Reinforcement Learning I

CSE 4711: Artificial Intelligence

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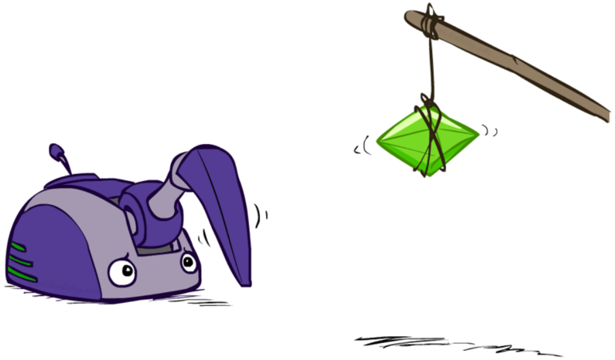
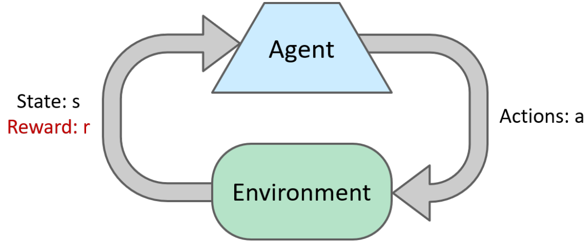
Islamic University of Technology

Reinforcement Learning



Reinforcement Learning

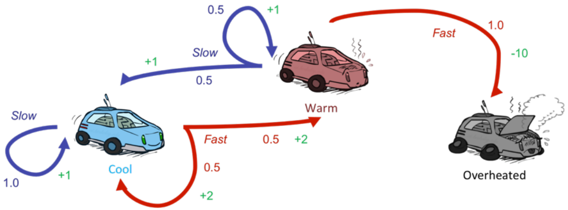


- Receive feedback in the form of **rewards**
- Agent's utility is defined by the reward function
- Must (learn to) act so as to **maximize expected rewards**
- All learning is based on observed samples of outcomes!

Video: AIBO - Initial, AIBO - Training, AIBO - Finished, SNAKE, Toddler, Crawler

Reinforcement Learning

- Still assume a Markov Decision Process (MDP):
  - A set of states $s \in S$
  - A set of actions (per state) A
  - A model $T(s, a, s')$
  - A reward function $R(s, a, s')$
- Still looking for a policy $\pi(s)$



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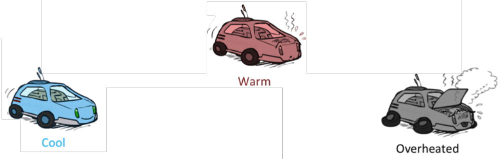
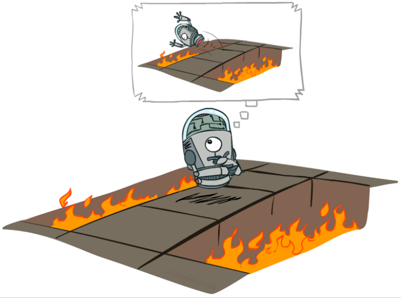


- New twist: **don't know** T **or** R
  - i.e. we don't know which states are good or what the actions do
  - Must try out actions and states to learn

Offline (MDPs) vs. Online (RL)



Offline Solution

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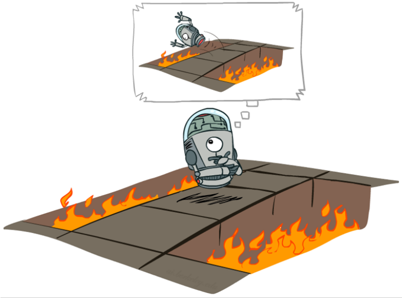
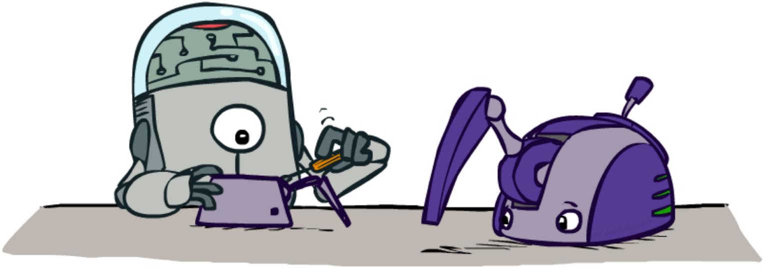


