Planarity & Dual Graph

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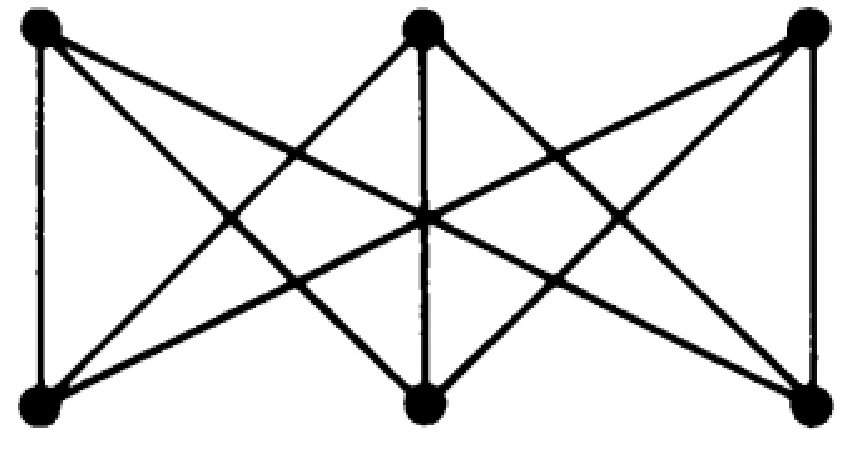
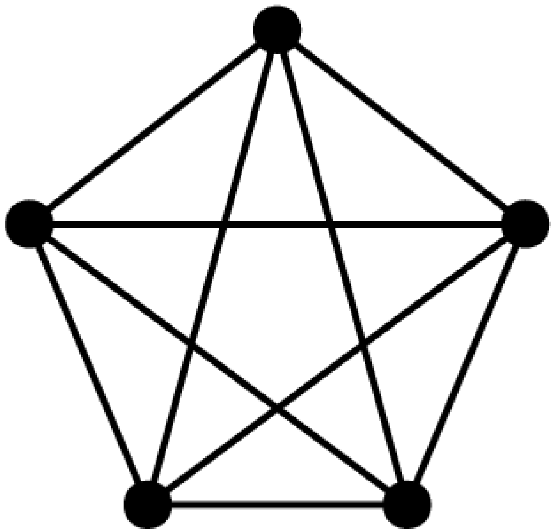
Kuratowski's Two Graphs



$K_{3,3}$

Nonplanar with minimal edges

Properties:
- Regular
- Nonplanar
- Removal of **one** edge/vertex makes them planar



K_5

Nonplanar with minimal vertices

Detection of planarity

How to check planarity? By drawing.

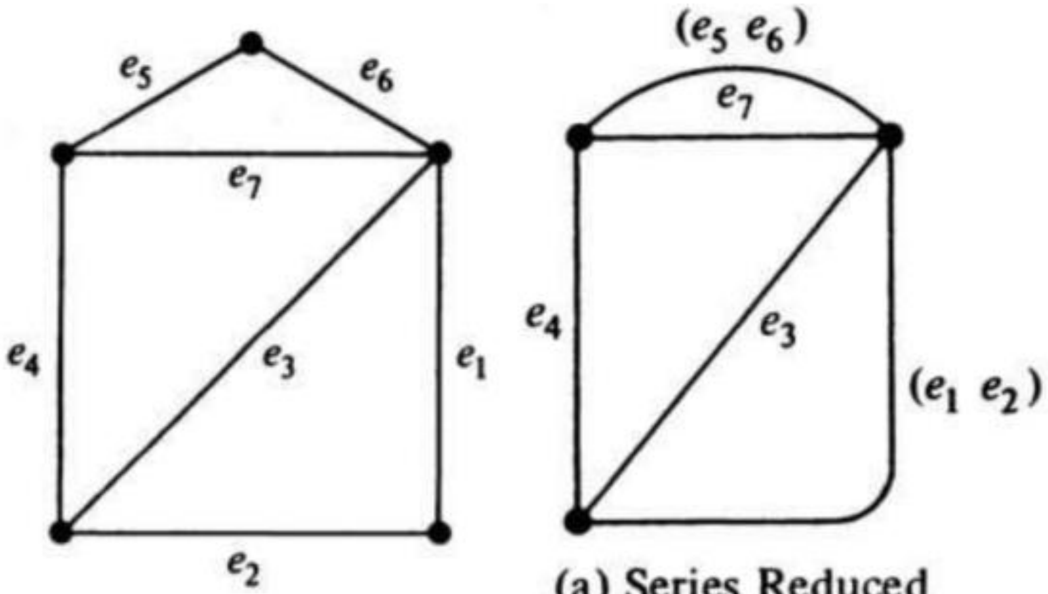
Formal approach:

- ➢Step 1: Remove self-loops
- ➢Step 2: Remove parallel edges, keeping one
- ➢Step 3: Eliminate all edges in series

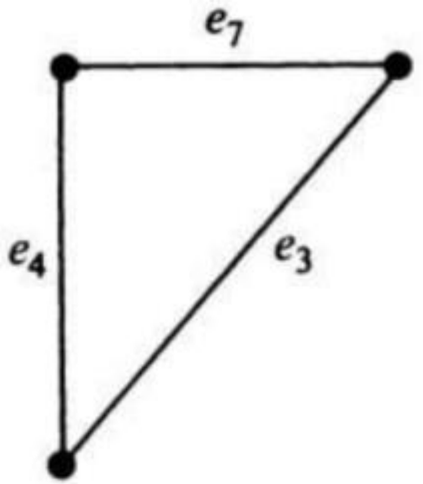
* {Resultant graph will be either:

- A single edge
- Complete graph of 4 vertices
- Non-separable simple graph with $n \geq 5$ and $e \geq 7$}

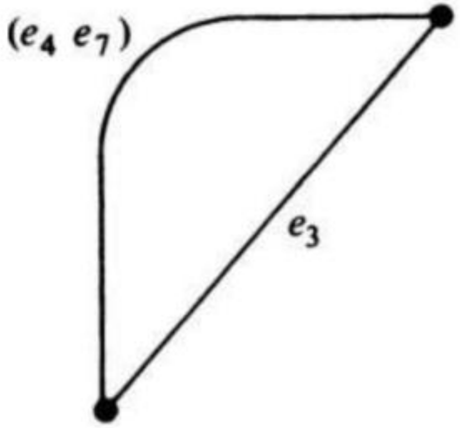
- ➢Step 4: Check $e \leq 3n\text{-}6$, if not satisfied then nonplanar



