CSE 4631

Chapter 9 - Smith

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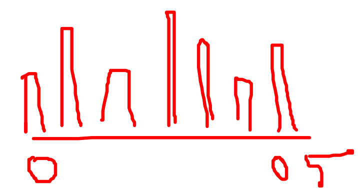
Applications of The Discrete Fourier Transform

$N \quad N$

$x \rightarrow X =$



1 • Spectral Analysis
  - Finding out the frequency spectrum of a signal.

2 • Finding out the frequency response of a system given the impulse response. A different way to describe a system.

system S

3 • Allows to perform convolution faster. O(N^2) to O(NlgN)
  - By using a variant of DFT called FFT (Fast Fourier Transform)

N^2 NlgN

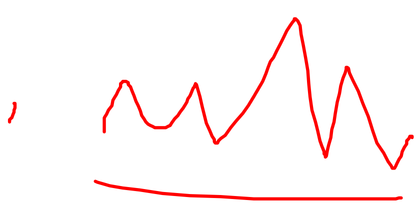
$\boxed{x * h = y}$

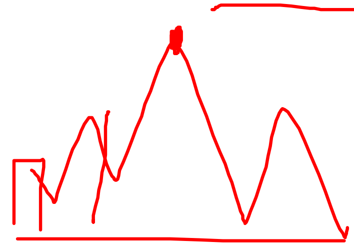
$X \otimes H = Y$

NlgN

Spectral Analysis of Signals

- Given a signal, we want to find out what are the component signals.
- This would allow us to guess from which sources the signals are coming from.
- For example, a signal which shows a spike of 60 Hz in its frequency spectrum, might originate from the electric power lines.

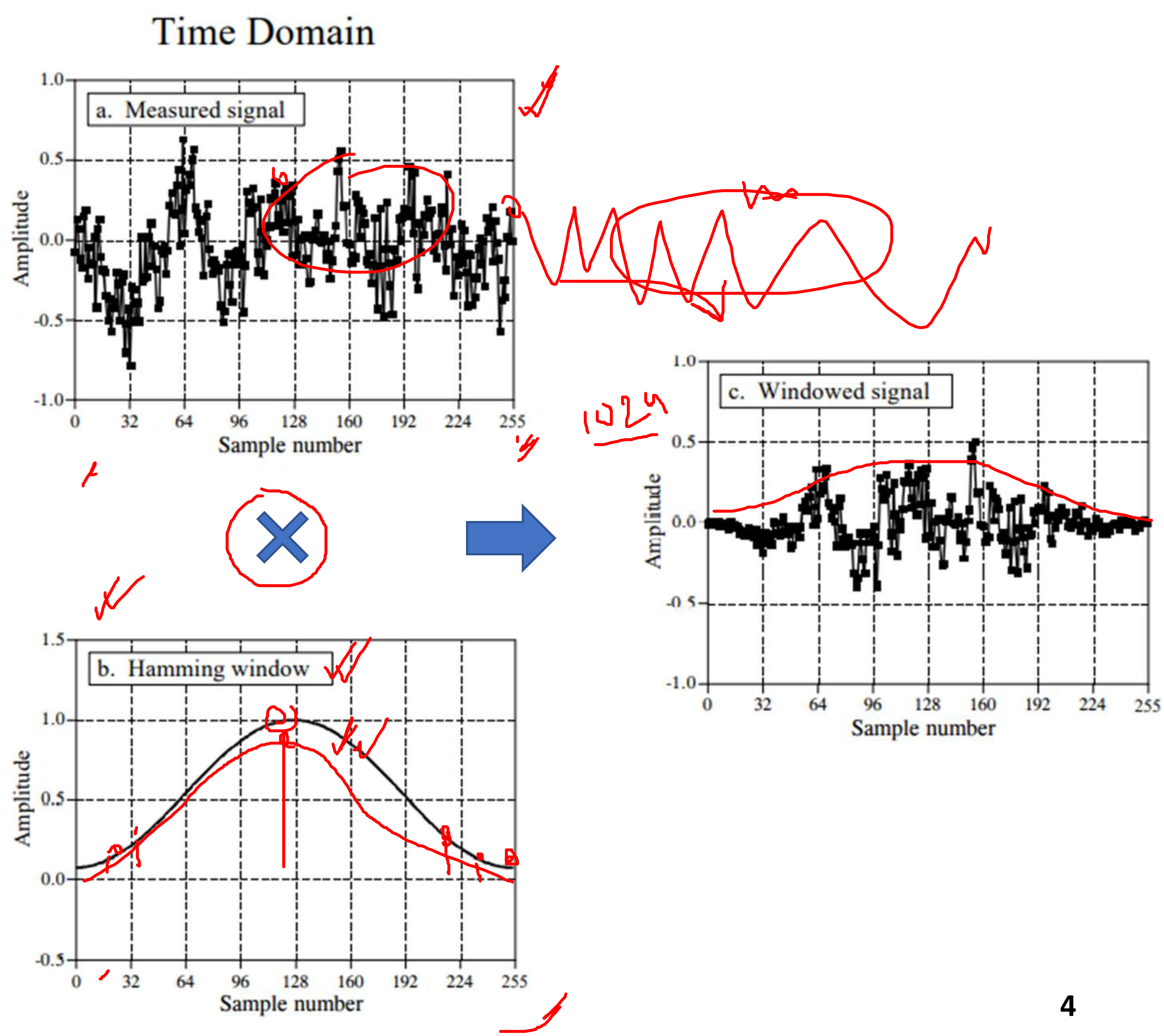




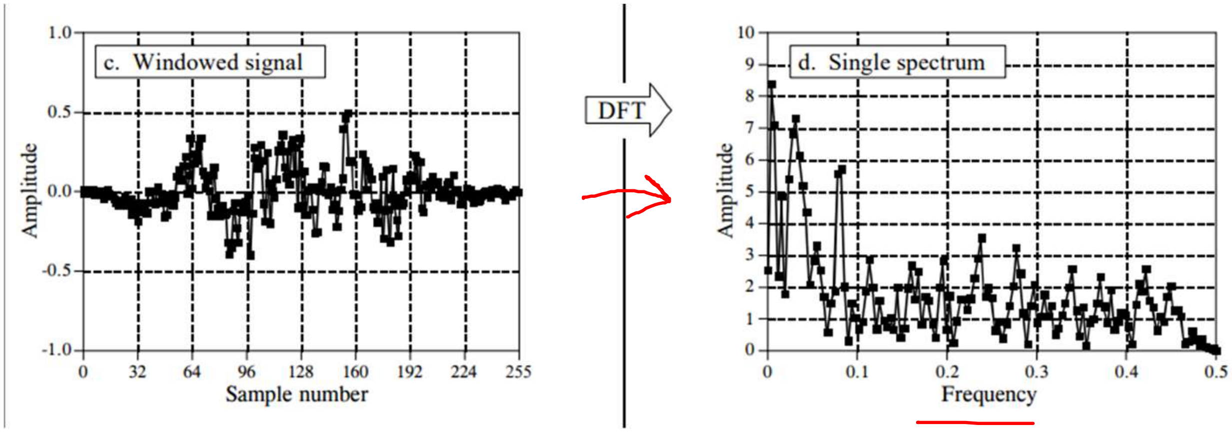
0·3 X S_F

Spectral Analysis

- Given a signal in time domain captured from ~~deep sea,~~ before converting it into frequency domain, first we need to element-wise multiply with a smooth signal called hamming window.
- Why? (you will find the answer in the upcoming slides)



Spectral Analysis



≥ 1024

- Now, let's convert it to frequency domain using DFT.
- Oh no! so much noise! What to do?
  - Option (A) Take a longer signal and do DFT
  - Option (B) Take a longer signal, divide it in smaller parts, then perform DFT on each small part, then find the average of all the outputs → 256
- Out of (A) and (B), which one will remove noise better?

longer

Spectral Analysis

- Turns out option (B) is the appropriate way to remove noise.

