CSE 4205
Digital Logic Design

Simplification of Boolean Functions

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The Map Method

- Complexity of the digital logic gates depends on the complexity of algebraic expression.
- Simplification – truth table can be used – but no specific rules
- Veitch / Karnaugh Map – simple straightforward procedure
- Made of squares – represent minterms

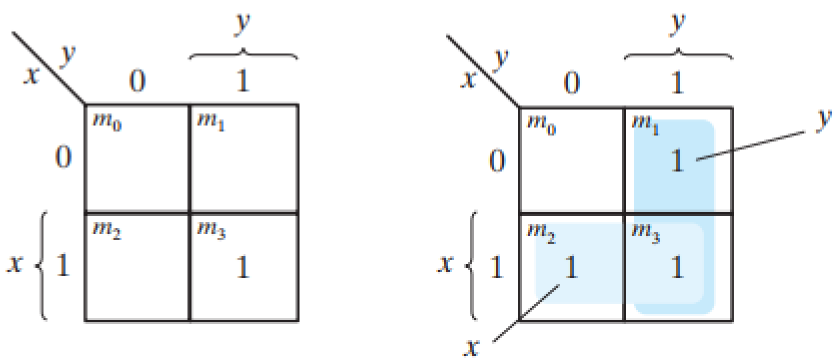
2 or 3 Variables Map

- There are 4 minterms for 2 variables and 8 minterms for 3 variables where variables are appeared in primed or not-primed form.
- Minterms are in binary sequence but in a sequence similar to the reflected code
- Only one bit changes in value from one adjacent column to the next
  - Any two adjacent squares in the map differ by only one variable

$$m_5 + m_7 = xy'z + xyz = xz(y' + y) = xz$$

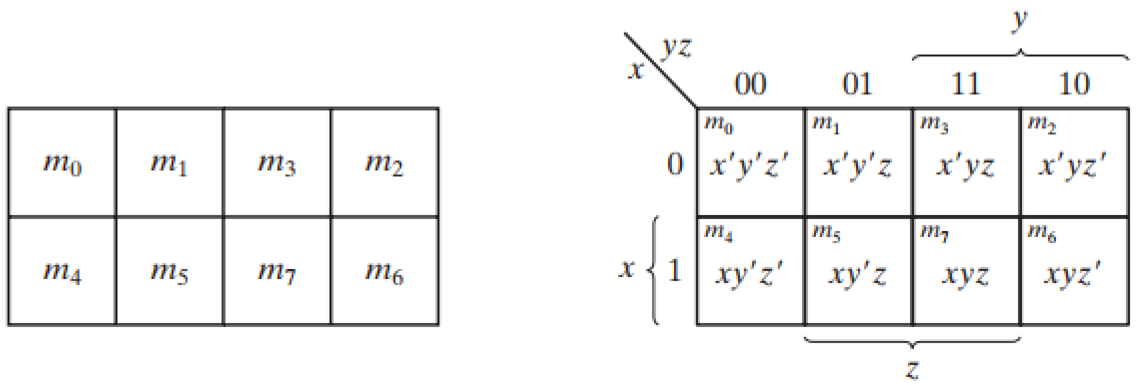
- A ***prime implicant*** is a product term obtained by combining the maximum possible number of adjacent squares in the map. (number = 2^n)

2 or 3 Variables Map



(a) xy (b) $x + y$

Two variables K-map



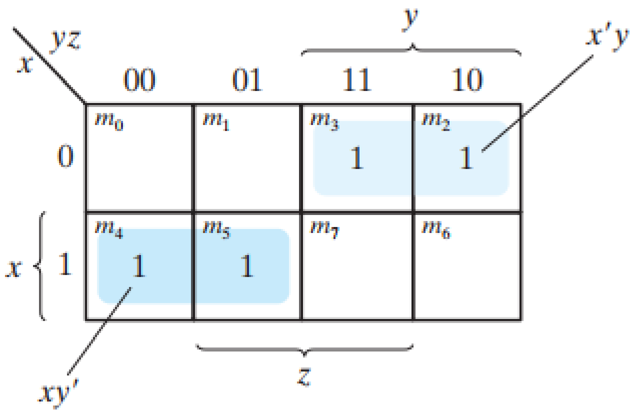
(a) (b)

