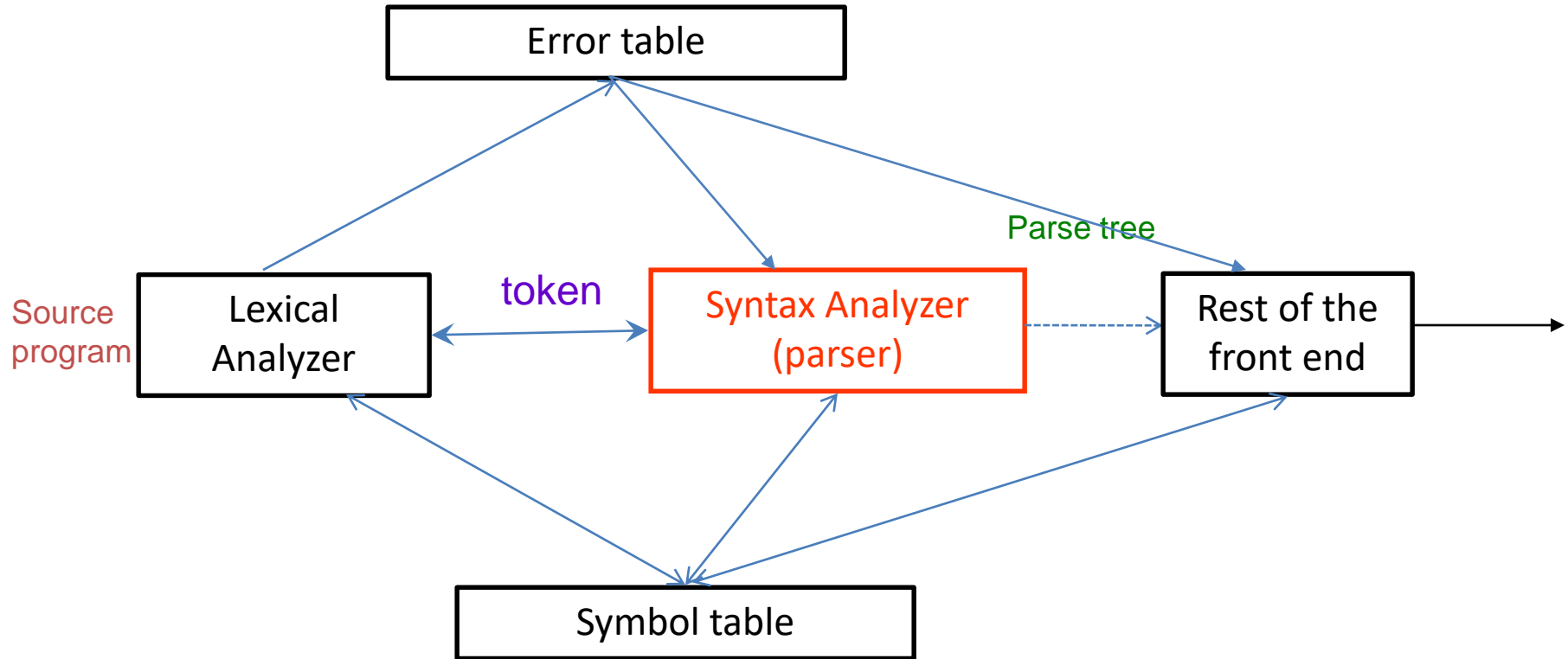


CSE 4801

Syntax Analysis

(Parsing)

Role of the Syntax Analyzer



Syntax Error Handling

Syntactic Errors: invalid order of tokens.

Example:

- Arithmetic expression with unbalanced parenthesis
- Two consecutive operands or operators
- A statement without end marker (; for TC), etc.

Goals of syntax error handler

- Report the presence of syntactic errors clearly and accurately
- Recover from each error quickly (optional)
- Should not slow down the entire compilation

Syntax Error Recovery

Recovery Strategies

1. Panic mode
2. Phrase level
3. Error productions
4. Global correction

Formal Definition of a Context-Free Grammar

A context-free grammar (grammar for short) consists of a set of terminals, a set of non-terminals, a start symbol, and a set of productions.

- **Terminals** are the basic symbols from which strings are formed.
- **Nonterminals** are syntactic variables that denote sets of strings.
- In a grammar, one nonterminal is distinguished as the **start symbol**, and the set of strings it denotes is the language generated by the grammar. Conventionally, the productions for the start symbol are listed first.
- The **productions** of a grammar specify the manner in which the terminals and nonterminals can be combined to form strings.

Productions

1. $E \rightarrow E + E$
2. $E \rightarrow E * E$
3. $E \rightarrow -E$
4. $E \rightarrow (E)$
5. $E \rightarrow \text{id}$
6. $E \rightarrow \text{num}$

- Terminals: +, *, -, (,), **id**, **num**
- Non-terminals: **E**
- Start Symbol: **E**
- Productions: 1..6

A **production** can be treated as a rewriting rule in which the nonterminal on the left can be replaced by the string on the right side of the production or vice-versa.

This replacement process is known as **derivation**.

Notational conventions

(context-free grammars)

1. Terminals

- early lower case letters: a, b, c, \dots
- Operator symbols: $+, -, *, \dots$
- Punctuation symbols: comma, parenthesis, \dots
- Digits: $0, 1, 2, \dots, 9$
- Boldface strings: **id**, **if**

Notational conventions

(context-free grammars)



2. Non-terminals

- early upper case letters: *A, B, C, ..*
- The letter *S*, usually the start symbol.
- Lower case italic names such as *expr* or *stmt*

3. Upper case letters, late in the alphabet, such as *X, Y, Z*, represent a grammar symbol, either ***terminal*** or ***non-terminal***.

4. Lower case letters, late in the alphabet, such as *x, y, z*, represent ***string of terminals***.

Notational conventions



(context-free grammars)

5. Lower case Greek letters, α , β , γ , represent **strings of grammar symbols**.
6. If $A \rightarrow \alpha_1$, $A \rightarrow \alpha_2$, ..., $A \rightarrow \alpha_k$ are all productions with A on the left side, then we may write $A \rightarrow \alpha_1 | \alpha_2 | \dots | \alpha_k$
7. Unless stated otherwise, the left side of the first production is the start symbol.

Derivation and Parse Tree

For input **id+id*id**

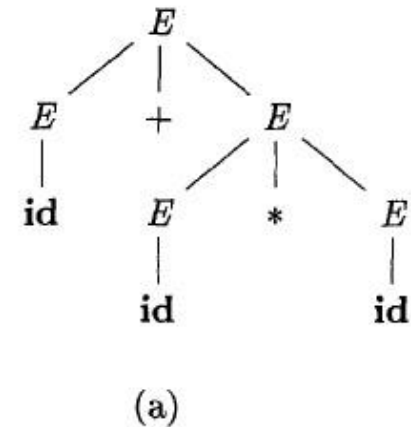
Grammar

1. $E \rightarrow E + E$
2. $E \rightarrow E * E$
3. $E \rightarrow -E$
4. $E \rightarrow (E)$
5. $E \rightarrow \text{id}$
6. $E \rightarrow \text{num}$

Derivations

$E \rightarrow E + E$
 $\rightarrow \text{id} + E$
 $\rightarrow \text{id} + E * E$
 $\rightarrow \text{id} + \text{id} * E$
 $\rightarrow \text{id} + \text{id} * \text{id}$

Parse Tree



Parse tree is a graphical representation for a set of derivation. A parse tree for input **id+id*id** is presented above.

Grammar Ambiguity

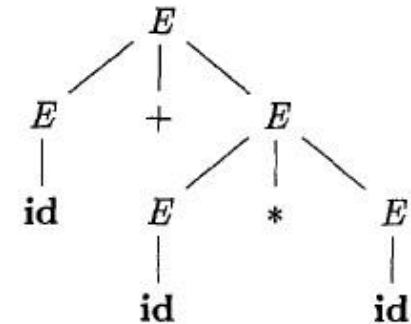
Grammar

1. $E \rightarrow E + E$
2. $E \rightarrow E * E$
3. $E \rightarrow -E$
4. $E \rightarrow (E)$
5. $E \rightarrow \text{id}$
6. $E \rightarrow \text{num}$

Derivations

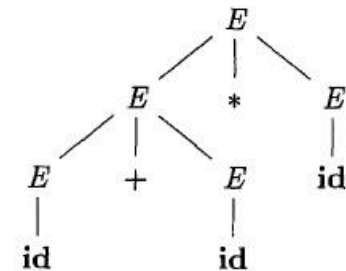
$$\begin{aligned}
 E &\rightarrow E + E \\
 &\rightarrow \text{id} + E \\
 &\rightarrow \text{id} + E * E \\
 &\rightarrow \text{id} + \text{id} * E \\
 &\rightarrow \text{id} + \text{id} * \text{id}
 \end{aligned}$$

Parse Tree



(a)

Alternate Parse Tree



(b)

