Homework-02

(a) State Variables

$l(t)$ → no. of newspapers at time t.

Output Variables

(i) average ~~ordering~~ purchase cost
(ii) average holding and shortage cost
(iii) average revenue from selling
(iv) average revenue from scrap
(v) average profit

System Variables

S_H → inventory capacity
S_L → ordering threshold
S_D → newspaper demand

(b) Set of events

(i) purchase newspaper (P)
(ii) sell newspaper (S)
(iii) sell scrap (C)
(iv) termination event (T)
(v) Evaluation (E)

(c) State equations

$[10Z = S_D]$

$$l(t^+) = \begin{cases} l(t) + 10Z & \text{for purchase} \\ l(t) - 1 & \text{for sell newspaper} \\ l(t) & \text{o/w} \end{cases}$$

Z → no of bundles

When purchased, it is done by bundles of 10.

Let, Z be a natural number of bundles of newspaper purchased.

When newspaper are sold at the end of the day, the inventory will be empty

Output Equations:

(i) Average Purchase Cost: (Job-average)

Given, the purchase cost of 1 newspaper is 50 cents.

When $\ell(t) < S_L$, the event will be called.

$S_D \longrightarrow$ Newspaper demand @ (no. to be bought)

$S_D = 0, 10, 20 \dots\dots < S_M$.

$K \longrightarrow$ ordering cost

Purchase cost, $C_0 = C_0 + (K + 50 S_D)$

Average purchase cost, $\bar{c} = \dfrac{C_0}{n}$ [No of evaluation months]

(ii) Average Shortage Cost: (Time Average)

Shortage cost is 17 cents.

Let, $\ell^-(t) = \max(-\ell(t), 0)$

No. of newspapers shortage in $(\tau, 0)$ is $\bar{\ell}^- = \dfrac{\int_0^\tau \ell^-(t)\,dt}{\tau}$

Average shortage cost is

$$17\bar{\ell}^- = 17 \times \frac{\int_0^\tau \ell^-(t)\,dt}{\tau}$$

(iii) Average Holding Cost: (Time Average)

Let, holding cost per newspaper be h.

Positive ~~line~~ inventory is $\ell^+(t) = \max(\ell(t), 0)$

In $(\tau, 0)$ time, average no. of newspapers in inventory is $\bar{\ell}^+ = \dfrac{\int_0^\tau \ell^+(t)\,dt}{\tau}$.

So, average holding cost, $h\bar{\ell}^+ = h \times \dfrac{\int_0^\tau \ell^+(t)\,dt}{\tau}$

(iv) Average Revenue from Selling Newspaper (Job Average)

Assuming every customer buys 1 newspaper,

revenue from selling newspapers,

If $l(t) > 0$, $r_0 = r_0 + 50$

Average revenue, $\bar{r}_0 = \frac{r_0}{n}$ [n = no. of evaluation months]

(v) Average Scrap Price (Job average)

The scrap price of each newspaper is 5 cents.

If $l(t) > 0$,

scrap price, $s_0 = s_0 + 5l(t)$.

Or, in general we can write, without condition,

$$S_0 = s_0 + 5 \max(l(t), 0)$$

Average scrap price,

$$\bar{S} = \frac{S_0}{n}$$ [n = no. of evaluation months]

(vi) Average ~~Revenue~~ Profit,

Average Profit, $\bar{p} = \bar{r} + \bar{s} - \bar{c} - h\bar{l} - 17\,\bar{l}^{-}$ cents.

Profit is the summation of revenue generated from selling newspapers and scraps minus ordering cost, holding cost and shortage cost.

