# Outline

Airlines Graph

Knight Transposition

Seven Bridges of Königsberg

# Airlines Graph

Consider a small country with five cities: $A, B, C, D, E$.
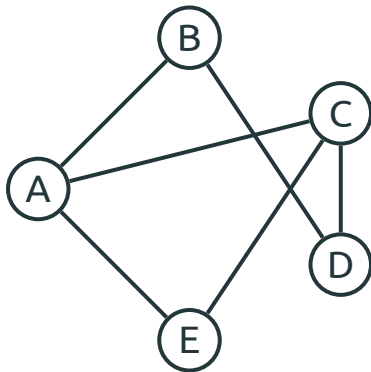
There are six flights:

$A - B, A - C, A - E,$

$B - D, C - D, C - E.$

Is there a direct flight from A to D? With one stop? With two stops?

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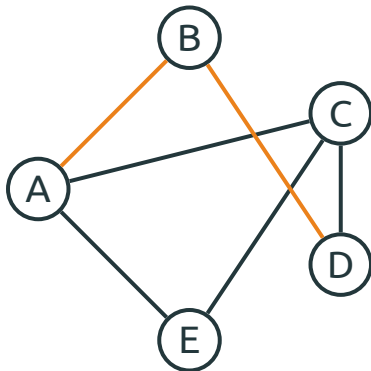
There are six flights:

  $A - B, A - C, A - E,$

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Is there a direct flight from A to D? With one stop? With two stops?

# Airlines Graph

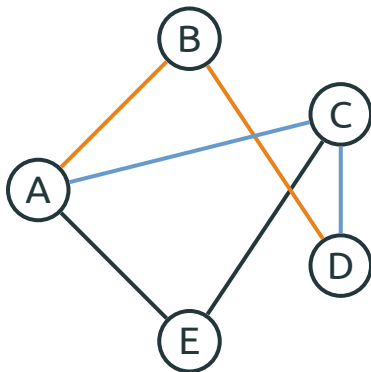Consider a small country with five cities: $A, B, C, D, E.$

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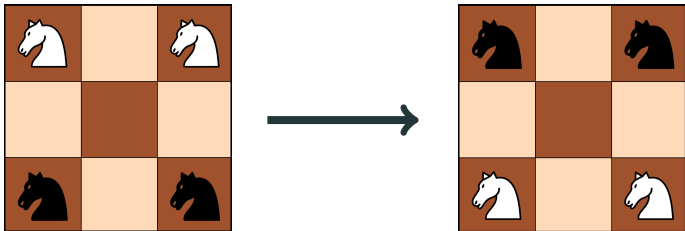Is there a direct flight from A to D? With one stop? With two stops?

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country with five
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Is there a direct
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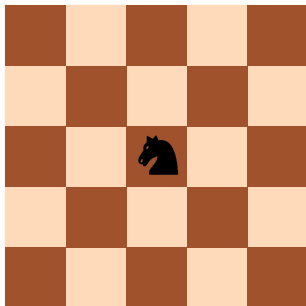With one stop?
With two stops?

# Outline

# Guarini's Puzzle



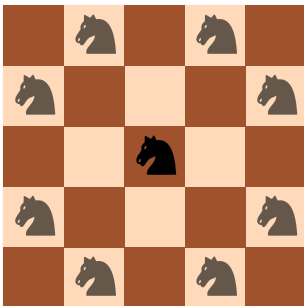Exchange the places of the white knights and the black knights.

