~~Lec-3-4~~

More of Exercise-1

Ex-1 (a) $\frac{(2+i)(3-2i)(1+2i)}{(1-i)^2}$

$= \frac{(6-4i+3i+2)(1+2i)}{(1-i)^2}$

$= \frac{(8-i)(1+2i)}{1-2i-1} = \frac{8+2-i+16i}{-2i} = \frac{10+15i}{-2i}$

$= 5i - \frac{15}{2}$ (Ans.) +verified

(b) $(2i-1)^2\left(\frac{4}{1-i} + \frac{2-i}{1+i}\right)$

$= (1-4i-4)\left(\frac{4+4i+2-2i-i-1}{1-i^2}\right)$

$= (-3-4i)\left(\frac{5+i}{2}\right)$

$= \frac{-15-20i-3i+4}{2} = \frac{-11}{2} - \frac{23}{2}i$ (Ans.) +verified

(c) $\frac{i^4+i^9+i^{16}}{2-i^5+i^{10}-i^{15}}$

$= \frac{1+i+1}{2-i-1+i} = 2+i$ (Ans.) +verified

(d) $|z_1\bar{z}_2 + z_2\bar{z}_1|$

$= |(x_1+iy_1)(x_2-iy_2) + (x_2+iy_2)(x_1-iy_1)|$

$= |x_1x_2 + y_1y_2 + i(y_1x_2 - y_2x_1) + x_2x_1 + y_1y_2$
$+ i(y_2x_1 - y_1x_1)|$

$= |2(x_1x_2 + y_1y_2)| = 2(x_1x_2 + y_1y_2)$ (Ans)

$\cancel{(e) \left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|}$

$\cancel{Q\ (d)\ |z_1\bar{z}_2 + \bar{z}_2\bar{z}_1|}$

$\cancel{= \overline{z_1\bar{z}_2} + \overline{z_2\bar{z}_1}}$

$\cancel{= \bar{z}_1 z_2 + z_2}$

(e) $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$

$= \frac{|x_1+x_2+1+i(y_1+y_2)|}{|x_1-x_2+i(y_1-y_2+1)|}$

$= \sqrt{\frac{(x_1+x_2+1)^2+(y_1+y_2)^2}{(x_1-x_2)^2+(y_1-y_2+1)^2}}$ (Ans.)

(f) $\frac{1}{2}\left(\frac{z_3}{\overline{z_3}} + \frac{\overline{z_3}}{z_3}\right)$

$$= \frac{1}{2}\left(\frac{z_3^2 + \overline{z_3}^2}{\overline{z_3}\, z_3}\right)$$

$$= \frac{2(z_3^2 + \overline{z_3}^2)}{4 z_3 \overline{z_3}}$$

$$= \frac{(z_3+\overline{z_3})^2 + (z_3-\overline{z_3})^2}{(z_3+\overline{z_3})^2 - (z_3-\overline{z_3})^2}$$

$$= \frac{(2x_3)^2 + (2y_3)^2}{(2x_3)^2 - (2y_3)^2}$$

$$= \frac{x_3^2 + y_3^2}{x_3^2 - y_3^2}$$ (Ans.)

Ex-2 $2x - 3iy + 4ix - 2y - 5 - 10i = (x+y+2) - (y-x+3)i$

$\Rightarrow (2x-2y-5) + (4x-3y-10)i = (x+y+2) + (x-y-3)i$

Comparing real and imaginary parts,

$2x - 2y - 5 = x + y + 2$

$\Rightarrow x - 3y = 7$ ——— (i)

and, $4x - 3y - 10 = x - y - 3$

$\Rightarrow 3x - 2y = 7$ ——— (ii)

