Solutions to Problems in Chapter 12 of

Simulation Modeling and Analysis, 5th ed., 2015, McGraw-Hill, New York

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12.1. We used a single stream of the linear congruential generator defined by $Z_i = 16{,}807Z_{i-1} \bmod (2^{31} - 1)$ with $Z_0 = 12{,}345{,}678$ for all sources of randomness. Let m be the code for the machine type and r be the code for the repairman type. Using the same design matrix as in Table 12.2 (but with factors s, m, and r, in that order), we replicated the entire design 5 times (with 8 simulations per replication), making all 40 runs independently, and used the t distribution with 4 df to obtain

Effect	90 % confidence interval for expected effect
s	23.13 ± 1.68
m	3.35 ± 2.37
r	3.96 ± 2.56
$s \times m$	3.37 ± 2.54
$s \times r$	6.07 ± 2.53
$m \times r$	-0.88 ± 1.66
$s \times m \times r$	0.29 ± 2.87

The strong (and statistically significant) positive main effect of s indicates that hiring 4 rather than 2 repairmen would markedly increase costs. The main effects of m and r are also significant and positive (though not as strong as that of s), indicating that it would be better to buy standard machines and hire standard repairmen. Since the $s \times m$ interaction is significant and positive, the increase in cost due to buying deluxe rather than standard machines is steeper when we have more repairmen. (Alternatively, hiring more repairmen increases costs more rapidly when the machines are deluxe rather than standard.) The relatively strong $s \times r$ interaction means that the increase in cost due to hiring expert rather than standard repairmen is steeper when there are more repairmen to hire (obviously); alternatively, hiring more repairmen increases costs by more when they are expert rather than standard (just as obvious). Neither the $m \times r$ nor the $s \times m \times r$ interaction is statistically significant. (We also ran this design for 20 replications and reached conclusions similar to those above, except that the $s \times m$ interaction was weaker: 1.97 ± 1.49.) Finally, since interactions are present, we should be careful not to extrapolate or interpolate the main effects beyond the particular factor combinations in this experiment; see the discussion at the end of Sec. 12.2.

12.2. Let p denote the processing policy (original round robin, as in Sec. 2.5, or priority as in Prob. 2.18). The coding chart is

Factor	–	+
q	0.05	0.40
p	Original	Priority

and the design matrix is the same as Table 12.4, with q in place of s and p in place of d. We used the simlib code from Prob. 2.18, with the same stream assignments, and made 10 replications:

Replication	R_1	R_2	R_3	R_4	e_q	e_p	e_{qp}
1	14.79	4.32	11.61	4.37	–8.85	–1.57	1.61
2	14.12	4.47	8.31	4.99	–6.48	–2.65	3.16
3	12.54	9.50	9.46	5.22	–3.64	–3.68	–0.60
4	14.73	5.37	12.75	4.64	–8.74	–1.36	0.63
5	10.56	5.10	9.04	4.82	–4.84	–0.90	0.62
6	11.45	6.37	9.95	4.05	–5.49	–1.91	–0.42
7	11.16	6.38	9.64	4.55	–4.93	–1.68	–0.16
8	10.07	6.36	9.52	5.07	–4.09	–0.92	–0.37
9	12.72	6.27	11.18	7.10	–5.27	–0.35	1.19
10	12.04	7.59	8.00	4.58	–3.93	–3.53	0.51
90% c.i.	12.42±0.98	6.18±0.89	9.95±0.86	4.94±0.48	–5.63±1.08	–1.85±0.65	0.62±0.67

Since the experimental response is the average response time of a job, smaller is better. Thus, as the main effects of both q and p are significantly negative, it appears that they both should be at their "+" levels, i.e., the effects of increasing the quantum and adopting the priority discipline are both to improve average response time, with the quantum being the more important factor. The interaction between the two is comparatively small, and not quite statistically significant here.