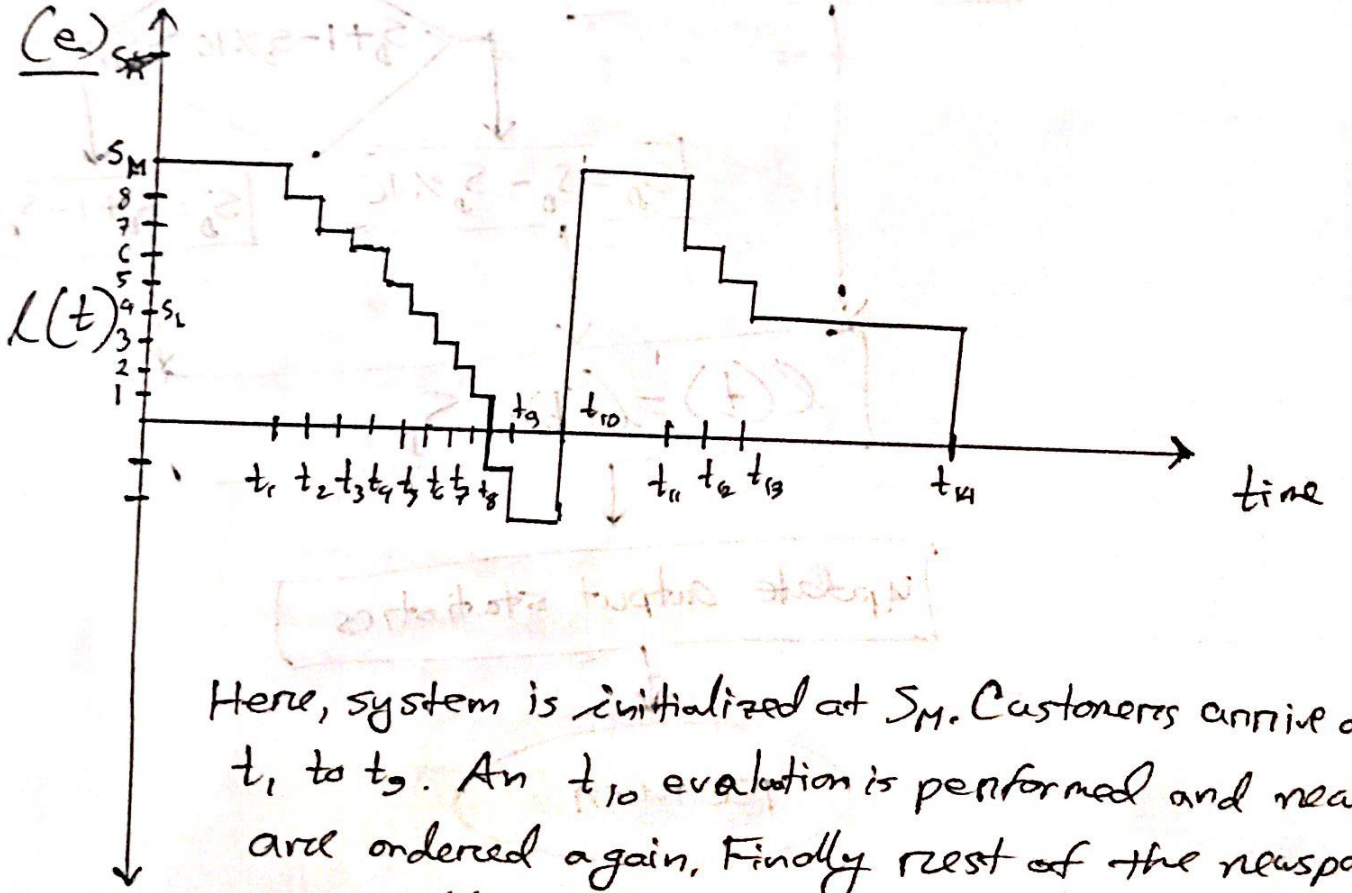
(d) State Space:

The state variable is $l(t)$.