Offline Solution

Online Learning

Model-Based Learning



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  - Normalize to give an estimate of $\hat{T}(s, a, s')$
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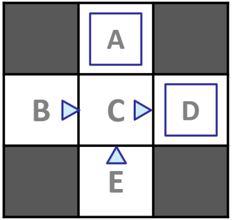
- Step 2: Solve the learned MDP
  - For example, use value iteration, as before

Example: Model-Based Learning

Input Policy π



Assume: $\gamma = 1$

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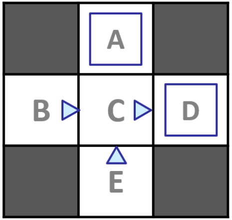

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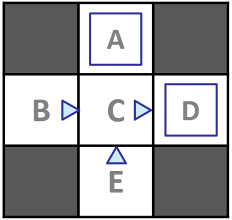

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Learned Model

$T(s, a, s')$

T(B, east, C) = 1.00
T(C, east, D) = 0.75
T(C, east, A) = 0.25
...

$\hat{R}(s, a, s')$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
...

Example: Expected Age

Goal: Compute expected age of students

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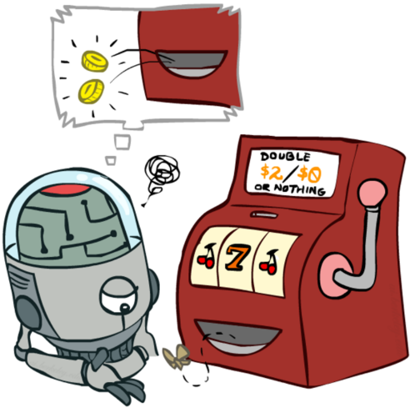
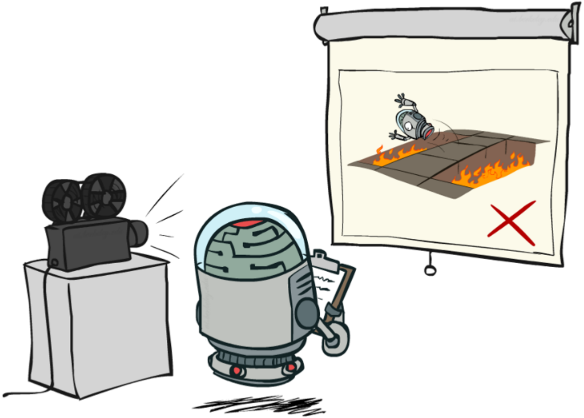
Why does this work? Because samples appear with the right frequencies.

Model-Free Learning

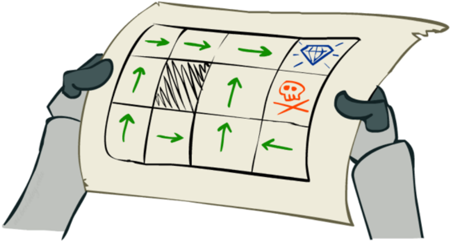


Passive Reinforcement Learning



Passive Reinforcement Learning

- Simplified task: policy evaluation
  - Input: a fixed policy $\pi(s)$
  - You don't know the transitions $T(s, a, s')$
  - You don't know the rewards $R(s, a, s')$
  - **Goal: learn the state values**



