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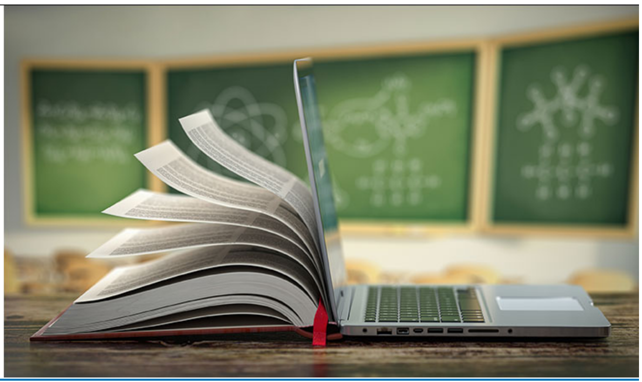

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A study on single and multi server queuing models using interval number

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Abstract: Queuing theory is a quantitative technique which consists in constructing mathematical models of various types of queuing systems. These models can be used for making predictions about how the system can adjust with demands. Queuing theory deals with analysis of queues and queuing behavior. In this paper, we proposed the single and multi server queuing model with interval numbers to deals with uncertain parameters. The Arrival rate and service rates are considered as interval number and we have used new interval arithmetic procedure to study the characteristics of queuing models. Numerical illustration is also given to validate the proposed model.

1. Introduction:

Queuing model [1] is a mathematical model and it has wide applications in service organizations such as healthcare services as well as manufacturing firms, in that various types of customers are serviced by various types of servers according to specific queue disciplines. In computer systems, queuing theory is quite useful to estimate the value of some computer performance measures.

Queuing theory is the mathematical study of waiting lines, or queues. In queuing theory a model is constructed so that queue lengths and waiting times can be predicted. Queuing theory is generally considered a branch of operations research because the results are often used when making business decisions about the resources needed to provide a service.

Queuing theory has its origins in research by A K Erlang when he created models to describe the Copenhagen telephone exchange. The ideas have since seen applications including telecommunications, traffic engineering, computing and the design of factories, shops, offices and hospitals.

Fuzzy queuing model was first introduced by R J Lie and E S Lee in 1989 further developed this model by many authors J J Buckley [4] in 1990, S Thamotharan [8] in 2013, , G Ramesh and K Ganesan [9] in 2015, G Ramesh and K Ganesan[10] in 2012, Ming Ma,Menahem friedman and Abraham Kandel[13] in 83-90, K.Ganesan and P Veeramani[14] in 2005,S shanmugasundaram,S thamotharanans M Ragapriya[16] in 2015.

Our aim of this paper, we proposed the single and multi server queuing model with interval number to deals with uncertain parameters.

2. Preliminaries

The aim of this section is to present some notations, notions and results which are of useful in our further consideration.

Let IR= { $\tilde{a} = [a_1, a_2] : a_1 \leq a_2$ and $a_1, a_2 \in R\}$ be the set of all proper intervals and $\overline{IR} = \{\tilde{a} = [a_1, a_2] : a_1 > a_2$ and $a_1, a_2 \in R\}$ be the set of all improper intervals on the real line R. If $\tilde{a} = a_1 = a_2 = a$, then $\tilde{a} = [a, a] = a$ is a real number (or a degenerate interval). We shall use the terms "interval" and "interval number" interchangeably. The mid-point and width (or half-width) of an interval number are defined as the mid-point and width (or half-width) of an interval number $\tilde{a} = [a_1, a_2]$ are defined as $m(\tilde{a}) = \left(\frac{a_1 + a_2}{2}\right)$ and $w(\tilde{a}) = \left(\frac{a_2 - a_1}{2}\right)$. The interval number $\tilde{a}$ can also be expressed in terms of its midpoint and width as $\tilde{a} = [a_1, a_2] = \langle m(\tilde{a}), w(\tilde{a})\rangle$

3. New interval arithmetic

Ming Ma et.al. [12] Have proposed a new fuzzy arithmetic based upon both location index and fuzziness index function. The location index number is taken in the ordinary arithmetic, where as the fuzziness index functions are considered to follow the lattice rules which are the least upper bound and greatest lower bound in the lattice L. That is for a, b $\in$ L we define a$\vee$b=max {a, b} and a$\vee$b=min {a, b}.