(a) Series Reduced



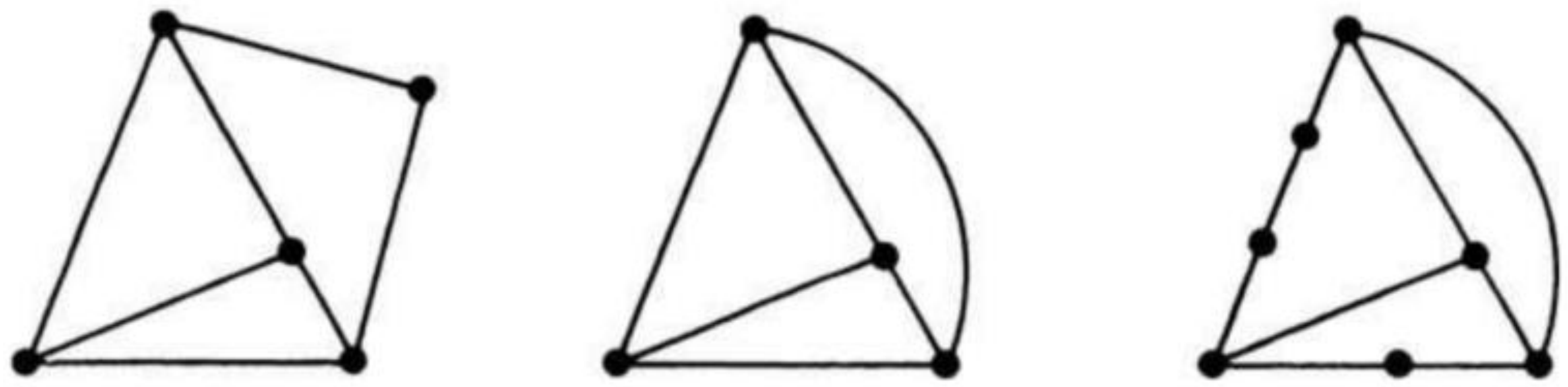
(b) Parallel Reduced



(c) Series Reduced

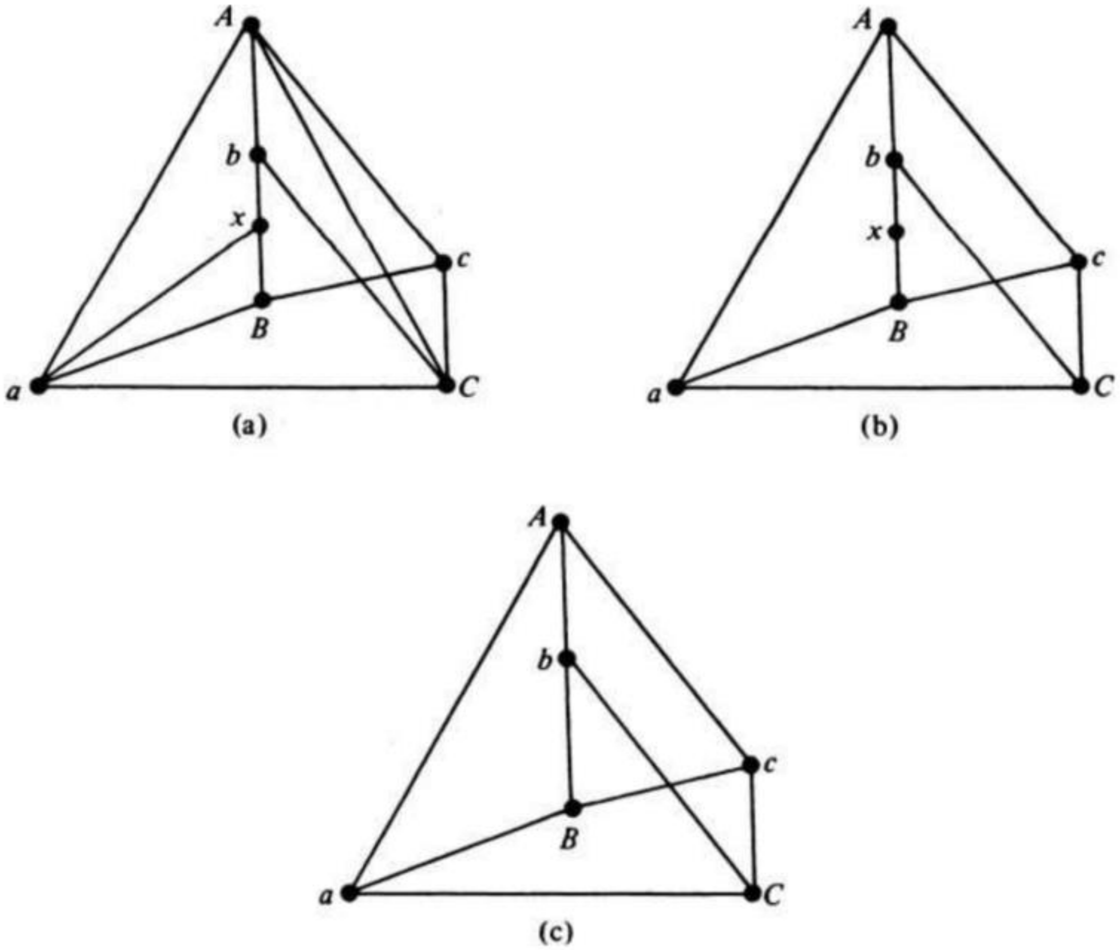
Homeomorphism

- one graph can be obtained from the other by the creation of edges in series (i.e., by insertion of vertices of degree two) or by the merger of edges in series.