Input: **id+id*id**

$$\begin{aligned}
 E &\rightarrow E * E \\
 &\rightarrow E + E * E \\
 &\rightarrow \text{id} + E * E \\
 &\rightarrow \text{id} + \text{id} * E \\
 &\rightarrow \text{id} + \text{id} * \text{id}
 \end{aligned}$$

Ambiguity in branching statement



Stmt -> if expr then stmt
| if expr then stmt else stmt
| other

If **E1** then if **E2** then **S1** else **S2**

Stmt -> if expr then **stmt** (by production 1)
-> if expr then *if expr then stmt else stmt* (by production 2)

Stmt -> if expr then **stmt** else stmt (by production 2)
-> if expr then *if expr then stmt* else stmt (by production 1)

Eliminating Ambiguity



stmt -> matched_stmt
 | unmatched_stmt

matched_stmt -> if expr then matched_stmt else matched_stmt
 | other

unmatched_stmt -> if expr then stmt
 | if expr then matched_stmt else unmatched_stmt

Types of Parsing

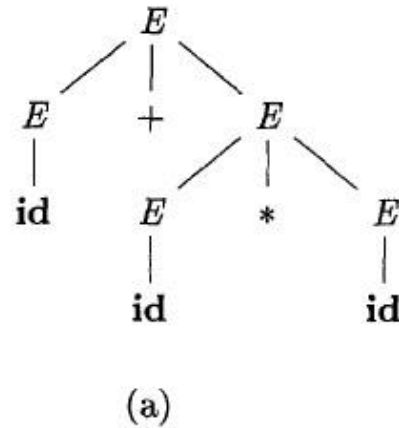
- **Top-Down Parsing (LL)**
 - Recursive Descent Parsing
 - Nonrecursive Predictive Parsing
- **Bottom-Up Parsing (LR)**
 - Simple LR (SLR)
 - Canonical LR (CLR)
 - Look Ahead LR (LALR)

LL: Scan the input from Left to Right, Use Left most derivation

LR: Scan the input from Left to Right, Use Right most derivation in Reverse order

Types of Parsing

Top-Down Parsing
(LL Parser)



Bottom-Up Parsing
(LR Parser)



LL Parser

LL

Scan or read source/input text
from *Left to Right*

Use *Left most derivation*

Grammar

1. $E \rightarrow E + E$
2. $E \rightarrow E * E$
3. $E \rightarrow -E$
4. $E \rightarrow (E)$
5. $E \rightarrow \text{id}$
6. $E \rightarrow \text{num}$

Input: **id+id*id**

Derivations

$E \rightarrow E + E$
 $\rightarrow \text{id} + E$
 $\rightarrow \text{id} + E * E$
 $\rightarrow \text{id} + \text{id} * E$
 $\rightarrow \text{id} + \text{id} * \text{id}$

LR Parser

L R

Scan source/input text
from *Left to Right*

Use *Right most derivation*
in *reverse order*

Grammar

1. $E \rightarrow E + E$
2. $E \rightarrow E * E$
3. $E \rightarrow -E$
4. $E \rightarrow (E)$
5. $E \rightarrow \text{id}$
6. $E \rightarrow \text{num}$

Input: **id+id*id**

Derivations

$E \rightarrow E + E$
 $\rightarrow \text{id} + E$
 $\rightarrow \text{id} + E * E$
 $\rightarrow \text{id} + \text{id} * E$
 $\rightarrow \text{id} + \text{id} * \text{id}$

Sentential Form

- A **sentential form** is any string derivable from the start symbol.
Note that this includes the forms with non-terminals at intermediate steps as well.
- A **right-sentential** form is a sentential form that occurs in a step of rightmost derivation (RMD).
- A **left-sentential** form is a sentential form that occurs in a step of leftmost derivation (RMD).
- A **sentence** is a sentential form consisting only of terminals.

Top-Down Parsing (LL)

Examples of Top-Down Parsing (LL)

- Recursive Descent Parsing
- Nonrecursive Predictive Parsing

LL Parser: Recursive Descent Parsing

Select a production to derive a non-terminal randomly during parsing. If any dead end is there before fully parsing the input do backtrack.

Grammar

1. $E \rightarrow E + E$
2. $E \rightarrow E * E$
3. $E \rightarrow -E$
4. $E \rightarrow (E)$
5. $E \rightarrow \text{id}$
6. $E \rightarrow \text{num}$

Input: **id+id*id**

E $\rightarrow E * E$ (using production no. 2; taken randomly)

.
.br/>.

LL Parser: Recursive Descent Parsing

Select a production to derive a non-terminal randomly during parsing. If any dead end is there before fully parsing the input do backtrack.

Grammar

1. $E \rightarrow E + E$
2. $E \rightarrow E * E$
3. $E \rightarrow -E$
4. $E \rightarrow (E)$
5. $E \rightarrow id$
6. $E \rightarrow num$

Input: **id+id*id**

Mismatch

$E \rightarrow E * E$ (using production no. 2)
 $E \rightarrow id * E$ (using production no. 5; backtrack needed)
 $E \rightarrow E * E$ (undo derivation by production no. 5)
 $E \rightarrow E + E * E$ (using production no. 1)
 $E \rightarrow id + E * E$ (using production no. 5)
 $E \rightarrow id + id * E$ (using production no. 5)
 $E \rightarrow id + id * id$ (using production no. 5)

Lots of backtracking may be needed in between these steps.

LL Parser: Predictive Parsing

During parsing select a unique production for derivation based on present input token and present symbol to derive.

Input: **id+id*id**

Grammar

1. $E \rightarrow E + E$
2. $E \rightarrow E * E$
3. $E \rightarrow -E$
4. $E \rightarrow (E)$
5. $E \rightarrow \text{id}$
6. $E \rightarrow \text{num}$

E \rightarrow ??

Modify a grammar for predictive parsing

- **Eliminate Left Recursion**
- **Do Left Factoring**

Left Recursion

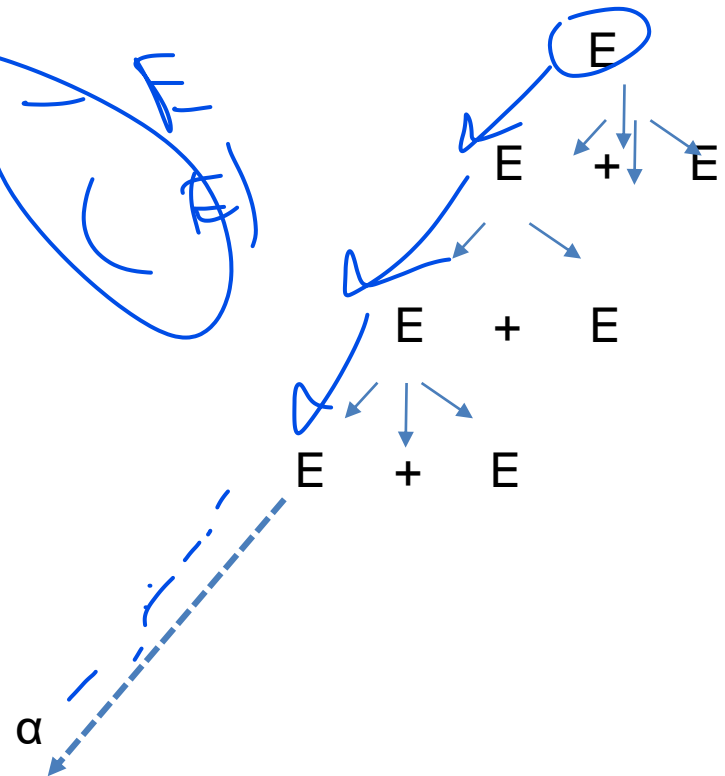
Grammar

1. $E \rightarrow E + E$
2. $E \rightarrow E * E$
3. $E \rightarrow -E$
4. $E \rightarrow (E)$
5. $E \rightarrow \text{id}$
6. $E \rightarrow \text{num}$

Input: **id+id*id**

$E \rightarrow E + E$
 $E \rightarrow E * E$
 $E \rightarrow E + E + E$

$A \rightarrow A +$



To implement top-down parsing left recursion need to be eliminated without affecting the language represented by the grammar.

