Constraint Satisfaction Problem I

CSE 4711: Artificial Intelligence

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What is Search for?

- Assumptions about the world
  - Single agent $\rightarrow$ No adverseries
  - Deterministic actions
  - Fully observed state
  - Discrete state space

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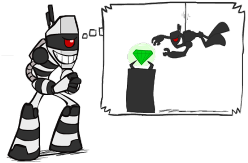
  - Heuristics give problem-specific guidance



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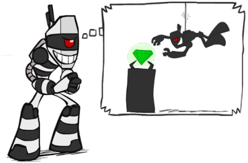
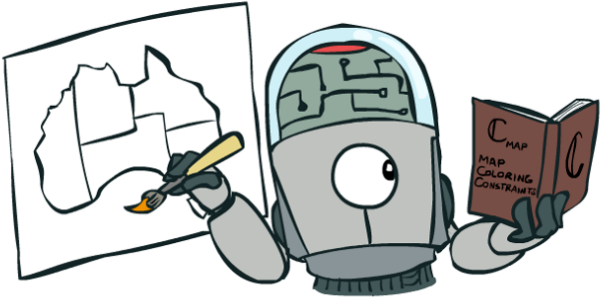
  - Heuristics give problem-specific guidance
- Identification: assignments to variables
  - The goal itself is important, not the path
  - All paths at the same depth (for some formulations)
  - CSPs are a specialized class of identification problems



Constraint Satisfaction Problems



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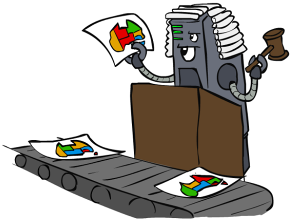
  - Goal test can be any function over states
  - Successor function can also be anything



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  - State is defined by variables X_i with values from a domain D (sometimes D depends on i)
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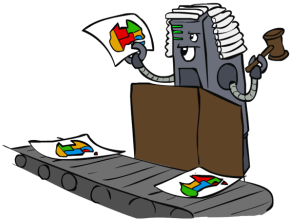

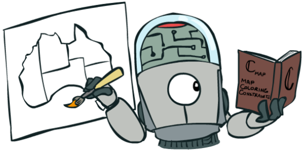

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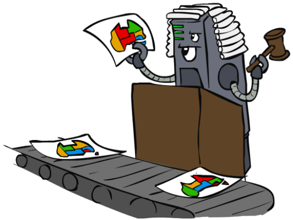

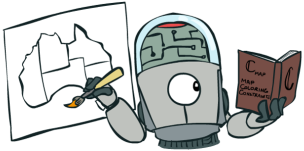

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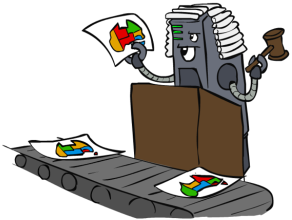
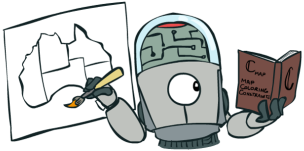
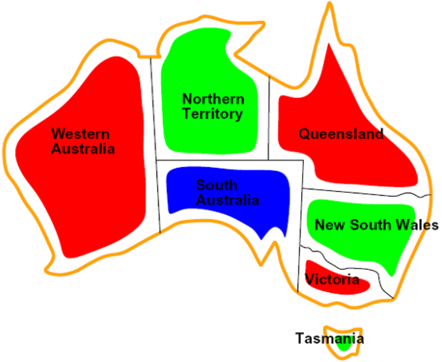
- Simple example of a *formal representation language*
- Allows useful general-purpose algorithms with more power than standard search algorithms



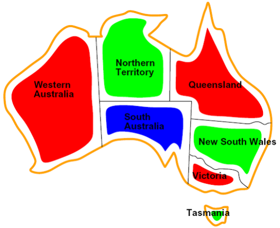


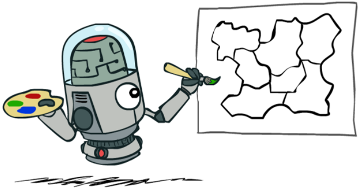
CSP Examples



Example: Map Coloring

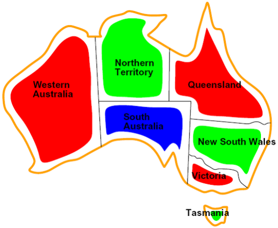
- Variables:

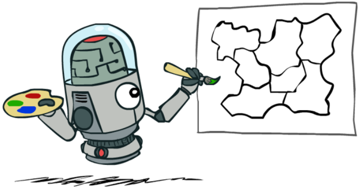




Example: Map Coloring

- Variables:
  - WA, NT, Q, NSW, V, SA, T





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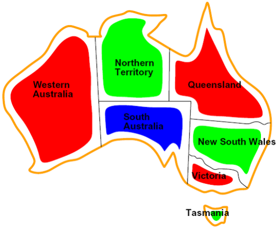

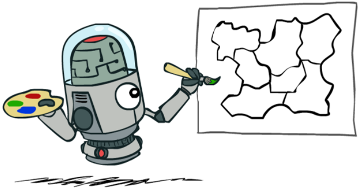

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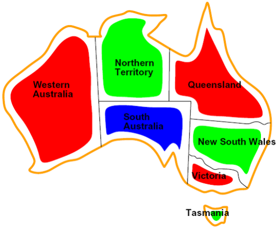
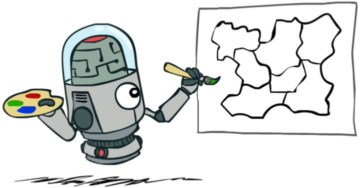
  - $D = \{\text{red}, \text{green}, \text{blue}\}$





