Cut-Sets & Cut-Vertices

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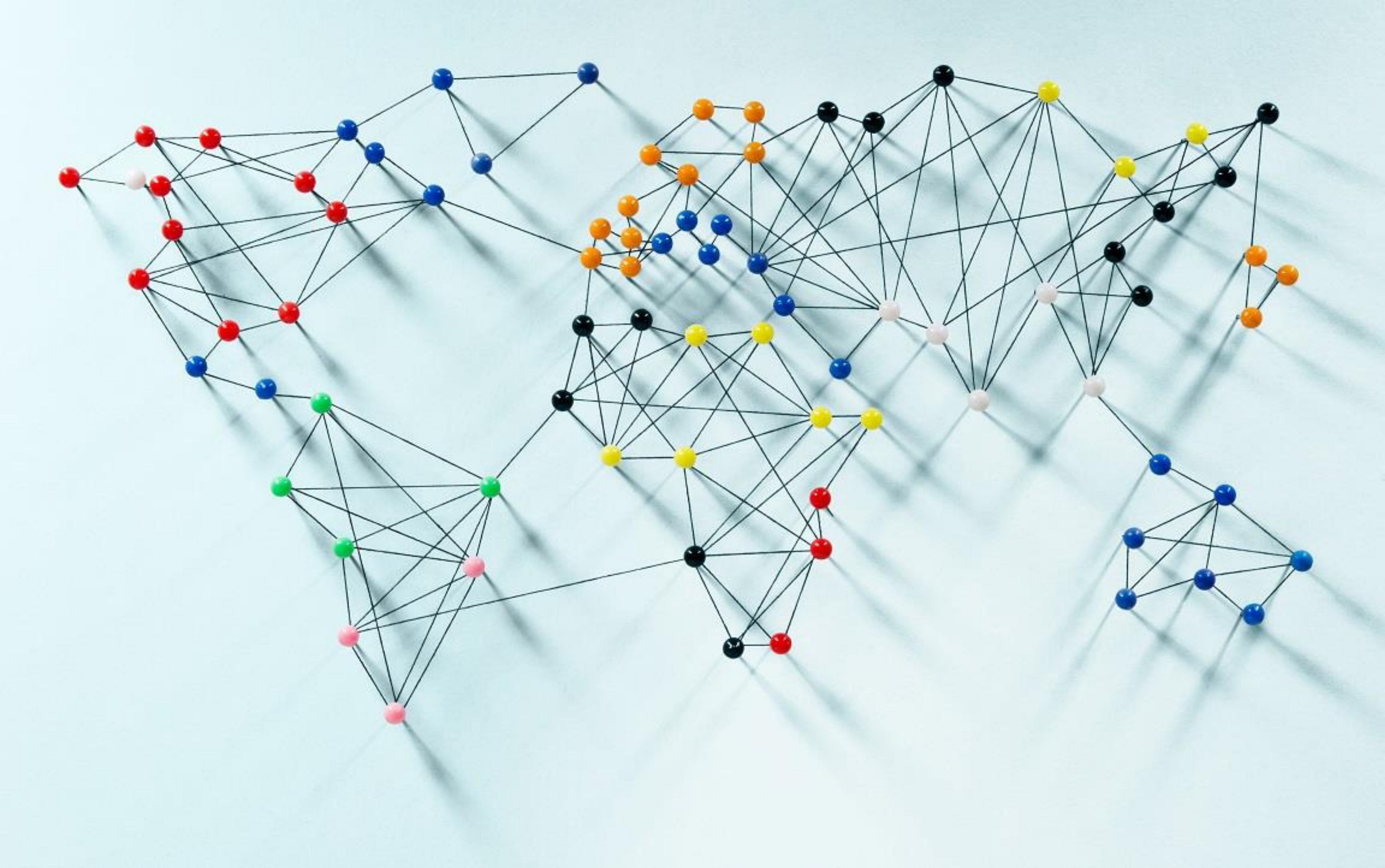
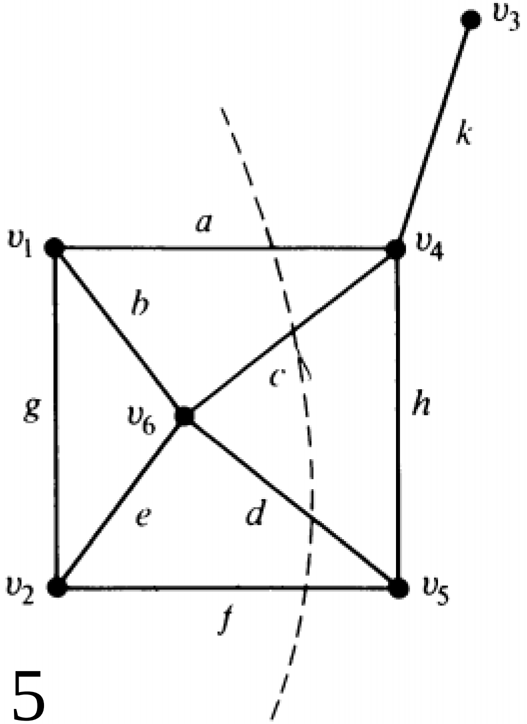