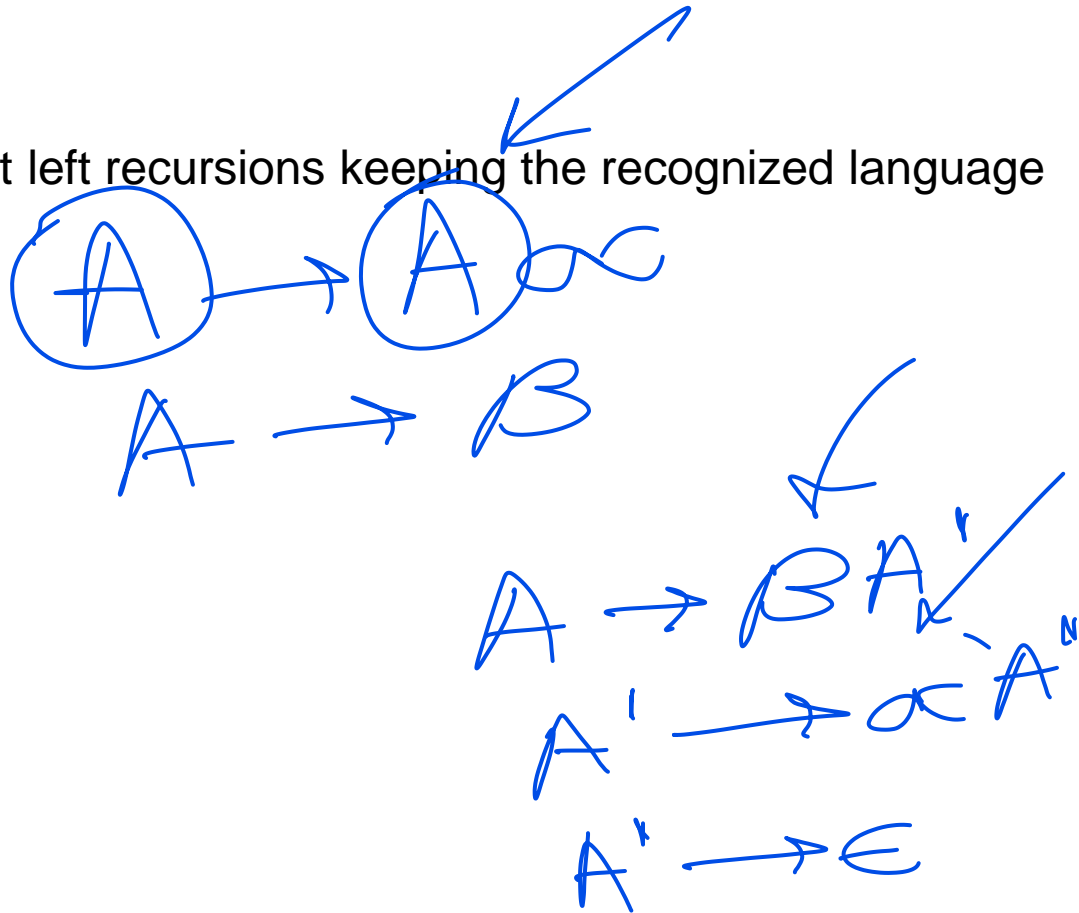
Elimination of Left Recursion

A pair of “A productions” with left recursion:

$$\begin{aligned} A &\rightarrow A\alpha \\ A &\rightarrow \beta \end{aligned}$$

Modified productions without left recursions keeping the recognized language intact:

$$\begin{aligned} A &\rightarrow \beta A' \\ A' &\rightarrow \alpha A' \\ A' &\rightarrow \epsilon \end{aligned}$$



Elimination of Left Recursion(2)

Grammar:

$$A \rightarrow A\alpha$$
$$A \rightarrow \beta$$

Above grammar can generate strings of format: $\beta\alpha^*$ ($\beta, \beta\alpha, \beta\alpha\alpha, \beta\alpha\alpha\alpha, \dots$)

Same set of strings will be generated by the following grammar:

$$A \rightarrow \beta A'$$
$$A' \rightarrow \alpha A'$$
$$A' \rightarrow \epsilon$$
$$E \rightarrow E + T$$
$$E \rightarrow T$$

>>

$$E \rightarrow T E'$$
$$E' \rightarrow + T E'$$
$$E' \rightarrow \epsilon$$

Elimination of Left Recursion (3)



A set of “A productions” with left recursion:

$$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid A\alpha_3 \mid \dots \mid A\alpha_n$$

$$A \rightarrow \beta_1 \mid \beta_2 \mid \beta_3 \mid \dots \mid \beta_k$$

Modified productions without left recursions keeping the recognized language intact:

$$A \rightarrow \beta_1 A' \mid \beta_2 A' \mid \beta_3 A' \mid \dots \mid \beta_k A'$$

$$A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \alpha_3 A' \mid \dots \mid \alpha_n A'$$

$$A' \rightarrow \varepsilon$$

Left Factoring

Consider a set of A-productions where first symbol(s) of the right side are exactly same:

$$\begin{array}{ll} A \rightarrow bC & A \rightarrow baC \\ A \rightarrow bD & \text{or} \quad A \rightarrow baD \\ A \rightarrow bE & A \rightarrow baE \end{array}$$

.

These form of ambiguity must be removed by “left factoring” process as follows:

$$\begin{array}{ll} A \rightarrow bH & \text{or} \quad A \rightarrow baH \\ H \rightarrow C \mid D \mid E & \end{array}$$

Predictive Parser (Top-down or LL)

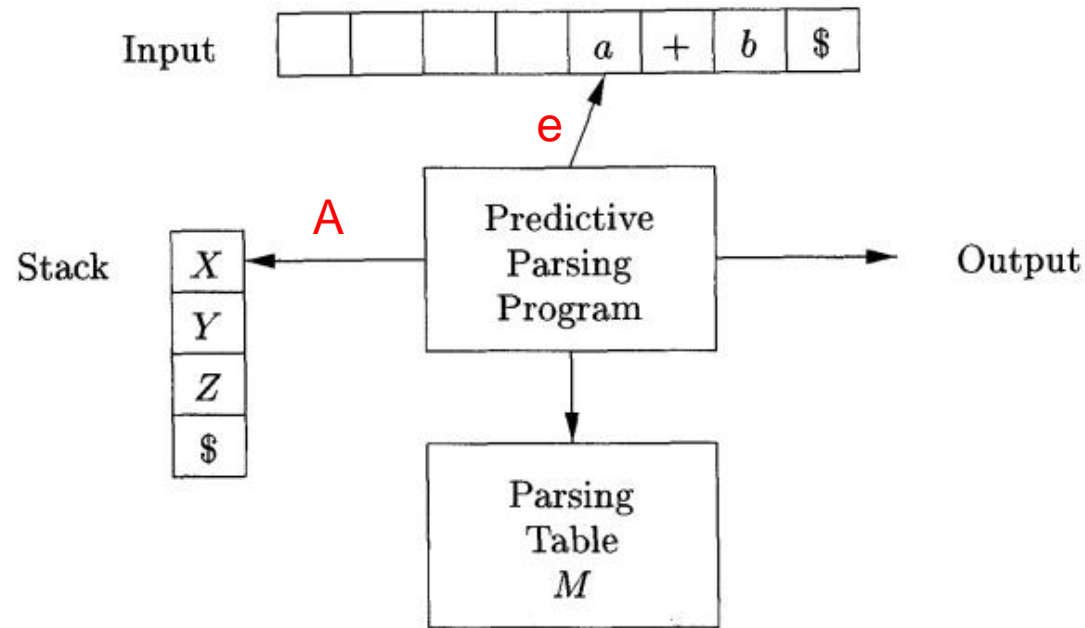


Fig: General structure of a predictive parser

Parsing Algorithm

do while 1

1. If $A = e = \$$, parsing is complete
2. else if $A = e \neq \$$ pop A from stack and advance the input pointer
3. else if A is a nonterminal, check entry at $M[A, e]$. If the entry is an A production then replace A by right side of A production in reverse order.
4. else report an error to error handler for present input symbol e .

Predictive Parser: Parse Table, M

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \text{id}$			$F \rightarrow (E)$		

Predictive parser: parsing steps

Modified Grammar

1. $E \rightarrow TE'$
2. $E' \rightarrow +TE'$
3. $E' \rightarrow \epsilon$
4. $T \rightarrow FT'$
5. $T' \rightarrow *FT' \mid \epsilon$
6. $T' \rightarrow \epsilon$
7. $F \rightarrow (E)$
8. $F \rightarrow id$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

Parse Table

Stack
E' \$
X

Input
id

MATCHED	STACK	INPUT	ACTION
	$E\$$	$id + id * id \$$	
	$TE' \$$	$id + id * id \$$	output $E \rightarrow TE'$
	$\rightarrow FT'E' \$$	$id + id * id \$$	output $T \rightarrow FT'$
	$id T'E' \$$	$id + id * id \$$	output $F \rightarrow id$
id	$T'E' \$$	$+ id * id \$$	match id
id	$E' \$$	$+ id * id \$$	output $T' \rightarrow \epsilon$
id	$+ TE' \$$	$+ id * id \$$	output $E' \rightarrow + TE'$
$id +$	$TE' \$$	$id * id \$$	match $+$
$id +$	$FT'E' \$$	$id * id \$$	output $T \rightarrow FT'$
$id +$	$id T'E' \$$	$id * id \$$	output $F \rightarrow id$
$id + id$	$T'E' \$$	$* id \$$	match id
$id + id$	$* FT'E' \$$	$* id \$$	output $T' \rightarrow * FT'$
$id + id *$	$FT'E' \$$	$id \$$	match $*$
$id + id *$	$id T'E' \$$	$id \$$	output $F \rightarrow id$
$id + id * id$	$T'E' \$$	$\$$	match id
$id + id * id$	$E' \$$	$\$$	output $T' \rightarrow \epsilon$
$id + id * id$	$\$$	$\$$	output $E' \rightarrow \epsilon$

Parsing Algorithm

do while 1

1. If $A = e = \$$, parsing is complete
2. else if $A = e \neq \$$ pop A from the stack and advance the input pointer
3. else if A is a nonterminal, check entry at $M[A, e]$. If the entry is an A production then replace A by right side of A production in reverse order.
4. else report an error to error handler for present input symbol e .