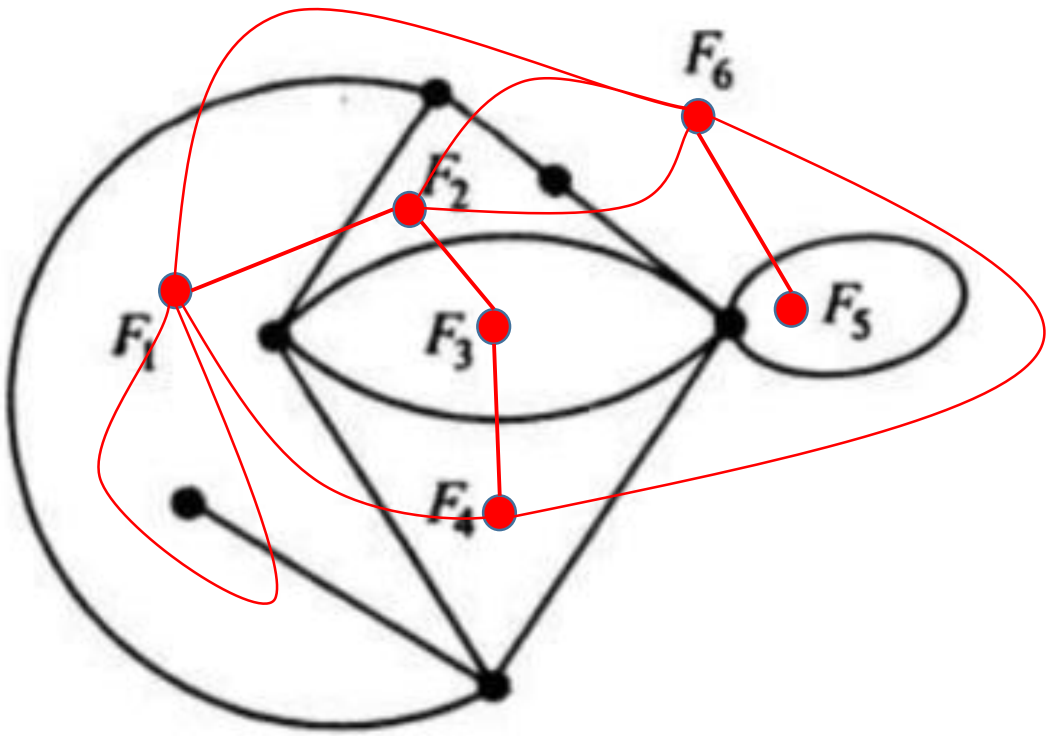


Detection of planarity

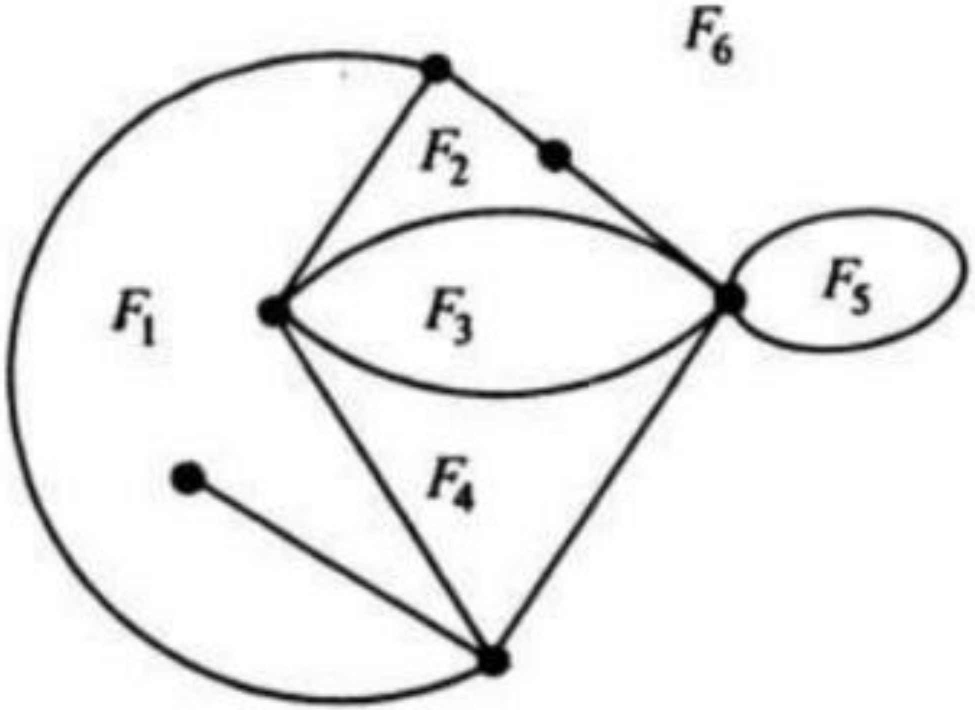
- Graph G is planar *G* does not contain either of Kuratowski's two graphs or any graph homeomorphic to either of them.