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- Variables:
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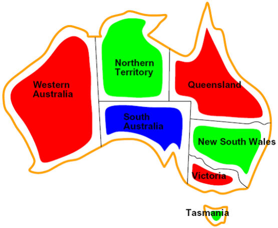
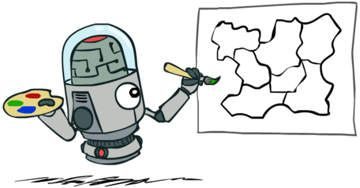
- Constraints: adjacent regions must have different colors





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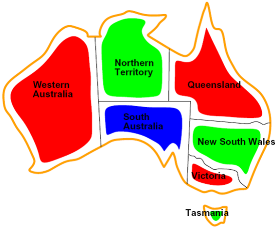

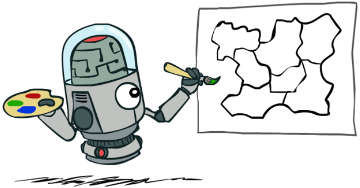

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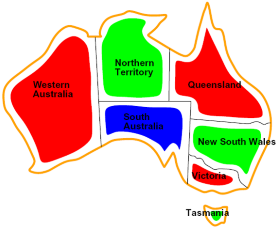

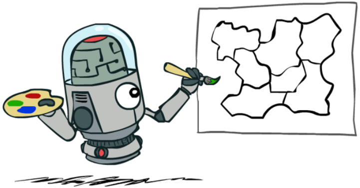

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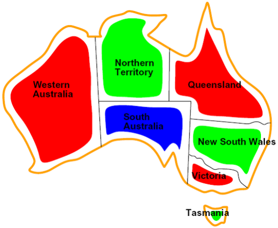
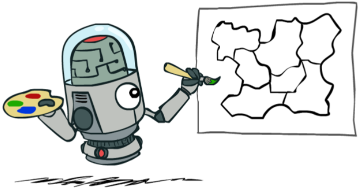
- Solutions are assignments satisfying all constraints





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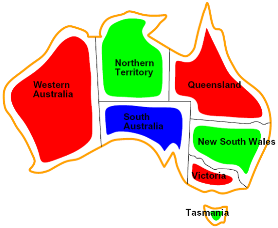

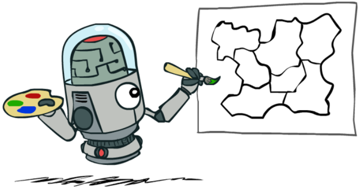

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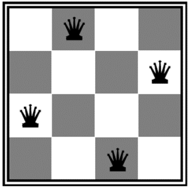

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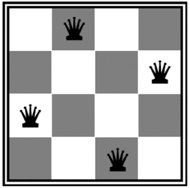

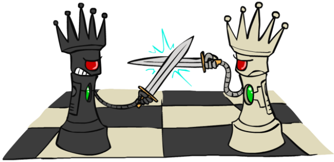

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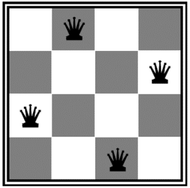

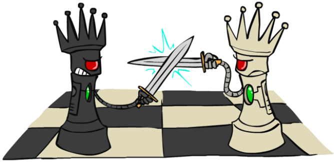

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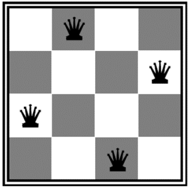

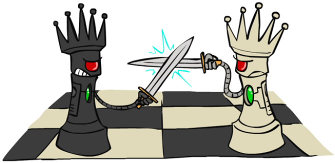

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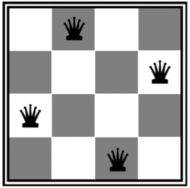
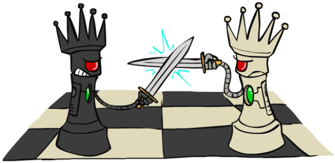
- Formulation 1:
  - Variables: X_{ij}
  - Domains: $\{0, 1\}$





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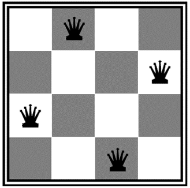

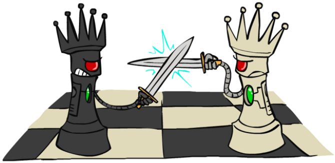

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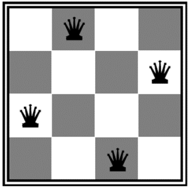

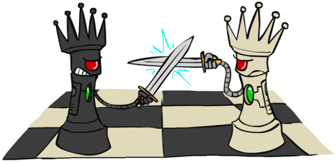

$$\forall i, j, k \; (X_{ij}, X_{ik}) \in \{(0,0), (0,1), (1,0)\}$$

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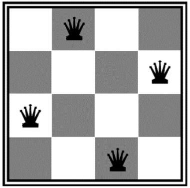

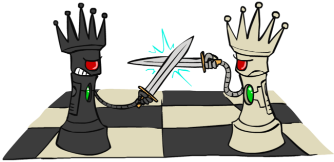

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$$\forall i, j, k\ (X_{ij}, X_{kj}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k\ (X_{ij}, X_{i+k,j+k}) \in \{(0,0), (0,1), (1,0)\}$$
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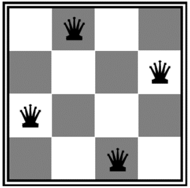

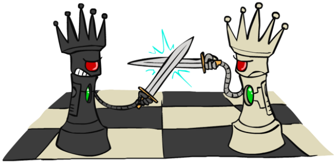

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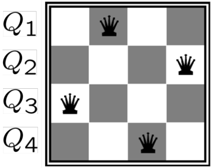
$$\forall i, j, k \; (X_{ij}, X_{i+k,j-k}) \in \{(0,0), (0,1), (1,0)\}$$

$$\sum_{i,j} X_{ij} = N$$

Example: N-Queens

- Formulation 2:
  - Variables: Q_k
  - Domains: $\{1, 2, 3, \ldots, N\}$
  - Constraints:



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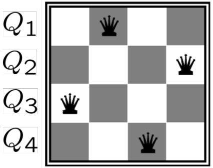

- Implicit: $\forall i, j$ non-threatening (Q_i, Q_j)

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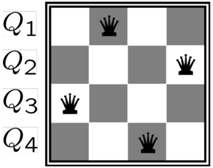
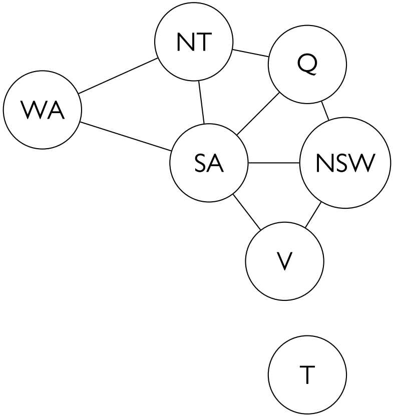


- Implicit: $\forall i, j$ non-threatening (Q_i, Q_j)
- Explicit: $(Q_i, Q_j) \in \{(1,3), (1,4), \ldots\}$

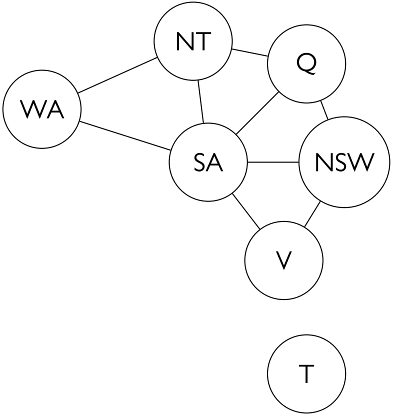
  $\ldots$

Constraint Graphs



Constraint Graphs

- Binary CSP: each constraint relates (at most) two variables
- Binary constraint graph: nodes are variables, arcs show constraints
- General-purpose CSP algorithms use the graph structure to speed up search. E.g., Tasmania is an independent subproblem!



Applet: CSP - fiveQueens

Example: Cryptarithmetic

- Variables
- Domains:
- Constraints:

$$\begin{array}{cccc} & T & W & O \\ + & T & W & O \\ \hline F & O & U & R \end{array}$$

Example: Cryptarithmetic

- Variables
  - $F, T, U, W, R, O, X_1, X_2, X_3$
- Domains:
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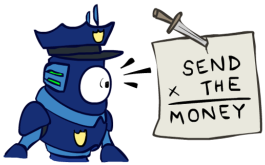
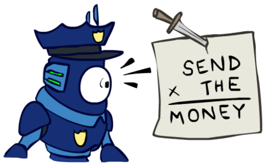


Example: Cryptarithmetic

- Variables
  - $F,\ T,\ U,\ W,\ R,\ O,\ X_1,\ X_2,\ X_3$
- Domains:
  - $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- Constraints:

$$\begin{array}{r} T\ W\ O \\ +\ T\ W\ O \\ \hline F\ O\ U\ R \end{array}$$

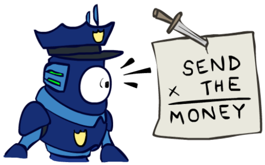


Example: Cryptarithmetic

- Variables
  - $F,\ T,\ U,\ W,\ R,\ O,\ X_1,\ X_2,\ X_3$
- Domains:
  - $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- Constraints:

  alldiff(F, T, U, W, R, O)

$$\begin{array}{cccc} & T & W & O \\ + & T & W & O \\ \hline F & O & U & R \end{array}$$



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```
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+   T W O
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```

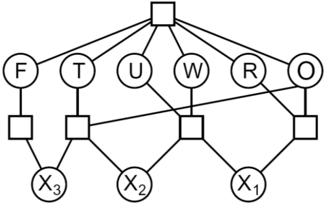

Example: Sudoku

- Variables
  - Each (open) square
- Domains
  - $\{1, 2, \ldots, 9\}$
- Constraints

					8			4
	8	4		1	6			
			5			1		
1		3	8			9		
6		8				4		3
		2			9	5		1
		7			2			
			7	8		2	6	
2			3					

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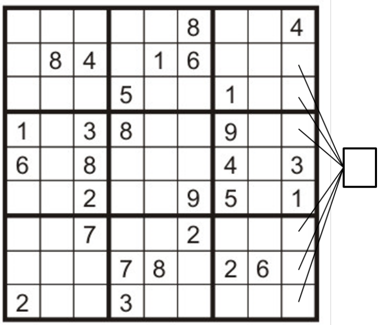

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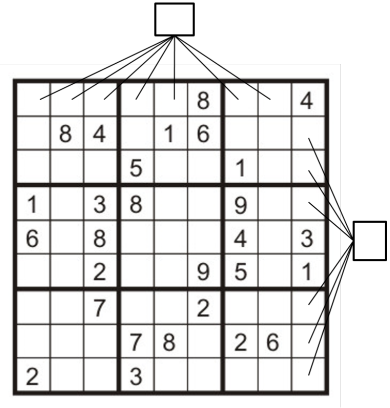

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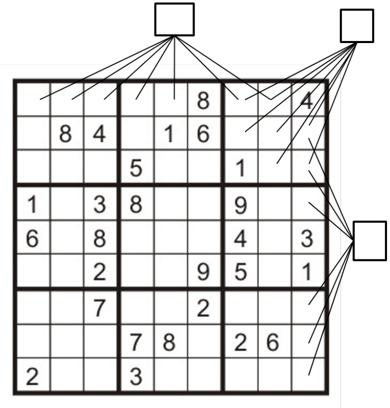