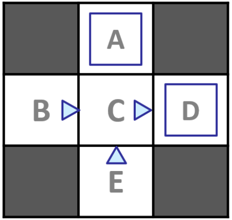
- In this case:
  - Learner is "along for the ride"
  - No choice about what actions to take
  - Just execute the policy and learn from experience
  - This is NOT offline planning! You actually take actions in the world

Direct Evaluation

- Goal: Compute values for each state under π
- Idea: Average together observed sample values
  - Act according to π
  - Every time you visit a state, write down what the sum of discounted rewards turned out to be
  - Average those samples

Example: Direct Evaluation

Input Policy π



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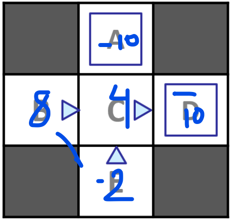

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$\frac{-}{4} \quad -5$

$\frac{27-11}{4} = 4$

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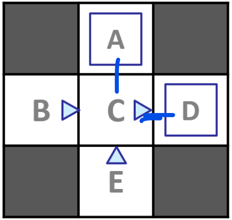

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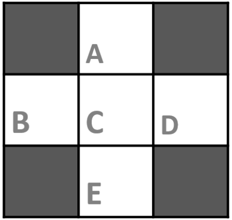
Output Values



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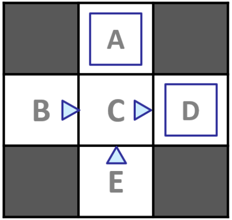

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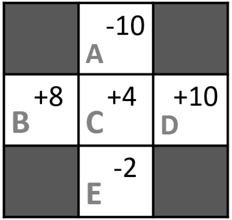

Pros and Cons of Direct Evaluation

- Pros
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  - It doesn't require any knowledge of T, R
  - It eventually computes the correct average values, using just sample transitions

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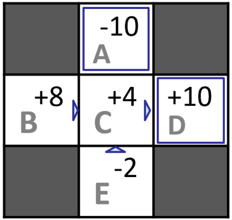

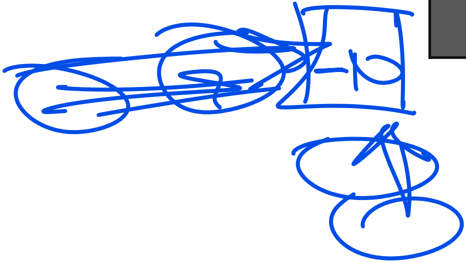

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  - Each state must be learned separately $\rightarrow$ takes a long time to learn

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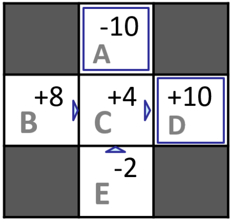

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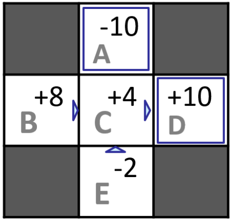
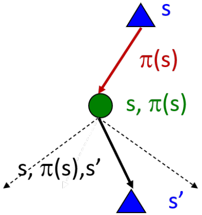


If B and E both go to C under this policy, how can their values be different?

Why Not Use Policy Evaluation?

- Simplified Bellman updates calculate V for a fixed policy:



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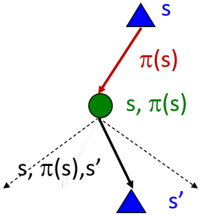
  - Each round, replace V with a one-step look-ahead layer over V
    $V_0^\pi(s) = 0$
    $V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s,\pi(s),s')\,[R(s,\pi(s),s') + \gamma V_k^\pi(s')]$
  - This approach fully exploited the connections between the states



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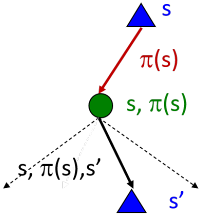
  - Unfortunately, we need T and R to do it!



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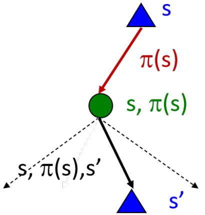

- Key question: how can we do this update to V without knowing T and R?
  - In other words, how to we take a weighted average without knowing the weights?

