(b)

$E_0 = FFF$

$E_1 = FFD$

$\cancel{DFD}\ FDF$

DFF

$E_2 = FDD$

DFD

DDF

$E_3 = DDD$

$P[E_0] = P[FFF] = (1-p)^3$

$P[E_1] = P[FFD] + P[FDF] + P[DFF] = 3p(1-p)(1-p)$

$P[E_2] = P[FDD] + P[DFD] + P[DDF] = 3p^2(1-p)$

$P[E_3] = P[DDD] = p^3.$

(c)

$S = \{D, F_1D, F_1F_2D, F_1F_2F_3D, \ldots\ldots, F_1 \ldots F_{(n-1)}D\}\ ;\ n \to \infty.$

$P[F_1 \ldots F_{(t-1)}D] = (1-p)^{(t-1)} \cdot p$ (Ans).

(d) Let, $E_i \equiv$ 'event where we get 3 successes after i attempts.

$$E = \{E_3, E_4, E_5, \ldots\ldots, E_n\} \quad ; \quad n \to \infty$$

So, if we take an event E_t where t is between 3 and n:

Size of event $E_t = {}^{(t-1)}C_2$

* $(t-1)$ because the last position is occupied by D. So, we have $(t-1)$ positions in which to place 2 other D's. This 2 is why we have ${}^{(t-1)}C_2$.

$$\therefore \; P[E_t] = {}^{(t-1)}C_2 \cdot (1-p)^{(t-3)} \cdot p^3$$

(Ans).

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(2)

~~The chance that~~

Since Chittagong is not an option total possibilities are 11.

So, to choose 6 from these 11:

$$\frac{11}{14} \cdot \frac{10}{13} \cdot \frac{9}{12} \cdot \frac{8}{11} \cdot \frac{7}{10} \cdot \frac{6}{9} = 0.154 \text{ (Ans)}.$$

or, $\frac{{}^{11}C_6}{{}^{14}C_6} = 0.154$ (Ans).

(3)

$$P[R_1 B_1 R_2 B_2] = P[R_1] \cdot P[B_1 | R_1] \cdot P[R_2 | R_1 B_1] \cdot P[B_2 | R_1 B_1 R_2]$$

$$= \frac{r}{(r+b)} \cdot \frac{b}{(r+b-1)} \cdot \frac{r-1}{(r+b-2)} \cdot \frac{b-1}{(r+b-3)}$$

$$= \frac{r(r-1) \cdot b(b-1)}{(r+b) \cdot (r+b-1) \cdot (r+b-2) \cdot (r+b-3)}$$

(4)

	Coin 1 (G)	Coin 2	Coin 3
Pick chance: ~~[illegible]~~	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$.
Heads chance:	.75	.5	.5
~~[illegible]~~ Chance to get 3 heads in a row:	$(.75)^3 = 0.42$	$(.5)^3$ 0.125	$(.5)^3$ (0.125).

$B_i \equiv$ ~~chance of~~ pick ~~pta~~ coin # i.

$A \equiv$ get 3 heads in a row.

So, $P[B_1] = \frac{1}{3}$ $P[B_2] = P[B_3] = \frac{1}{3}$.

$P[A|B_1] = 0.42$

$P[A|B_2] = 0.125$

$P[A|B_3] = 0.125$

$$P[A] = P[A|B_1] \cdot P[B_1]$$

$$+ P[A|B_2] \cdot P[B_2]$$

$$+ P[A|B_3] \cdot P[B_3]$$

$$= (0.42) \cdot \frac{1}{3} + (0.125) \cdot \frac{1}{3} + (0.125) \cdot \frac{1}{3}$$

$$= 0.22 .$$

~~Now, the probability that~~

Now, if I get 3 heads, the probability that the coin is biased is:

$$P[B_1|A] = \frac{P[AB_1]}{P[A]}$$

$$= \frac{P[A|B_1] \cdot P[B_1]}{P[A]} = \frac{(0.42) \cdot \left(\frac{1}{3}\right)}{0.22}$$