Construction of Parse Table



Steps:

- Eliminate *Left-Recursion* from the Grammar
- If necessary perform *Left Factoring*
- Find set of *First(X)* and *Follow(X)*
- Build the parse table

Predictive parser: Eliminate left recursion

Grammar

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \text{id}$

Modified Grammar

1. $E \rightarrow T E'$
2. $E' \rightarrow +TE'$
3. $E' \rightarrow \epsilon$
4. $T \rightarrow FT'$
5. $T' \rightarrow *FT' \mid \epsilon$
6. $T' \rightarrow \epsilon$
7. $F \rightarrow (E)$
8. $F \rightarrow \text{id}$

Sentential Form

A **sentential form** is any string derivable from the start symbol of a grammar. Note that this includes the forms with non-terminals at intermediate steps as well.

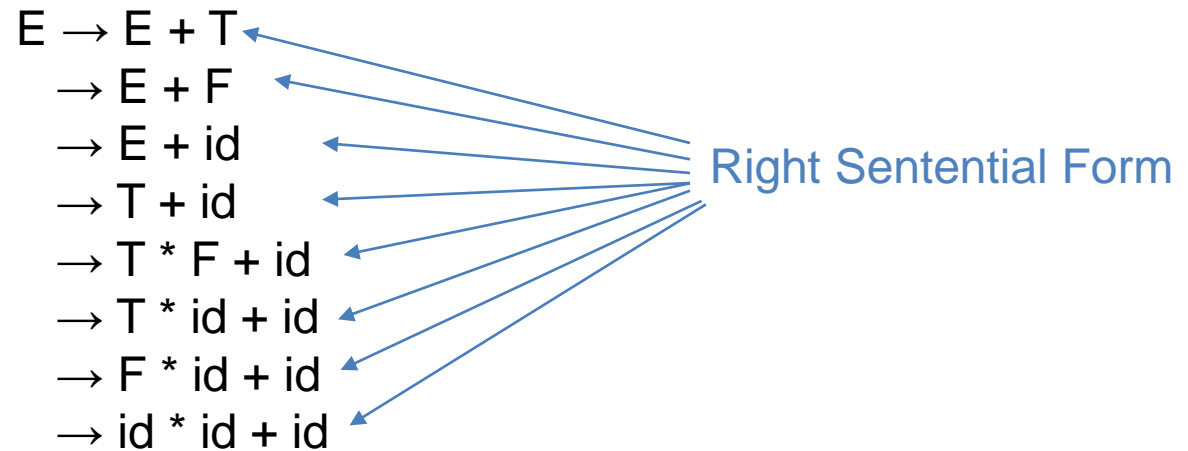
1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \text{id}$

$$\begin{aligned} E &\rightarrow E + T \\ &\rightarrow T + T \\ &\rightarrow T * F + T \\ &\rightarrow F * F + T \\ &\rightarrow \text{id} * F + T \\ &\rightarrow \text{id} * \text{id} + T \\ &\rightarrow \text{id} * \text{id} + F \\ &\rightarrow \text{id} * \text{id} + \text{id} \end{aligned}$$

Right Sentential Form

A **right sentential form** is a sentential form that occurs in steps of rightmost derivation (RMD).

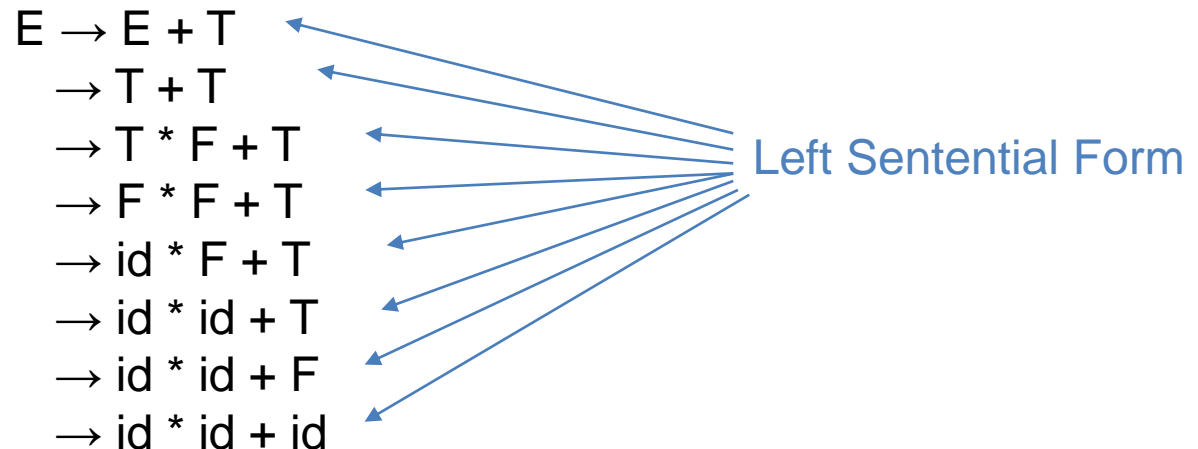
1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \mathbf{id}$



Left Sentential Form

A **left sentential form** is a sentential form that occurs in steps of leftmost derivation (LMD).

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \text{id}$



First (X) and Follow(X)

If X is a non-terminal then First(X) is the set of terminals that can appear at the beginning of its sentential form. if X is terminal, First (X) is X itself.

$A \rightarrow bC \mid Ce$
 $\rightarrow C \rightarrow d \mid f$

\subseteq

$\text{First}(bC) = \{b\}$
 $\text{First}(Ce) = \{d, f\}$

$A = \{b, d, f\}$
 $C = \{d, f\}$

$A \rightarrow bC \rightarrow bd$

$A \rightarrow Ce \rightarrow de$

$A \rightarrow Ce \rightarrow fe$

$\text{First}(A) = \{b, d, f\}$

$\text{First}(C) = \{d, f\}$

bd

Follow(X) is a set of terminals that may appear after X in a sentential form.

$A \rightarrow Ce \rightarrow de$

$A \rightarrow Ce \rightarrow fe$

$\text{Follow}(C) = \{e\}$

First (X)

1. If X is a terminal, then $\text{FIRST}(X) = \{X\}$.
2. If $X \rightarrow \epsilon$ is a production, then add ϵ to $\text{FIRST}(X)$.
3. If X is a nonterminal and $X \rightarrow Y$, then everything in $\text{FIRST}(Y)$ is in $\text{FIRST}(X)$.
4. If X is a nonterminal and $X \rightarrow Y_1 Y_2 \cdots Y_k$ is a production for some $k \geq 1$, then place a in $\text{FIRST}(X)$ if for some i , a is in $\text{FIRST}(Y_i)$, and ϵ is in all of $\text{FIRST}(Y_1), \dots, \text{FIRST}(Y_{i-1})$; that is, $Y_1 \cdots Y_{i-1} \xRightarrow{*} \epsilon$. If ϵ is in $\text{FIRST}(Y_j)$ for all $j = 1, 2, \dots, k$, then add ϵ to $\text{FIRST}(X)$. For example, everything in $\text{FIRST}(Y_1)$ is surely in $\text{FIRST}(X)$. If Y_1 does not derive ϵ , then we add nothing more to $\text{FIRST}(X)$, but if $Y_1 \xRightarrow{*} \epsilon$, then we add $\text{FIRST}(Y_2)$, and so on.

~~modify~~

Predictive parser: First (X)

Modified Grammar

1. $E \rightarrow T E'$
2. $E' \rightarrow + T E'$
3. $E' \rightarrow \epsilon$
4. $T \rightarrow F T'$
5. $T' \rightarrow * F T'$
6. $T' \rightarrow \epsilon$
7. $F \rightarrow (E)$
8. $F \rightarrow \underline{id}$

First

$E = \{ (, id \}$
 $E' = \{ +, \epsilon \}$
 $T = \{ (, id \}$
 $T' = \{ *, \epsilon \}$
 $F = \{ (, id \}$

First (E) = First (T) = First (F) = { (, id }

First (E') = { +, ϵ }

First (T') = { *, ϵ }

Predictive parser: First (X)

For a production $A \rightarrow \alpha$; $\text{First}(A)$ and $\text{First}(\alpha)$ may not be equal. Because the grammar may contain multiple A productions like follows-

$A \rightarrow \alpha$

$A \rightarrow \beta$

.....

$$\text{First}(A) = \text{First}(\alpha) \cup \text{First}(\beta)$$

So, for a production $A \rightarrow \alpha$; $\text{First}(\alpha) \subseteq \text{First}(A)$.