Three variables K-map

2 or 3 Variables Map…

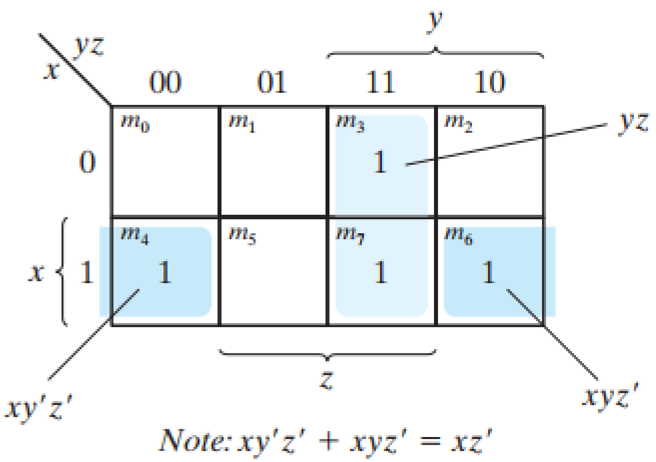
F = x’yz + x’yz’ + xy’z’ + xy’z

$$F(x, y, z) = \Sigma(2, 3, 4, 5)$$



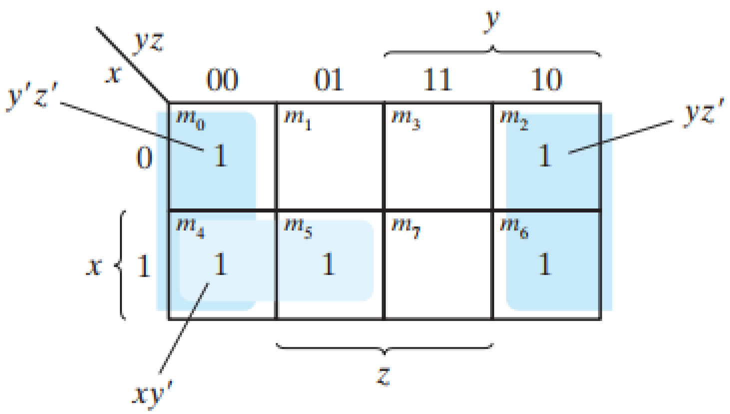
F = x’yz + xy’z’ + xyz + xyz’

$$F(x, y, z) = \Sigma(3, 4, 6, 7)$$



2 or 3 Variables Map…

$$F(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$



Note: $y'z' + yz' = z'$

Solve these ?

$F = x'yz + x'yz' + xy'z' + xy'z$

$F = A'C + A'B + AB'C + BC$

4 Variables Map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

(a)

$wx \backslash yz$	00	01	11	10
00	m_0 $w'x'y'z'$	m_1 $w'x'y'z$	m_3 $w'x'yz$	m_2 $w'x'yz'$
01	m_4 $w'xy'z'$	m_5 $w'xy'z$	m_7 $w'xyz$	m_6 $w'xyz'$
11	m_{12} $wxy'z'$	m_{13} $wxy'z$	m_{15} $wxyz$	m_{14} $wxyz'$
10	m_8 $wx'y'z'$	m_9 $wx'y'z$	m_{11} $wx'yz$	m_{10} $wx'yz'$

y (columns 11, 10); x (rows 01, 11); w (rows 11, 10); z (columns 01, 11)

(b)

Remember:

1 square = 4 literals term

2 squares = 3 literals term

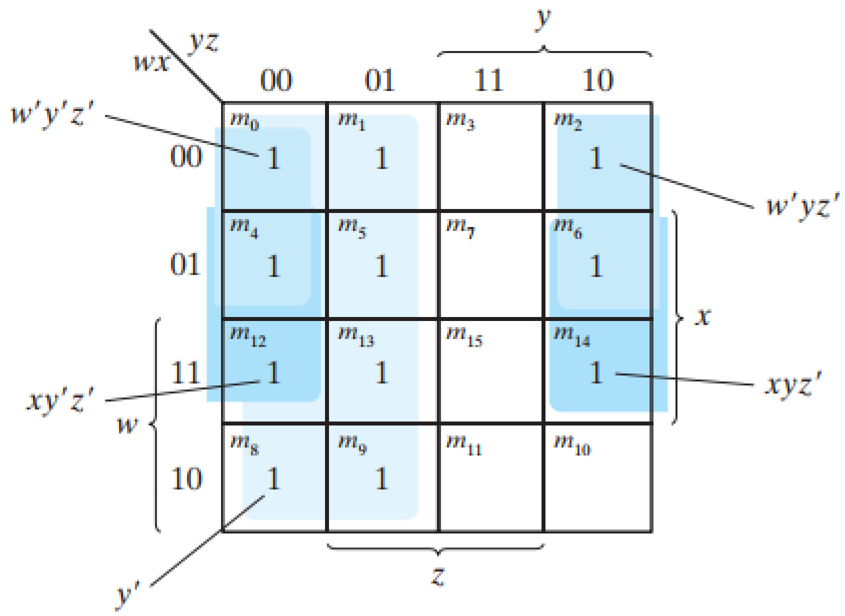
4 squares = 2 literals term

8 squares = 1 literal term

16 squares = 1

4 Variable Map

$$F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

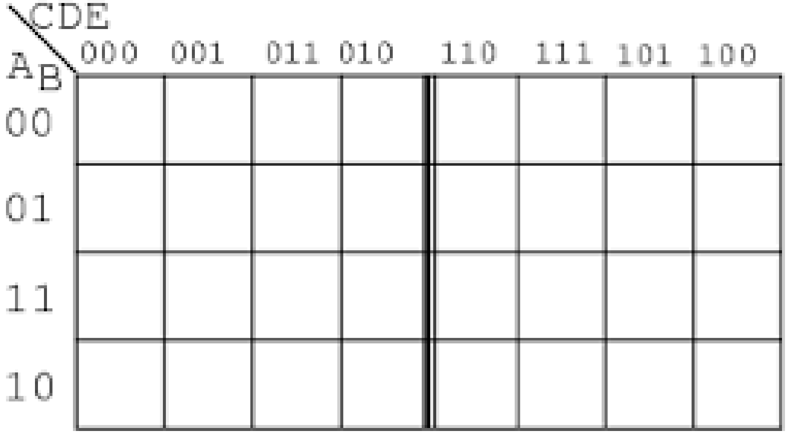


Note: $w'y'z' + w'yz' = w'z'$
$xy'z' + xyz' = xz'$

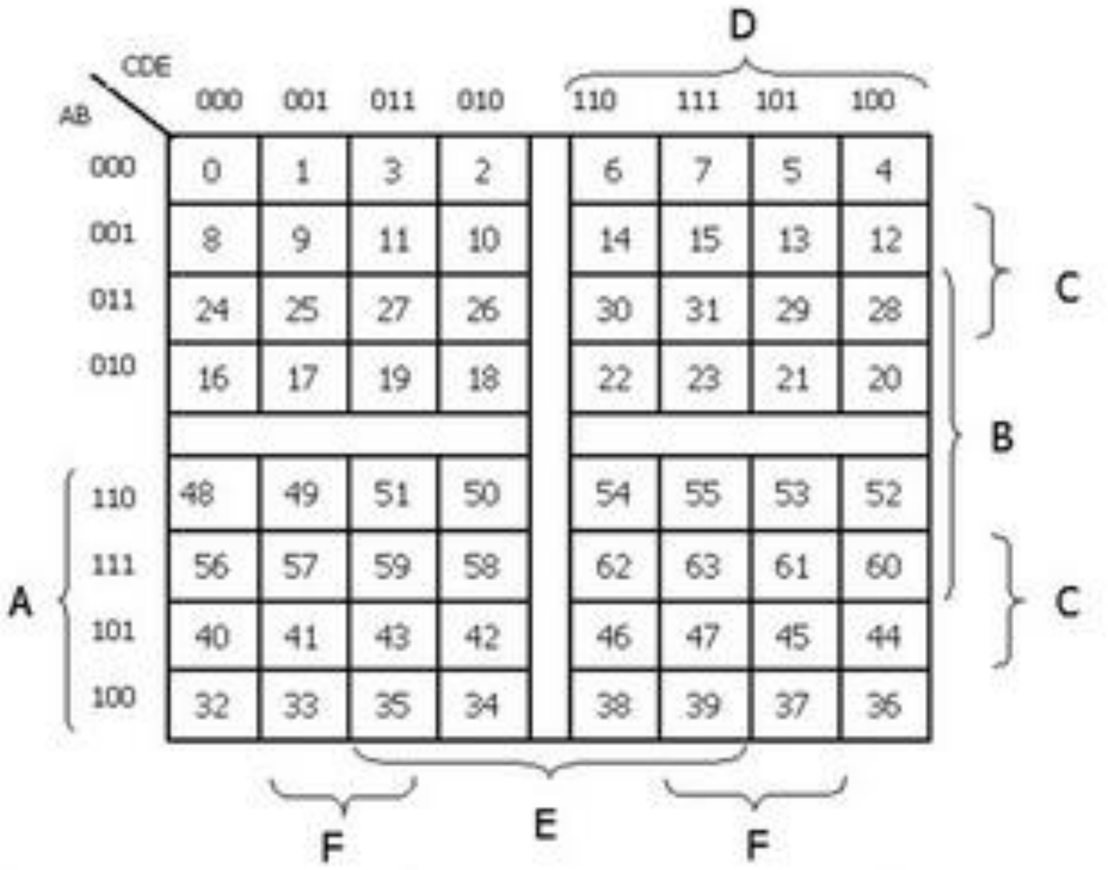
Solve this:

$$F = A'B'C' + B'CD' + A'BCD' + AB'C'$$

5 or 6 Variables Map



5- variable Karnaugh map (Gray code)

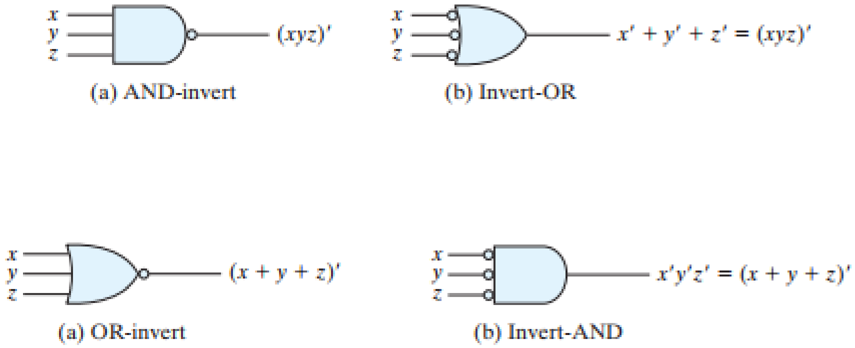


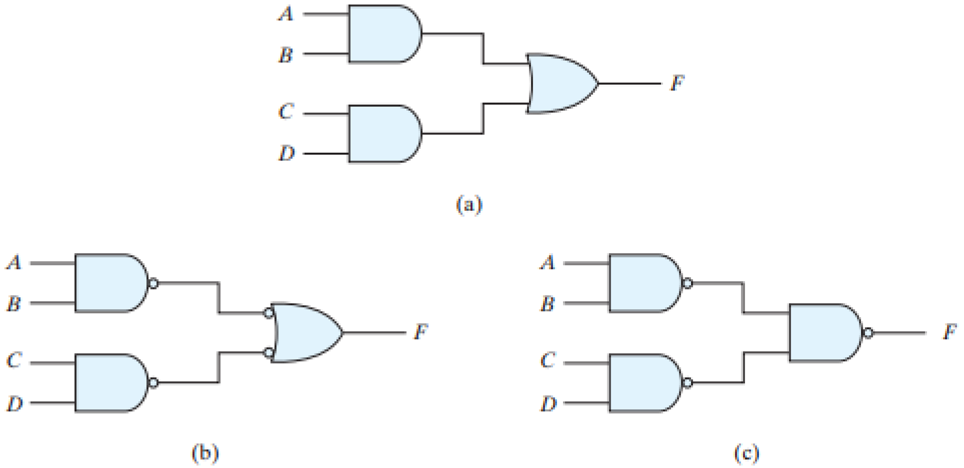
POS & SOP Simplification

Last Class

NAND & NOR implementation

- NAND and NOR are easier to implement





NAND implementation

Home Task

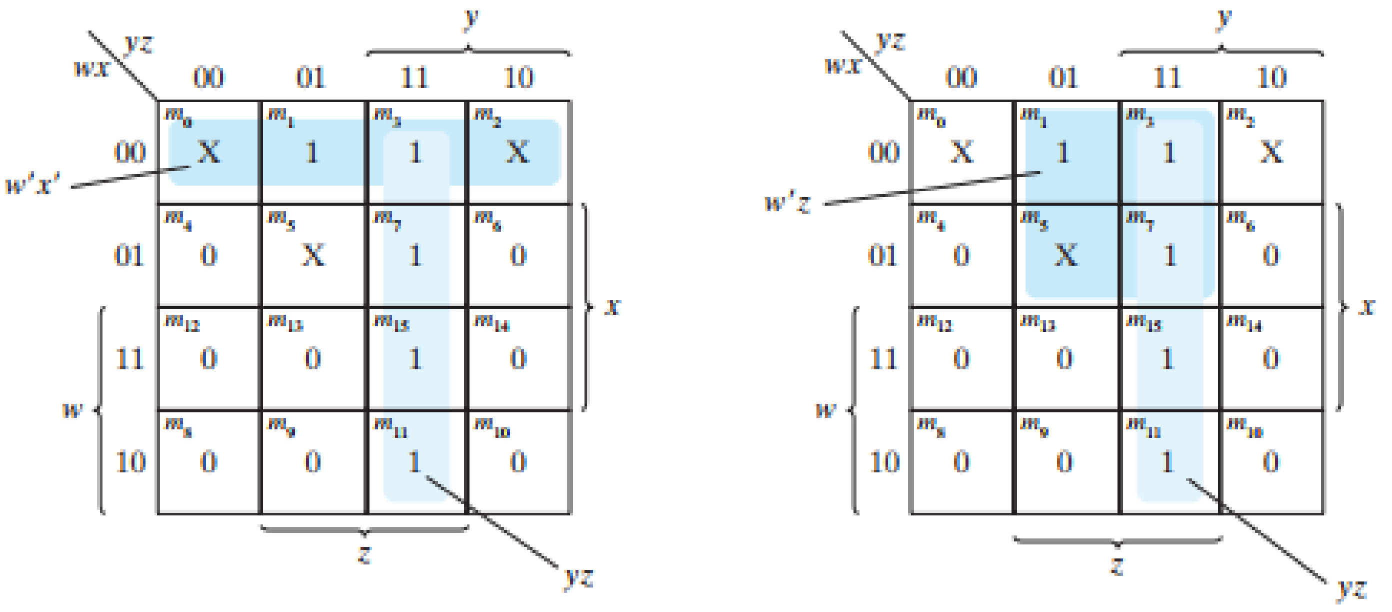
NOR Implementation

Home Task

Don’t Care Condition

$$F(w,x,y,z) = \Sigma(1,3,7,11,15)$$

$$d(w,x,y,z) = \Sigma(0,2,5)$$



(a) $F = yz + w'x'$

(b) $F = yz + w'z$

The Tabular Method / Quine-McCluskey Method

$$F = \Sigma(0, 1, 2, 8, 10, 11, 14, 15)$$

(a)

	w x y z	
0	0 0 0 0	√
1	0 0 0 1	√
2	0 0 1 0	√
8	1 0 0 0	√
10	1 0 1 0	√
11	1 0 1 1	√
14	1 1 1 0	√
15	1 1 1 1	√

(b)

	w x y z	
0, 1	0 0 0 –	
0, 2	0 0 – 0	√
0, 8	– 0 0 0	√
2, 10	– 0 1 0	√
8, 10	1 0 – 0	√
10, 11	1 0 1 –	√
10, 14	1 – 1 –	√
11, 15	1 – 1 1	√
14, 15	1 1 1 –	√

(c)

	w x y z
0, 2, 8, 10	– 0 – 0
0, 8, 2, 10	– 0 – 0
10, 11, 14, 15	1 – 1 –
10, 14, 11, 15	1 – 1 –