Solving, we get,

$$\begin{array}{r} 3x - 9y = 21 \\ 3x - 2y = 7 \\ \hline -7y = 14 \end{array}$$

$\therefore y = -2$

So, $x + 6 = 7$

$\therefore x = 1$

Ans: $(1, -2)$

Ex-4 $\omega = 3iz - z^2 = 3i(x+iy) - (x+iy)^2$

$= 3ix - 3y - x^2 + y^2 - 2xyi$

$= -x^2 + y^2 - 3y + i(3x - 2xy)$

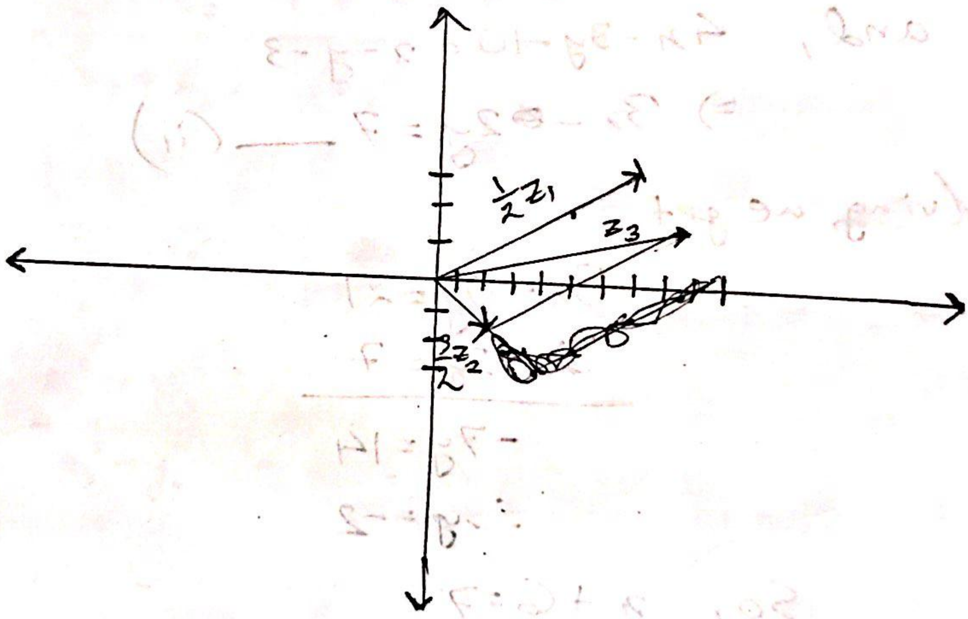
$|\omega|^2 = (-x^2 + y^2 - 3y)^2 + (3x - 2xy)^2$

$= x^4 + y^4 + 9y^2 - 6y^3 - 2x^2y^2 + 6x^2y$

$+ 9x^2 + 4x^2y^2 - 12x^2y$

$= x^4 + y^4 - 6y^3 + 2x^2y^2 - 6x^2y + 9x^2 + 9y^2$ (Ans)

Ex-5(b) $\frac{1}{2}(4-3i) + \frac{3}{2}(5+2i)$



case
centre is (0,1)

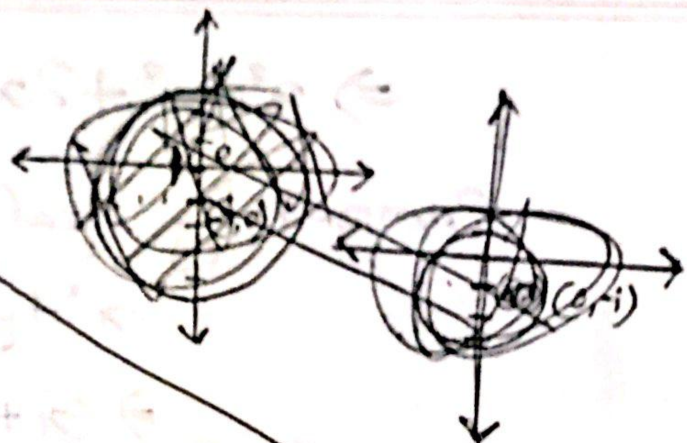
case $x^2+y^2=r^2$, So, radius of circle is 2