Sample-Based Policy Evaluation

- We want to improve our estimate of V by computing these averages:
$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s,\pi(s),s')\left[R(s,\pi(s),s') + \gamma V_k^{\pi}(s')\right]$

100

V^*

$$\frac{\sum \text{samples}}{n}$$

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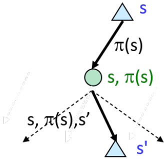

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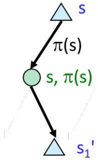
$$sample_1 = R(s, \pi(s), s_1') + \gamma V_k^{\pi}(s_1')$$



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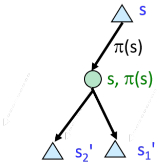

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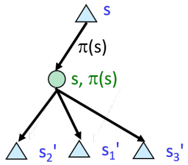

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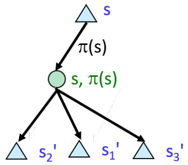
$$V^{\pi}_{k+1}(s) \leftarrow \frac{1}{n}\sum_i sample_i$$



Sample-Based Policy Evaluation

- We want to improve our estimate of V by computing these averages:
  $V^{\pi}_{k+1}(s) \leftarrow \sum_{s'} T(s,\pi(s),s')\left[R(s,\pi(s),s') + \gamma V^{\pi}_{k}(s')\right]$
- Idea: Take samples of outcomes s' (by doing the action!) and average

$$
\begin{aligned}
sample_1 &= R(s,\pi(s),s'_1) + \gamma V^{\pi}_{k}(s'_1)\\
sample_1 &= R(s,\pi(s),s'_2) + \gamma V^{\pi}_{k}(s'_2)\\
&\dots\\
sample_n &= R(s,\pi(s),s'_n) + \gamma V^{\pi}_{k}(s'_n)\\
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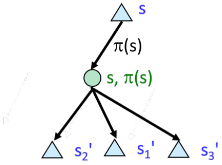
\end{aligned}
$$



Almost! But we can't rewind time to get sample after sample from state s.

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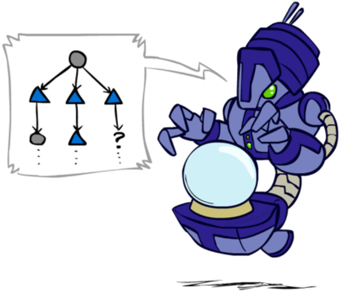
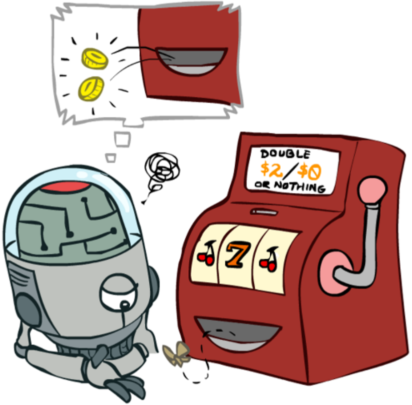
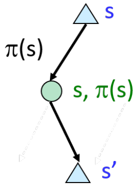


Temporal Difference Learning



Temporal Difference Learning

- Big idea: learn from every experience!
  - Update $V(s)$ each time we experience a transition (s, a, s', r)
  - Likely outcomes s' will contribute updates more often



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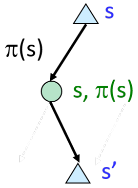
  - Policy still fixed, still doing evaluation!
  - Move values toward value of whatever successor occurs: running average



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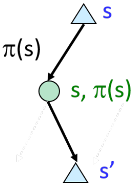

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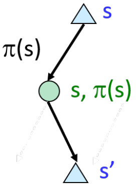
    $$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + (\alpha)sample$$