# Chess Knight

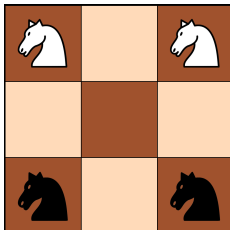A chess knight can move in an **L** shape in any direction

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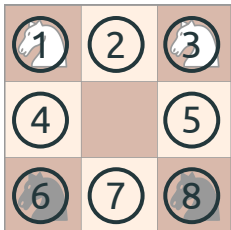
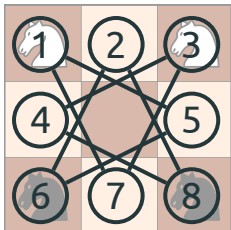# Guarini's Puzzle. Solution



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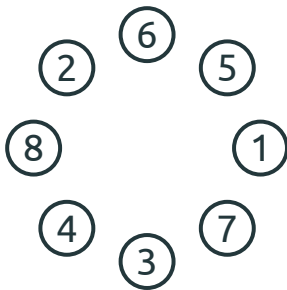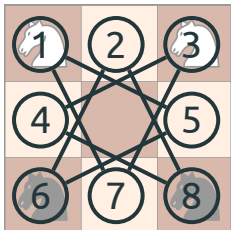
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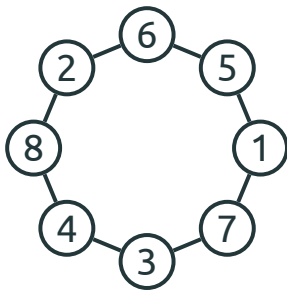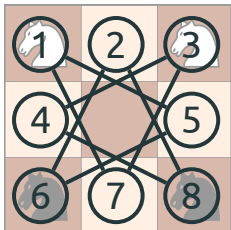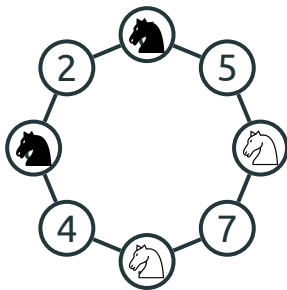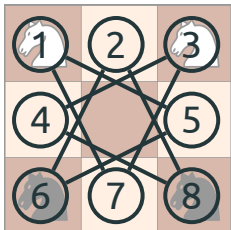
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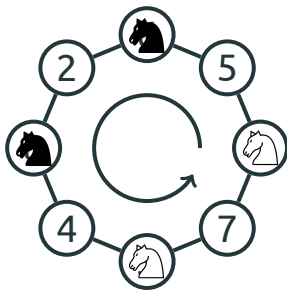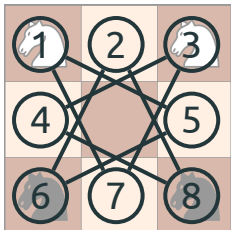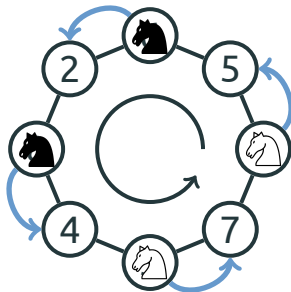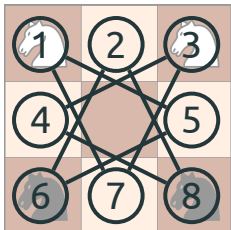# Guarini's Puzzle. Solution



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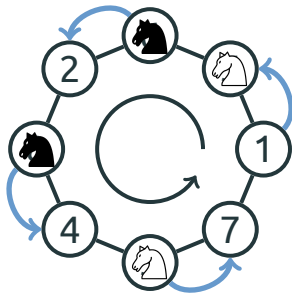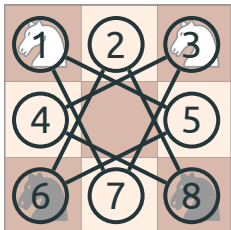# Guarini's Puzzle. Solution



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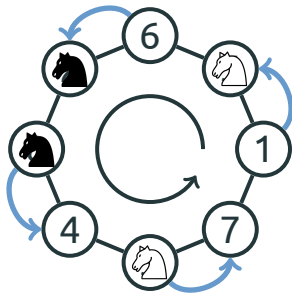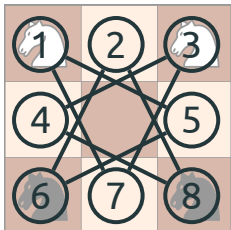# Guarini's Puzzle. Solution