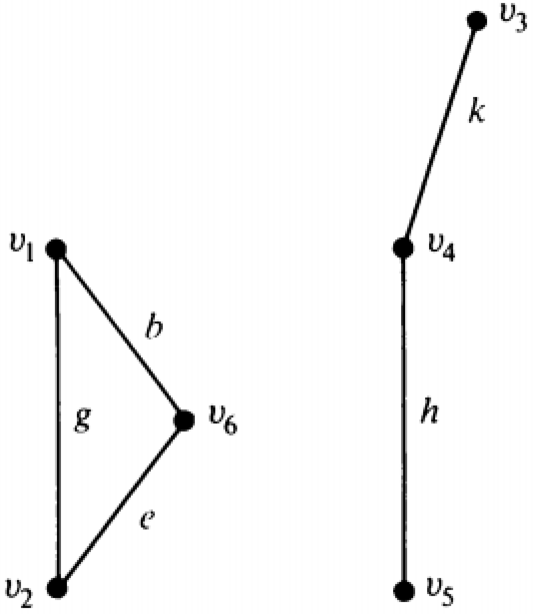
Cut-Sets

- In a connected graph *G*, a *cut-set* is a set of edges whose removal from *G* leaves *G* disconnected, provided removal of no proper subset of these edges disconnects *G*.
- Minimal cut-set/Proper cut-set/simple cut-set/cocycle
- Cut-set always cuts the graph in two.
- Removal of cut-set reduces the rank of graph by one.