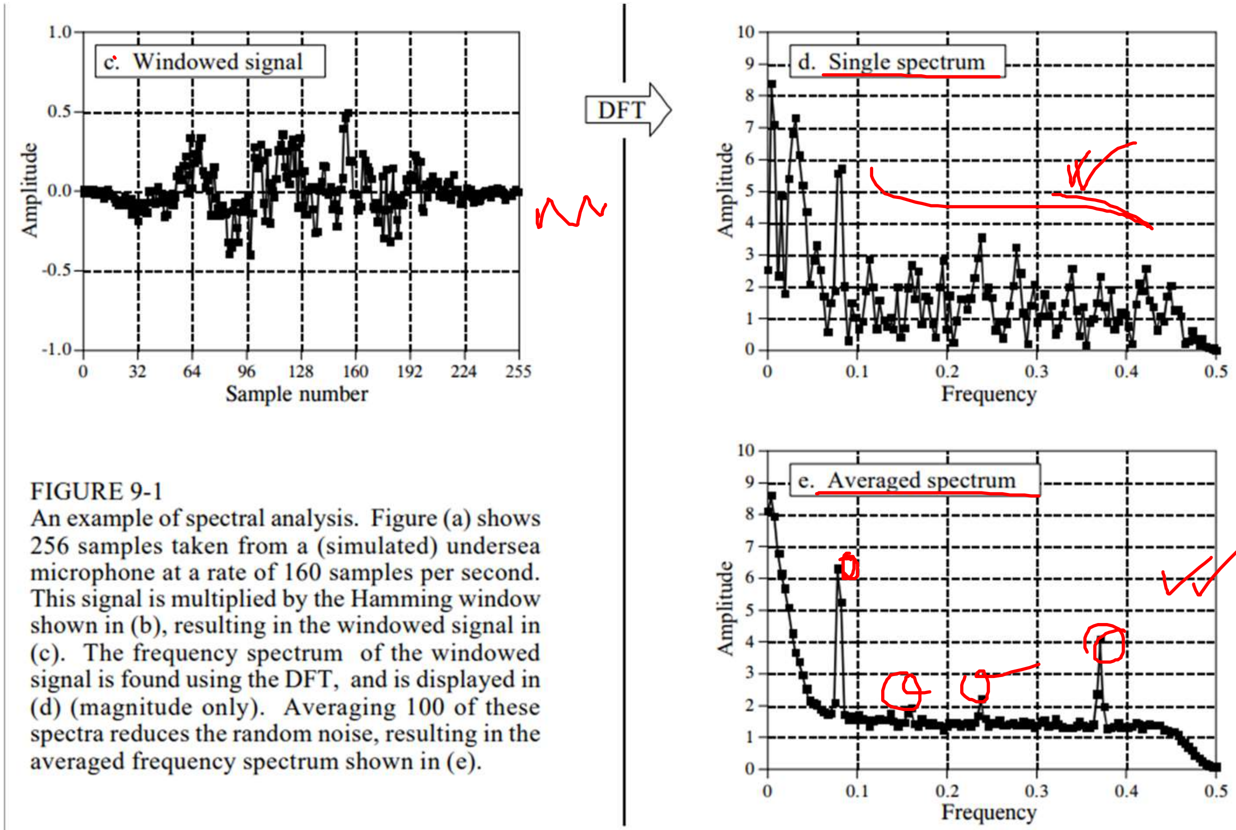


FIGURE 9-1
An example of spectral analysis. Figure (a) shows 256 samples taken from a (simulated) undersea microphone at a rate of 160 samples per second. This signal is multiplied by the Hamming window shown in (b), resulting in the windowed signal in (c). The frequency spectrum of the windowed signal is found using the DFT, and is displayed in (d) (magnitude only). Averaging 100 of these spectra reduces the random noise, resulting in the averaged frequency spectrum shown in (e).

Spectral Analysis

Why option (B) is better approach?

- Even though longer signal contain more information, the greater number of samples in the spectrum dilutes the information by the same factor.
- Longer DFTs provide better frequency resolution, but the same noise level.
- It's better to use more of the original signal in a way that doesn't increase the number of points in the frequency spectrum. This can be done by breaking the longer input signal into many 256 point segments. Each of these segments is multiplied by the Hamming window, run through a 256 point DFT, and converted to polar notation. The resulting frequency spectra are then averaged to form a single 129 point frequency spectrum.

Spectral Analysis

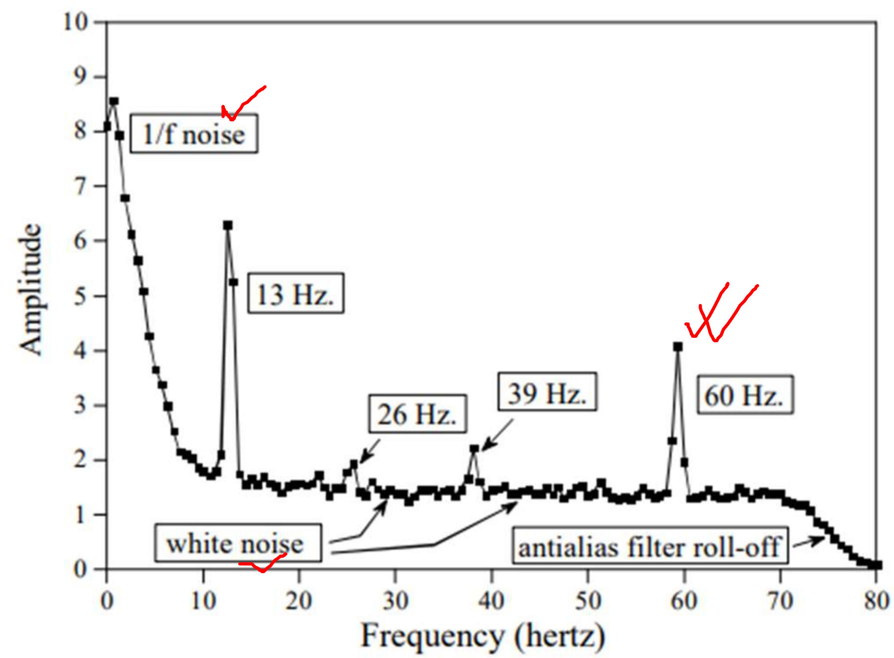
- Another method to reduce the noise in frequency spectrum –
  - Using a long DFT and applying a low-pass filter in the output to smoothen it out.