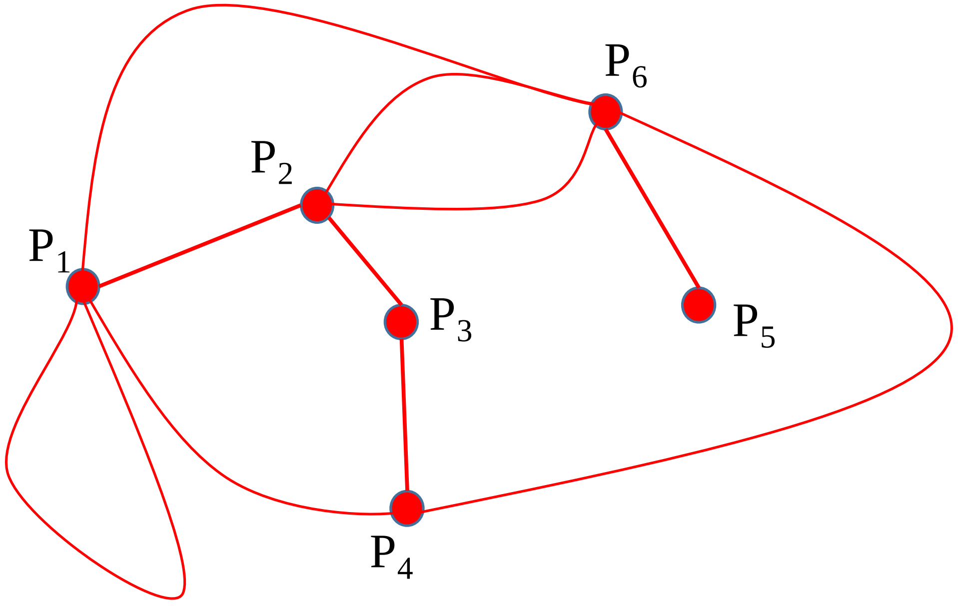


Dual Graph



Dual Graph

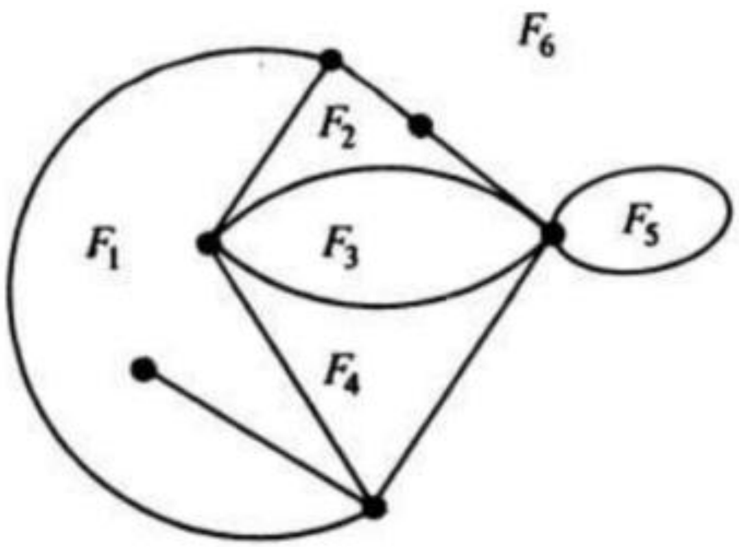




Dual Graph

Some Properties:

- Self-loop yields a pendant edge (Vice-versa)
- Edges in series yields parallel edges (Vice-versa)
- G* is also planar