Cut-Sets



Rank 5



Rank 4

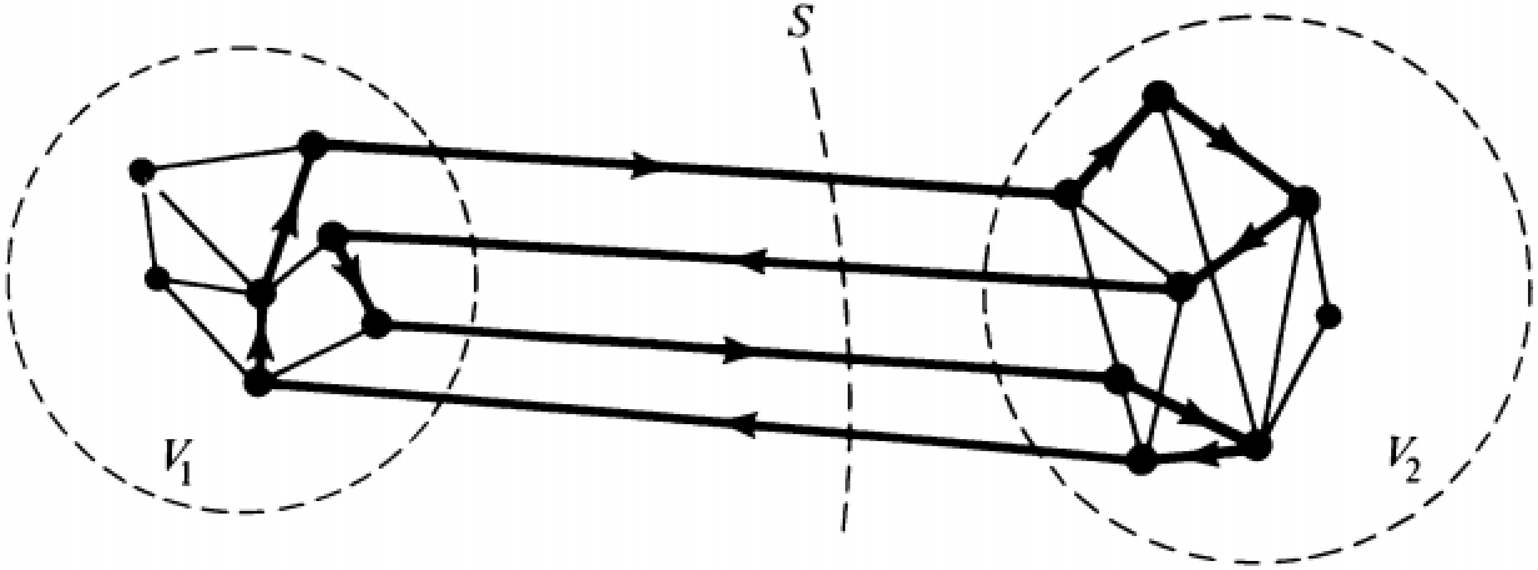
- For tree every edge is a cut-set

Properties of a Cut-Set

- Theorem 4.1 – Every cut-set in a connected graph G must contain at least one branch of every spanning tree of G.
- Theorem 4.2 – In a connected graph G, any minimal set of edges containing at least one branch of every spanning tree of G is a cut-set.

Properties of a Cut-Set

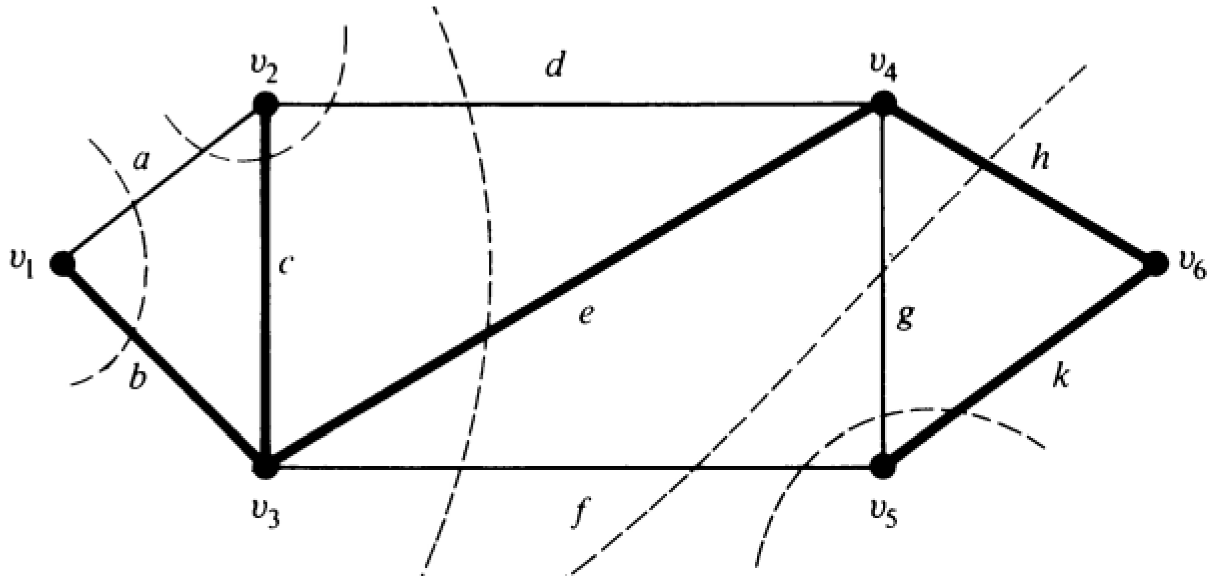
- Theorem 4.3 – Every circuit has an even number of edges in common with any cut-set.



Circuit Γ shown in heavy lines, and is traversed along the direction of the arrows

All Cut-Sets in a graph

- Just as a spanning tree is essential for defining a set of fundamental circuits, so is a spanning tree essential for a set of *fundamental cut-sets*.
- A cut-set S containing exactly one branch of a tree T is called a *fundamental cut-set* with respect to T.