$$= 0.64 \quad \text{(Ans)}$$

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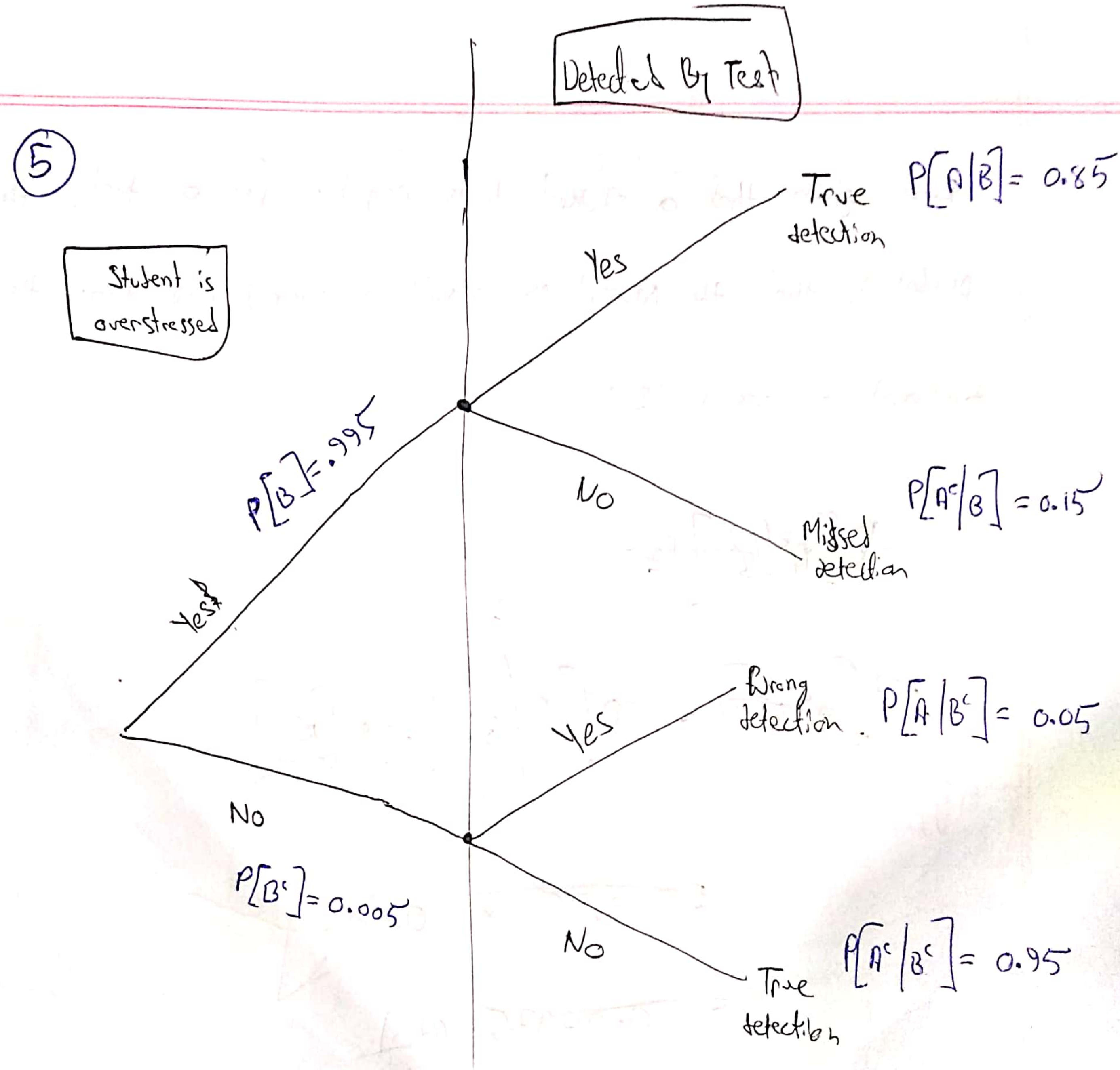
5



$B \equiv$ student is overstressed

$B^c \equiv$ student is not overstressed.

$A \equiv$ detected by test

$A^c \equiv$ not detected by test.

Now, given that a student tests negative for a test, the probability that the ~~student's~~ result is correct and that the student is not overstressed is:

$$\cancel{P\left[A^c \middle| B\right] =}$$

$$\cancel{P\left[A^c B^c\right] = P\left[A^c \middle| B^c\right] \cdot P\left[B^c\right]}$$

