ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **SUMMER SEMESTER, 2019-2020**

DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

CSE 4803: Graph Theory

Answer all the questions. Figures in the right margin indicate marks.

Note: Write **Student-ID** and **Name** on top of the **first page** and write **Student-ID** and **Page-Number** in every page of the answer script. Submission pdf should be named as **"Full_Student_ID<space>Course Code.pdf"**

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1. JP Hammond is an alien pet hobbyist and has a number of pet aliens at his personal zoo. For redecoration of his zoo, he needs to transport the aliens to a nearby zoo. During this transfer, he wants to minimize the number of cages needed. But some of the aliens cannot be kept together as they have predator-prey relationship as shown below:

Alphas – Betos
Certas – Delatos
Certas – Espas
Fetas – Espas
Delatos – Fetas
Espas – Betos
Alphas – Certas
Betos – Certas

a) What is the minimum number of cages needed for the transfer? [Hint: use graph coloring] **10**

b) Find a possible transfer arrangement of the aliens. **5**

c) What are the chromatic numbers for the following graphs: **10**

i. W_n
ii. K_n
iii. $K_{5,3}$
iv. A tree with 28 vertices
v. A path of length 19
vi. r-Regular graph
vii. Q_3
viii. Complete multipartite graphs, $K_{r1,r2,...,rt}$
ix. A maximal planar graph with 5 vertices
x. A closed walk of length n

2. a) Determine a cut-set matrix and a circuit matrix for the graph in question 1(a). **8+5**

b) Is it possible to determine the diameter of a connected graph from the adjacency matrix? If yes, then give an example. If not, then discuss the reason behind it. **7**
[Hint: The diameter of a connected graph is defined as the largest distance between two vertices in the graph.]

c) Circuit matrix and cut-set matrix do not provide unique representation of graphs. But they have their own benefits. Discuss some of their uses. **5**

3. a) Show that, a *clique* in a graph corresponds to an *independent set* in the compliment of the graph. **5**

b) Draw a single graph and highlight the followings: **6+2**
i. maximal clique
ii. maximal independent set
What are the *clique number* and *independence number* of the graph?

c) Draw two non-isomorphic, connected, simple, and non-separable graphs G_1 and G_2, with as small a number of edges as you can, such that the circuit matrices $B(G_1) = B(G_2)$. **6**

d) For the graph drawn for question 3(b), find a fundamental cut-set matrix (consider a spanning tree as you like). How does this matrix reflect the rank and nullity of the graph? **6**