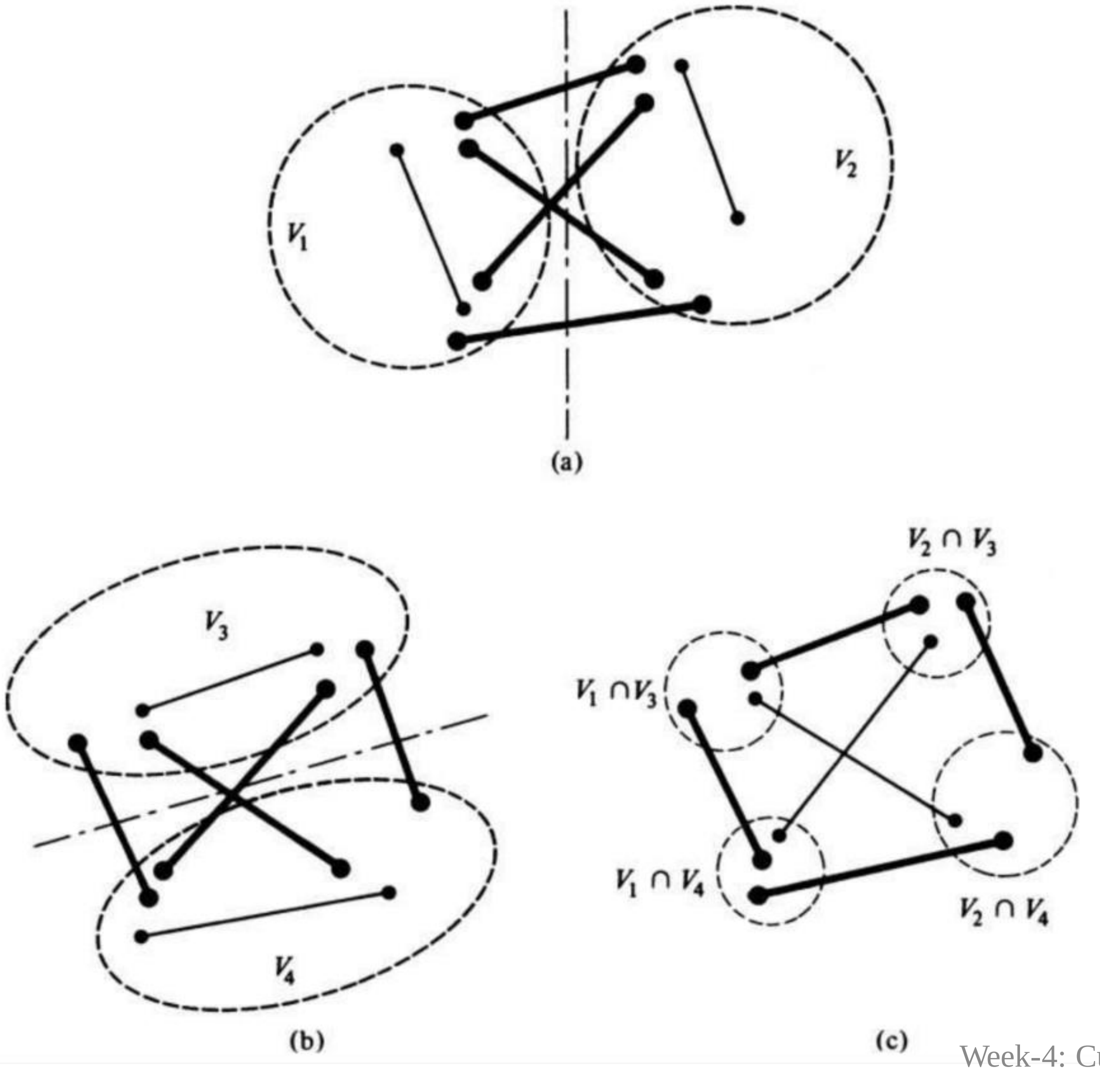
All Cut-Sets in a graph

- Just as every chord of a spanning tree defines a *unique* fundamental circuit, every branch of a spanning tree defines a *unique* fundamental cut-set.

- Both fundamental cut-set and fundamental circuit has meaning only with respect to a *given* spanning tree.

- Theorem 4.4 – The ring sum of any two cut-sets in a graph is either a third cut-set or an edge-disjoint union of cut-sets.

