

## Assignment 04

# Divide and Conquer

Please submit your solutions in PDF format. The PDF must be typed, NOT handwritten. Solution for each problem must start on a new page. The solutions should be concise; complicated and half-witted solutions might receive low marks, even when they are correct. Solutions should be submitted on the course website.

### Problem 1: Collaborators

[2 points]

List the name of the collaborators for this assignment. If you did not collaborate with anyone, write “None” (without the quotes).

**Problem 2: Go Off on a Tangent**

[5 points]

Assume that you are running Convex Hull Algorithm (as taught in class) on a set of points  $S$ . For  $CH(A)$  ( $a_1, a_2, \dots, a_p$ ), let  $a_1$  be the point with maximum  $x$ . For  $CH(B)$  ( $b_1, b_2, \dots, b_q$ ), let  $b_1$  be the point with minimum  $x$ . Let  $L$  as the vertical line that separates  $A$  and  $B$ . We define  $y(i, j)$  as the  $y$ -coordinate of the intersection between  $L$  and the line segment  $(a_i, b_j)$ . Argue that  $(a_i, b_j)$  is the uppertangent if and only if it maximizes  $y(i, j)$ .

**Problem 3: GiveIn: Shakes, Fries, Burgers**

[33 points]

Consider that your friend wants to setup a burger joint chain, *GiveIn*. *GiveIn* restaurants will be opened on different locations of a city. The city can be considered as an undirected graph  $G = (V, E)$ , where each potential location is denoted by the set of vertices. The adjacent locations are connected via the edges. Now, to avoid competition between two *GiveIn* restaurants, they won't be opened on adjacent vertices. Each vertex  $u$  has an integer  $p_u (\geq 0)$  associated with it, denoting the profit of opening restaurant in that location. Your goal is to design an algorithm to find out a set of vertices  $O$  that maximizes the total profit  $\sum_{u \in O} p_u$ . Consider that  $G$  is acyclic.

(a) [5 points] Consider the following greedy approach of opening *GiveIn*:

- Set  $O = \emptyset$
- Sort the vertices in the descending order of their profit.
- Repeat the following steps until  $V$  is empty.
  - Pick the first vertex  $u$  (that has the highest profit) from  $V$  and add it to  $O$ .
  - Remove  $u$  and all of its neighbors from  $V$ .

Draw an example graph where the algorithm will not work.

(b) [10 points] Provide an efficient algorithm to solve the problem.

(c) [8 + 4 points] Assume that all the potential locations are equally good. So the goal is to find out the largest set of vertices to open the restaurant. Provide a simple greedy algorithm to solve this problem and argue its correctness.

(d) [6 points] Assume that the graph is not necessarily acyclic. Provide an algorithm to solve the problem.