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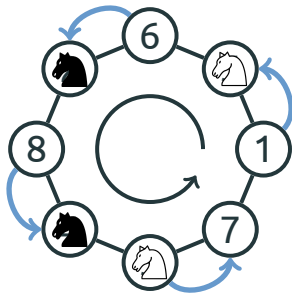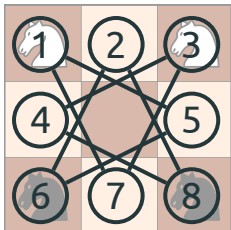# Guarini's Puzzle. Solution



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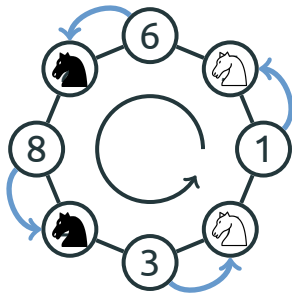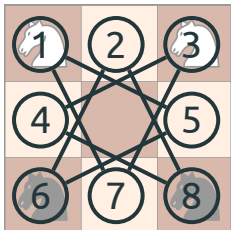# Guarini's Puzzle. Solution



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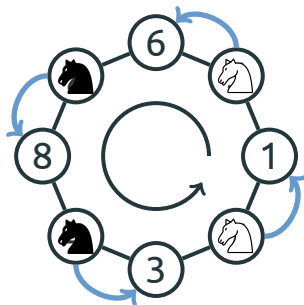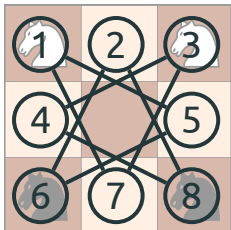# Guarini's Puzzle. Solution



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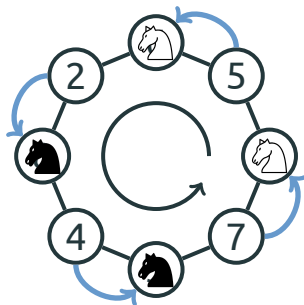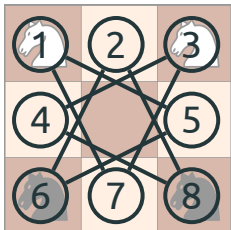# Guarini's Puzzle. Solution



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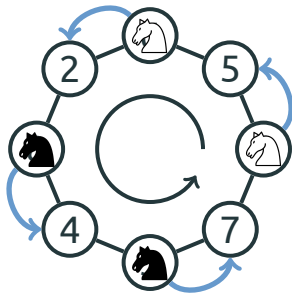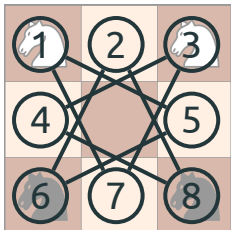# Guarini's Puzzle. Solution



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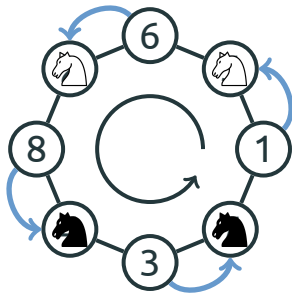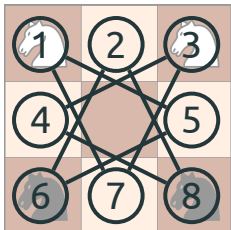# Guarini's Puzzle. Solution