Follow (X)

Follow(X) to be the set of terminals that can appear immediately to the right of Non-Terminal X in some sentential form.

Algorithm:

1. Place \$ in FOLLOW(S), where S is the start symbol, and \$ is the input right endmarker.
2. If there is a production $A \rightarrow \alpha B \beta$, then everything in FIRST(β) except ϵ is in FOLLOW(B).
3. If there is a production $A \rightarrow \alpha B$, or a production $A \rightarrow \alpha B \beta$, where FIRST(β) contains ϵ , then everything in FOLLOW(A) is in FOLLOW(B).
- 4. Follow(X) never contains ϵ .

Predictive parser: Follow (X)

$E = \{ \text{), } \$ \}$
 $E' = \{ \text{), } \$ \}$
 $T = \{ \text{+}, \text{), } \$ \}$
 $T' = \{ \text{+}, \text{), } \$ \}$

Modified Grammar

1. $E \rightarrow TE'$
2. $E' \rightarrow +TE'$
3. $E' \rightarrow \epsilon$
4. $T \rightarrow FT'$
5. $T' \rightarrow *FT' \mid \epsilon$
6. $T' \rightarrow \epsilon$
7. $F \rightarrow (E)$
8. $F \rightarrow \text{id}$

$F = \{ \text{+}, \text{), } \$ \}$

1. Place \$ in FOLLOW(S), where S is the start symbol, and \$ is the input right endmarker.
2. If there is a production $A \rightarrow \alpha B \beta$, then everything in FIRST(β) except ϵ is in FOLLOW(B).
3. If there is a production $A \rightarrow \alpha B$, or a production $A \rightarrow \alpha B \beta$, where FIRST(β) contains ϵ , then everything in FOLLOW(A) is in FOLLOW(B).

Follow(E) = { \$, ... }

Follow(T) \leftarrow First (E') except ϵ [+]

Follow(F) \leftarrow First (T') except ϵ [*]

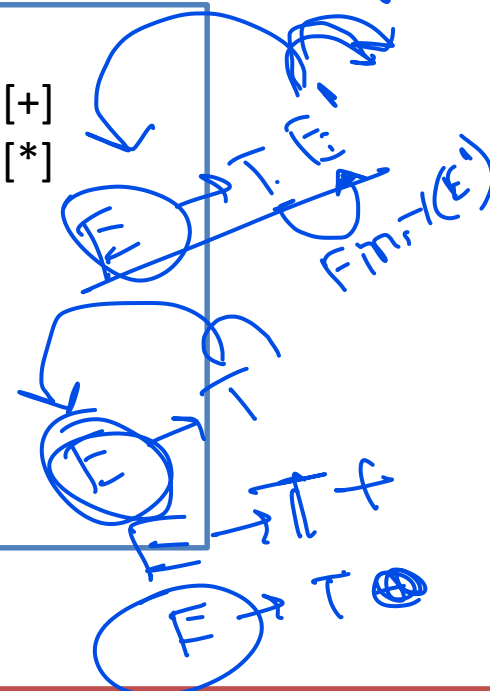
Follow(E) \leftarrow ')

Follow(E') \leftarrow Follow(E)

Follow(T') \leftarrow Follow(T)

Follow(T) \leftarrow Follow(E)

Follow(F) \leftarrow Follow(T)



First (E) = First (T) = First (F) = { (, id }

First (E') = { +, ϵ }

First (T') = { *, ϵ }

Follow (E) = Follow (E') = {), \$ }

Follow (T) = Follow (T') = { +,), \$ }

Follow(F) = { +, *,), \$ }

Predictive parser: Generating parse table

For each production $A \rightarrow \alpha$, do the following.

1. For each terminal a in $\text{FIRST}(\alpha)$, add $A \rightarrow \alpha$ to $M[A, a]$.
2. If ϵ is in $\text{FIRST}(\alpha)$, then for each terminal b in $\text{FOLLOW}(A)$, add $A \rightarrow \alpha$ to $M[A, b]$. If ϵ is in $\text{FIRST}(\alpha)$ and $\$$ is in $\text{FOLLOW}(A)$, add $A \rightarrow \alpha$ to $M[A, \$]$ as well.

If, after performing the above, there is no production at all in $M[A, a]$, then set $M[A, a]$ to **error** (which we normally represent by an empty entry in the table).

Algorithm to generate parse table

First (E) = First (T) = First (F) = { (, id }
First (E') = { +, ϵ }
First (T') = { *, ϵ }

Follow (E) = Follow (E') = { }, \$ }
Follow (T) = Follow (T') = { +,), \$ }
Follow (F) = { +, *,), \$ }

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \text{id}$			$F \rightarrow (E)$		

Types of Parsing

- **Top-Down Parsing (LL)**
 - Recursive Descent Parsing
 - Nonrecursive Predictive Parsing
- **Bottom-Up Parsing (LR)**
 - Simple LR (SLR)
 - Canonical LR (CLR)
 - Look Ahead LR (LALR)

LL: Scan the input from Left to Right, Use Left most derivation

LR: Scan the input from Left to Right, Use Right most derivation in Reverse order

Some Definitions for LR Parser

- Viable Prefix
- Handle
- Handle Pruning

Viable Prefix

The prefix of a right sentential form which may appear in parser stack is known as **viable prefix**.

Handle and handle pruning

A Handle is a substring of a right sentential form that matches the body (right side) of a production. Handle always appears at top of the stack.

Handle reduction is a step in the reverse of rightmost derivation. A rightmost derivation in reverse can be obtained by handle pruning.

Grammar

1. $E \rightarrow E + E$
2. $E \rightarrow E * E$
3. $E \rightarrow -E$
4. $E \rightarrow (E)$
5. $E \rightarrow \text{id}$
6. $E \rightarrow \text{num}$

Input: **id+id*id**

$E \rightarrow \text{id} + \text{id} * \text{id}$

$E \rightarrow \text{id} + \text{id} * E$

$E \rightarrow \text{id} + E * E$

$E \rightarrow \text{id} + E$

handle

handle pruning

Bottom-up parsing (LR)

L = Scan the input from Left to Right

R = Rightmost derivation in Reverse Order

Types of LR Parsing

- Operator-precedence parsing
- Simple LR (SLR)
- Canonical LR (CLR)
- Look Ahead LR (LALR)

LR Parser

An LR parser makes shift-reduce decisions by maintaining states to keep track of where we are in a parse. States are generated from the grammar.

Shift - shift to another state.

Reduce – apply a reduction operation using handle.

States represent sets of "items."

LR Parser States

States represent sets of "items."

An LR(0) item (item for short) of a grammar G is a production of G with a dot at some position of the body. Thus, production $A \rightarrow XYZ$ yields the four items-

$A \rightarrow .XYZ$

$A \rightarrow X.YZ$

$A \rightarrow XY.Z$

$A \rightarrow XYZ.$

The production $A \rightarrow \text{empty}$ generates only one item, $A \rightarrow .$