The state space, $X_l = \{\ldots -2, -1, 0, 1, 2 \ldots \ldots S_M\}$

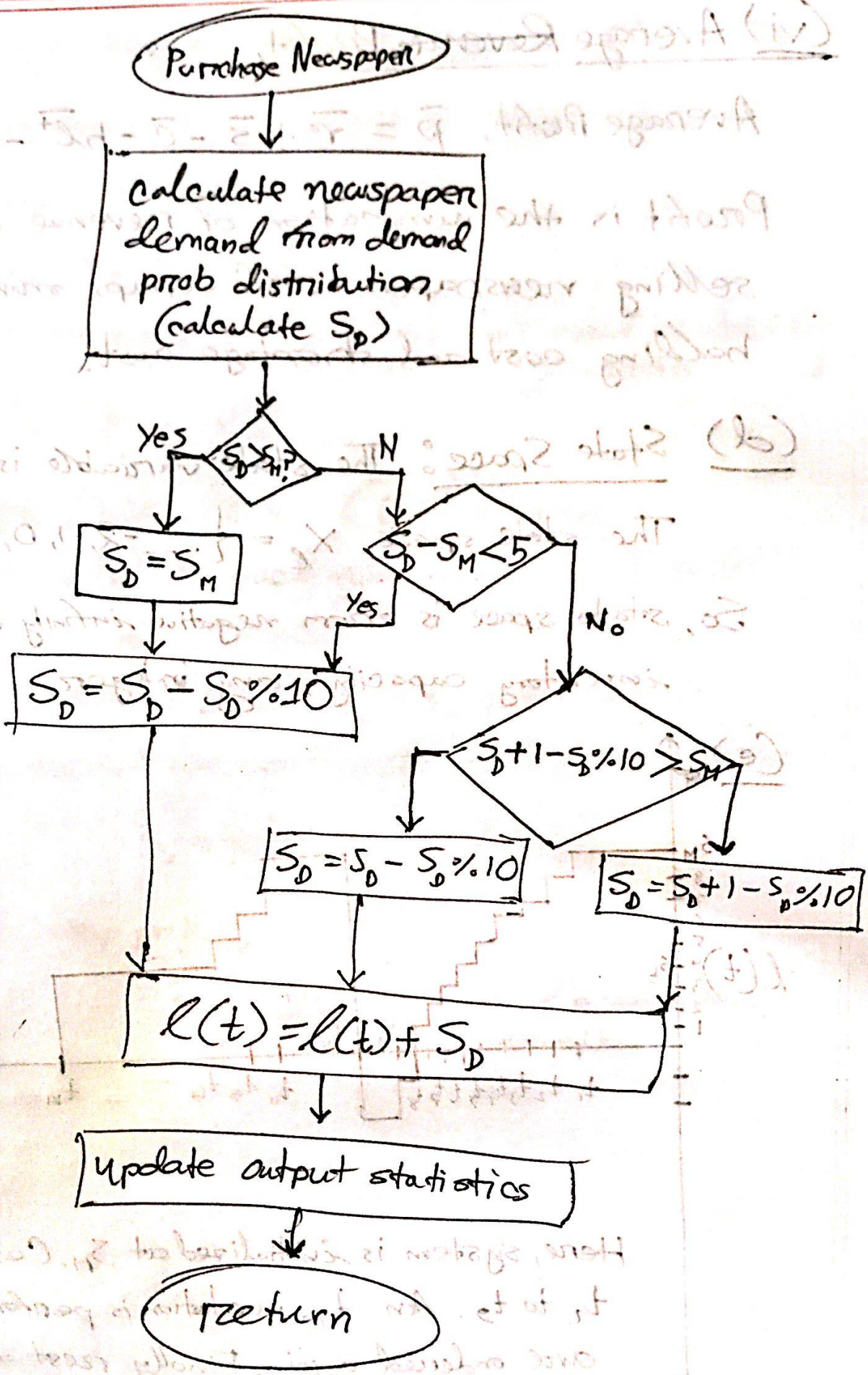
So, state space is from negative infinity upto maximum inventory capacity; any integers.

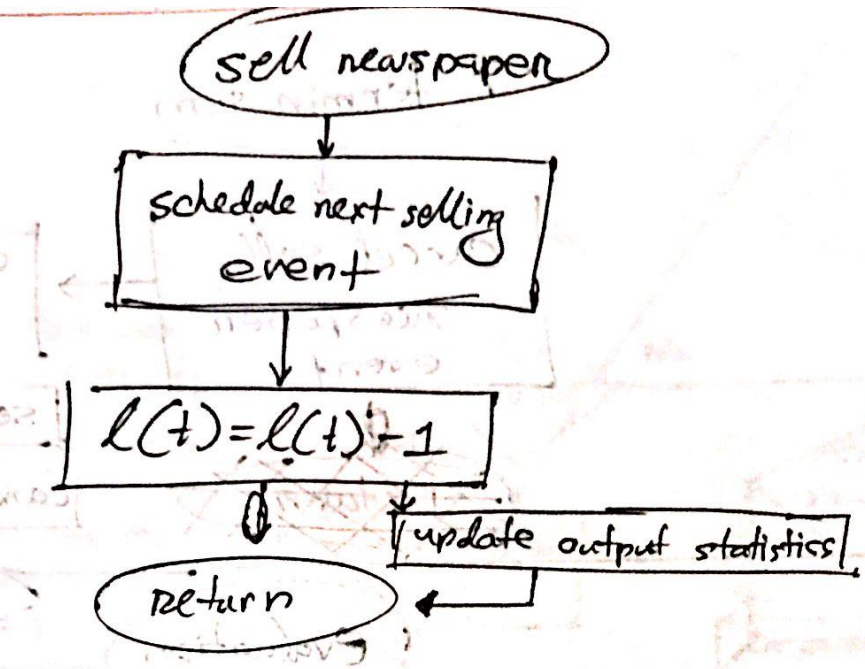
(e)