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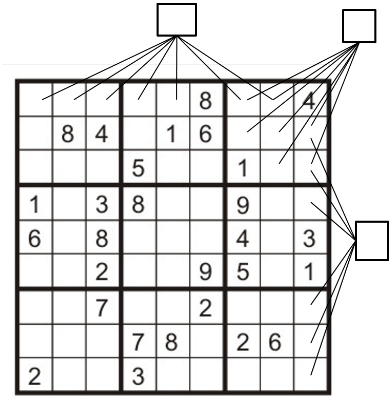

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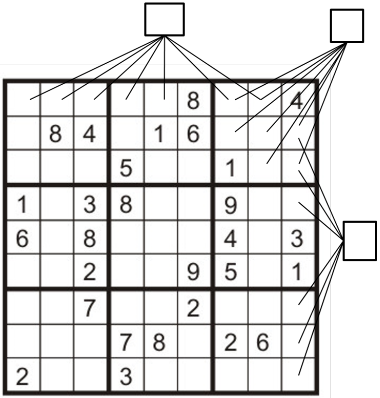
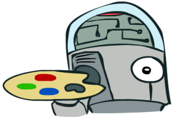
  - Can also have a bunch of pairwise inequalities



Varieties of CSPs and Constraints

Varieties of CSPs

- Discrete Variables
  - Finite domains
    - Size d means $O(d^n)$ complete assignments
    - E.g., Boolean CSPs, including Boolean satisfiability (NP-complete)



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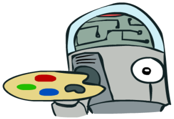

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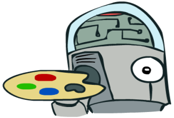

- Continuous Variables
  - E.g., start/end times for Hubble Telescope observations
  - Linear constraints solvable in polynomial time by LP methods

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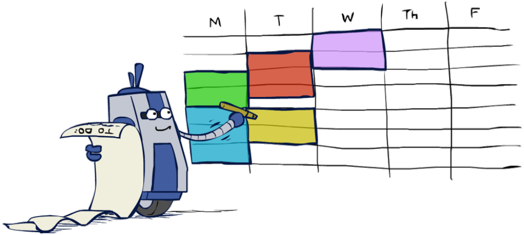
- Higher-order constraints involve 3 or more variables, e.g., cryptarithmetic column constraints
- Preferences (soft constraints)
  - E.g., red is better than green
  - Often representable by a cost for each variable assignment
  - Gives constrained optimization problems
  - (We'll ignore these until we get to Bayes' nets)

Real-World CSPs

- Scheduling problem
- Timetabling problem
- Assignment problem
- Hardware configuration
- Transportation scheduling
- Factory scheduling
- Circuit layout
- Fault diagnosis
- … lots more!

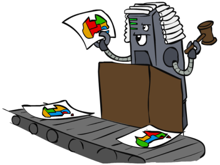


- Many real-world problems involve real-valued variables…

Solving CSPs

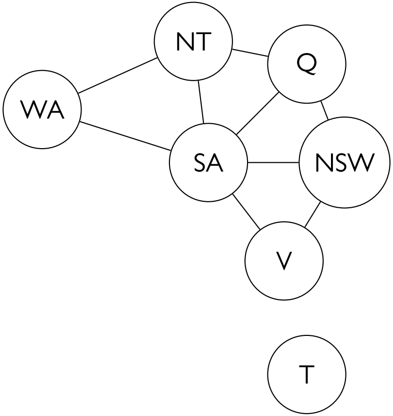
Standard Search Formulation

- States defined by the values assigned so far (partial assignments)
  - Initial state: the empty assignment, {}
  - Successor function: assign a value to an unassigned variable
  - Goal test: the current assignment is complete and satisfies all constraints



Search Methods

- What would BFS do?



Website: simple - naive

Search Methods

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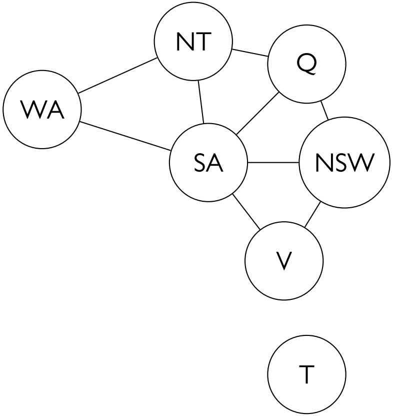
- What would DFS do?



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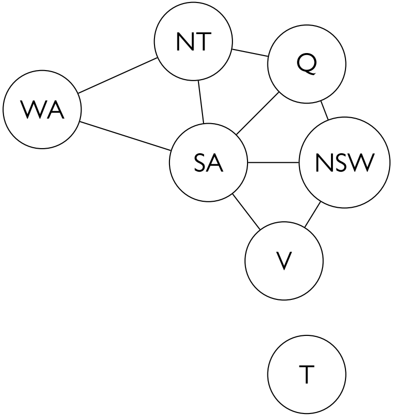
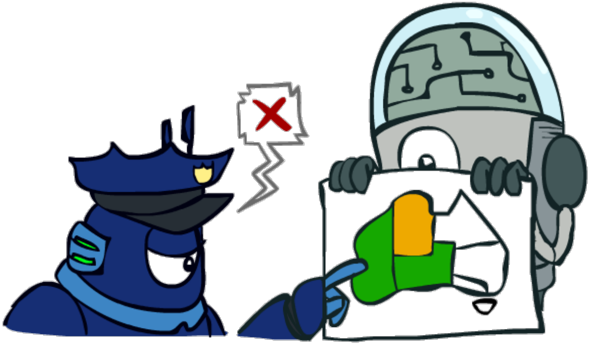


- What would BFS do?
- What would DFS do?
- What problems does naïve search have?