There is also a second method for reducing spectral noise. Start by taking a very long DFT, say 16,384 points. The resulting frequency spectrum is high resolution (8193 samples), but very noisy. A low-pass digital filter is then used to *smooth* the spectrum, reducing the noise at the expense of the resolution. For example, the simplest digital filter might average 64 adjacent samples in the original spectrum to produce each sample in the filtered spectrum. Going through the calculations, this provides about the same noise and resolution as the first method, where the 16,384 points would be broken into 64 segments of 256 points each.

Spectral Analysis

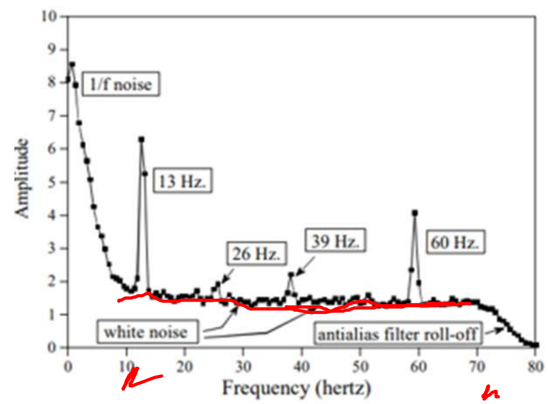
- Now that we have smoothened out the frequency spectrum, we can examine it.

FIGURE 9-2
Example frequency spectrum. Three types of features appear in the spectra of acquired signals: (1) random noise, such as white noise and 1/f noise, (2) interfering signals from power lines, switching power supplies, radio and TV stations, microphonics, etc., and (3) real signals, usually appearing as a fundamental plus harmonics. This example spectrum (magnitude only) shows several of these features.



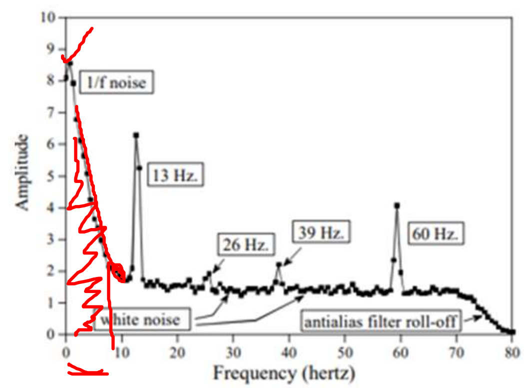
White Noise

Between 10 and 70 hertz, the signal consists of a relatively flat region. This is called white noise because it contains an equal amount of all frequencies, the same as white light. It results from the noise on the time domain waveform being uncorrelated from sample-to-sample. That is, knowing the noise value present on any one sample provides no information on the noise value present on any other sample. For example, the random motion of electrons in electronic circuits produces white noise. As a more familiar example, the sound of the water spray hitting the shower floor is white noise



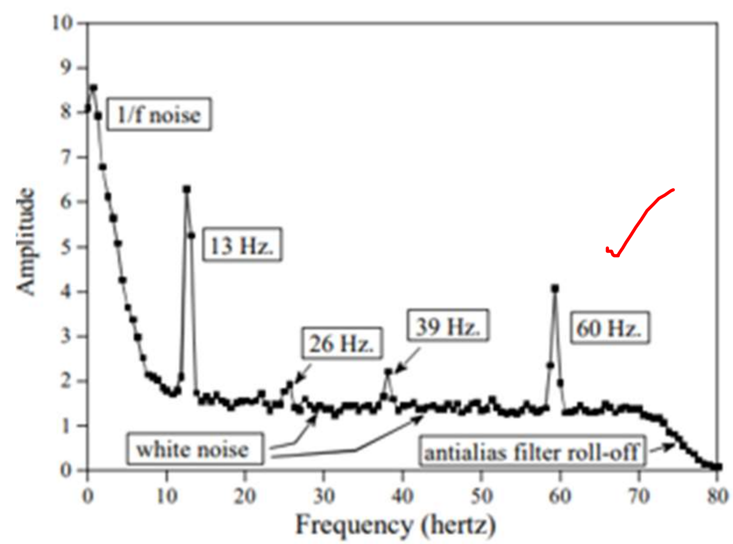
1/f Noise

Below about 10 hertz, the noise rapidly increases due to a curiosity called **1/f noise** (one-over-f noise). 1/f noise is a mystery. It has been measured in very diverse systems, such as traffic density on freeways and electronic noise in transistors. It probably could be measured in all systems, if you look low enough in frequency. In spite of its wide occurrence, a general theory and understanding of 1/f noise has eluded researchers. The cause of this noise can be identified in some specific systems; however, this doesn't answer the question of why 1/f noise is everywhere. For common analog electronics and most physical systems, the transition between white noise and 1/f noise occurs between about 1 and 100 hertz.



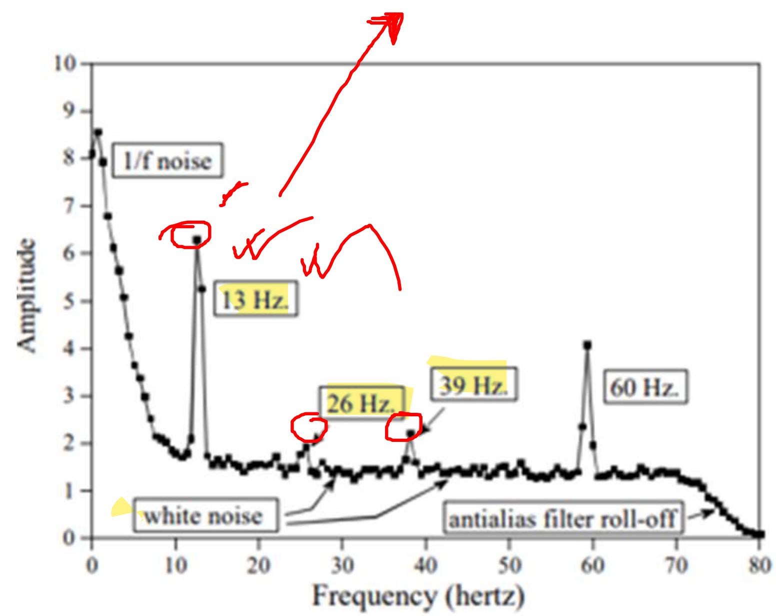
60 Hz

- Now we come to the sharp peaks in Fig. 9-2. The easiest to explain is at 60 hertz, a result of electromagnetic interference from commercial electrical power.



Actual signals

- Now we come to the actual signals. There is a strong peak at **13 hertz**, with weaker peaks at **26 and 39 hertz**. This is the frequency spectrum of a nonsinusoidal periodic waveform.
- The peak at 13 hertz is called the fundamental frequency, while the peaks at 26 and 39 hertz are referred to as the second and third harmonic respectively.
- This 13 hertz signal might be generated, for example, by a submarines's three **bladed propeller turning at 4.33 revolutions per second**. This is the basis of passive sonar, identifying undersea sounds by their frequency and harmonic content.