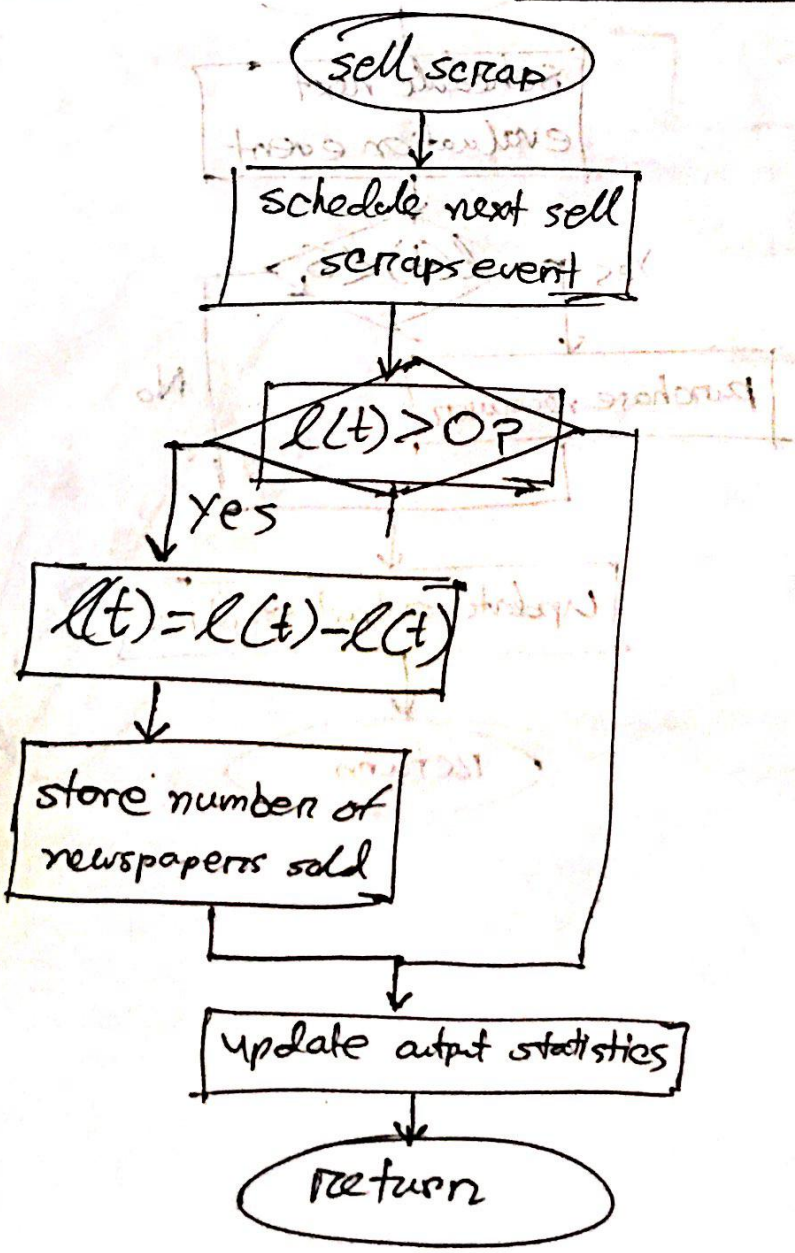


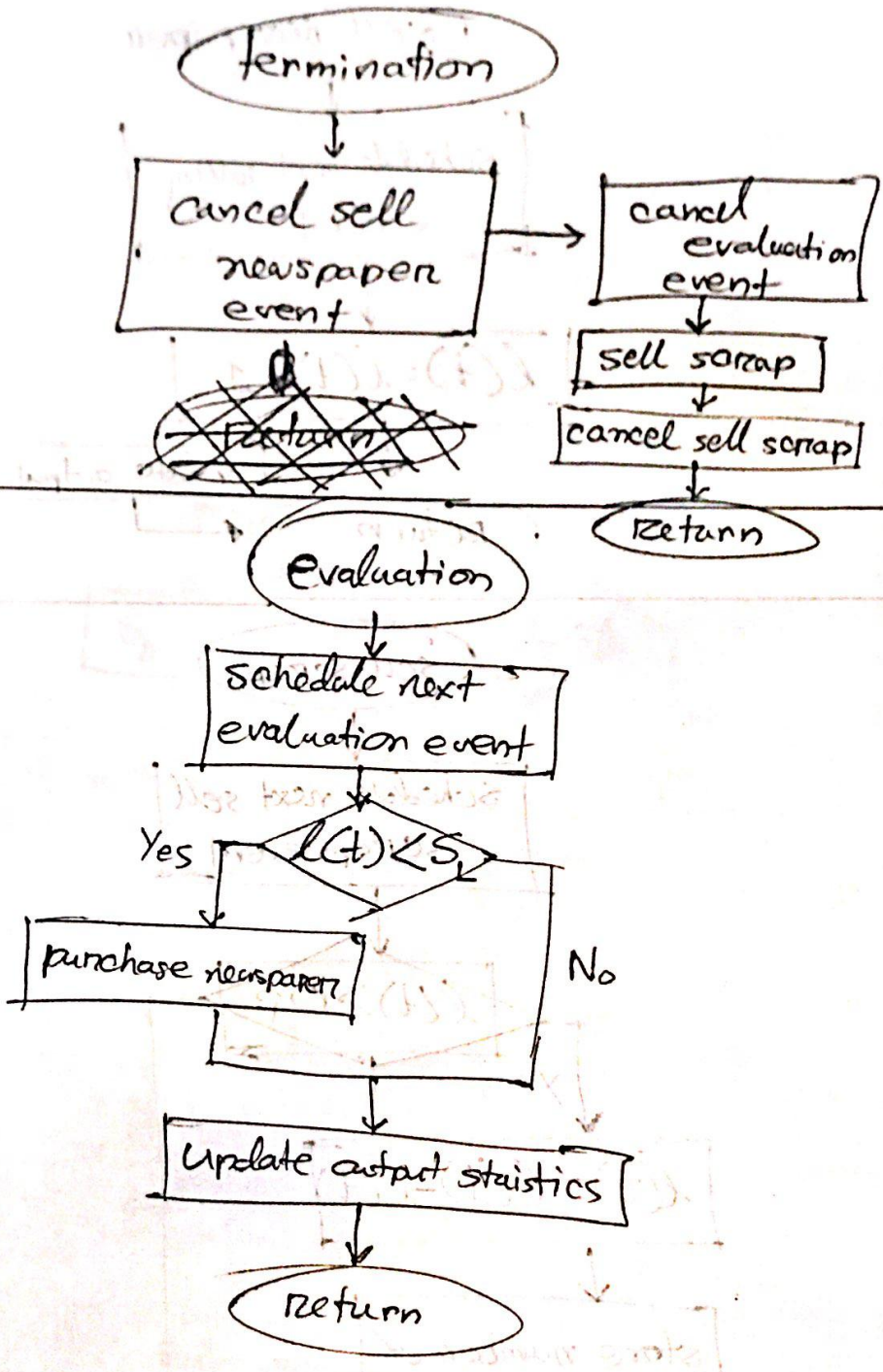
Here, system is initialized at S_M. Customers arrive at t_1 to t_9. An t_{10} evaluation is performed and newspapers are ordered again. Finally rest of the newspapers are sold as scrap at the end of the day.

(f)