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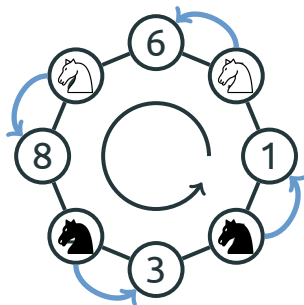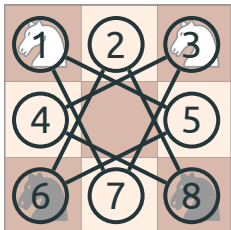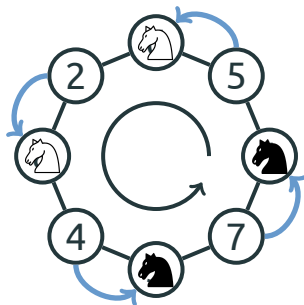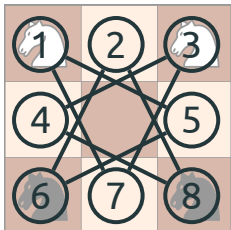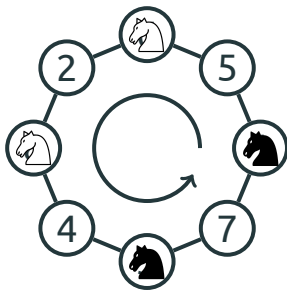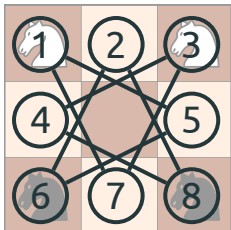# Guarini's Puzzle. Solution



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Königsberg, Prussia, 1735

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Walk through Königsberg

Cross each bridge
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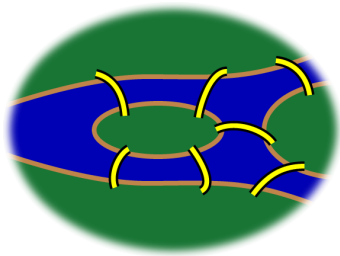Walk through Königsberg

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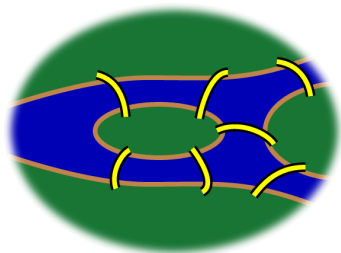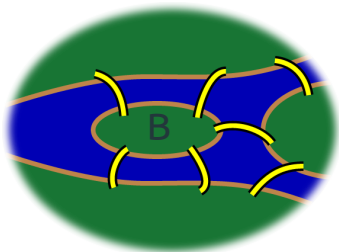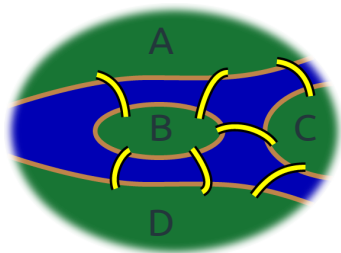


Impossible!

Leonhard Euler

# Bridges of Königsberg. Graph

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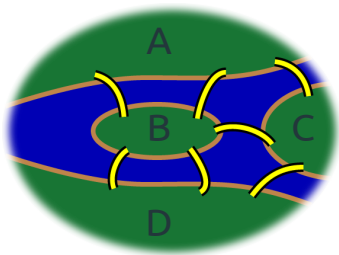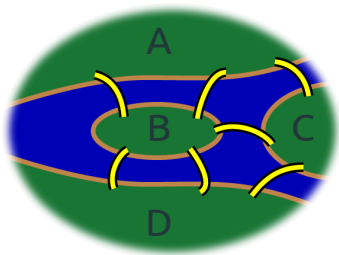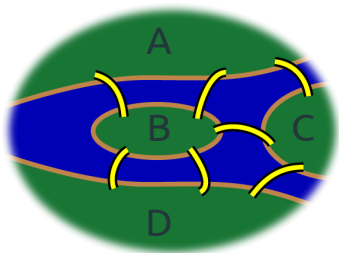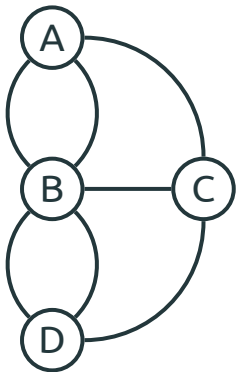
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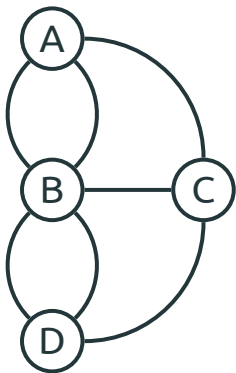# Bridges of Königsberg. Graph