SLR Parser

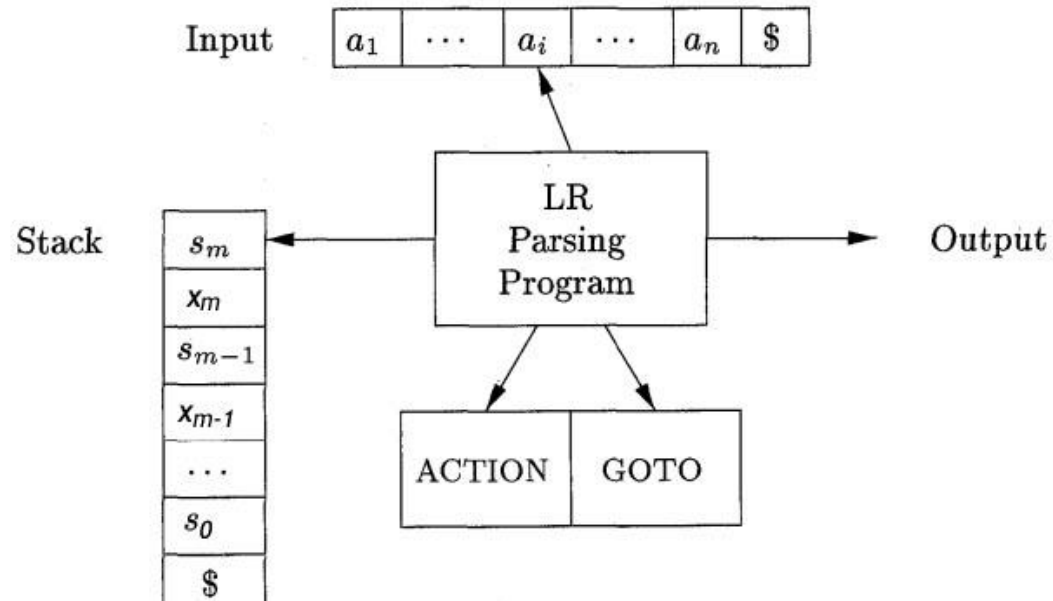
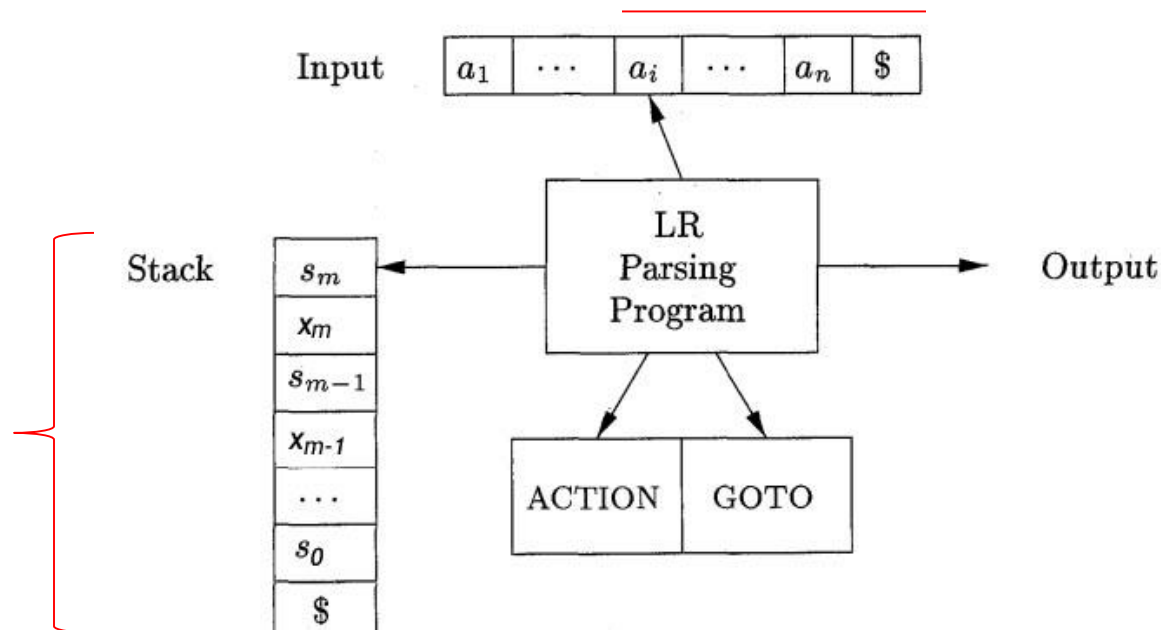


Fig: General Diagram of an SLR parser

Configuration of an SLR parser

A pair whose first component is the stack and second component is the unprocessed (unexpanded) input.



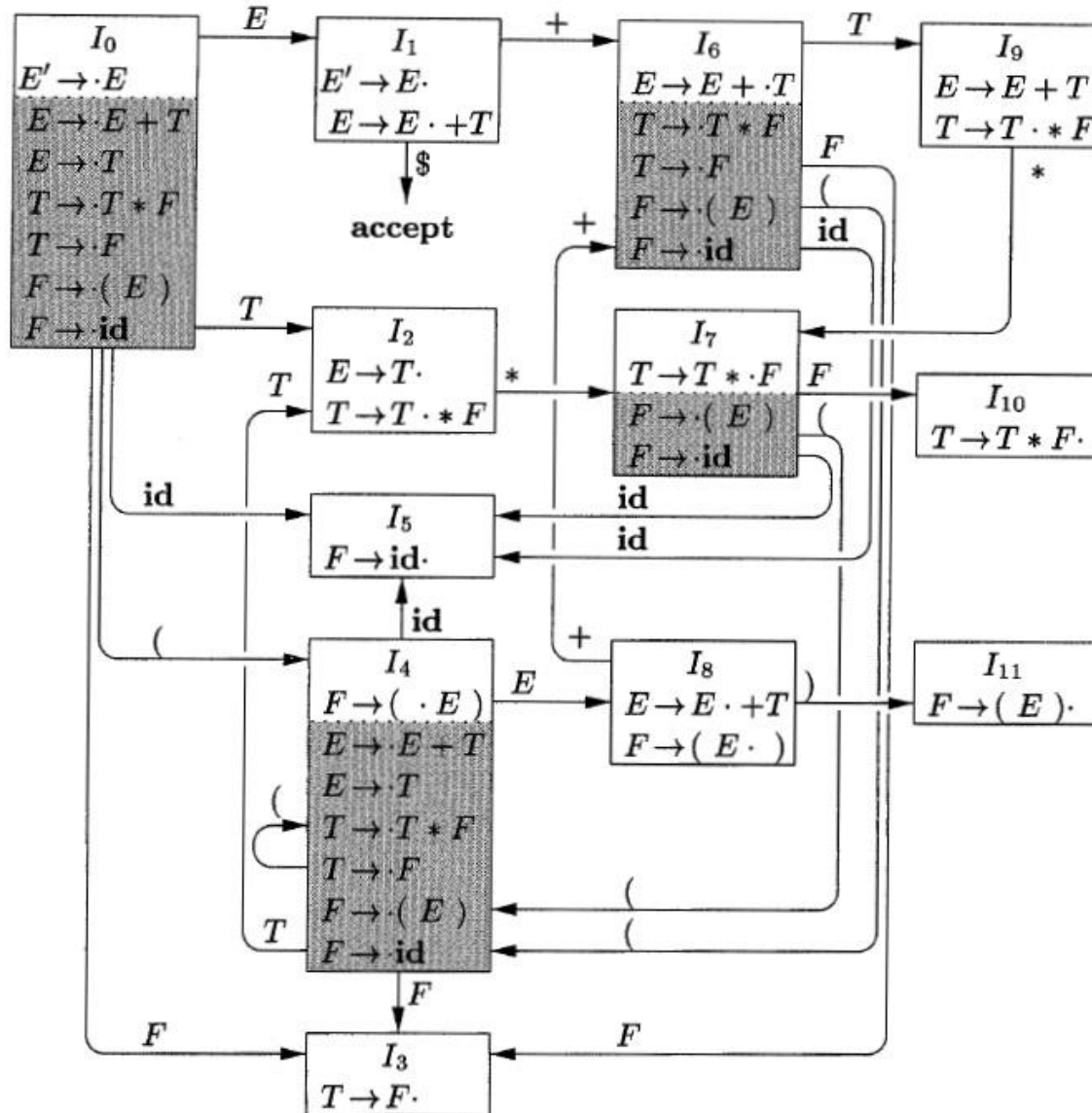
SLR: Transition diagram for LR(0) items

G

$E \rightarrow E + T$
 $E \rightarrow T$
 $T \rightarrow T * F$
 $T \rightarrow F$
 $F \rightarrow (E)$
 $F \rightarrow id$

G'

$E' \rightarrow E$
 $E \rightarrow E + T$
 $E \rightarrow T$
 $T \rightarrow T * F$
 $T \rightarrow F$
 $F \rightarrow (E)$
 $F \rightarrow id$



Closure of Item Sets

If I is a set of items for a grammar G , then $\text{CLOSURE}(I)$ is the set of items constructed from I by the two rules:

1. Initially, add every item in I to $\text{CLOSURE}(I)$.
2. If $A \rightarrow \alpha \cdot B \beta$ is in $\text{CLOSURE}(I)$ and $B \rightarrow \gamma$ is a production, then add the item $B \rightarrow \cdot \gamma$ to $\text{CLOSURE}(I)$, if it is not already there. Apply this rule until no more new items can be added to $\text{CLOSURE}(I)$.

SLR Parse Table Generation

1. Construct $C = \{I_0, I_1, \dots, I_n\}$, the collection of sets of LR(0) items for G' .
2. State i is constructed from I_i . The parsing actions for state i are determined as follows:
 - (a) If $[A \rightarrow \alpha \cdot a \beta]$ is in I_i and $\text{GOTO}(I_i, a) = I_j$, then set $\text{ACTION}[i, a]$ to “shift j .” Here a must be a terminal.
 - (b) If $[A \rightarrow \alpha \cdot]$ is in I_i , then set $\text{ACTION}[i, a]$ to “reduce $A \rightarrow \alpha$ ” for all a in $\text{FOLLOW}(A)$; here A may not be S' .
 - (c) If $[S' \rightarrow S \cdot]$ is in I_i , then set $\text{ACTION}[i, \$]$ to “accept.”

If any conflicting actions result from the above rules, we say the grammar is not SLR(1). The algorithm fails to produce a parser in this case.