Effect of taking longer DFT

- Better resolution; close peaks are better separated.

time

DFT

N

N/2

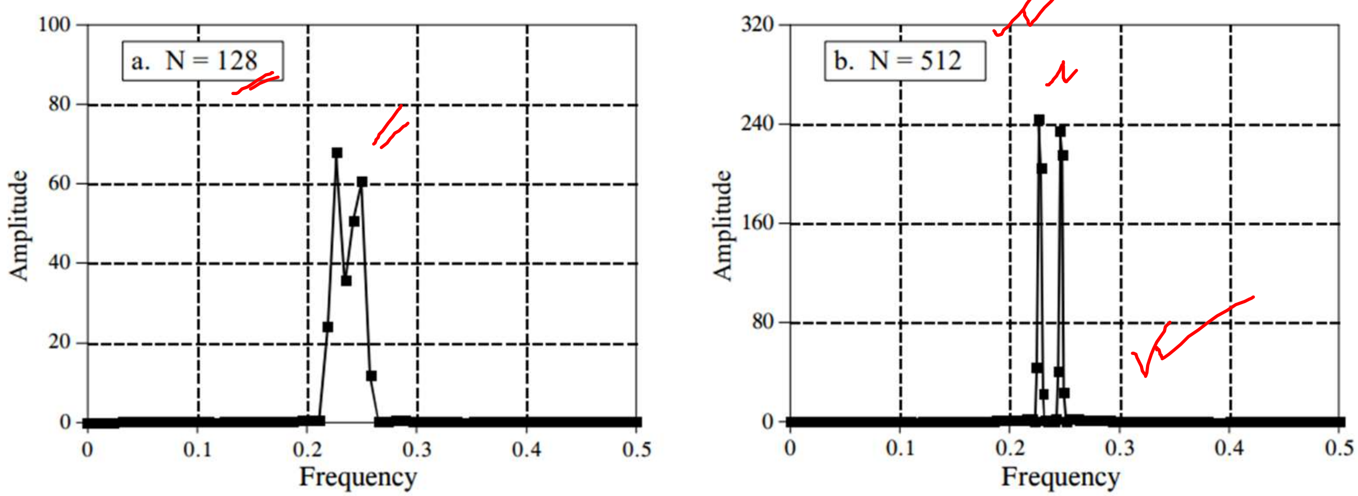


FIGURE 9-3
Frequency spectrum resolution. The longer the DFT, the better the ability to separate closely spaced features. In these example magnitudes, a 128 point DFT cannot resolve the two peaks, while a 512 point DFT can.

Effect of using Hamming Window

- Removes the tails (Good), Broadens the peaks (Bad). [trade-off]

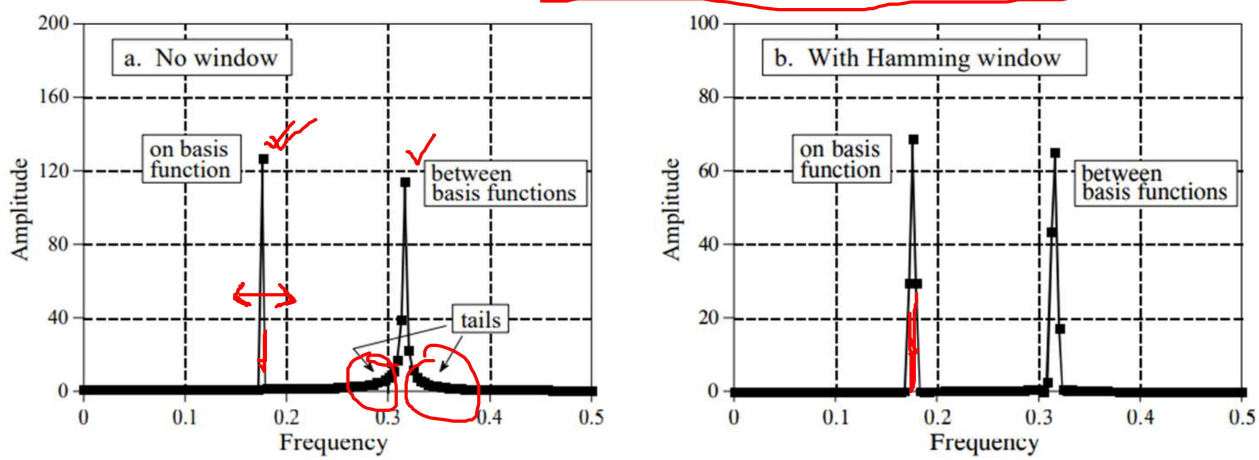
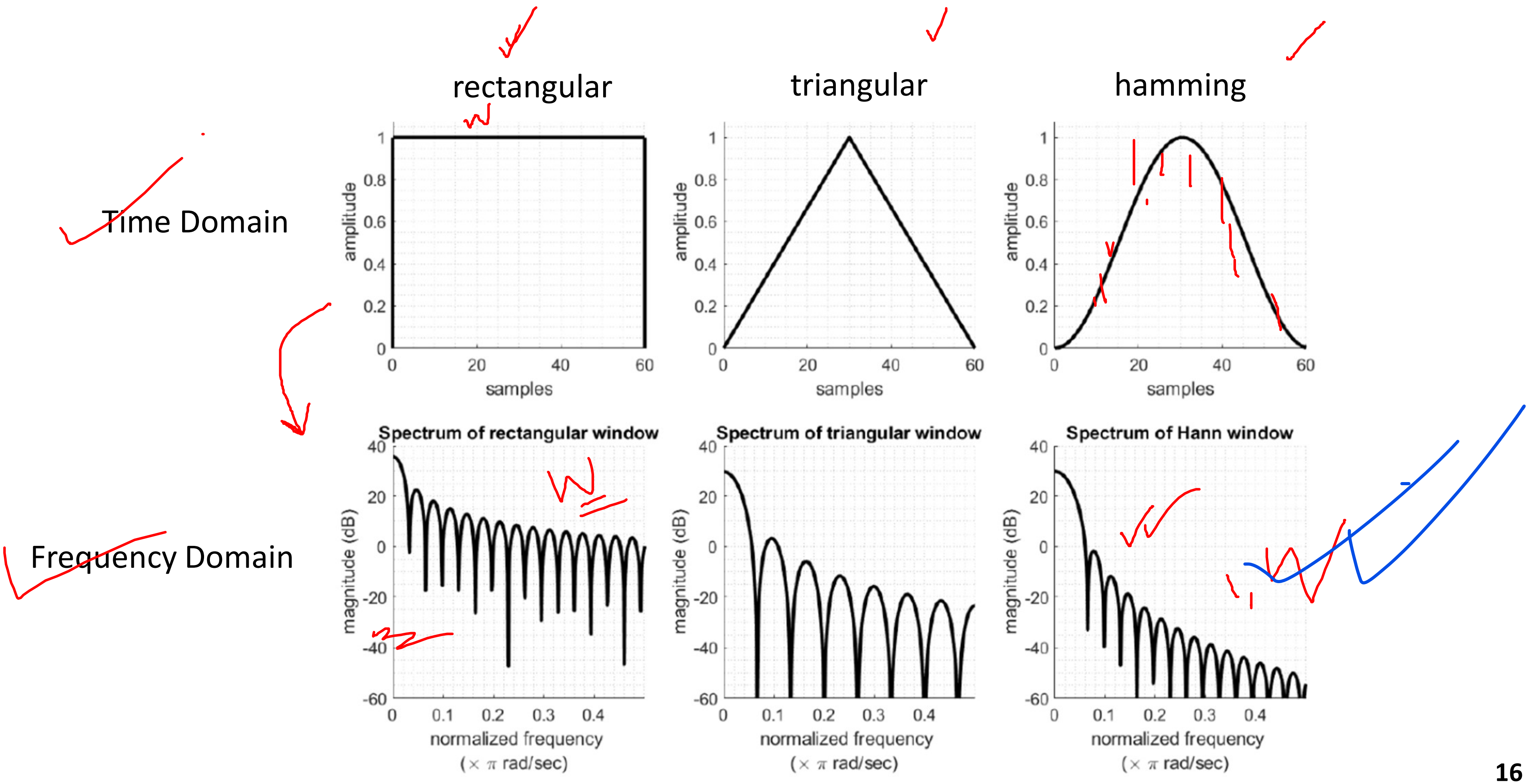


