Math 4441: Probability and Statistics

Tutorial Class 1: Introduction to probability, conditional probability and Bayes theorem

Following problems will be discussed in the first tutorial class.

1. Assume that an application from a computer (i.e., the sender) is sending data packets to another computer (i.e., the receiver). The probability that a particular packet is received by the second computer is $p$. Find the following probability models:
   a) Three packets are sent by the sender and the sequence of the success (denote it by $D$) and failure (denote it by $F$) will be observed.
   b) Three packets are sent by the sender and the number of success(es) is/are observed.
   c) A single packet is sent repeatedly until it is successfully received by the receiver. The number of attempts required to send the packet is observed.
   d) The sender sends packets one after another until 3 packets are successfully received by the receiver. The number of packets sent are observed.

2. There are 14 marketing firms hiring new graduates. Suppose you randomly found the recruitment advertisement of six of these firms and sent them your resume. If three of these Marketing firms are in Chittagong, what is the probability that you did not apply to a marketing firm in Chittagong?

3. Suppose that four balls are selected one at a time, without replacement, from a box containing $r$ red balls and b blue balls ($r \geq 2, b \geq 2$). Determine the probability of obtaining the sequence of outcomes red, blue, red, blue.

4. In an article about confusion of eye witnesses, John Allen Paulos cites the problem of three coins, one of which is biased so that it runs up heads $75\%$ of the time. If you randomly select one of the coins, toss it three times, and obtain three heads, what is the probability that this is the biased coin?

5. A new test has been developed to determine whether a given student is overstressed. This test is 95% accurate if the student is not overstressed, but only 85% accurate if the student is in fact overstressed. It is known that 99.5% of all students are overstressed. Given that a particular student tests negative for stress, what is the probability that the test results are correct, and that this student is not overstressed?

6. The advantage of certain blood test is that 90% of the time it is positive for patients having a certain disease. Its disadvantage is that 25% of the time it is also positive in healthy people. In a certain location 30% of the people have a disease, and anybody with a positive blood test is given a drug that cures the disease. If 20% of the time the drug produces a characteristic rash, what is the probability that a person from this location who has the rash had the disease in the first place?

7. Suppose that a box contains 10 balls. At the start, 3 are white and 7 are blue. Whenever a ball is selected from the box, a layer of blue paint is applied to it, so blue balls stay blue, and white balls become blue; afterward, the ball is returned to the box, so that there are always 10 balls in the box.
   a) Find the probability that a blue ball is chosen at the beginning of round 2.
   b) Suppose at the start of round 2 (i.e., before any painting in round 2 is performed), a blue ball is selected. Find the probability that the blue ball was originally blue (it was blue at the beginning).
   c) Find the probability that the $6^{th}$ ball to be drawn from the box is white.