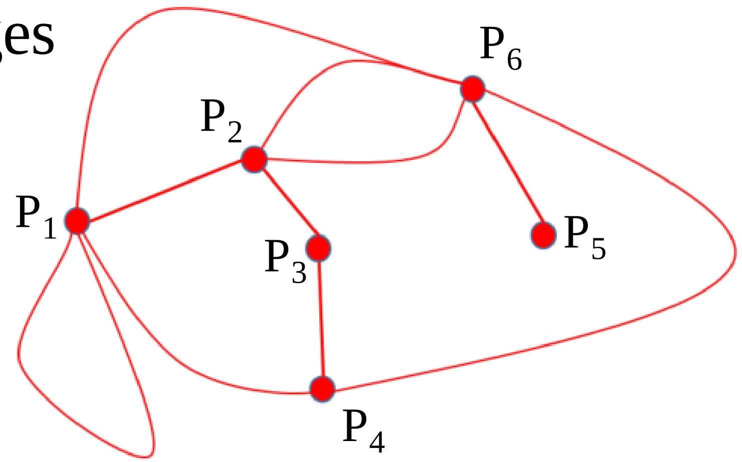


G

{*n*, *e*, *f*, r, *μ*}

- ***$n^* = f$,***
  $e^* = e$,
  $f^* = n$.

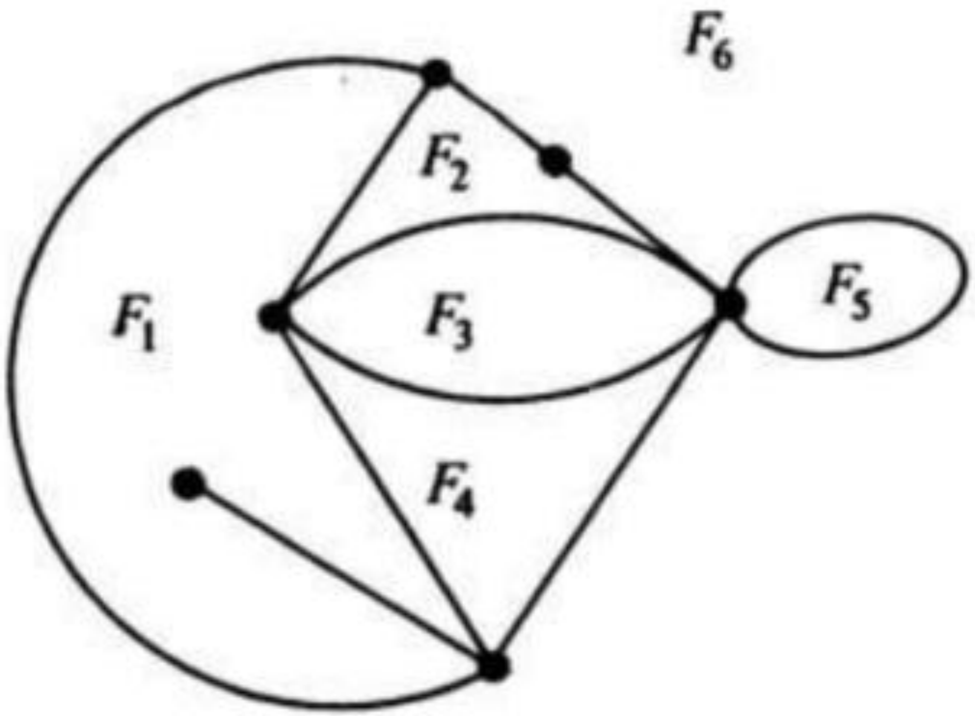
- ***$r^* = \mu$,***
  $\mu^* = r$.