FIGURE 9-4
Example of using a window in spectral analysis. Figure (a) shows the frequency spectrum (magnitude only) of a signal consisting of two sine waves. One sine wave has a frequency exactly equal to a basis function, allowing it to be represented by a single sample. The other sine wave has a frequency *between* two of the basis functions, resulting in *tails* on the peak. Figure (b) shows the frequency spectrum of the same signal, but with a Hamming window applied before taking the DFT. The window makes the peaks look the same and reduces the tails, but broadens the peaks.

Computers can't do computations with an infinite number of data points, so all signals are "cut off" at either end. This causes the ripple on either side of the peaks..
The hamming window reduces this ripples, giving you a **more accurate idea of the original signal's frequency spectrum**.

Different Windows

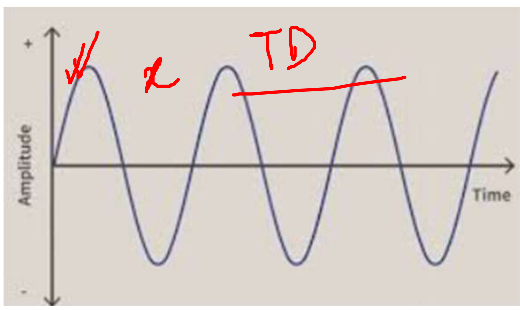


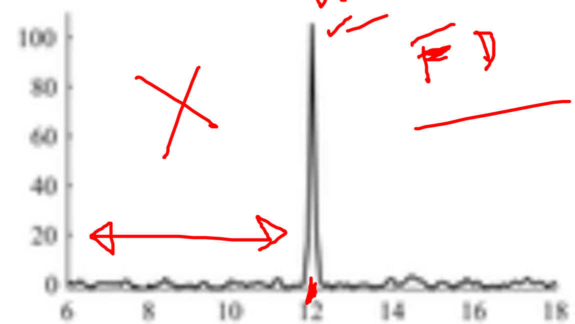
Effect of different windows

- To understand the effect of different windows on our signal, let's take a sinusoidal signal infinite in both directions. Its frequency domain version will be a single spike somewhere from 0 to 0.5 depending on the freq. of the sinusoid.

$x \circledast w$

$X * W$

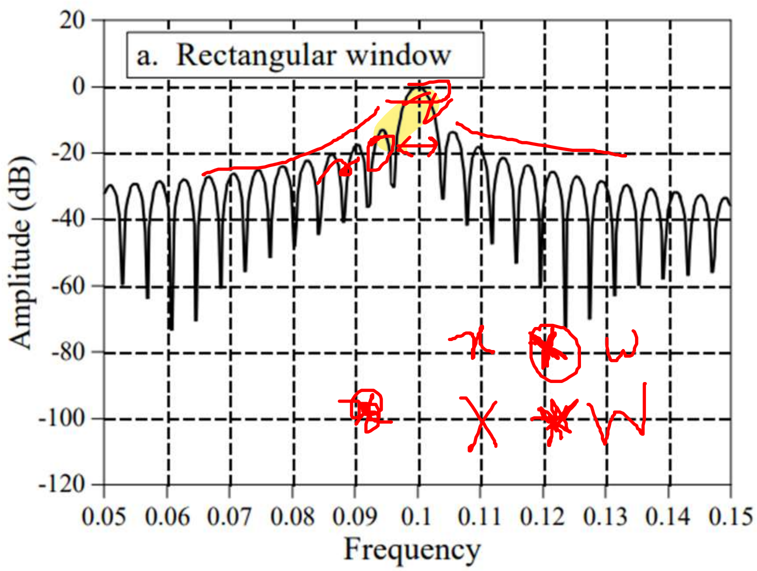


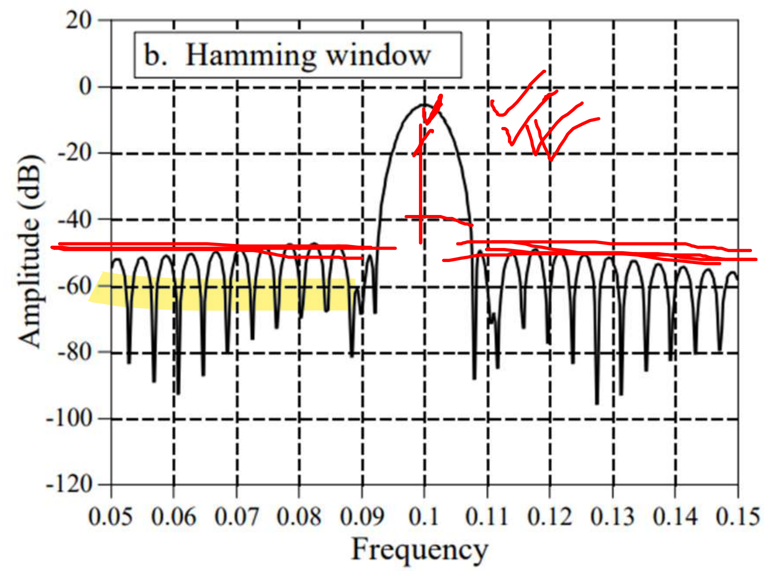


- Now, how will this spike change if, right before converting it to freq. domain, first we apply different windows over the infinite time domain sinusoidal signal?
- (applying window means making everything outside window zero and multiplying the windowed samples with corresponding sample value in the window)

Effect of different windows over the example signal's frequency domain

Detailed view of a spectral peak using various windows. Each peak in the frequency spectrum is a central lobe surrounded by tails formed from side lobes. By changing the window shape, the amplitude of the side lobes can be reduced at the expense of making the main lobe wider. The rectangular window, (a), has the narrowest main lobe but the largest amplitude side lobes. The Hamming window, (b), and the Blackman window, (c), have lower amplitude side lobes at the expense of a wider main lobe. The flat-top window, (d), is used when the amplitude of a peak must be accurately measured. These curves are for 255 point windows; longer windows produce proportionately narrower peaks.