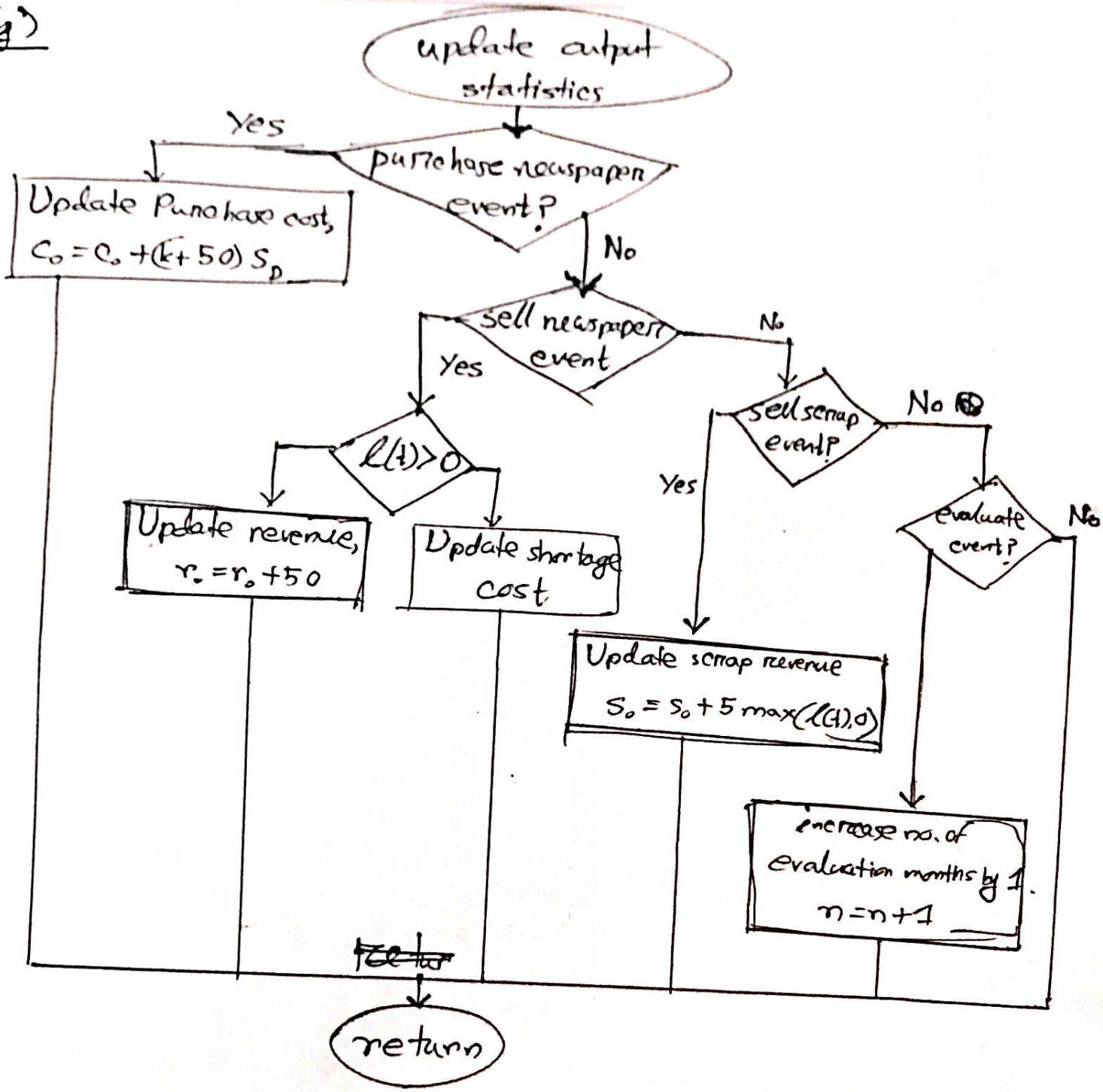

Purchase Newspaper
Calculate newspaper demand from demand prob distribution. (calculate S_D)
$S_D > S_M$?
Yes
N
$S_D = S_M$
$S_D - S_M < 5$
Yes
No
$S_D = S_D - S_D \% 10$
$S_D + 1 - S_D \% 10 > S_M$
$S_D = S_D - S_D \% 10$
$S_D = S_D + 1 - S_D \% 10$
$\ell(t) = \ell(t) + S_D$
Update output statistics
return


sell newspaper
schedule next selling event
l(t)=l(t)-1
update output statistics
return


sell scrap
schedule next sell scraps event
l(t)>0?
yes
l(t)=l(t)-l(t)
store number of newspapers sold
update output statistics
return


termination
cancel sell newspaper event
cancel evaluation event
sell scrap
cancel sell scrap
return
return
Evaluation
schedule next evaluation event
Yes
I(t) < S
No
purchase newspaper
Update output statistics
return

(g)


update output statistics
purchase newspaper event?
Yes
No
Update Purchase cost, $C_0 = C_0 + (k+50) S_D$
sell newspaper event
Yes
No
$\ell(1) > 0$
Update revenue, $r_0 = r_0 + 50$
Update shortage cost
sell scrap event?
Yes
No
Update scrap revenue $S_0 = S_0 + 5 \max(\ell(1), 0)$
evaluate event?
No
increase no. of evaluation months by 1. $n = n + 1$
return