Solutions to Problems in Chapter 10 of
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10.1. Let $w_Z(100)$ and $w_K(100)$ be the expected average total time in system of the first 100 customers for the one-zippy and two-klunky configurations, respectively. Then $w_Z(100) = d_Z(100) + 0.9 = 4.13 + 0.9 = 5.03$ and $w_K(100) = d_K(100) + 1.8 = 3.70 + 1.8 =$ 5.50, so with regard to this performance measure, one Zippy would be better (contrary to the conclusion based on expected average delay in queue). A rational consumer would probably prefer the shorter total time in system (and thus the one-Zippy configuration), but this would mean a longer delay in queue (universally upsetting) as well as a longer queue.

10.2. System 1 is the one-Zippy model and system 2 is the two-Klunky model. The three parts of the problem were done in a single run without resetting the seed.

(*a*) We got $\bar{Z}(5) = 1.03, \widehat{\mathrm{Var}}\left[\bar{Z}(5)\right] = 0.52$, and from Table T1, $t_{4,0.95} = 2.132$. Thus, the 90 percent confidence interval is 1.03 ± 1.54 or $[-0.51, 2.57]$. Since this interval contains 0, no significant difference was detected here at the 0.10 level; a sample size of $n = 5$ was simply too small to detect the difference for the amount of variation present.

(*b*) We will use the Welch approach. We obtained the following results:

$$\bar{X}_1(5) = 4.26, S_1^2(5) = 4.62, \bar{X}_2(10) = 3.18, S_2^2(10) = 3.49, \text{ and } \hat{f} = 7.14.$$

From Excel we get that $t_{7.14,0.95} = 1.89$, so the 90 percent confidence interval is

$$1.08 \pm 1.89\sqrt{\frac{4.62}{5} + \frac{3.49}{10}} \quad \text{or} \quad [-1.05, 3.21]$$

which also contains 0.

(*c*) We obtained the following table ($h_1 = 1.896$ from Table 10.11):

i	$X_i^{(1)}(20)$	$S_i^2(20)$	N_i	$\bar{X}_i^{(2)}(N_i - 20)$	W_{i1}	W_{i2}	$\tilde{X}_i(N_i)$
1	4.31	14.53	327	4.30	0.072	0.928	4.31
2	3.57	4.46	101	3.92	0.234	0.766	3.84

Thus, we would select system 2 (two Klunkies) as being the configuration producing the smallest expected average delay in queue of the first 100 customers, which is the correct answer. The sample sizes (327 and 101) required to make this selection are much larger than those specified in parts (*a*) and (*b*); this is consistent with our failure in (*a*) and (*b*) to detect any difference between the configurations.

10.3. The configuration numbers were defined so that number 1 was the original quantum (q = 0.10), and numbers 2, 3, and 4 have quanta 0.05, 0.20, and 0.40, respectively. We ran the two parts of the problem consecutively, i.e., did not restart the random-number generator for the second part; for this reason the values of $\bar{X}_i^{(1)}(20)$ and $S_i^2(20)$ for a given i are different for the two parts.

(a) We got the following table ($h_1 = 2.583$ from Table 10.11):

i	$\bar{X}_i^{(1)}(20)$	$S_i^2(20)$	N_i	$\bar{X}_i^{(2)}(N_i - 20)$	W_{i1}	W_{i2}	$\tilde{X}_i(N_i)$
1	8.88	1.92	27	8.16	0.822	0.178	8.75
2	12.21	3.09	43	11.99	0.537	0.463	12.11
3	6.30	0.82	21	6.54	1.153	-0.153	6.26
4	5.57	1.58	22	6.84	0.953	0.047	5.63

Thus, we would select system 4 (with the longest quantum, q = 0.40) as being the one that would provide the smallest steady-state mean response time.

(b) We got the following table ($h_2 = 1.601$ from Table 10.12):

i	$\bar{X}_i^{(1)}(20)$	$S_i^2(20)$	N_i	$\bar{X}_i^{(2)}(N_i - 20)$	W_{i1}	W_{i2}	$\tilde{X}_i(N_i)$
1	7.72	1.58	21	8.47	1.216	-0.216	7.56
2	11.43	1.03	21	13.39	1.314	-0.314	10.82
3	6.47	1.35	21	6.50	1.251	-0.251	6.46
4	5.62	0.41	21	5.92	1.588	-0.588	5.44

The selected subset consists of systems 3 and 4 (with the two longest quanta, q = 0.20 and q = 0.40), i.e., the conclusion is that the best configuration is either 3 or 4.

10.4. Let configurations 1, 2, and 3 be the models resulting from adding a machine to workstations 1, 2, and 4, respectively. We obtained the following table (from Table 10.11, $h_1 = 2.342$):

i	$\bar{X}_i^{(1)}(20)$	$S_i^2(20)$	N_i	$\bar{X}_i^{(2)}(N_i - 20)$	W_{i1}	W_{i2}	$\tilde{X}_i(N_i)$
1	9.54	14.36	79	11.46	0.276	0.724	10.93
2	8.23	14.62	81	8.63	0.291	0.709	8.51
3	7.68	6.16	34	8.84	0.628	0.372	8.11

We thus select configuration 3, i.e., add the new machine to station 4. Looking back at Fig. 2.46 and Table 2.1 (the single-run "analysis" of this question), we might have come to this conclusion anyway, but now we have a much firmer justification.

This entire study could be greatly improved by *not* assuming on the basis of the *single* run reported in Fig. 2.46 that workstations 1, 2, and 4 *are* the bottlenecks that should be considered for capacity expansion. One approach would have been to consider instead $k = 5$ alternative configurations, adding a machine to *each* station one at a time. (Each of these five configurations would contain 14 machines.) Since this could involve making a large number of replications of configurations that will end up being discarded anyway, we might instead use the method of Sec. 10.4.2 to select, say, a subset of 3 configurations that contains the best system, and then select the best of these three as done above. Since we would be compounding two statistical procedures, we might want to use a higher value of P^*, at least for the initial subset selection.

