty.
    - (the set of pixels that are 4-neighbors of both p and q whose values are from V)



Example of Connectivity

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Path 8 neighbors m neighbors

- ➢ m-connectivity eliminates the multiple path connections that arise in 8-connectivity.

Path

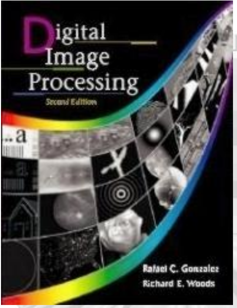
- A path from pixel p with coordinates (x,y) to pixel q with coordinates (s,t) is a sequence of distinct pixels with coordinates

$$(x_0,y_0),(x_1,y_1),\ldots(x_n,y_n)$$

where $(x_0,y_0) = (x,y)$, $(x_n,y_n) = (s,t)$ and (x_i,y_i) is adjacent to (x_{i-1},y_{i-1})

and *n* is the length of the path

- we can define 4-,8-, or m-paths depending on type of adjacency specified.



Adjacent

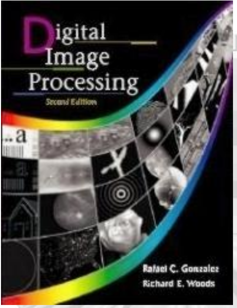
- A pixel p is adjacent to a pixel q if they are connected.
- Two image area subsets R1 and R2 are adjacent if some pixel in R1 is adjacent to some pixel R2.

$$\left.\begin{matrix}1&1&1\\1&0&1\\0&1&0\end{matrix}\right\}R_i$$
$$\left.\begin{matrix}0&0&1\\1&1&1\\1&1&1\end{matrix}\right\}R_j$$

Distance Measures

- For pixel p, q and z with coordinates (x,y), (s,t) and (u,v), respectively,

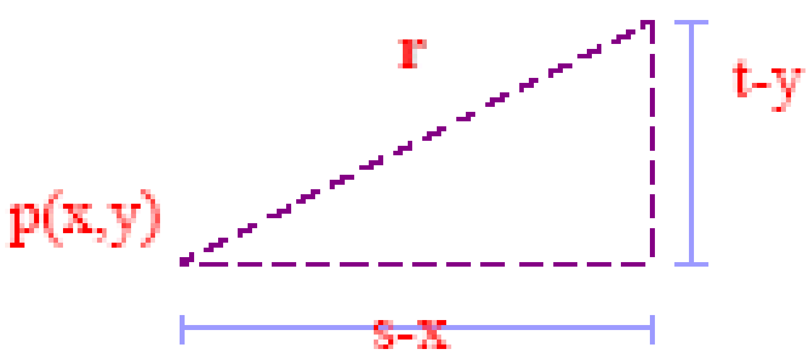
  D is a distance function or metric if
  - ✓ (a) $D(p,q) \geq 0$; $D(p,q) = 0$ iff $p=q$
  - ✓ (b) $D(p,q) = D(q,p)$
  - ✓ (c) $D(p,z) \leq D(p,q) + D(q,z)$



Euclidean Distance

$$D_e(p,q) = \left[(x-s)^2 + (y-t)^2\right]^{1/2}$$

q(s,t)



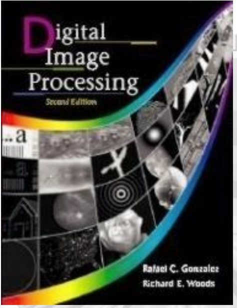
radius (r) centered at (x,y)

City-Block Distance

$$D_4(p,q) = |x-s| + |y-t|$$

$$\begin{array}{ccccc} & & 2 & & \\ & 2 & 1 & 2 & \\ 2 & 1 & 0 & 1 & 2 \\ & 2 & 1 & 2 & \\ & & 2 & & \end{array}$$

- diamond centered at (x,y)
- D_4 =1 are 4-neighbors of (x,y)

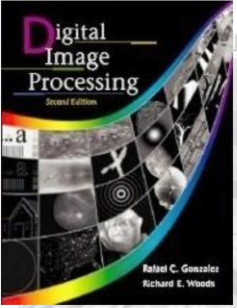


Chessboard Distance

$$D_8(p,q) = \max(|x - s|, |y - t|)$$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

square centered at (x,y)

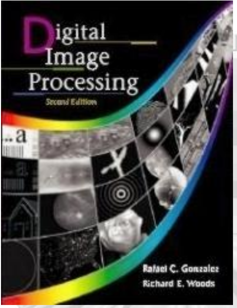


M-connectivity distances

- distances of m-connectivity of the path between 2 pixels depends on values of pixels along the path.
- e.g., if only connectivity of pixels valued 1 is allowed. find the m-distance between p and p_4

	p_3	p_4	0	1	1	1	1	1
p_1	p_2		0	1	0	1	1	1
p			1		1		1	

$d = 2$ $d=3$ $d=4$



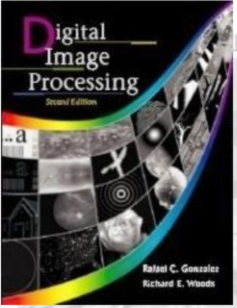
Linear and Nonlinear Operations

- An operator H is said to be a linear operator if it satisfy the following conditions.

$$\begin{aligned} H\left[a_i f_i(x,y) + a_j f_j(x,y)\right] &= a_i H\left[f_i(x,y)\right] + a_j H\left[f_j(x,y)\right] \\ &= a_i g_i(x,y) + a_j g_j(x,y) \end{aligned}$$

  - ✓The sum of two images is identical to applying the operator to the image individually, multiplying the results by the constants, and adding those results.

- An operator that fails the test of above equation is nonlinear. Ex) $|af_i - bf_j|$



Arithmetic Operations

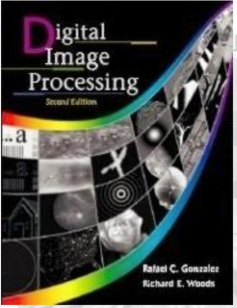
- Image arithmetic involves images of the same size.

$$s(x, y) = f(x, y) + g(x, y)$$

$$d(x, y) = f(x, y) - g(x, y)$$

$$p(x, y) = f(x, y) \times g(x, y)$$

$$v(x, y) = f(x, y) \div g(x, y)$$



Logical Operations

- When dealing with binary images, it is common practice to refer to union, intersection, and complement as the OR, AND, and NOT logical operations,
  - where "logical" arises from logic theory in which 1 and 0 denote true and false,respectively.
  - think of foreground(1-valued) and background (0-valued) sets of pixels.

FIGURE 2.33 Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.