Website: simple - naive

Backtracking Search



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- Depth-first search with these two improvements is called *backtracking search* (not the best name)
- Can solve n-queens for $n \approx 25$

Backtracking Search

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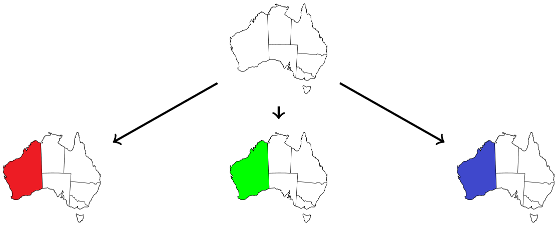

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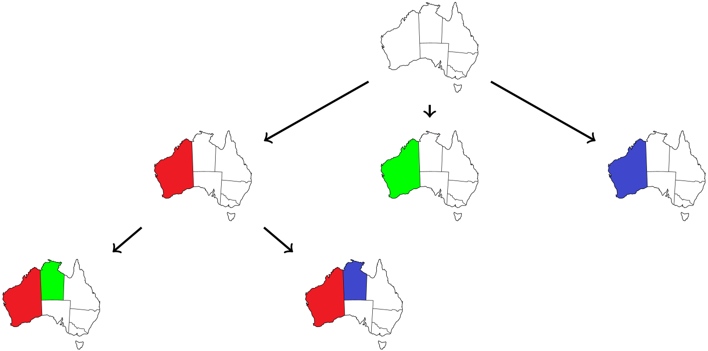

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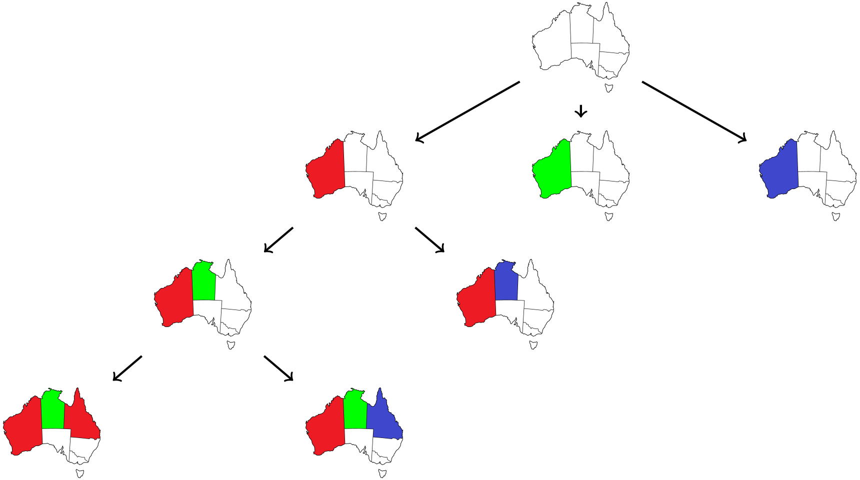


Backtracking Search

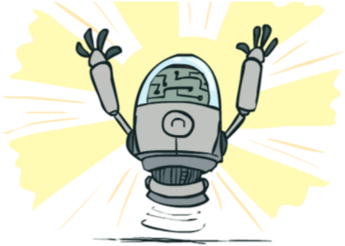
```
function BACKTRACKING-SEARCH(csp) returns a solution, or failure
   return RECURSIVE-BACKTRACKING({}, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns a solution, or failure
   if assignment is complete then return assignment
   var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
   for each value in ORDER-DOMAIN-VALUE(var, assignment, csp) do
      if value is consistent with assignment given CONSTRAINTS[csp] then
         add {var = value} to assignment
         result ← RECURSIVE-BACKTRACKING(assignment, csp)
         if result ≠ failure then return result
         remove {var = value} from assignment
   return failure
```

- Backtracking = DFS + variable-ordering + fail-on-violation

Website: simple - backtracking

Improving Backtracking

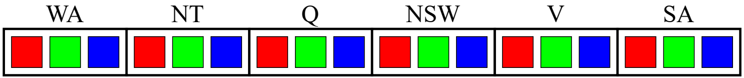
- General-purpose ideas give huge gains in speed
- Filtering: Can we detect inevitable failure early?
- Ordering:
  - Which variable should be assigned next?
  - In what order should its values be tried?
- Structure: Can we exploit the problem structure?



Filtering

Filtering: Forward Checking

- Filtering: Keep track of domains for unassigned variables and cross off bad options
- Forward checking: Cross off values that violate a constraint when added to the existing assignment



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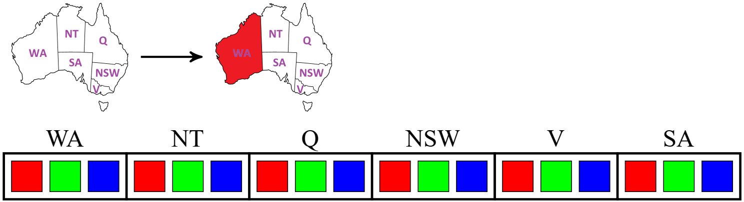

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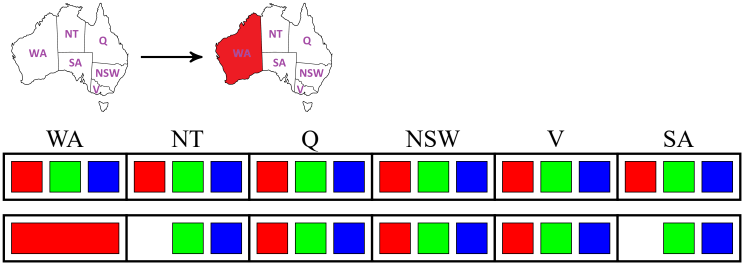

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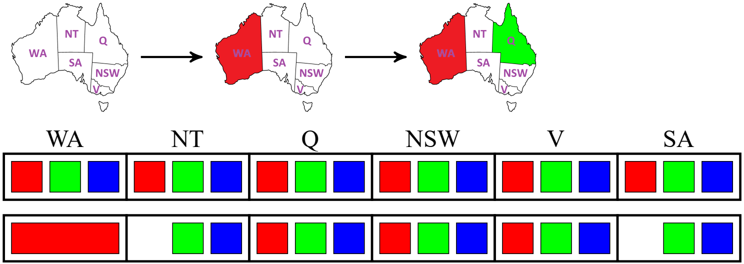