12.3. (*a*) Let $v = \sum_{i=1}^{4} \text{Var}(R_i)$, which is not affected by whether we use CRN. Then

$$\text{Var}(e_s) = \frac{v}{4} + \frac{-C_{12} + C_{13} - C_{14} - C_{23} + C_{24} - C_{34}}{2}$$

$$\text{Var}(e_d) = \frac{v}{4} + \frac{C_{12} - C_{13} - C_{14} - C_{23} - C_{24} + C_{34}}{2}$$

$$\text{Var}(e_{sd}) = \frac{v}{4} + \frac{-C_{12} - C_{13} + C_{14} + C_{23} - C_{24} - C_{34}}{2}$$

Each of the above contains two C_{ij}'s that enter the formula positively [C_{13} and C_{24} for Var(e_s), C_{12} and C_{34} for Var(e_d), and C_{14} and C_{23} for Var(e_{sd})] as well as four C_{ij}'s that enter the formula negatively; since we are assuming that CRN works everywhere, all the C_{ij}'s are positive, so the negative entries are helpful and the positive entries are harmful. Whether the overall effect is to reduce or increase the variances of the effects estimates depends on the particular numerical values of these covariances. To illustrate that CRN can actually be either helpful or harmful overall, suppose that Var(R_i) = 10 for $1 \le i \le 4$; in this case the variance of each of the three effects estimates above would be 10 if we did independent sampling throughout, i.e., did not use CRN at all. If we used CRN across all 4 design points and the covariance matrix of the random vector $(R_1, R_2, R_3, R_4)^T$ (a superscript T denotes transposition) were actually

$$\mathbf{\Sigma} = \begin{bmatrix} 10 & 1 & 1 & 1 \\ 1 & 10 & 1 & 1 \\ 1 & 1 & 10 & 1 \\ 1 & 1 & 1 & 10 \end{bmatrix}$$

then the variance of each of the three effects estimates would be reduced to 9, i.e., CRN would indeed reduce the variance of all effects estimates. (The eigenvalues of Σ are 13, 9, 9, and 9, which are all positive, so Σ is positive definite and thus is a legitimate covariance matrix.) But if the covariance matrix were instead

$$\mathbf{\Sigma} = \begin{bmatrix} 10 & 1 & 3 & 1 \\ 1 & 10 & 1 & 3 \\ 3 & 1 & 10 & 1 \\ 1 & 3 & 1 & 10 \end{bmatrix}$$

then Var(e_s) would be increased to 11, i.e., CRN backfires from the point of view of estimating the main effect of s. Similarly, if

$$\mathbf{\Sigma} = \begin{bmatrix} 10 & 3 & 1 & 1 \\ 3 & 10 & 1 & 1 \\ 1 & 1 & 10 & 3 \\ 1 & 1 & 3 & 10 \end{bmatrix}$$

then Var(e_d) would be increased to 11, and if

$$\mathbf{\Sigma} = \begin{bmatrix} 10 & 1 & 1 & 3 \\ 1 & 10 & 3 & 1 \\ 1 & 3 & 10 & 1 \\ 3 & 1 & 1 & 10 \end{bmatrix}$$

Var(e_{sd}) would rise to 11. [The eigenvalues of the last three Σ's are all positive (15, 11, 7, and 7 in each case), so each of these Σ's is positive definite and is thus a legitimate covariance matrix.] Thus, CRN may or may not reduce the variances of the effects estimates, so should be used cautiously.

