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B.Sc. Engg. CSE 2nd Semester　　　　23 October 2018

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION　　　　**SUMMER SEMESTER, 2017-2018**

DURATION: 3 Hours　　　　**FULL MARKS: 200**

Math 4241: Integral Calculus and Differential Equations

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 8 (eight) questions. Answer any 6 (six) of them.

Give figure(s) where necessary. Figures in the right margin indicate marks.

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1. a) Write the fundamental theorem of Calculus. Find dy/dx of $y = \int_0^{sinx} \frac{dt}{\sqrt{1-t^2}}$ i. by using Fundamental theorem, ii. by evaluating the integral and then differentiating the result. 5+16.33

   b) Find the total area between the region and the x-axis formed by the curve $y = 3x^2 - 3$, $-2 \le x \le 2$ 12

2. a) Evaluate the followings: 3×5

   i. $\int_{-4}^{4} |x-2|\, dx$, ii. $\int_1^{e^{\pi/4}} \frac{4}{x(1+ln^2 x)}\, dx$, iii. $\int_{-1}^{-1/2} x^{-2} \sin^2(1+\frac{1}{x})\, dx$

   b) Find the area of the regions enclosed by the lines and the curves as follows: 9+9.33

   i. $y = 7 - x^2$ and $y = x^2 + 4$

   ii. $x - y^2 = 0$ and $x + 2y^2 = 3$

3. a) The solid lies between the planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross-sections perpendicular to the x-axis are circular disks whose diameter run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$. Find the volume of the solid. 18.33

   b) Find the volume of the solid generated by revolving the regions bounded by the given curve and the lines $y = 2\sqrt{x}$, $y = 2$, $x = 0$ about x-axis. 15

4. a) Define length of curve. Find the length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$. 5.33+7

   b) Find the surface area generated by revolving the curve $x = \left(\frac{1}{3}\right) y^{3/2} - y^{1/2}$, $1 \le y \le 3$, about y- axis. 10

   c) Find the lateral surface area of the cone generated by revolving the line segment $y = x/2$, $0 \le x \le 4$, about y-axis. Check your answer with the following formula: 7+4
   Lateral surface area = (1/2) × base circumference × slant height.

5. a) Using Trapezoidal and Simpson's rules with n = 4 and 8, find the approximate value of $\int_0^3 \sqrt{x+1}\, dx$. Finally compare your results with true value and comments on it. 10+5.33

   b) Define proper and improper integrals with examples. Evaluate the following integrals and then state whether they are convergent or not : 4+2×7

   i. $\int_{-\infty}^{\infty} \frac{1}{e^x + e^{-x}}\, dx$,　　ii. $\int_0^{ln2} x^{-2} e^{-1/x}\, dx$

6. a) Define ordinary and partial differential equations with examples. Form an ordinary differential equation corresponding to the family of curves $y = k(x-k)^2$, where k is an arbitrary constant. Finally, identify it. 4+10+3

b) Define is exact differential equation and write its necessary condition. Test whether the following differential equations are exact or not. 4.33+4×3

i. $(2y\sin x\cos x + y^2\sin x)dx + (\sin^2 x + 2y\cos x)dy = 0$, ii. $(y^2 + 2xy)dx - x^2\,dy = 0$

iii. $(4x + 3y^2)dx + 2xy\,dy = 0$, iv. $\left(\frac{x}{y^2} + x\right)dx + \left(\frac{x^2}{y^3} + y\right)dy = 0$

7. a) Determine the constant A such that the given equation is an exact differential equation (DE) and then solve it. 5.33+12

$$\left(\frac{Ay}{x^3} + \frac{y}{x^2}\right)dx + \left(\frac{1}{x^2} - \frac{1}{x}\right)dy = 0$$

b) Solve the following DEs: 2×8

i. $4xy\,dx + (x^2+1)dy = 0$, ii. $(x^2 + 3y^2)dx - 2xydy = 0$

8. a) What is first order linear differential equation? Explain with examples, when Bernoulli's DE becomes a first order linear DE. 4.33+2

b) Solve the following initial value problems: 3×9

i. $x\frac{dy}{dx} - 2y = 2x^4$, $y(2)=8$, ii. $\frac{dy}{dx} - y = \sin 2x$, $y(0)=0$ iii. $\frac{dy}{dx} + y = xy^3$, $y(0)=1$

Engg. /HD CSE 2^{nd} Semester

28 October 2017

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

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Department of Computer Science and Engineering (CSE)

ESTER FINAL EXAMINATION — SUMMER SEMESTER, 2016-2017

RATION: 3 Hours — FULL MARKS:200

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a) Evaluate the following integrals and then graph the integrands. Finally find the area using appropriate formula from geometry: 20

i. $\int_{-1}^{1}(1-|x|)dx$ ii. $\int_{-3}^{3}\sqrt{9-x^2}dx$

b) Define average value of a function f(x) on [a, b]. Graph the functions and find its average value over the given intervals: 13.33

i. $f(x) = -3x^2 - 1$ on $[0,1]$ ii. $f(x) = (x-1)^2$ on $[0, 3]$

a) Find dy/dx of $y = \int_0^{\tan x} \frac{dt}{1+t^2}$ i) by using Fundamental theorem, ii) by evaluating the integral and then differentiating the result. 12

b) Evaluate the integral i) $\int_{-4}^{4}|x|\,dx$, ii) $\int_0^{1/2}\frac{4}{\sqrt{1-x^2}}dx$ 10

c) Find the total area between the region and the x-axis formed by the curve $y = -x^2 - 2x$, $-3 \le x \le 2$ 11.33

a) Define even and odd functions with examples. If f is a continuous function on the symmetric interval [−a, a] then prove that: 17.33

i. $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$, for even function,

ii. $\int_{-a}^{a} f(x)dx = 0$, for odd function.