$\alpha \rightarrow$ sample weight

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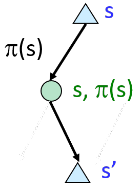
    Same update:

    $$V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$$



Exponential Moving Average

- The running interpolation update:

$$\bar{x}_n = (1 - \alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$$

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$$\bar{x}_n = \frac{x_n + (1-\alpha) \cdot x_{n-1} + (1-\alpha)^2 \cdot x_{n-2} + \dots}{1 + (1-\alpha) + (1-\alpha)^2 + \dots}$$

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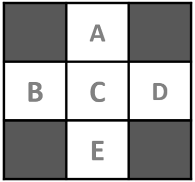
- Forgets about the past (distant past values were wrong anyway)
- Decreasing learning rate (alpha) can give converging averages

Example: Temporal Difference Learning

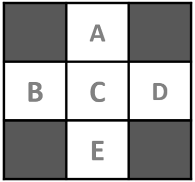
States

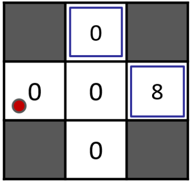


Assume: $\gamma = 1$, $\alpha = 1/2$

Example: Temporal Difference Learning

States





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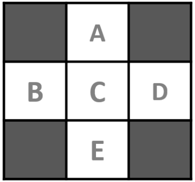
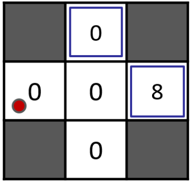


Observed Transitions



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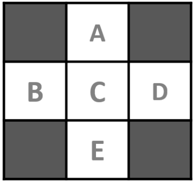

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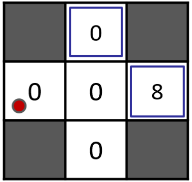
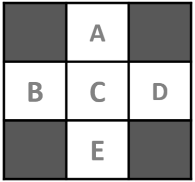
Observed Transitions

B, east, C, -2



Example: Temporal Difference Learning

States



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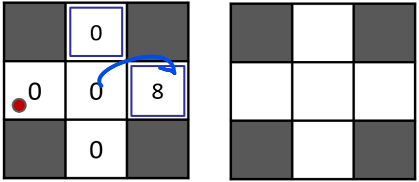
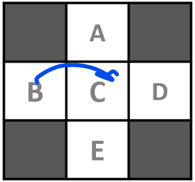


Example: Temporal Difference Learning

States



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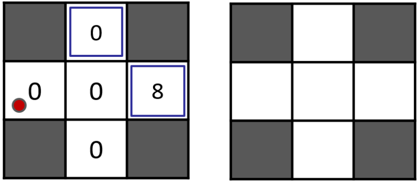
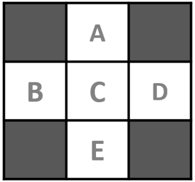


$$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + \alpha\left[R(s,\pi(s),s') + \gamma V^{\pi}(s')\right]$$

$$\frac{0-2}{2} = -1$$

Example: Temporal Difference Learning

States



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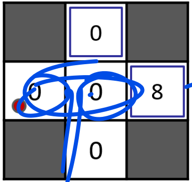
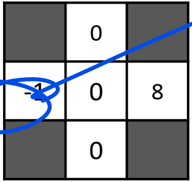
B, east, C, -2





$-(0.5)$

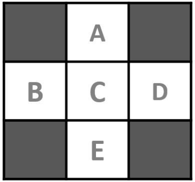
$\frac{1}{4}$

$$V^\pi(s) \leftarrow (1-\alpha)V^\pi(s) + \alpha\left[R(s,\pi(s),s') + \gamma V^\pi(s')\right]$$

$$= -0.5 \quad \frac{1}{2}\left(\frac{3}{4}\times 0 + \frac{1}{4}(-2)\right)$$

Example: Temporal Difference Learning

States



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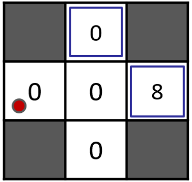
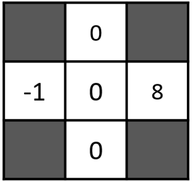
Observed Transitions