(*b*) If we want to reduce Var(e_s) and Var(e_d) as expressed in (*a*), we see that e_s and e_d conflict in terms of the desired signs of C_{12}, C_{13}, C_{24} and C_{34}, but they agree that C_{23} and C_{14} should be positive. Thus, we could use CRN in the simulations generating R_2 and R_3, making $C_{23} > 0$. We could then use a *separate* set of CRN to run the simulations for both R_1 and R_4, making $C_{14} > 0$. Now since R_1 and R_2 were obtained from separate random-number sets, they are independent and so $C_{12} = 0$; similarly, $C_{13} = C_{24} = C_{34} = 0$. Thus, we can arrange the runs to use CRN part of the time to get the positive covariances that have the effect of reducing both Var(e_s) and Var(e_d), and use independent sampling part of the time to avoid covariances on which there is a conflict. Note that this all works to the disadvantage of Var(e_{sd}), which would like C_{23} and C_{14} both to be negative, so we will for sure hurt the precision of the interaction-effect estimate in this scheme.

12.4. To be consistent with Example 9.29, we use a run length of 65 seconds and a warmup period of 5 seconds. We also use a constant processing time of 2.5 milliseconds (the correct value), rather than the 3 milliseconds stated in Example 9.26 (see the errata for the book). In the table below, we give 90 percent confidence intervals for the 4 expected main effects and the 6 expected two-factor interaction effects.

Expected main effect	**90 percent confidence interval**	**Expected interaction effect**	**90 percent confidence interval**
$E(e_1)$	-1.489 ± 0.055	$E(e_{12})$	-0.036 ± 0.009
$E(e_2)$	-1.279 ± 0.034	$E(e_{13})$	-0.008 ± 0.005
$E(e_3)$	-0.456 ± 0.016	$E(e_{14})$	-0.000 ± 0.004
$E(e_4)$	-0.394 ± 0.010	$E(e_{23})$	-0.000 ± 0.005
-	-	$E(e_{24})$	-0.002 ± 0.004
-	-	$E(e_{34})$	-0.0054 ± 0.0047

The four main effects are statistically significant, and it appears that a third processor should be added to SP-1. Furthermore, three interaction effects are statistically significant, but all are small in magnitude. If we make each confidence interval at level 99 percent (so that the *overall* confidence level is 90 percent), then only one interaction effect is statistically significant.

12.5. We implemented common random by generating all five service times when a job arrived, and we used a different random-number stream for each of the six sources of randomness.

(*a*) In the table below, we give 90 percent confidence intervals for the 5 expected main effects and the 10 expected two-factor interaction effects.

Expected main effect	90 percent confidence interval	Expected interaction effect	90 percent confidence interval
$E(e_1)$	-0.528 ± 0.059	$E(e_{12})$	-0.002 ± 0.004
$E(e_2)$	-0.147 ± 0.007	$E(e_{13})$	-0.012 ± 0.024
$E(e_3)$	-1.344 ± 0.124	$E(e_{14})$	-0.004 ± 0.003
$E(e_4)$	-0.264 ± 0.014	$E(e_{15})$	-0.000 ± 0.002
$E(e_5)$	-0.077 ± 0.002	$E(e_{23})$	-0.006 ± 0.007
-	-	$E(e_{24})$	-0.001 ± 0.002
-	-	$E(e_{25})$	-0.001 ± 0.004
-	-	$E(e_{34})$	-0.000 ± 0.004
-	-	$E(e_{35})$	-0.000 ± 0.002
-	-	$E(e_{45})$	-0.000 ± 0.004

The five main effects are statistically significant. All two-factor interactions are small in magnitude, with only the 1×4 interaction being statistically significant. A second server should be added at station 3, where the mean service is the largest.

(*b*) The utilization factors for the five stations are 0.8, 0.6. 0.9. 0.7, and 0.5, respectively. Note that station 3 has the largest utilization and also the largest main effect in magnitude, etc.

(*c*) Since we can only add one more server to the entire system, it is sufficient to make simulation runs at only five design points (i.e., add a server to station 1, add a server to station 2, etc.)

12.6. (*a*) If we have 2 machines in station 2, then the maximum revenue for the manufacturing line is \$576,000 = (4 parts/hour)(24 hours/day)(30 days)(\$200). The total cost for 1, 2, 1, and 1 machines in stations 1 through 4, respectively, and 1 queue position for each of stations 2 through 4 is \$128,000. Thus, if there are 2 machines in station 2, then the mean profit cannot exceed \$448,000 = \$576,000 - \$128,000. However, we achieved a profit of \$578,400 in Example 12.8 with 3 machines in station 2.