Imp

Ex-6 (a) $|z-i|=2$

$\Rightarrow |x+iy-i|=2$

$\Rightarrow x^2+(y-1)^2=4=2^2$

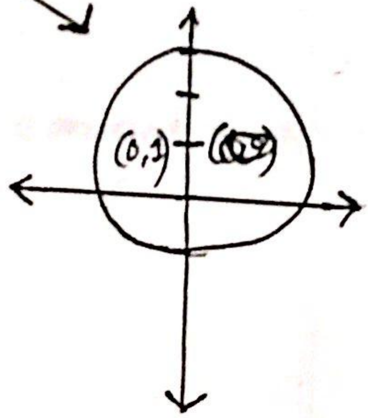


(b) (d) $z(\bar{z}+2)=3$

$\Rightarrow (x+iy)(x-iy+2)=3$

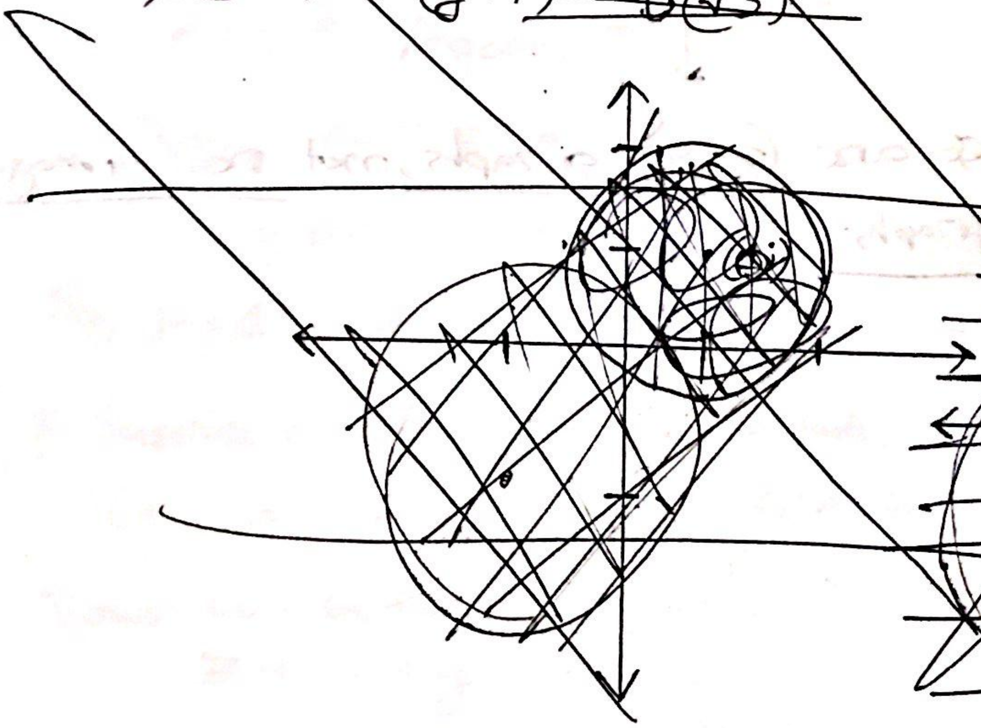
$\Rightarrow (x+iy)(x-iy)+2(x+iy)=3$

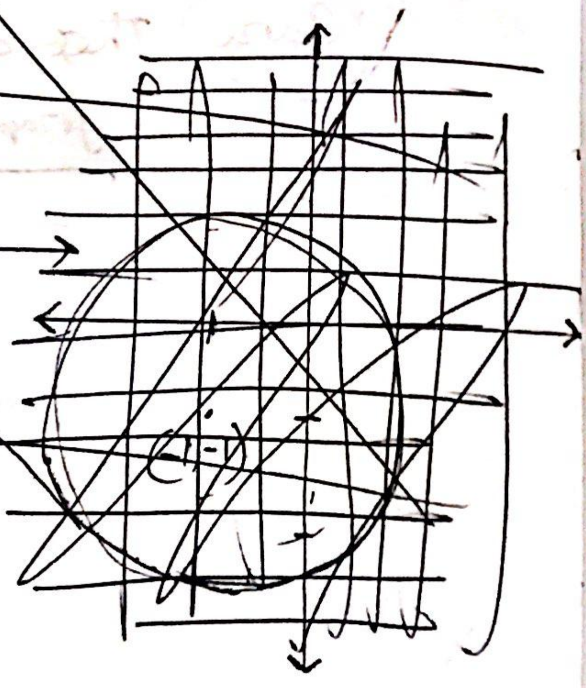
$\Rightarrow x^2+y^2+2x+2iy=3$



$\cancel{\Rightarrow x^2+2x+1+y^2+2yi+i^2=3}$

$\cancel{\Rightarrow (x+1)^2+(y+i)^2=3(\sqrt{3})^2}$



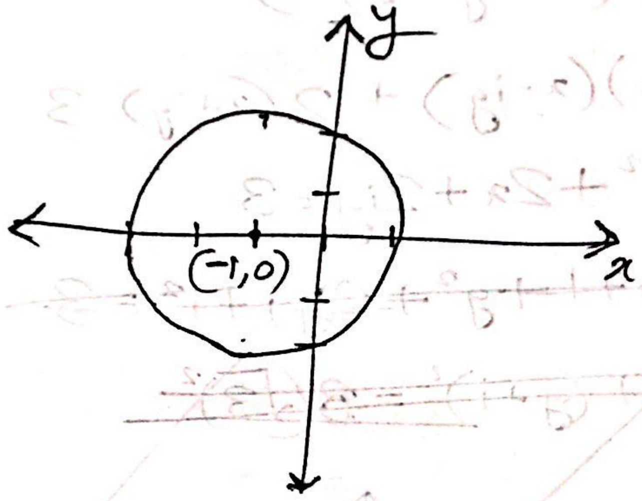


$\Rightarrow x^2+y^2+2x-3+2yi=0$

Comparing real parts, we get,

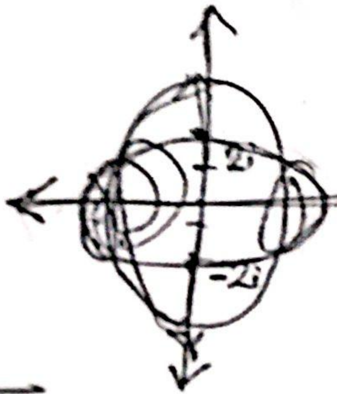