Effect of different windows over the example signal's frequency domain

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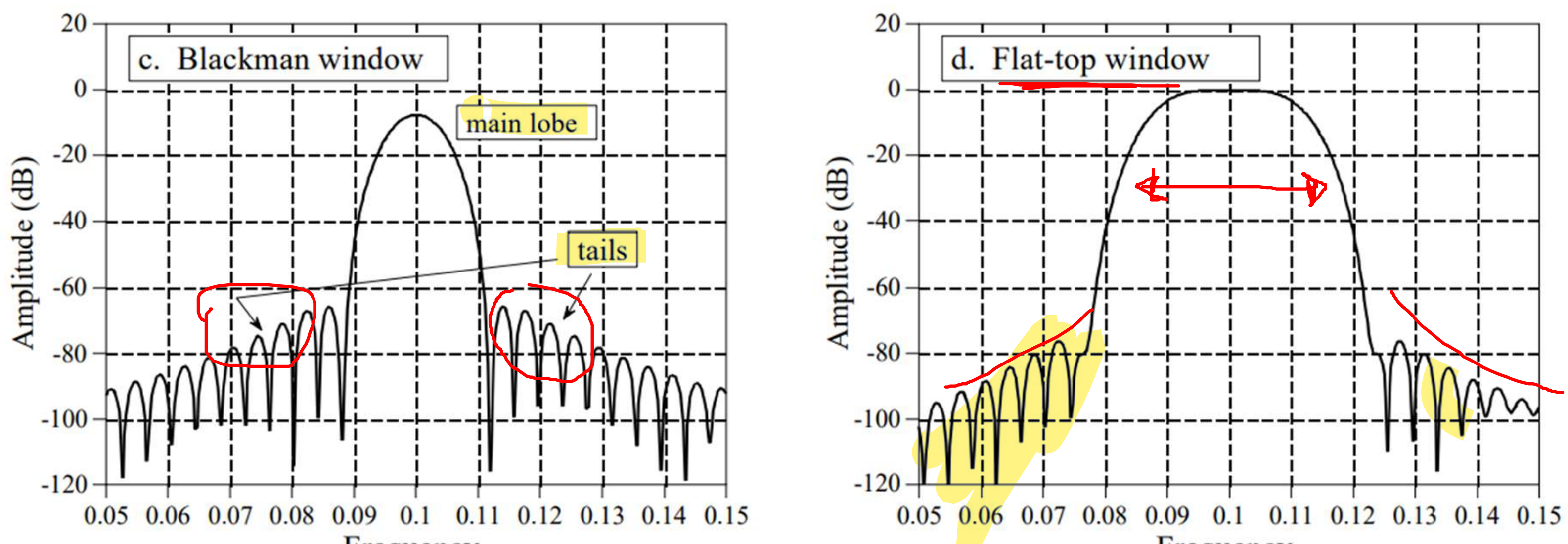
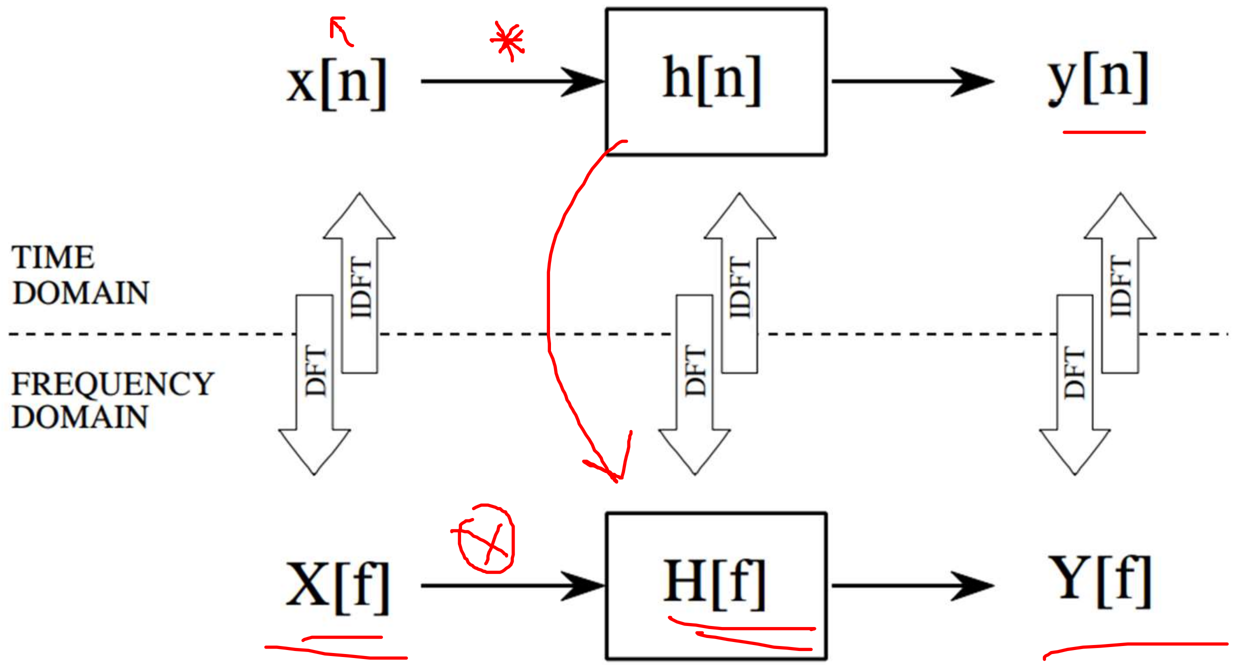