For any two intervals $\tilde{a} = [a_1, a_2], \tilde{b} = [b_1, b_2] \in IR$ and for $* \in \{+, \_, * \div\}$, the arithmetic operations on $\tilde{a}$ And $\tilde{b}$ are defined as:

$$\tilde{a} * \tilde{b} = [a_1, a_2] * [b_1, b_2] = \langle m(\tilde{a}), w(\tilde{a})\rangle * \langle m(\tilde{b}), w(\tilde{b})\rangle = \langle m(\tilde{a}) * m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b})\}\rangle$$

In particular

Addition: $\tilde{a} + \tilde{b} = \langle m(\tilde{a}), w(\tilde{a})\rangle + \langle m(\tilde{b}), w(\tilde{b})\rangle = \langle m(\tilde{a}) + m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b})\}\rangle$

Subtraction: $\tilde{a} - \tilde{b} = \langle m(\tilde{a}), w(\tilde{a})\rangle - \langle m(\tilde{b}), w(\tilde{b})\rangle = \langle m(\tilde{a}) - m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b})\}\rangle$

Multiplication: $\tilde{a} \times \tilde{b} = \langle m(\tilde{a}), w(\tilde{a})\rangle \times \langle m(\tilde{b}), w(\tilde{b})\rangle = \langle m(\tilde{a}) \times m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b})\}\rangle$

Division: $\tilde{a} \div \tilde{b} = \langle m(\tilde{a}), w(\tilde{a})\rangle \div \langle m(\tilde{b}), w(\tilde{b})\rangle = \langle m(\tilde{a}) \div m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b})\}\rangle$, provided $m(\tilde{b}) \approx \tilde{0}$

4. Symbols and notations:

n = Total number of customers in the system, both waiting and in service.
μ = Average number of customers being serviced per unit of time.
λ =Average number of customers arriving per unit of time.
C= Number of parallel service channels.
L_S = The average number of customers in the system, both waiting in the service.
L_q = The Average number of customers waiting in the queue.

W_S = Average waiting time of a customer in the system both waiting and in service.

W_q = Average waiting time of a customer in the queue.

P_n = Probability that there are n customers in the queue.

5. THE (FM/FM/1) :(∞/FCFS) QUEUE

In this paper we consider an infinite source population with first come first served discipline where both the inter arrival time λ and service time μ follow an exponential distribution.

Traffic intensity $(P) = \frac{\lambda}{\mu}$

Probability of system is ideal $(P_0) = 1 - P = P_0 = 1 - \frac{\lambda}{\mu}$

Probability of n units in the system (waiting time and service time) $P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$

The expected number of customer in the system $L_s = \frac{\lambda}{\mu - \lambda}$

The expected number of customers in the queue $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$

The average waiting time in the system $W_s = \frac{1}{\mu - \lambda}$

The average waiting time of a customer in the queue $W_q = \frac{\lambda}{\mu(\mu - \lambda)}$

$\text{Expected length of non-empty quie } \left(L_{neq}\right) = \frac{\mu}{\mu - \lambda}$

$\text{The probability track that K or more than K customers in the system } P \geq K = \left(\frac{\lambda}{\mu}\right)^k$

$\text{The probabilitythat more than K customers are in the system } (P > K) = \left(\frac{\lambda}{\mu}\right)^{K+1}$

$\text{The probability that atleast one customer is s tan ding in quie } P = K = \left(\frac{\lambda}{\mu}\right)^2$

6. The (FM/FM/C) :(FCFS/∞/∞)

Multi server queue has two or more service facility in parallel providing identical service. All the customers in the waiting line can be served by more than one station. The arrival time and the service time follow poison and exponential distribution.