3. The goto transitions for state i are constructed for all nonterminals A using the rule: If $\text{GOTO}(I_i, A) = I_j$, then $\text{GOTO}[i, A] = j$.
4. All entries not defined by rules (2) and (3) are made “error.”
5. The initial state of the parser is the one constructed from the set of items containing $[S' \rightarrow \cdot S]$.

SLR parse table

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow id$

STATE	ACTION						GOTO		
	id	+	*	()	\$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

s6: Shift present token to stack and move to state 6

r6: reduce by production 6 and [move to a new state](#) (use goto section)

SLR parsing algorithm

```
let  $a$  be the first symbol of  $w\$$ ;  
while(1) { /* repeat forever */  
    let  $s$  be the state on top of the stack;  
    if ( ACTION[ $s, a$ ] = shift  $t$  ) {  
        push  $t$  onto the stack;  
        let  $a$  be the next input symbol;  
    } else if ( ACTION[ $s, a$ ] = reduce  $A \rightarrow \beta$  ) {  
        pop  $|\beta|$  symbols off the stack;  
        let state  $t$  now be on top of the stack;  
        push GOTO[ $t, A$ ] onto the stack;  
        output the production  $A \rightarrow \beta$ ;  
    } else if ( ACTION[ $s, a$ ] = accept ) break; /* parsing is done */  
    else call error-recovery routine;  
}
```

Figure 4.36: LR-parsing program

Moves of SLR parser on $\text{id}*\text{id}+\text{id}\$$

	STACK	SYMBOLS	INPUT	ACTION
(1)	0	\$	$\text{id} * \text{id} + \text{id} \$$	shift
(2)	0 5	\$ id	$* \text{id} + \text{id} \$$	reduce by $F \rightarrow \text{id}$
(3)	0 3	\$ F	$* \text{id} + \text{id} \$$	reduce by $T \rightarrow F$
(4)	0 2	\$ T	$* \text{id} + \text{id} \$$	shift
(5)	0 2 7	\$ $T *$	$\text{id} + \text{id} \$$	shift
(6)	0 2 7 5	\$ $T * \text{id}$	$+ \text{id} \$$	reduce by $F \rightarrow \text{id}$
(7)	0 2 7 10	\$ $T * F$	$+ \text{id} \$$	reduce by $T \rightarrow T * F$
(8)	0 2	\$ T	$+ \text{id} \$$	reduce by $E \rightarrow T$
(9)	0 1	\$ E	$+ \text{id} \$$	shift
(10)	0 1 6	\$ $E +$	$\text{id} \$$	shift
(11)	0 1 6 5	\$ $E + \text{id}$	\$	reduce by $F \rightarrow \text{id}$
(12)	0 1 6 3	\$ $E + F$	\$	reduce by $T \rightarrow F$
(13)	0 1 6 9	\$ $E + T$	\$	reduce by $E \rightarrow E + T$
(14)	0 1	\$ E	\$	accept

Moves of SLR parser on $id*id+id\$$

STATE	ACTION						GOTO		
	id	+	*	()	\$	E	T	F
0	s5				s4		1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5				s4		8	2	3
5		r6	r6		r6	r6			
6	s5				s4			9	3
7	s5				s4				10
8		s6				s11			
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Stack:

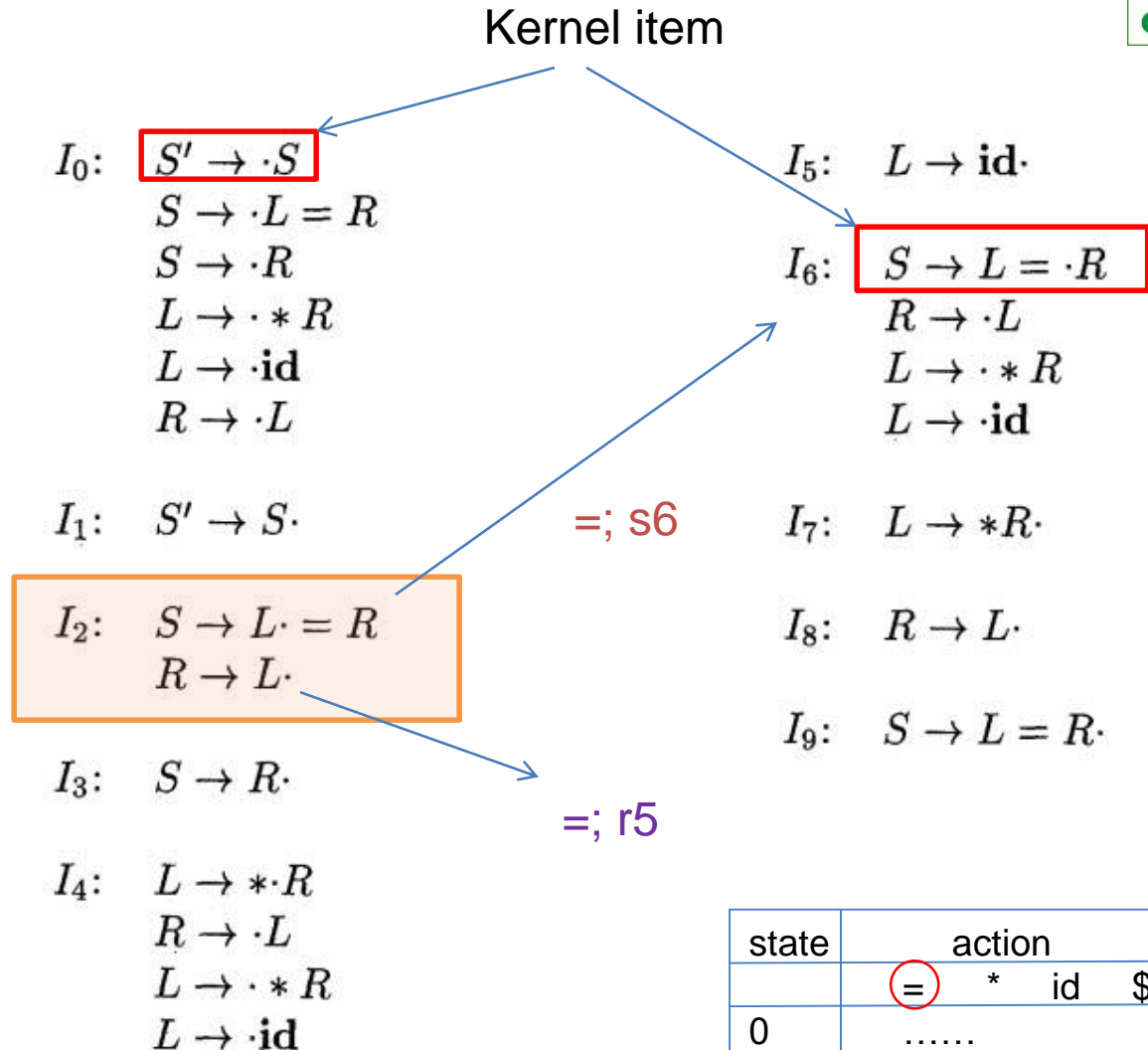
Symbols:

- $E \rightarrow E + T$
- $E \rightarrow T$
- $T \rightarrow T * F$
- $T \rightarrow F$
- $F \rightarrow (E)$
- $F \rightarrow id$

	STACK	SYMBOLS	INPUT	ACTION
(1)	0	\$	$id * id + id \$$	shift
(2)	0 5	$\$ id$	$* id + id \$$	reduce by $F \rightarrow id$
(3)	0 3	$\$ F$	$* id + id \$$	reduce by $T \rightarrow F$
(4)	0 2	$\$ T$	$* id + id \$$	shift
(5)	0 2 7	$\$ T *$	$id + id \$$	shift
(6)	0 2 7 5	$\$ T * id$	$+ id \$$	reduce by $F \rightarrow id$
(7)	0 2 7 10	$\$ T * F$	$+ id \$$	reduce by $T \rightarrow T * F$
(8)	0 2	$\$ T$	$+ id \$$	reduce by $E \rightarrow T$
(9)	0 1	$\$ E$	$+ id \$$	shift
(10)	0 1 6	$\$ E +$	$id \$$	shift
(11)	0 1 6 5	$\$ E + id$	\$	reduce by $F \rightarrow id$
(12)	0 1 6 3	$\$ E + F$	\$	reduce by $T \rightarrow F$
(13)	0 1 6 9	$\$ E + T$	\$	reduce by $E \rightarrow E + T$
(14)	0 1	$\$ E$	\$	accept

Weakness of SLR Parser

Grammar	
1.	$S \rightarrow L = R$
2.	$S \rightarrow R$
3.	$L \rightarrow * R$
4.	$L \rightarrow \text{id}$
5.	$R \rightarrow L$



state	action
	$=$ * id \$
0
1
2	<u>s6/r5</u>
.
.