B, east, C, -2

C, east, D, -2





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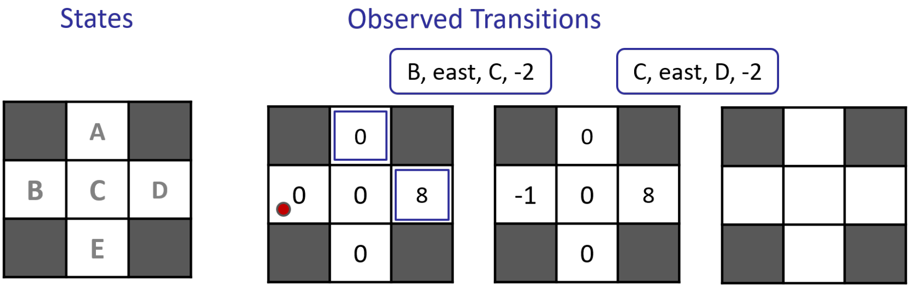

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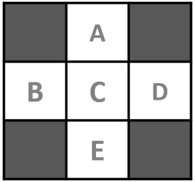

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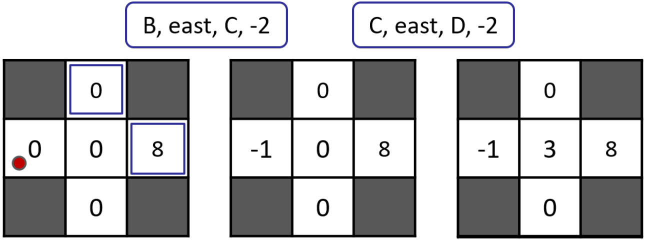

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Problems with TD Value Learning

- TD value leaning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages
- However, if we want to turn values into a (new) policy, we're sunk:

$$\pi(s) = \operatorname{argmax} \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V(s') \right]$$

argmax Q

100

50

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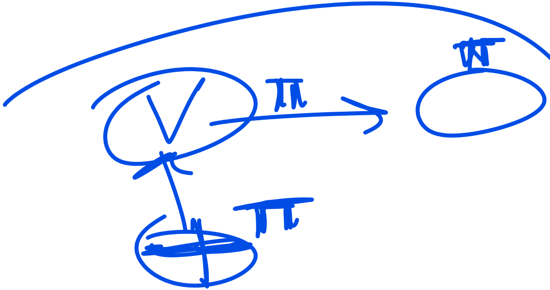
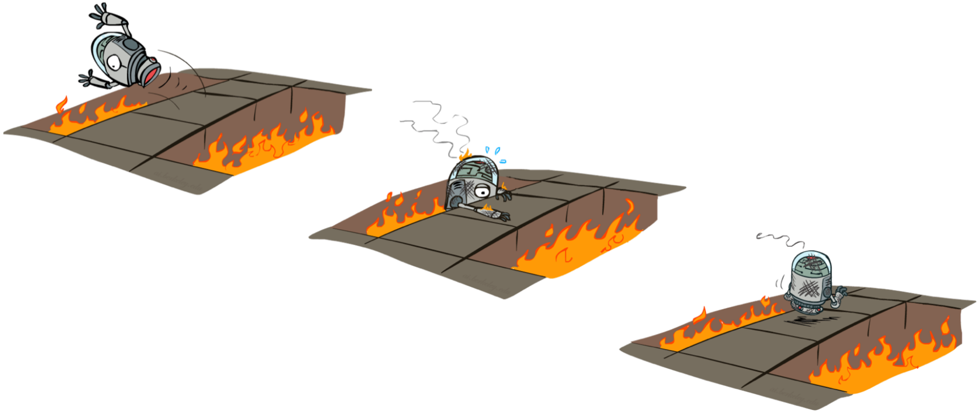
- Idea: learn Q-values, not values
- Makes action selection model-free too!