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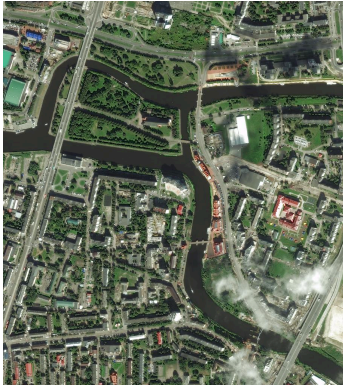# Bridges of Königsberg. Graph
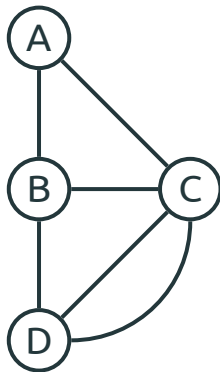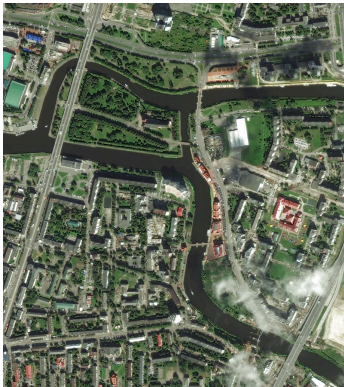
# Bridges of Königsberg. Graph



Is there a path which visits every edge exactly once?

Eulerian Path

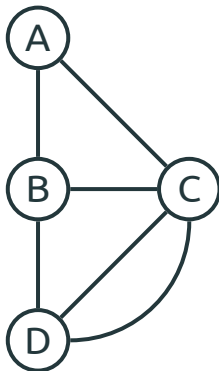# Six Bridges of Kaliningrad

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B and D have <span style="color:orange">odd</span> number of neighbors

If there exists an Eulerian path, B and D must be START and FINISH

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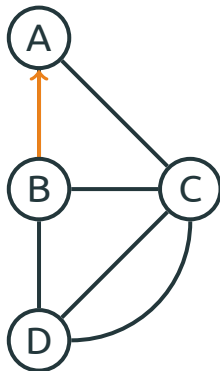# Six Bridges of Kaliningrad

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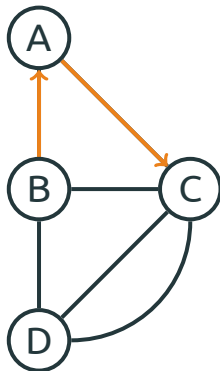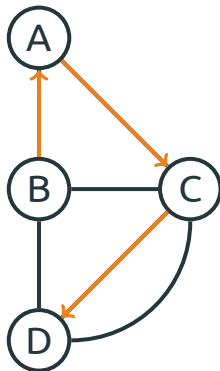# Six Bridges of Kaliningrad

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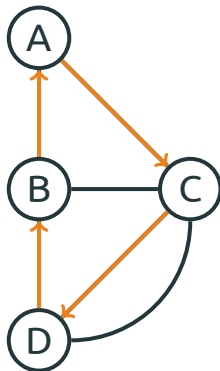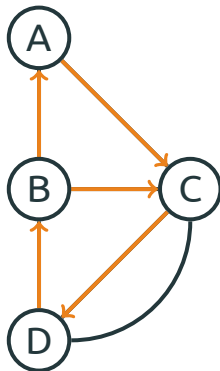# Six Bridges of Kaliningrad

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