$$F = w'x'y' + x'z' + wy$$

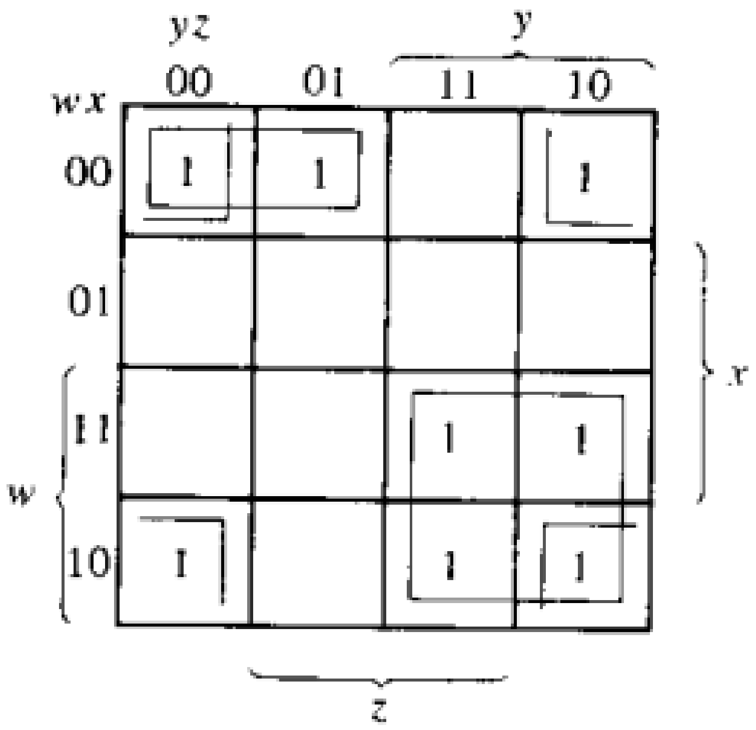
The Tabular Method (Alternative Way)

TABLE 3-6
Determination of Prime Implicants of Example 3-13 with Decimal Notation

(a)		(b)		(c)
0	√	0, 1 (1)		0, 2, 8, 10 (2, 8)
		0, 2 (2)	√	0, 2, 8, 10 (2, 8)
1	√	0, 8 (8)	√	
2	√			10, 11, 14, 15 (1, 4)
8	√	2, 10 (8)	√	10, 11, 14, 15 (1, 4)
		8, 10 (2)	√	
10	√			
		10, 11 (1)	√	
		10, 14 (4)	√	
11	√			
14	√			
		11, 15 (4)	√	
15	√	14, 15 (1)	√	

The Tabular Method

- Verification using K-map:



Prime Implicants Determination

- **Another Example:**

$$F(w, x, y, z) = \Sigma(1, 4, 6, 7, 8, 9, 10, 11, 15)$$

Determination of Prime Implicants for Example 3-14

(a)			(b)			(c)
0001	1	✓	1, 9	(8)		8, 9, 10, 11 (1, 2)
0100	4	✓	4, 6	(2)		8, 9, 10, 11 (1, 2)
1000	8	✓	8, 9	(1)	✓	
			8, 10	(2)	✓	
0110	6	✓	6, 7	(1)		
1001	9	✓	9, 11	(2)	✓	
1010	10	✓	10, 11	(1)	✓	
0111	7	✓	7, 15	(8)		
1011	11	✓	11, 15	(4)		
1111	15	✓				

Prime Implicants Selections

Prime implicants

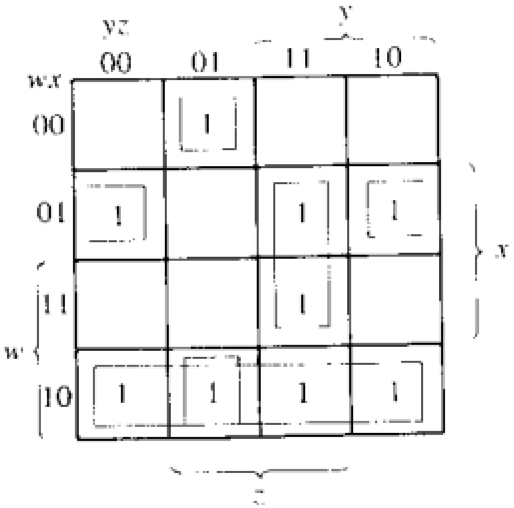
Decimal	w	x	y	z	Term
1, 9 (8)	–	0	0	1	$x'y'z$
4, 6 (2)	0	1	–	0	$w'xz'$
6, 7 (1)	0	1	1	–	$w'xy$
7, 15 (8)	–	1	1	1	xyz
11, 15 (4)	1	–	1	1	wyz
8, 9, 10, 11 (1, 2)	1	0	–	–	wx'

Prime Implicant Table for Example 3-15

			1	4	6	7	8	9	10	11	15
√	$x'y'z$	1, 9	X					X			
√	$w'xz'$	4, 6		X	X						
	$w'xy$	6, 7			X	X					
	xyz	7, 15				X					X
	wyz	11, 15								X	X
√	wx'	8, 9, 10, 11					X	X	X	X	
			√	√	√		√	√	√	√	

The Tabular Method

Verification using K-map:



$$F = x'y'z + w'xz' + wx' + xyz$$