(*b*) If we put 2 machines in station 3, its potential overall processing rate is 10 = 2(5), which is larger than the potential overall processing rate of 6 = 3(2) for station 2, which is the bottleneck. Adding a third machine to station 3 would give us more unneeded capacity and cost us \$25,000.

(*c*) If we have 2 rather than 3 machines in station 1, then station 2 is starved (idle) a larger percentage of the time. The revenue that we lose from this is larger than the \$25,000 cost for buying a third machine for station 1.

12.7. (*a*) We want to keep a large number of parts “staged” for station 2 (the bottleneck), so that it doesn’t become starved.

(*b*) We want to keep station 2 from becoming blocked.

12.8. Let $I_i = \begin{cases} 1 & \text{if } i\text{th confidence interval does not contain its measure of performance} \\ 0 & \text{otherwise} \end{cases}$

$N =$ number of confidence intervals that do not contain their respective measures

Then $N = \sum_{i=1}^{21} I_i$ and $E(N) = E\sum_{i=1}^{21} I_i = \sum_{i=1}^{21} E(I_i) = \sum_{i=1}^{21} 0.1 = 21(0.1) = 2.1$

12.9. Consider the linear equation $x_s = as + b$. If we want $x_s = -1$ when s = 20, then we get the following equation:

$$-1 = 20a + b \tag{1}$$

If we want $x_s = 1$ when s = 60, then we get the following equation:

$$1 = 60a + b \tag{2}$$

Solving equations (1) and (2) for *a* and *b*, we get *a* = 1/20 and *b* = -2.

12.10. The metamodel given by Eq. (12.7) only had to fit the response data at the *four* corner design points, whereas the metamodel given by Eq. (12.10) had to fit the response data at all *nine* design points.

12.11. Confidence intervals for the expected main effects are as follows:

Expected main effect	95 percent confidence interval
$E(e_s)$	7.76 ± 1.24
$E(e_d)$	-5.66 ± 0.55
$E(e_m)$	-1.09 ± 0.79
$E(e_P)$	31.40 ± 0.33

All main effects are statistically significant and, in particular, express delivery seems like a *bad idea*. Confidence intervals for expected two- and three-factor interactions are as follows:

Expected two-factor interaction effect	95 percent confidence interval
$E(e_{sd})$	14.23 ± 0.73
$E(e_{sm})$	-18.23 ± 0.60
$E(e_{sP})$	10.29 ± 0.37
$E(e_{dm})$	2.46 ± 0.67
$E(e_{dP})$	2.05 ± 0.26
$E(e_{mP})$	-5.59 ± 0.25

Expected three-factor interaction effect	95 percent confidence interval
$E(e_{sdm})$	7.28 ± 0.95
$E(e_{sdP})$	-4.48 ± 0.11
$E(e_{smP})$	1.94 ± 0.17
$E(e_{dmP})$	2.73 ± 0.29

All two- and three-factor interaction effects are statistically significant, and some are quite large in magnitude. A 95 percent confidence interval for $E(e_{sdmP})$ is -0.64 ± 0.20, so this four-factor interaction effect is statistically significant as well.

Since the main effect of m is only -1.09, it appears on the surface that this factor does not have much impact on average cost. However, since the $s \times m$ interaction is quite large, the situation needs to be looked at more closely. Suppose that $d = 50$ and P = Normal. If $s = 20$, then moving m from 1 to 2 increases the average cost by 16.48. On the other hand, if $s = 60$, then moving m from 1 to 2 increases the average cost by 20.41. Thus, moving factor m from 1 to 2 *does* have a significant impact on average cost, but the amount of the change depends on the level of factor s.