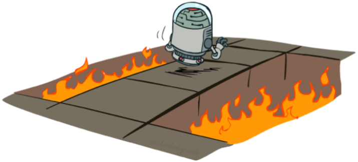


Active Reinforcement Learning



Active Reinforcement Learning

- Full reinforcement learning: optimal policies (like value iteration)
  - You don't know the transitions $T(s, a, s')$
  - You don't know the rewards $R(s, a, s')$
  - You choose the actions now
  - **Goal: learn the optimal policy/values**
- In this case:
  - Learner makes choices!
  - Fundamental tradeoff: exploration vs. exploitation
  - This is NOT offline planning! You actually take actions in the world and find out what happens...



Detour: Q-Value Iteration

- Value iteration: find successive (depth-limited) values
  - Start with $V_0(s) = 0$, which we know is right
  - Given V_k, calculate the depth $k+1$ values for all states

$$V_k + 1(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

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- But Q-values are more useful, so compute them instead
  - Start with $Q_0(s,a) = 0$, which we know is right
  - Given Q_k, calculate the depth $k+1$ q-values for all q-states:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')\right]$$

Q-Learning

- Q-Learning: sample-based Q-value iteration

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

Video: Q-gridworld, Q-auto, Q-crawler

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- Learn $Q(s,a)$ values as you go

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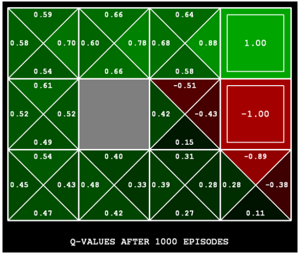

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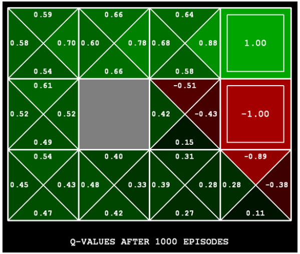
- Learn $Q(s,a)$ values as you go
  - Receive a sample (s,a,s',r)



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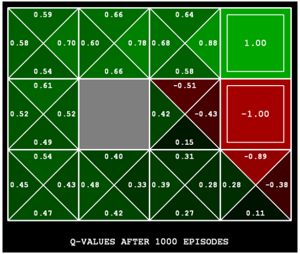

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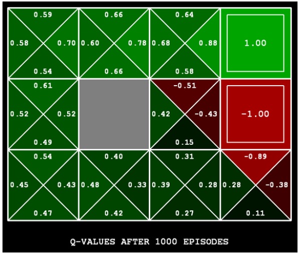

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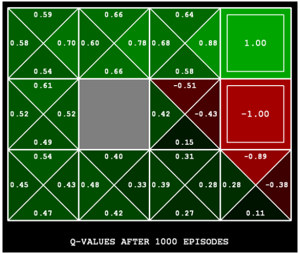
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:
    $sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$
  - Incorporate the new estimate into a running average:
    $Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)[sample]$



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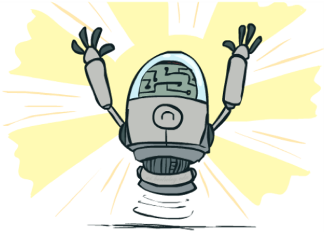
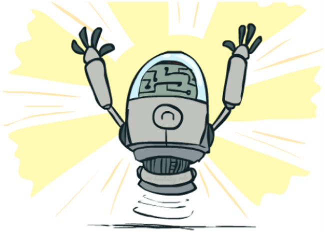
Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy - even if you're acting suboptimally!
- This is called **off-policy learning**



Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy - even if you're acting suboptimally!
- This is called **off-policy learning**
- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - ...but not decrease it too quickly
  - Basically, in the limit, it doesn't matter how you select actions (!)



Suggested Reading

- Russell & Norvig: Chapter 21
- Sutton and Barto: 6.1, 6.2, 6.5