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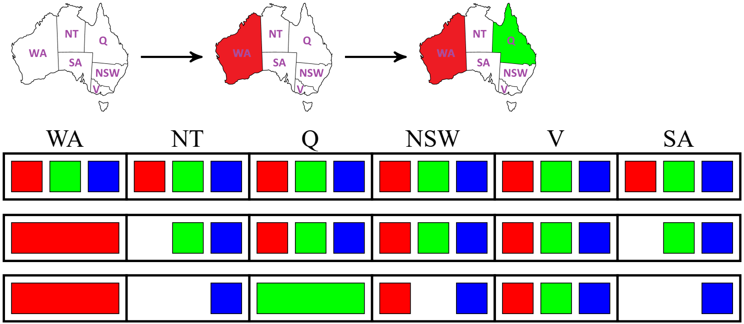

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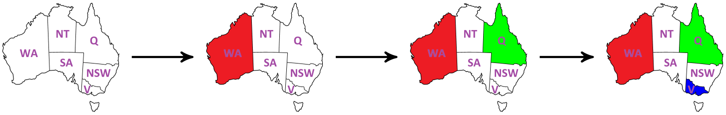

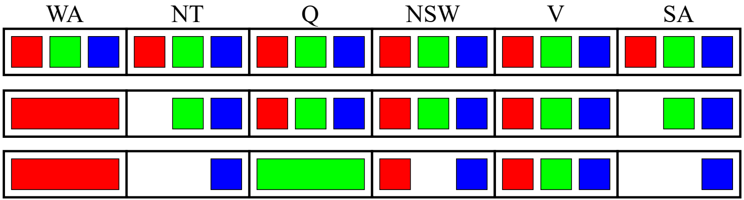

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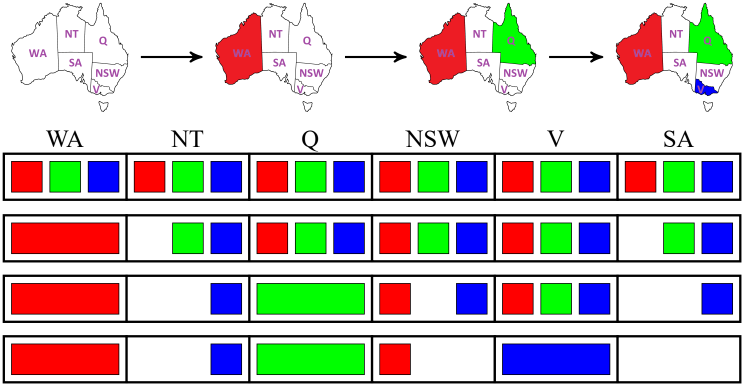

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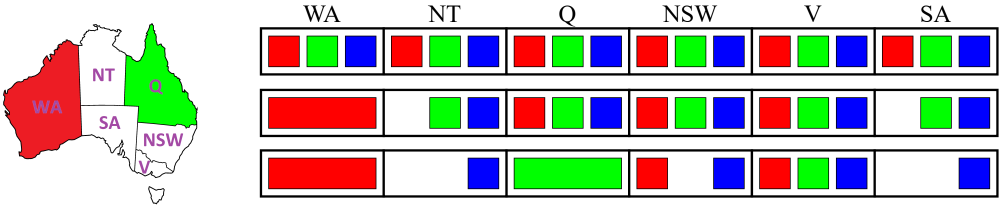

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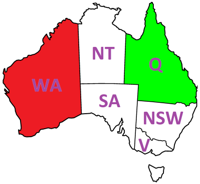

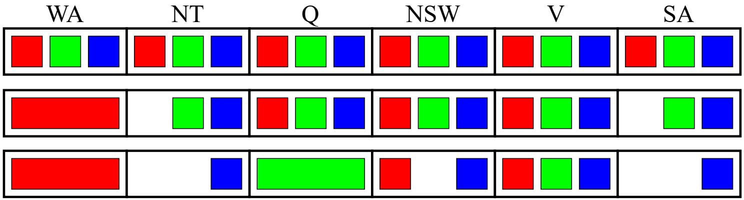

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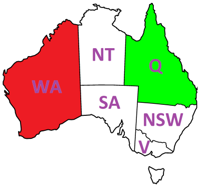

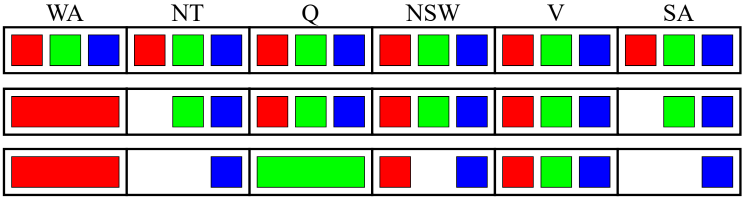

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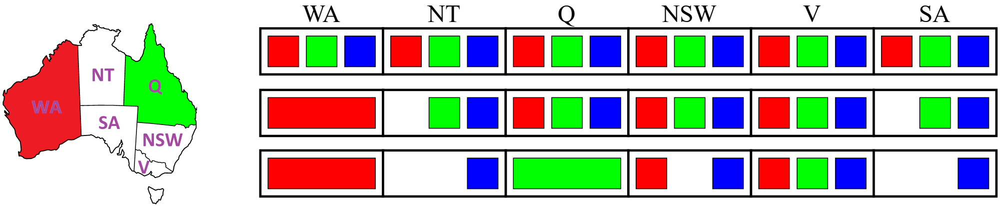