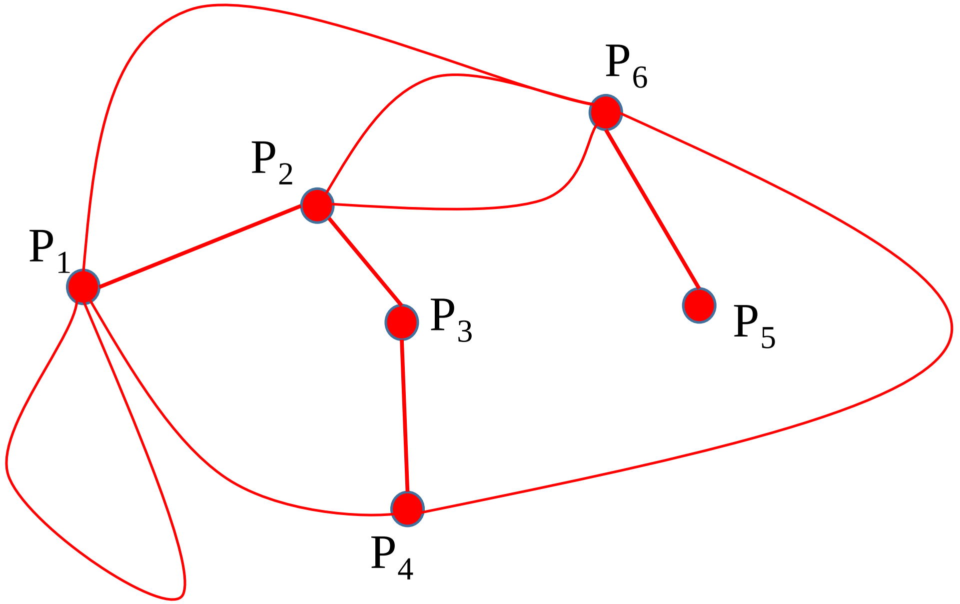


G*

{*n* *, *e* *, *f* *, r *, *μ* *}

Duality Properties





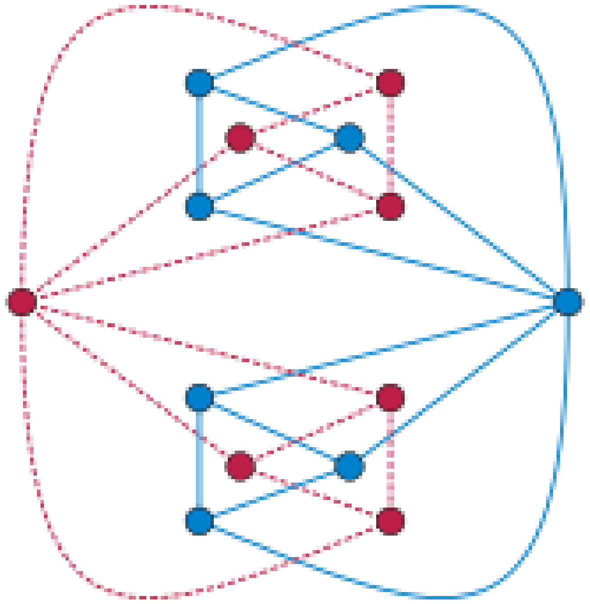
Thickness & Crossing

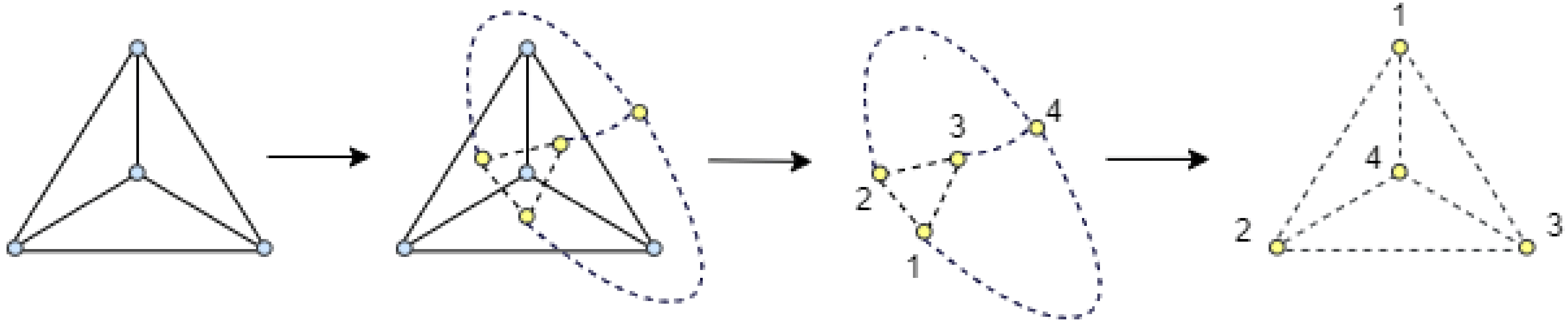
- *Thickness*: The least number of planar subgraphs whose union is the given graph G.
- Thickness of a planar graph is 1

- *Crossings*: the fewest number of crossings (or intersections) necessary in order to “draw” the graph in a plane?
- Crossing number of planar graph is 0
- Kuratowski’s graphs have a crossing number of 1.

Self dual graphs

- a planar graph G isomorphic to its own dual





Completely Regular planar graph

- A planar graph G is said to be completely regular if the degrees of all vertices of G are equal and every region is bounded by the same number of edges.