The performance measures of the multi-server queuing system are,

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Expected number of customers in the system $L_S = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^C}{(C-1)!(C\mu-\lambda)^2}P_0 + \frac{\lambda}{\mu}$

Expected number of customers waiting in the queue $L_q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^C}{(C-1)!(C\mu-\lambda)^2}P_0$

Average time a customer spends in the system $W_S = \frac{\mu\left(\frac{\lambda}{\mu}\right)^C}{(C-1)!(C\mu-\lambda)^2}P_0 + \frac{1}{\mu}$

Average waiting time of a customer in the queue $W_q = \frac{\mu\left(\frac{\lambda}{\mu}\right)^C}{(C-1)!(C\mu-\lambda)^2}P_0$

Here

$$P_0 = \frac{1}{\sum_{n=0}^{c-1}\frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^C}{C!}.\frac{C\mu}{C\mu-\lambda}}$$

7. Numerical example:

Consider a FM/FM/1 queue, where the both the arrival rate and service rate are interval numbers represented by $\lambda = [1,3]$ $\mu = [11,13]$

$$\tilde{a}+\tilde{b} = \langle m(\tilde{a}), w(\tilde{a})\rangle + \langle m(\tilde{b}), w(\tilde{b})\rangle = \langle m(\tilde{a}) + m(\tilde{b}), \max\{ w(\tilde{a}), w(\tilde{b})\}\rangle = \langle 2+12, \max\{1,1\}\rangle$$

$$\tilde{a}-\tilde{b} = \langle m(\tilde{a}), w(\tilde{a})\rangle - \langle m(\tilde{b}), w(\tilde{b})\rangle = \langle m(\tilde{a}) - m(\tilde{b}), \max\{ w(\tilde{a}), w(\tilde{b})\}\rangle = \langle -10,1\rangle$$

$$\tilde{a}\times\tilde{b} = \langle m(\tilde{a}), w(\tilde{a})\rangle \times \langle m(\tilde{b}), w(\tilde{b})\rangle = \langle m(\tilde{a}) \times m(\tilde{b}), \max\{ w(\tilde{a}), w(\tilde{b})\}\rangle = \langle 24,1\rangle$$

$$\tilde{a}\div\tilde{b} = \langle m(\tilde{a}), w(\tilde{a})\rangle \div \langle m(\tilde{b}), w(\tilde{b})\rangle = \langle m(\tilde{a}) \div m(\tilde{b}), \max\{ w(\tilde{a}), w(\tilde{b})\}\rangle\langle 0.1666,1\rangle$$

$$\text{Traffic intensity(p)} = \frac{\lambda}{\mu} = \frac{[1,3]}{[11,13]} = \frac{\langle 2,1\rangle}{\langle 12,1\rangle} = \langle 0.1666,1\rangle$$

Probability that the service facility is idle (probability of 0 units in system)

$$(P_0) = 1-p = 1-\frac{\lambda}{\mu} = 1-\langle 0.1666,1\rangle = \langle 1,0\rangle - \langle 0.1666,1\rangle = \langle 0.8334,1\rangle$$

Probability of n units in the system (waiting time and service time)

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0 = \langle 1.18173, 0.4167\rangle$$

The expected number of customer in the system

$$L_S = \frac{\lambda}{\mu-\lambda} = \frac{[1,3]}{[11,13]-[1,3]} = \frac{\langle 2,1\rangle}{\langle 12,1\rangle - \langle 2,1\rangle} = \frac{\langle 2,1\rangle}{\langle 10,1\rangle} = \langle 0.2,1\rangle$$

The expected number of customers waiting in the queue

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{[1,3]^2}{[11,13]([11,13]-[1,3])} = \frac{\langle 2,1\rangle^2}{[11,13](\langle 12,1\rangle - \langle 2,1\rangle)} = \frac{\langle 4,1\rangle}{\langle 12,1\rangle\langle 10,1\rangle} = \frac{\langle 4,1\rangle}{\langle 120,1\rangle} = \langle 0.0333,1\rangle$$

The average waiting time in the system

$$W_s = \frac{1}{\mu-\lambda} = \frac{1}{[11,13]-[1,3]} = \frac{\langle 1,0\rangle}{\langle 12,1\rangle - \langle 2,1\rangle} = \frac{\langle 1,0\rangle}{\langle 10,1\rangle} = \langle 0.1,1\rangle$$