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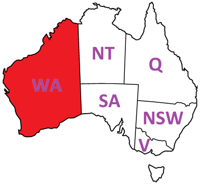
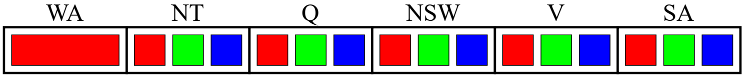
- Why didn't we detect this yet?
- *Constraint propagation:* reason from constraint to constraint

Consistency of A Single Arc

- An arc $X \rightarrow Y$ is consistent iff for *every* x in the tail there is *some* y in the head which could be assigned without violating a constraint





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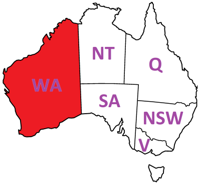

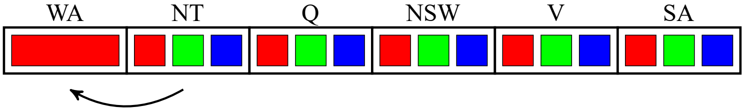

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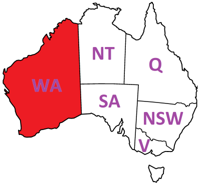

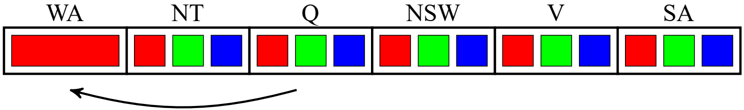

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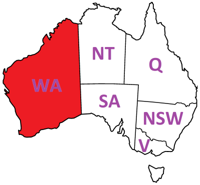

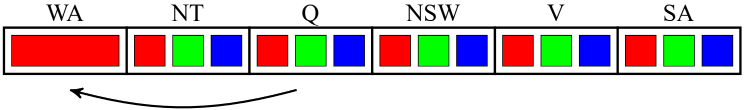

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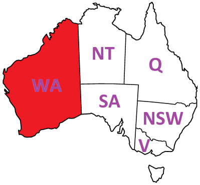
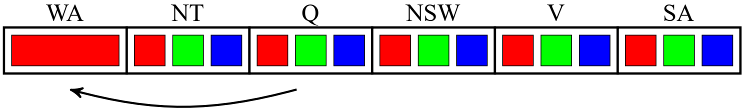
- An arc $X \rightarrow Y$ is consistent iff for *every* x in the tail there is *some* y in the head which could be assigned without violating a constraint





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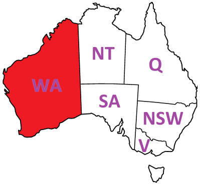

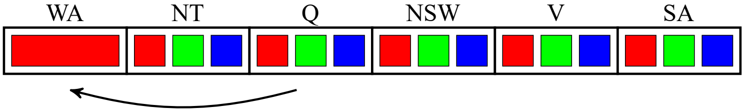

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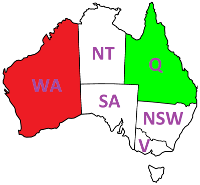
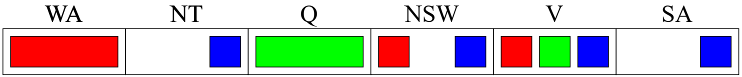
- Forward checking: Enforcing consistency of arcs pointing to each new assignment

Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:





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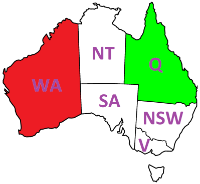

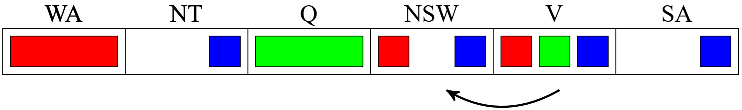

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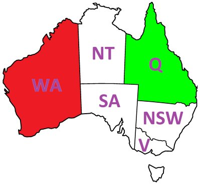

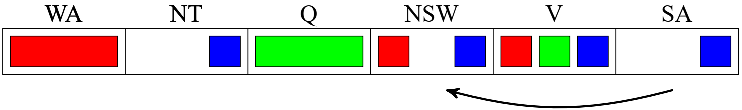

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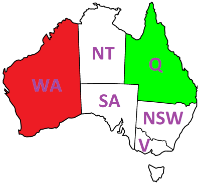

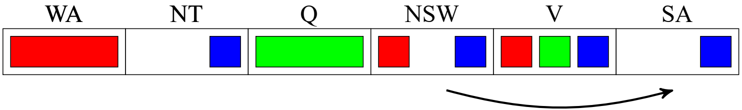

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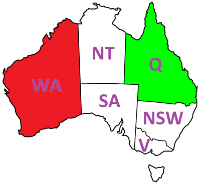

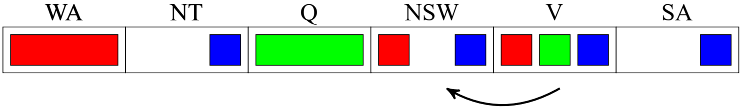

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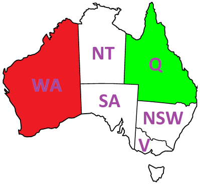

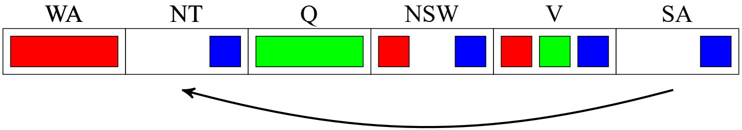

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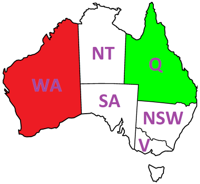
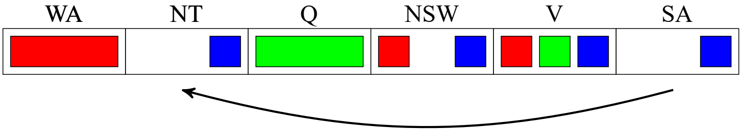




- Important: If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment
- What's the downside of enforcing arc consistency?