Fig: Canonical LR(0) Collections

shift-reduce conflict

CLR States and Transition diagram

$A \rightarrow \alpha.B\beta, a$

closure:

$B \rightarrow \cdot\gamma, \text{First}(\beta a)$

Grammar

1. $S \rightarrow CC$
2. $C \rightarrow cC$
3. $C \rightarrow d$

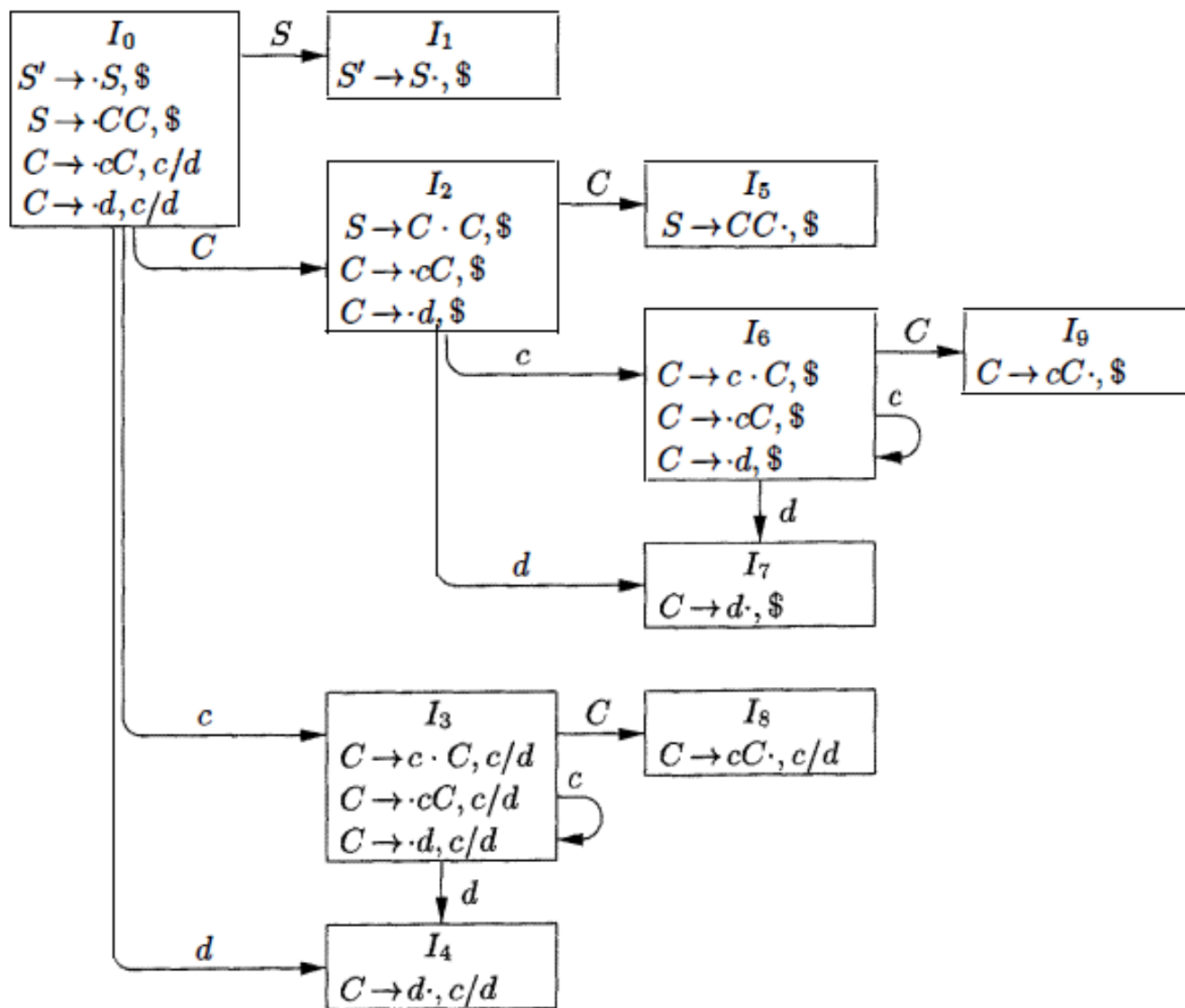


Fig: Canonical LR(1) Collections / GOTO Graph

CLR Parse Table

Grammar

1. $S \rightarrow CC$
2. $C \rightarrow cC$
3. $C \rightarrow d$

STATE	ACTION			GOTO	
	<i>c</i>	<i>d</i>	\$	<i>S</i>	<i>C</i>
0	s3	s4		1	2
1			acc		
2	s6	s7			5
3	s3	s4			8
4	r3	r3			
5			r1		
6	s6	s7			9
7			r3		
8	r2	r2			
9			r2		

Fig: CLR Parse Table

LALR from CLR states

Grammar

1. $S \rightarrow CC$
2. $C \rightarrow cC$
3. $C \rightarrow d$

I_{36} : $C \rightarrow c \cdot C, c/d/\$$
 $C \rightarrow \cdot cC, c/d/\$$
 $C \rightarrow \cdot d, c/d/\$$

I_{47} : $C \rightarrow d \cdot, c/d/\$$

I_{89} : $C \rightarrow cC \cdot, c/d/\$$

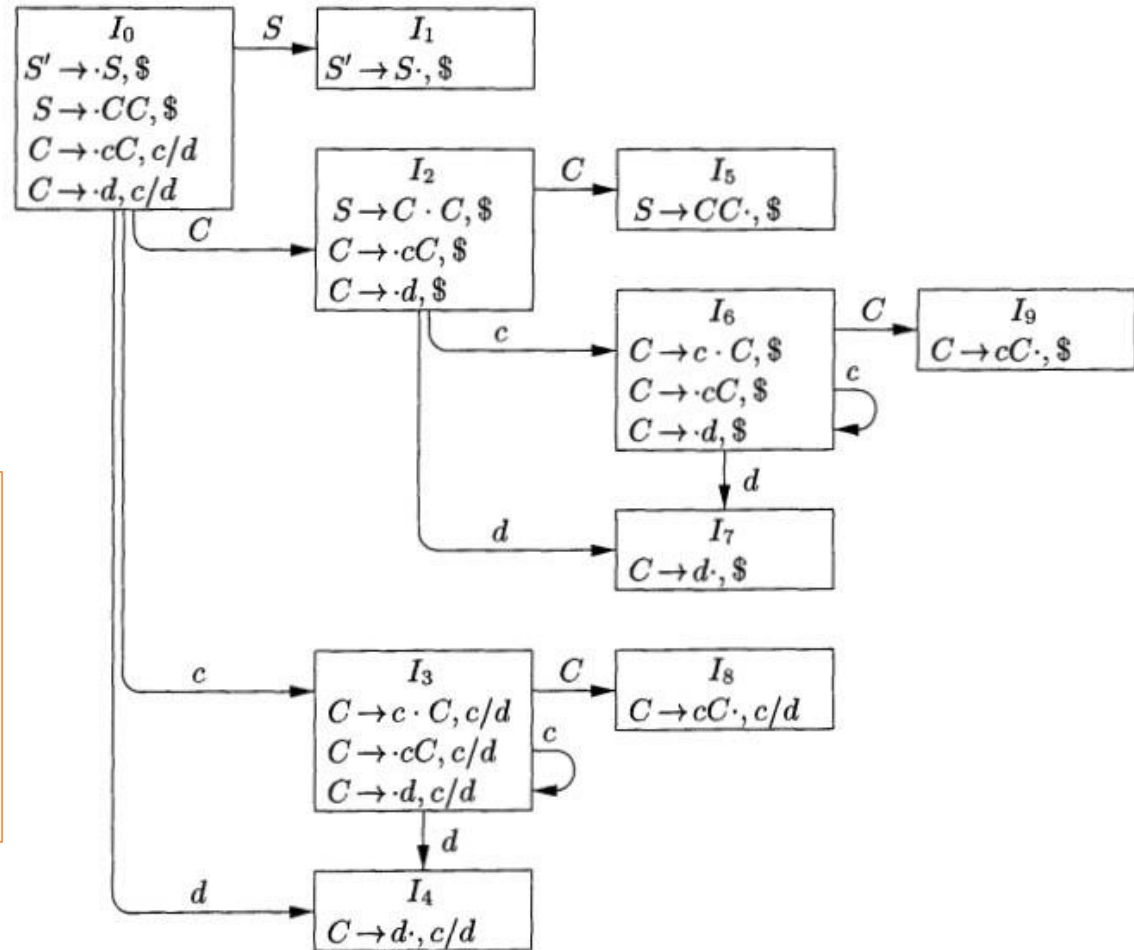


Fig: Canonical LR(1) Collections

LALR Parse Table

Grammar

1. $S \rightarrow CC$
2. $C \rightarrow cC$
3. $C \rightarrow d$

STATE	ACTION			GOTO	
	<i>c</i>	<i>d</i>	\$	<i>S</i>	<i>C</i>
0	s36	s47		1	2
1			acc		
2	s36	s47			5
36	s36	s47			89
47	r3	r3	r3		
5			r1		
89	r2	r2	r2		

Fig: LALR parse table