Frequency Response of Systems

Convolution

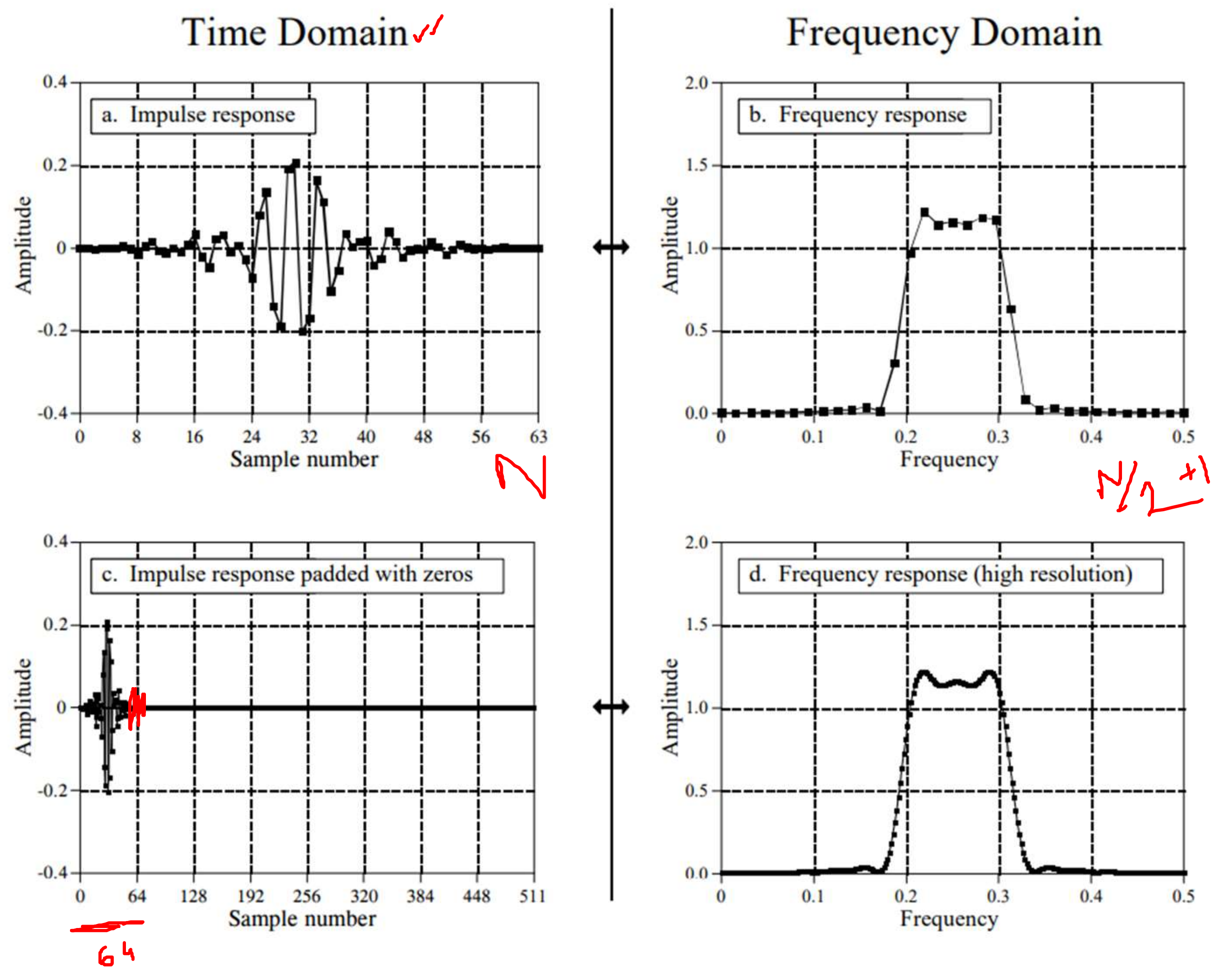


Multiplication

elementwise

How to get a smoother freq. response of a system?

- Answer:
  - Padding
    - (with zeros)



Convolution via Frequency Domain

- **Why do we need to learn to do convolution in a completely new way?**

- Reason 1: Gives us a fast implementation of convolution by utilizing a fast variant of DFT called FFT (Fast Fourier Transform)

- Reason 2: Deconvolution is easier in frequency domain. What is deconvolution? Oh, it's just, given one input signal and the output signal of a convolution operation, finding out the other input signal.
  - Given the values of h and Y where Y = x*h, finding out the value of x.
  - Why it is easier in freq. domain? Because in freq. domain, deconvolution is just division (elementwise).

Convolution via Frequency Domain

$X \circledast H = Y$

- X[f] × H[f] = Y[f] .
- In polar form, the magnitudes are multiplied:

MagY[f] = MagX[f] × MagH[f]

$O(N)$

and the phases are added:

PhaseY[f] = PhaseX[f] + PhaseH[f]

In rectangular form, a bit more complicated,

$$Re\,Y[f] = Re\,X[f]\,Re\,H[f] - Im\,X[f]\,Im\,H[f]$$

$$Im\,Y[f] = Im\,X[f]\,Re\,H[f] + Re\,X[f]\,Im\,H[f]$$

However, all of these are elementwise. So, O(N) operation, nice!

Now, Deconvolution.. Still, they are elementwise operations!

rectangular form. For instance, let's look at the *division* of one frequency domain signal by another. In polar form, the division of frequency domain signals is achieved by the inverse operations we used for multiplication. To calculate: $H[f] = Y[f]/X[f]$, divide the magnitudes and subtract the phases, i.e., $MagH[f] = MagY[f]/MagX[f]$, $PhaseH[f] = PhaseY[f] - PhaseX[f]$. In rectangular form this becomes:

EQUATION 9-2
Division of frequency domain signals in rectangular form, where: $H[f] = Y[f]/X[f]$.

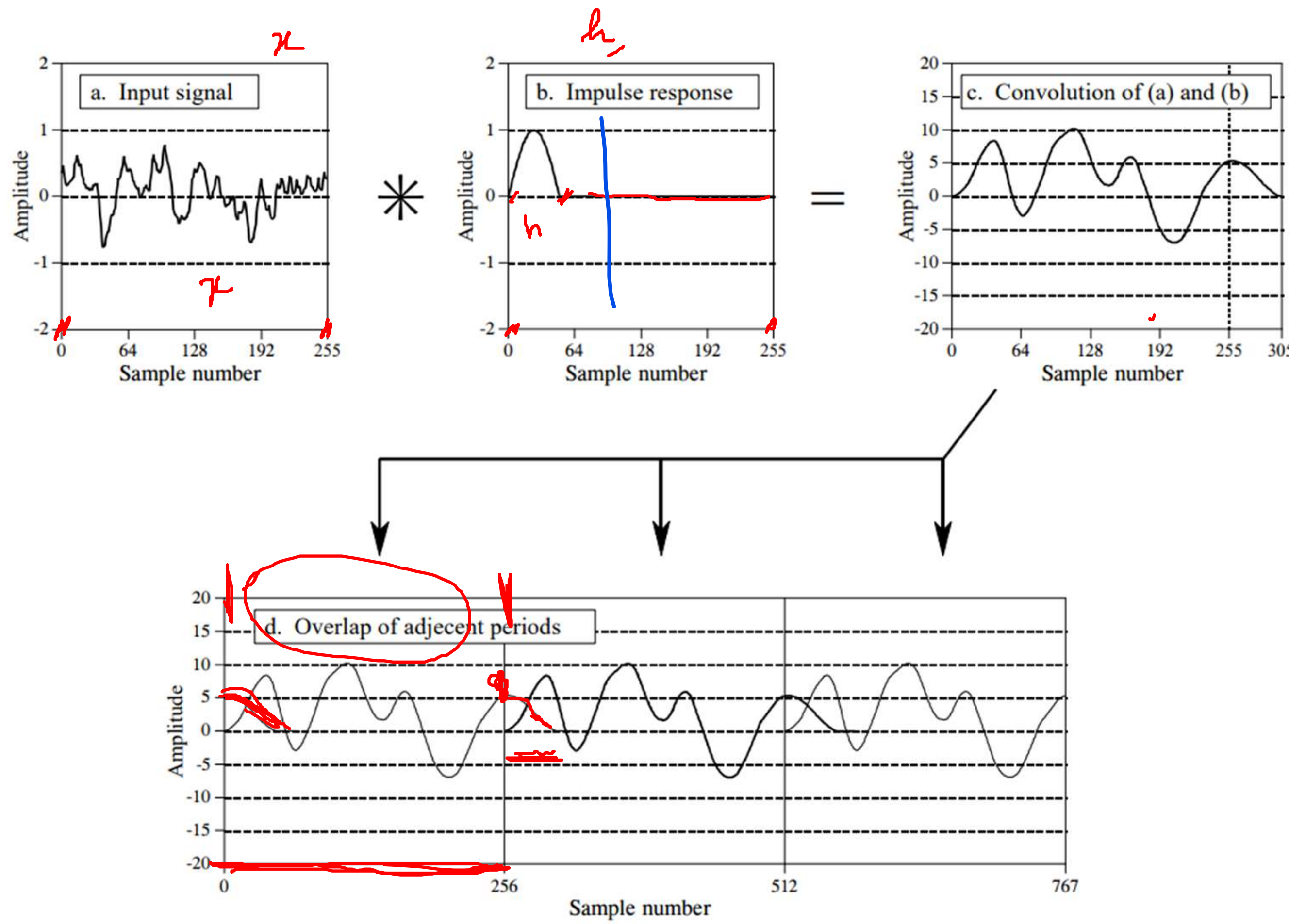
$$ReH[f] = \frac{ReY[f]\,ReX[f] + ImY[f]\,ImX[f]}{ReX[f]^2 + ImX[f]^2}$$