$$x^2+y^2+2x-3=0$$

$$\Rightarrow (x+1)^2+y^2=2^2$$



Care These are (x-y) graphs, not real-imaginary graphs.

Ex-6



(b) $|z+2i| + |z-2i| = 6$

$\Rightarrow |x+iy+2i| + |x+iy-2i| = 6$

$\Rightarrow \sqrt{x^2+(2+y)^2} = 6 + \sqrt{x^2+(y-2)^2}$

$\Rightarrow \cancel{x^2} + (2+y)^2 = 36 + 12\sqrt{x^2+(y-2)^2} + \cancel{x^2} + ($

$\Rightarrow (y+2)^2 - (y-2)^2 - 36 = 12\sqrt{x^2+(y-2)^2}$

$\Rightarrow 8y - 36 = 12\sqrt{x^2+(y-2)^2}$

$\Rightarrow \left(\frac{2}{3}y - 3\right)^2 = x^2 + (y-2)^2$

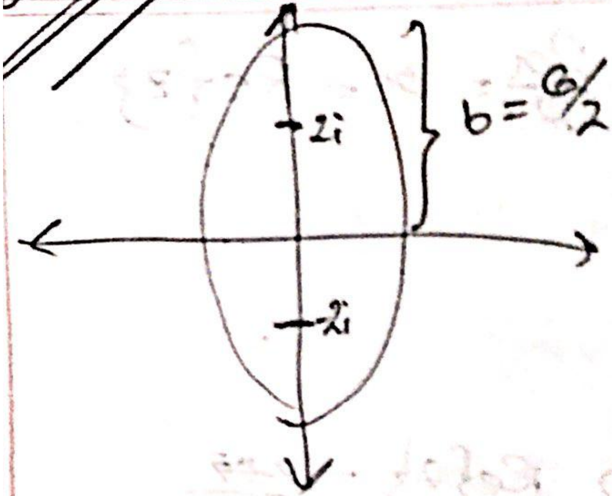
$\Rightarrow \frac{4}{9}y^2 - 4y + 9 = x^2 + (y-2)^2$