b) Find the area of the regions enclosed by the lines and the curves as follows: 16

i. $y = x^2$ and $y = -x^2 + 4x$

ii. $x - y^2 = 0$ and $x + 2y^2 = 3$

a) Find the volume of the given pyramid which has a square base of area 9 square meter and height 5 meters. 18.33

b) If the solid is generated by revolving the regions bounded by the given curves $y = x^2$ and the lines $y = 0$, $x = 2$ about x-axis then find its volume. 15

c) Find the length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$.

a) Find the surface area generated by revolving the curve $y = \sqrt{x+1}$, $1 \le x \le 5$, about x- axis. 15

b) Define proper and improper integrals with examples. Evaluate the following integrals and then state whether they are convergent or not : 18.33

i. $\int_{-\infty}^{\infty}\frac{1}{e^x + e^{-x}}\,dx$,

ii. $\int_0^{\ln 2} x^{-2}e^{-1/x}\,dx$

6. a) Define ordinary and partial differential equations. Explain with examples, order and degree of a differential equation, linear and non-linear ordinary differential equations.

   b) Form an ordinary differential equation from the curve $v = \frac{A}{r} + B$, where A and B are constants, and then identify it.

   c) Find the DE of the family of circles of variable radii r with center on x-axis.

7. a) What is exact differential equation and write its necessary condition. Determine whether the following differential equations are exact or not:

      i. $(3x + 2y)dx + (4x + y)dy = 0$

      ii. $(2y \sin x \cos x + y^2 \sin x)dx + (\sin^2 x + 2y \cos x)dy = 0$

   b) Determine the constant k such that the given equation is an exact DE and then solve it,
   $(x^2 + 3xy)dx + (kx^2 + 4y)dy = 0$

8. a) Solve the following differential equations:

      i. $(y + 2)dx + y(x + 4)dy = 0, \; y(-3) = -1$

      ii. $(x^2 + 3y^2)dx - 2xydy = 0, \; y(2) = 6$

   b) What is Bernoulli's differential equation? In what conditions, it becomes a first order linear DE, explain with examples. Is Bernoulli's DE linear or not? Finally, solve the IVP,
   $\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3}$, $y(1)=2$

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B.Sc. Engg. / HD CSE 2nd Semester (64)

19 October 2016

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1. a) What is the physical meaning of $\int_a^b f(x)dx$? Find the area under the curve represented by the following data: 12

X:	5	10	15	20	25	30
Y:	1.5	5.12	4.25	6.65	5.75	2.45

b) Evaluate and sketch the region whose area is represented by the integral $\int_{-a}^{a} \sqrt{a^2 - x^2}\, dx$ and then verify it using appropriate formula from geometry. 12

c) Find the total area between the curve $y = 1-x^2$ and the x-axis over the interval [0, 2] by using anti-derivative method. 9.33

2. a) Write the properties of improper integral with examples. Evaluate the integrals and state whether they are divergent or convergent: 15

i. $\int_0^1 \frac{1}{\sqrt{x}\ (x+1)} dx$, ii. $\int_{-1}^{\infty} \frac{x}{1+x^2} dx$

b) Define Beta and Gamma functions. Find the relationship between them. 10

c) Evaluate $\int_0^{\infty} e^{-x^2}\, dx$ and hence show that $\left\lceil \frac{1}{2} \right. = \sqrt{\pi}$ 8.33

3. a) Sketch the region enclosed by the curves $y^2=4x$ and $y=2x-4$ then find its area. 13.33

b) Find the volume of the solid that is obtained when the regions between the curves $f(x) = x^2 + 2$, and $g(x) = x$ over the interval [1, 3] is revolved about x-axis. 20

4. a) Define the arc length for a curve and for parametric equations. Find the circumference of a circle of radius 15 meters from the parametric equations $x = 15\cos\theta$ and $y = 15\sin\theta$, $0 \le \theta \le 2\pi$. 13.33

b) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ for $0 \le x \le 1$ about x-axis and for $0 \le y \le 3$ about y-axis. 20

5. a) Define linear and nonlinear ordinary differential equations (DE) with examples. Find the differential equations corresponding to the family of curves $y=k(x-k)^2$, where k is an arbitrary constant. 13.33

b) Determine the constant A such that the given DE $(Ax^2y+2y^2)dx + (x^3+4xy)dy=0$ is an exact and then solve it. 20

6. a) What is integrating factor? Consider the DE $(y^2 + 2xy)\, dx + x^2dy = 0$, find the integrating factor of the form y^n for which the given DE transformed into an exact DE, where n is an integer. 13.33

b) Solve the following differential equations:

i. $(xy + 2x + y + 2)dx + (x^2 + 2x)dy = 0$

ii. $(2xy + 3y^2)dx - (2xy + x^2)dy = 0$

7. a) Define Bernoulli's DE. State in what conditions the Bernoulli's DE reduces to a first order li DE, explain with examples.

b) Solve the following initial value problems:

i. $x\frac{dy}{dx} - 2y = 2x^4, \quad y(2) = 8$

ii. $\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3}, \quad y(1) = 2$

8. a) Define partial differential equation and solve the following PDE:

i. $(y + z)\frac{\partial z}{\partial x} + (z + x)\frac{\partial z}{\partial y} = x + y$

ii. $(x^2 - yz)\frac{\partial z}{\partial x} + (y^2 - zx)\frac{\partial z}{\partial y} = z^2 - xy$

b) Find the integral surface of the linear partial differential equation $x(y^2 + z)\frac{\partial z}{\partial x} - y(x^2 + z)\frac{\partial z}{\partial y} = (x^2 - y^2)z$, which passes through the curve $xz=a^3$, $y=0$.