$$ImH[f] = \frac{ImY[f]\,ReX[f] - ReY[f]\,ImX[f]}{ReX[f]^2 + ImX[f]^2}$$

A small caveat/side-effect of Convolution in frequency domain

$X \otimes H = Y \rightarrow y$

N N N N



Circular Convolution

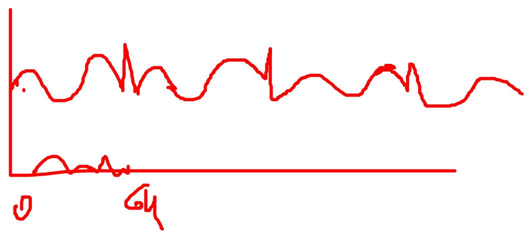
x h

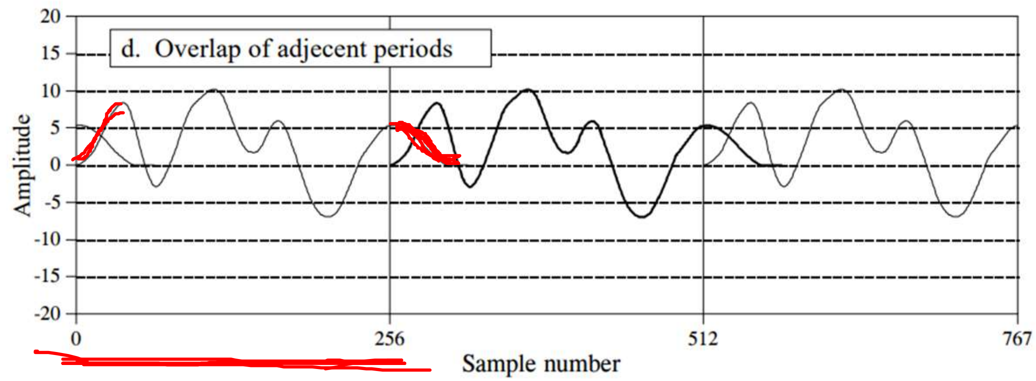
N m N

y

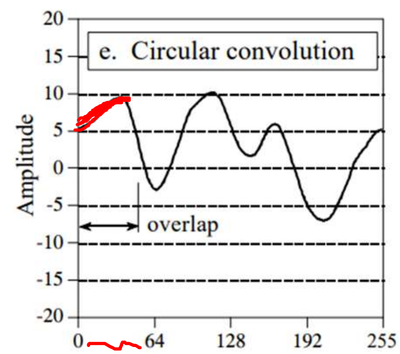
$\boxed{N + m - 1}$

Why is this happening?



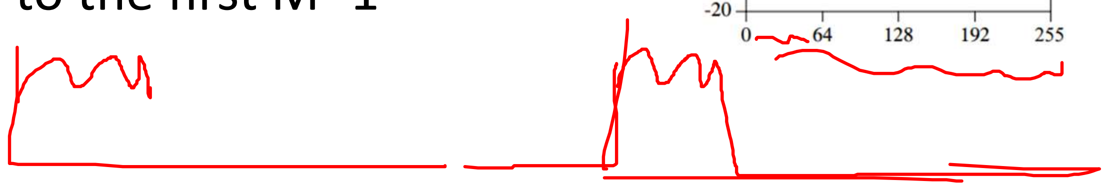


- Because, the output of convolution needs N+M-1 samples which don't fit in this N sampled output.



M - 1

- Secondly, **DFT views a signal as periodic with period N.** That's why the output that exceeds N wraps around and gets added to the first M -1 values.

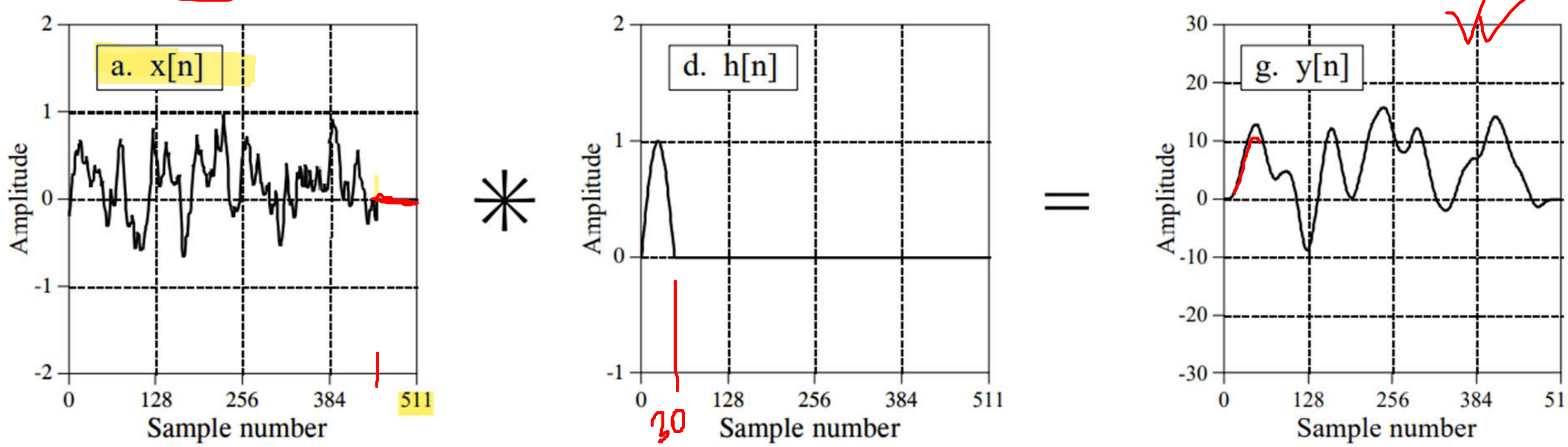


How to avoid this side-effect

- Pad **M-1** zeros to the signal X of size **N,** and make it equal to N+M-1. Then, perform convolution using DFT.

469

469



30

440 sample

$N = 440$

$M = 30$

$440 + 30 - 1$
$= 469$

Reading Assignment: Chapter **9**

Thank You.