$\Rightarrow x^2 + y^2 - 4y + 4 = \frac{4}{9}y^2 \cancel{-4y} + 9$

$\Rightarrow x^2 + \frac{5}{9}y^2 = 5$

$\Rightarrow \frac{x^2}{5} + \frac{y^2}{9} = 1$ (Ans.)

o in Rough



$b = 6/2 = 3$

$\therefore 2b = 6$

$\Rightarrow b = 3$

$be = 2$

$\Rightarrow e = 2/3$

$\Rightarrow \sqrt{1 - a^2/b^2} = 2/3$

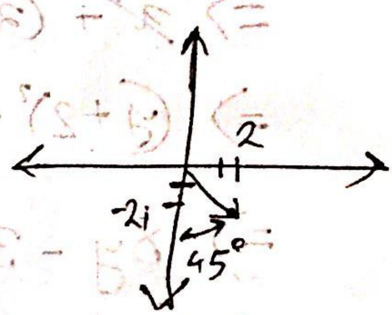
$\Rightarrow 1 - \frac{a^2}{9} = \frac{4}{9}$

$\Rightarrow a^2 = 5 \quad \therefore a = \sqrt{5}$

So, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{5} + \frac{y^2}{9} = 1$ (Ans:)

<u>Exercise-7</u>. (a) $2-2i$



$r = \sqrt{2^2+2^2} = 4$

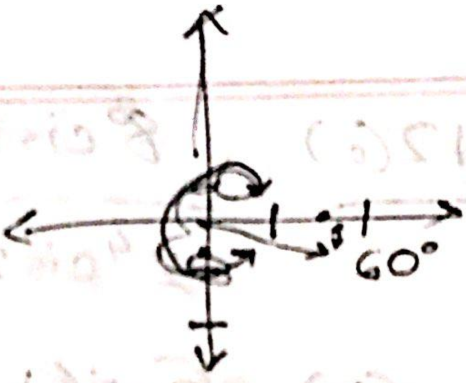
$\theta = \tan^{-1}\frac{-2}{2} = 2\pi - \pi/4$

$= 7\pi/4$

So, $2-2i = 4\cos 7\pi/4 + 4i\sin 7\pi/4$

$= r(\cos\theta + i\sin\theta) \longrightarrow$ always write formula first

(h) $\sqrt{3}/2 - 3i/2$



$r = \sqrt{\frac{9}{4} + \frac{9}{4}} = \sqrt{3}$

$\theta = \tan^{-1}\left(\frac{3/2}{-\sqrt{3}/2}\right) = -60°$

So, $\sqrt{3}/2 - \sqrt{3}i/2 = r\cos\theta + r\sin\theta$

$= \sqrt{3}\cos(-60°) + \sqrt{3}\sin(-60°)$ (Ans.)

Ex-8 $2+i$

Here, $r = \sqrt{4+1} = \sqrt{5}$ $\theta = \tan^{-1}\left(\frac{1}{2}\right)$

Now, $2+i = r(\cos\theta + i\sin\theta)$

From Euler's formula,

$e^{i\theta} = \cos\theta + i\sin\theta$

So, $2+i = re^{i\theta}$

$\Rightarrow 2+i = \sqrt{5}e^{i\tan^{-1}(1/2)}$ (proved)

Ex-11

$z_1 = 150\angle -45° = 150\cos 45 - 150\sin 45$

$z_2 = -100$

$z_3 = 225\angle 30° = 225\cos 30° + 225\sin 30°$

$z_4 = 200\angle 45° = 200\cos 45° + 200\sin 45°$

Add all,

12(a) $\dfrac{8^3 \operatorname{cis} 120^\circ}{2^4 \operatorname{cis} 240^\circ} = 32 \operatorname{cis}(-120^\circ)$

(b) $\dfrac{(3e^{\pi i/6})(2e^{-5\pi i/4})(6e^{5\pi i/3})}{(4e^{2\pi i/3})^2}$

$= \dfrac{36}{8} e^{i\left(\frac{\pi}{6} - \frac{5\pi}{4} + \frac{5\pi}{3} - \frac{4\pi}{3}\right)}$

$= \dfrac{9}{2} e^{i\left(-\frac{3}{4}\pi\right)}$ (Ans.)