The average waiting time of customer in the queue

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{[1,3]}{[11,13]([11,13]-[1,3])} = \frac{\langle 2,1\rangle}{\langle 12,1\rangle\langle 10,1\rangle} = \frac{\langle 2,1\rangle}{\langle 120,1\rangle} = \langle 0.2,1\rangle$$

Expected Length of Non-empty Queue

$$(L_{neq}) = \mu/(\mu-\lambda) = \frac{[11,13]}{([11,13]-[1,3])} = \frac{\langle 12,1\rangle}{(\langle 12,1\rangle - \langle 2,1\rangle)} = \frac{\langle 12,1\rangle}{\langle 10,1\rangle} = \langle 1.2,1\rangle$$

The Probability Track That k or More than K Customers in the System

$$P \geq K = \left(\frac{\lambda}{\mu}\right)^K = \left(\frac{[1,3]}{[11,13]}\right)^K = \left(\frac{\langle 2,1\rangle}{\langle 12,1\rangle}\right)^K = (\langle 0.1666,1\rangle)^K$$

The probability that more than k customers are in the system

$$(P > K) = \left(\frac{\lambda}{\mu}\right)^{K+1} = \left(\langle 0.1666,1\rangle\right)^{K+1}$$

The probability that at least one customer is standing in queue

$$P = K = \left(\frac{\lambda}{\mu}\right)^2 = \left(\frac{[1,3]}{[11,13]}\right)^2 = \left(\frac{\langle 2,1\rangle}{\langle 12,1\rangle}\right)^2 = \left(\langle 0.1666,1\rangle\right)^2 = \langle 0.0277,1\rangle$$

Consider a FM/FM/C queue, where the both the arrival rate and service rate are interval number represented by $\lambda = [16,18]$ And $\mu = [5,7]$

$$P_0 = \frac{1}{\sum_{n=0}^{c-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^C}{C!} \cdot \frac{C\mu}{C\mu-\lambda}}$$

$P_0 = \langle 0.0479,1\rangle$

Expected number of customers in the system $L_S = \dfrac{\lambda\mu\left(\dfrac{\lambda}{\mu}\right)^C}{(C-1)!(C\mu-\lambda)^2} P_0 + \dfrac{\lambda}{\mu}$

$$L_S = \frac{[16,18][5,7]\left(\dfrac{[16,18]}{[5,7]}\right)^4}{(4-1)!(4[5,7]-[16,18])^2}\langle 0.0479,1\rangle + \frac{[16,18]}{[5,7]} = \langle 3.903,1\rangle$$

$$L_q = \frac{\lambda\mu\left(\dfrac{\lambda}{\mu}\right)^C}{(C-1)!(C\mu-\lambda)^2} P_0 = \frac{[16,18][5,7]\left(\dfrac{[16,18]}{[5,7]}\right)^4}{(4-1)!(4[5,7]-[16,18])^2}\langle 0.0479,1\rangle = \langle 1.070,1\rangle$$

$$W_S = \frac{\mu\left(\dfrac{\lambda}{\mu}\right)^C}{(C-1)!(C\mu-\lambda)^2} P_0 + \frac{1}{\mu} = \frac{[5,7]\left(\dfrac{[16,18]}{[5,7]}\right)^4}{(4-1)!(4[5,7]-[16,18])^2}\langle 0.0479,1\rangle + \frac{1}{[5,7]} = \langle 0.222,1\rangle$$

Average waiting time of a customer in the queue

$$W_q = \frac{\mu\left(\dfrac{\lambda}{\mu}\right)^C}{(C-1)!(C\mu-\lambda)^2} P_0 = \frac{[5,7]\left(\dfrac{[16,18]}{[5,7]}\right)^4}{(4-1)!(4[5,7]-[16,18])^2}\langle 0.0479,1\rangle = \langle 0.062,1\rangle$$

8. Conclusion

In this paper, a new approach to solve the single and multi server queue is considered. The performance measures of single and multiserver queues are obtained as interval numbers using the new interval arithmetic. The solution procedure with the help of the numerical examples is illustrated.

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