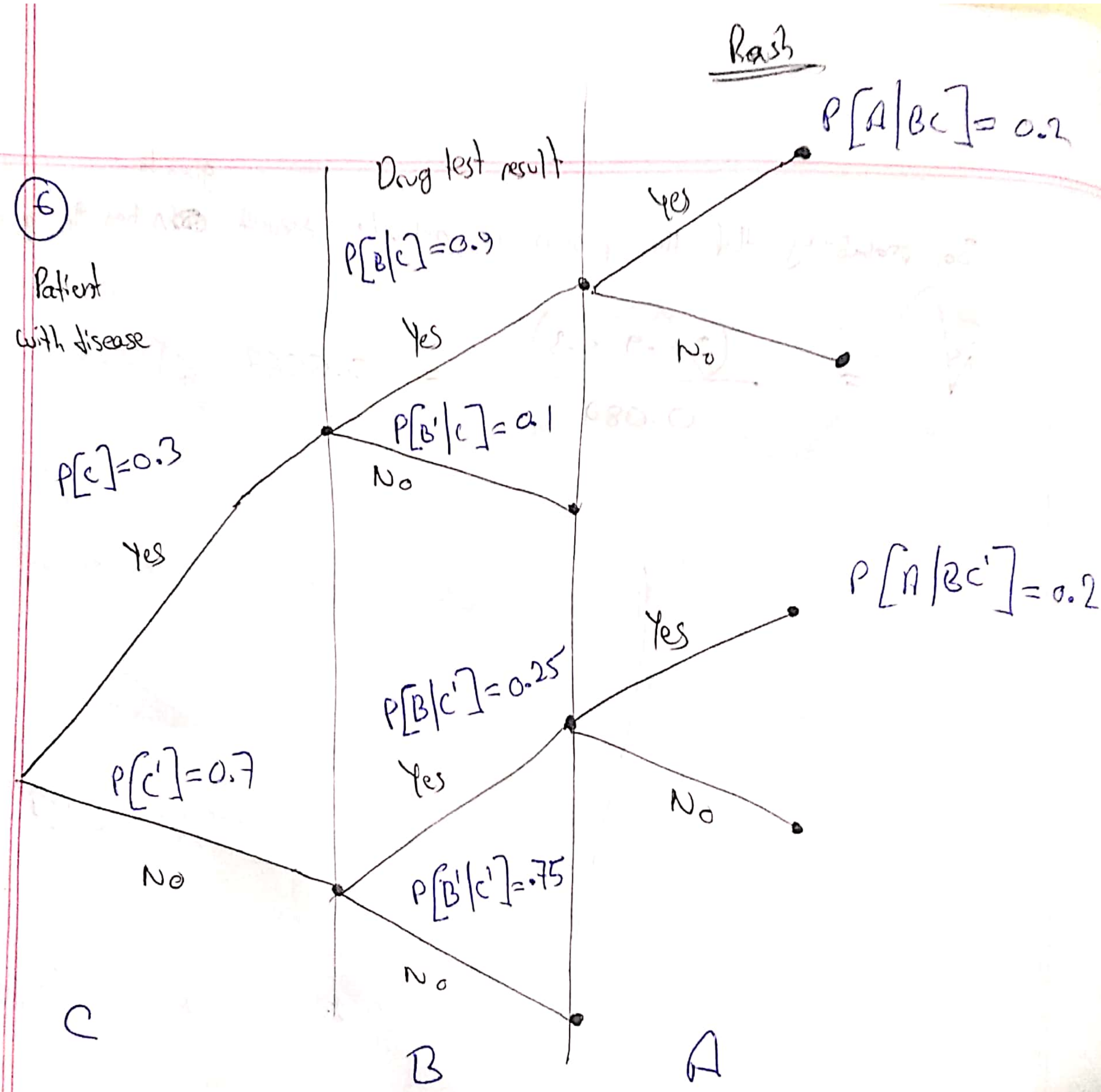
$$\cancel{= 0.95 \times 0.005}$$

$$\cancel{= 0.00475 \text{ (Ans)}}$$

$$P\left[B^c \middle| A^c\right] = \frac{P\left[A^c B^c\right]}{P\left[A^c\right]}$$

$$= \frac{P\left[B^c\right] \cdot P\left[A^c \middle| B^c\right]}{P\left[B^c\right] \cdot P\left[A^c \middle| B^c\right] + P\left[A^c \middle| B\right] \cdot P\left[B\right]}$$

6



$C \equiv$ patient has disease

$C^c \equiv$ patient does not have disease.

$B \equiv$ drug test ~~dode~~ positive.

$B^c \equiv$ drug test negative.

$A \equiv$ rash.

$A^c \equiv$ no rash

Probability that person who has ~~does tested~~ rash had the disease:

$$P[CB/A] = \frac{P[ABC]}{P[A]}$$

$$= \frac{P[C] \cdot P[B/C] \cdot P[A/BC]}{\left(P[A/BC] \cdot P[B/C] \cdot P[C]\right) + \left(P[A/BC'] \cdot P[B/C'] \cdot P[C']\right)}$$

$$= \frac{(0.2 \times 0.9 \times 0.3)}{(0.2 \times 0.9 \times 0.3) + (0.2 \times 0.25 \times 0.7)}$$

$$\cancel{= 0.06} = 0.6067 \text{ (Ans)}.$$

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(a)

Probability that the ball is blue at the begining of round 2:

$$\cancel{P[\text{blue at round 2}] = \frac{7}{10} + \frac{8}{10}}$$

$$\cancel{P[\text{blue at round 2}] = \frac{\left(\frac{7}{10}\right) + \frac{8}{10}}{2} = .75} \text{ (Ans).}$$

$$P[B] = P[B|W] \cdot P[W] + P[B|B] \cdot P[B] = \left(\frac{8}{10} \cdot \frac{3}{10}\right) + \left(\frac{7}{10} \cdot \frac{7}{10}\right) = 0.73 \text{ (Ans)}.$$

(b)

For the ball to not be originally blue the same white ball needs to be picked twice in a row. So, for a ball to originally be blue the probability is:

$$P[\text{originally blue}] = \left(1 - \frac{3}{10} \cdot \frac{1}{10}\right)$$

$$= \frac{97}{100} \text{ (Ans)}$$

(c)

$A \equiv$ originally white

$B \equiv$ a ball was never drawn

So ~~Pro~~ $P[6^{th}$ ball is white$] = \cancel{P[A\cap B]} \left(1-\frac{1}{10}\right)^5 \cdot \frac{3}{10}$ (Ans).