(c) $r = \sqrt{3+1} = 2$ $\quad r = \sqrt{1+1} = \sqrt{2}$

$\theta = \tan^{-1} \dfrac{-1}{\sqrt{3}} = -30^\circ$ $\quad \theta = \tan^{-1}\left(\dfrac{-1}{1}\right) = -45^\circ$

So, $\left\{\dfrac{2 \operatorname{cis}(-30^\circ)}{2 \operatorname{cis}(30^\circ)}\right\}^4 \cdot \left\{\dfrac{\operatorname{cis}(45^\circ)}{\operatorname{cis}(-45^\circ)}\right\}^5$

$= \operatorname{cis}(-240^\circ) \cdot \operatorname{cis}(450^\circ)$

$= \operatorname{cis} 210^\circ$

$= \cos 210^\circ + \sin 210^\circ$

$= -\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}$ (Ans.)

Roots of Complex Number:

13(b) $(-4+4i)^{1/5}$

$r = \sqrt{4^2+4^2} = 4\sqrt{2}$

$\theta = \tan^{-1}\frac{4}{-4} = \pi - \pi/4 = \frac{3\pi}{4}$

So, $(-4+4i) = r\cos\theta + ir\sin\theta$

$= 4\sqrt{2}\cos\left(\frac{3\pi}{4}+2\pi k\right) + i\,4\sqrt{2}\sin\left(\frac{3\pi}{4}+2\pi k\right)$

$= 4\sqrt{2}\, e^{\frac{3\pi i}{4}+2\pi i k}$

$(-4+4i)^{1/5} = (4\sqrt{2})^{1/5} \cdot e^{i\left(\frac{3\pi+8\pi k}{4}\right)/5}$

$= \sqrt{2} \cdot e^{i\frac{3\pi+8\pi k}{20}}, \quad k = 0, 1, 2, 3, 4$

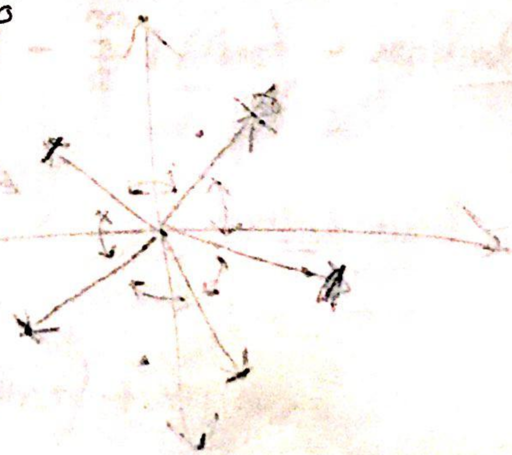
For $k=0$, $\sqrt{2} \cdot e^{i\frac{3\pi}{20}}$

$k=1$, $\sqrt{2} \cdot e^{i\frac{11\pi}{20}}$

$k=2$, $\sqrt{2}\, e^{i\frac{19\pi}{20}}$

$k=3$, $\sqrt{2}\, e^{i\frac{27\pi}{20}}$

$k=4$, $\sqrt{2}\, e^{i\frac{7\pi}{4}}$



Imp.

Ex-15 (b) $z^6 + 1 = \sqrt{3}i$

~~$\Rightarrow z^6 - \sqrt{3}i + 1 = 0$~~

~~$\Rightarrow z^6 - 2 \cdot \frac{\sqrt{3}}{2} \cdot i + \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} + 1 = 0$~~

~~$\Rightarrow \left(z^3 - \frac{\sqrt{3}}{2}\right)^2 = -\frac{1}{4}$~~

~~$\Rightarrow z^3 - \frac{\sqrt{3}}{2} = \pm \frac{i}{2}$~~

~~Either, $z^3 - \frac{\sqrt{3}}{2} = \frac{i}{2}$ $\quad z^3 - \frac{\sqrt{3}}{2} = -\frac{i}{2}$~~

$$z = \sqrt[6]{-1 + \sqrt{3}i}$$

then solve roots

Graphically the roots are always equally distributed