Remember: Delete from the tail!

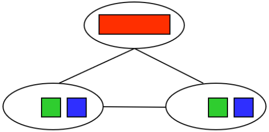
Enforcing Arc Consistency in a CSP

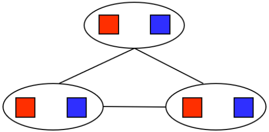
```
function AC-3(csp) returns the CSP, possibly with reduced domains
   inputs: csp, a binary CSP with variables {X_1, X_2, ..., X_N}
   local variables: queue, a queue of arcs, initially all the arcs in csp
   while queue is not empty do
      (X_i, X_j) ← REMOVE-FIRST(queue)
      if REMOVE-INCONSISTENT-VALUES(X_i, X_j) then
         for each X_k in NEIGHBORS[X_i] do
            add (X_k, X_i) to queue
function REMOVE-INCONSISTENT-VALUES(X_i, X_j) returns true iff succeeds
   removed ← false
   for each x in DOMAIN[X_i] do
      if no value y in DOMAIN[X_j] allows (x, y) to satisfy the constraint X_i ↔ X_j
         then delete x from DOMAIN[X_i]; removed ← true
   return removed
```

Applet: CSP - fiveQueens

Limitations of Arc Consistency

- After enforcing arc consistency:
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)





Website: complex - backtracking, forward, complex - backtracking, arc

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- After enforcing arc consistency:
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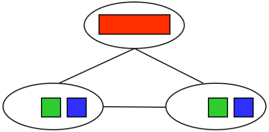
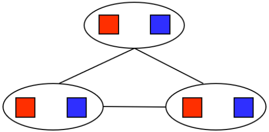
- Arc consistency still runs inside a backtracking search!





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Ordering

Ordering: Minimum Remaining Values

- Variable Ordering: Minimum remaining values (MRV):
    - Choose the variable with the fewest legal left values in its domain

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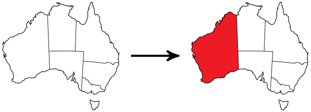

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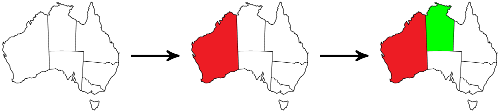

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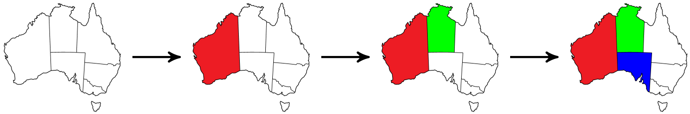

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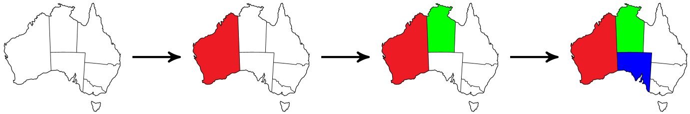

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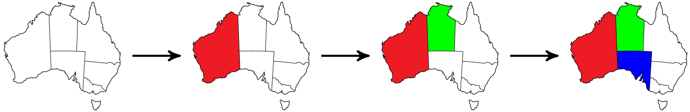

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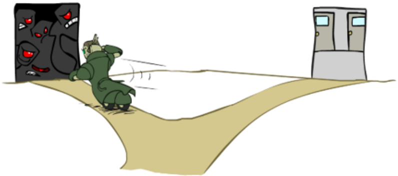

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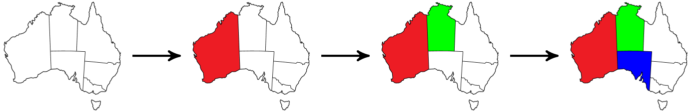
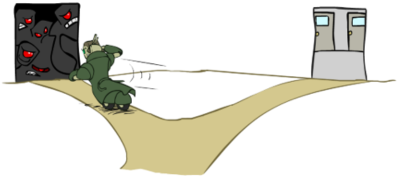


- Why min rather than max? $\rightarrow$ Fails faster
- Also called “most constrained variable”



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Ordering: Least Constraining Value

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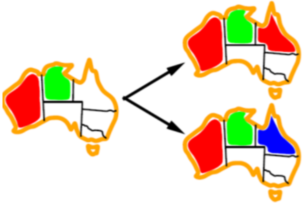
- i.e., the one that rules out the fewest values in the remaining variables
- Note that it may take some computation to determine this! (e.g., rerunning filtering)



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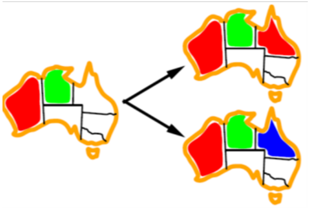

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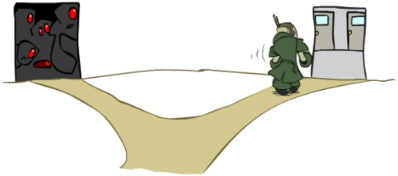

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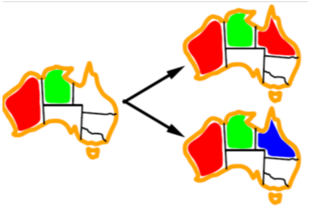
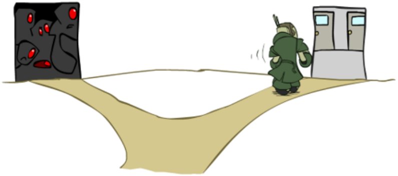
- Why least rather than most? $\rightarrow$ Leave more options for others
- Combining these ordering ideas makes 1000 queens feasible





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Suggested Reading

- Russell & Norvig: Chapter 6.1