All Cut-Sets in a graph

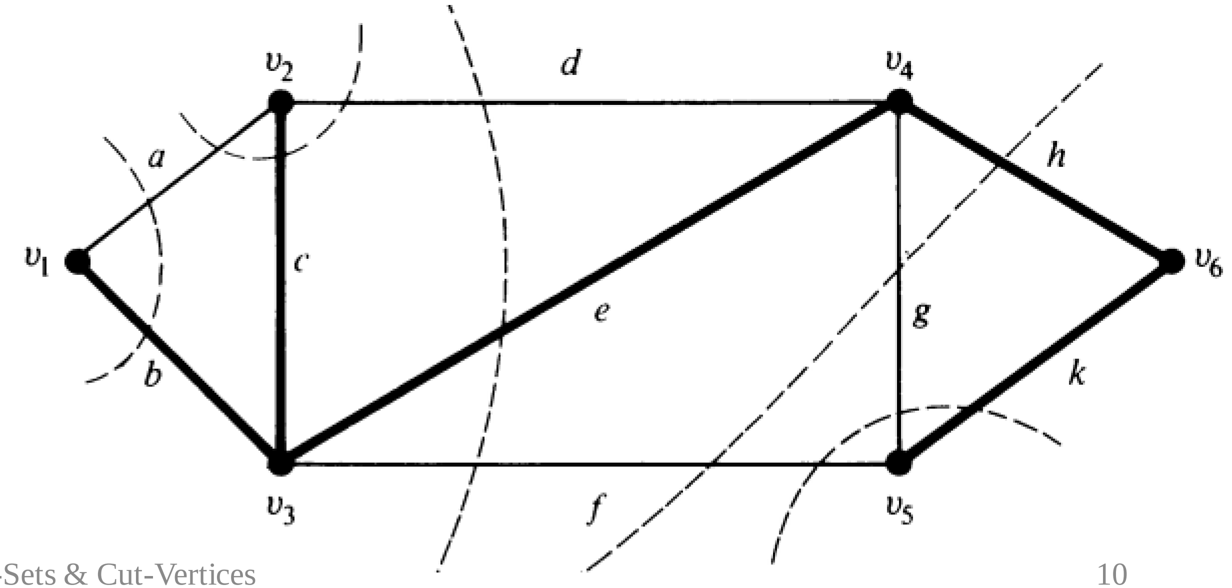


All Cut-Sets in a graph

- {d,e,f} ⊕ {f,g,h} = {d,e,g,h}; another cut-set
- {a,b} ⊕ {b,c,e,f} = {a,c,e,f}; another cut-set
- {d,e,g,h} ⊕ {f,g,k} = {d,e,f,h,k}

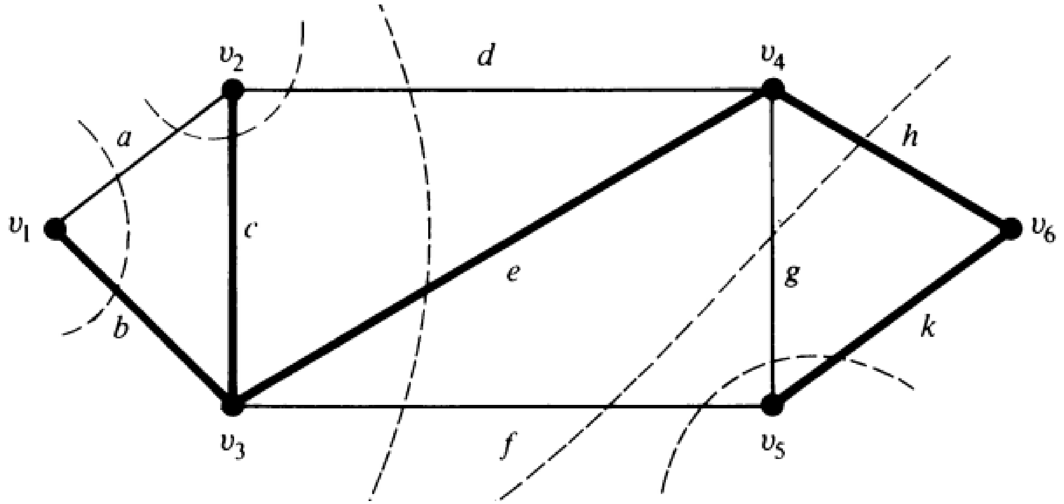
= {d,e,f} U {h,k}; an edge-disjoint union of cut-sets.

So, we found a way to generate more cut-sets



Fundamental Circuits & Cut-Sets

- Theorem 4.5 – With respect to a given spanning tree *T*, a chord c_i that determines a fundamental circuit Γ occurs in every fundamental cut-set associated with the branches in Γ and in no other.
- T = {*b*,*c*,*e*,*h*,*k*}
- Γ = {*f*,*e*,*h*,*k*}; taking *f*
- Cut-Sets produced
  - {*d*,*e*,*f*}
  - {*f*,*g*,*h*}
  - {*f*,*g*,*k*}



Fundamental Circuits & Cut-Sets

- Theorem 4.6 – With respect to a given spanning tree T, a branch b_i, that determines a fundamental cut-set S is contained in every fundamental circuit assosciated with the chords in S, an in no others.

- T = $\{b,c,e,h,k\}$
- Fundamental Cut-set by $e = \{d,e,f\}$
- F.C. for chord $d = \{d,c,e\}$
- F.C. for chord $f = \{e,h,k,f\}$