10.5. Let configuration 1 be the original FIFO queue discipline in Sec. 2.5, and let configuration 2 be the priority queue discipline of Prob. 2.18. We picked $P^* = 0.90$ and, looking at Fig. 2.24 for 30 and 40 terminals, chose $d^* = 1$ second. We obtained the following results (from Table 10.11, $h_1 = 1.896$):

i	$\bar{X}_i^{(1)}(20)$	$S_i^2(20)$	N_i	$\bar{X}_i^{(2)}(N_i - 20)$	W_{i1}	W_{i2}	$\tilde{X}_i(N_i)$
1	8.88	1.92	21	9.31	1.257	-0.257	8.77
2	8.49	2.38	21	6.48	1.209	-0.209	8.91

We choose the original FIFO rule as being the best, although the difference between the final $\tilde{X}_i(N_i)$ values is very small; in fact, their order is reversed from that of the $\bar{X}_i^{(1)}(20)$ values. Our guess is that μ_1 and μ_2 are within d^* of each other, so according to our indifference-zone criterion we do not care which configuration is chosen.

We repeated the above select-the-best procedure with 60 terminals and obtained final $\tilde{X}_i(N_i)$ values of 30.29 for the FIFO rule and 18.11 for the priority rule, suggesting clear superiority of the latter with 60 terminals. The queue is much shorter with 35 terminals (an average of 8.7 rather than 33.0 jobs, interpolating in Fig. 2.24 for the former value), and priorities do not matter much for short queues.

10.6. Using the procedure in Sec. 10.4.2, we obtained the following results (from Table 10.12, $h_2 = 1.243$, and $i = s$ is the number of repairmen):

i	$\bar{X}_i^{(1)}(20)$	$S_i^2(20)$	N_i	$\bar{X}_i^{(2)}(N_i - 20)$	W_{i1}	W_{i2}	$\tilde{X}_i(N_i)$
1	96.91	62.53	21	96.31	1.401	-0.401	97.15
2	75.16	20.02	21	72.72	1.804	-0.804	77.12
3	80.88	5.37	21	89.91	2.634	-1.634	66.13
4	90.33	13.98	21	92.86	1.980	-0.980	87.85
5	99.75	10.89	21	98.09	2.123	-1.123	101.62

Thus, the selected subset consists of the $m = 3$ systems with 2, 3, and 4 repairmen.

10.7. Defining configuration $i = 1$ to be the current system ($q = 0.10$) and configurations $i = 2, 3,$ and 4 to represent $q = 0.05, 0.20,$ and 0.40, respectively, we re-used the initial 20 replications from each configuration in Prob. 10.3(*a*), which were as follows:

Replication (*j*)	X_{1j}	X_{2j}	X_{3j}	X_{4j}
1	10.18	10.46	7.03	4.34
2	8.34	12.75	5.07	6.71
3	7.81	12.26	4.97	7.35
4	8.17	12.31	5.48	7.61
5	8.58	12.68	6.54	5.44
6	7.10	12.65	4.97	4.95
7	11.96	12.69	6.04	5.80
8	10.56	12.64	6.05	6.90
9	8.44	13.78	4.96	5.33
10	11.46	9.27	7.26	7.61
11	9.81	12.96	7.65	5.26
12	6.99	11.34	7.21	4.45
13	8.15	14.88	7.56	4.37
14	9.88	8.89	6.23	4.32
15	7.61	15.64	6.29	3.69
16	9.02	10.55	6.15	5.51
17	7.45	13.47	5.92	5.25
18	8.26	9.70	7.69	4.91
19	9.28	13.45	6.53	7.33

20	8.45	11.88	6.34	4.21
Mean	8.88	12.21	6.60	5.57
Variance	1.92	3.10	0.82	1.58

Since there are $k = 3$ confidence intervals, we make each at level 96.667 percent. Using both the paired-*t* and Welch approaches, we got the following table:

i	$\bar{X}_i(20) - \bar{X}_1(20)$	Paired-*t* confidence interval for $\mu_i - \mu_1$	Welch confidence interval for $\mu_i - \mu_1$
2	3.34	$(2.00, 4.67)$	$(2.23, 4.44)$
3	-2.58	$(-3.32, -1.84)$	$(-3.40, -1.76)$
4	-3.31	$(-4.11, -2.50)$	$(-4.23, -2.38)$

Since none of the confidence intervals contain 0, the mean of each proposed alternative configuration is statistically different than the mean of the current configuration. Moreover, it appears that configuration 2 would increase average response time, while configurations 3 and 4 would decrease it.

10.8. We ran 10 replications for each value of $s = i$ (the number of repairmen). The following are the individual-replication results for the average cost per hour:

Replication (j)	X_{1j}	X_{2j}	X_{3j}	X_{4j}	X_{5j}
1	96.81	74.19	76.32	86.49	104.61
2	102.94	68.75	76.97	90.93	97.71
3	111.17	72.91	78.40	94.02	95.96
4	98.82	78.23	78.57	88.06	98.98
5	95.65	70.08	77.01	88.63	104.40
6	104.59	76.21	76.09	90.95	102.39
7	107.12	77.51	78.19	89.37	98.48
8	106.73	73.04	80.58	94.54	97.50
9	93.76	71.11	80.20	92.04	96.83
10	102.67	74.57	81.47	93.69	96.97
Mean	102.03	73.66	78.38	90.87	99.38
Variance	31.78	9.74	3.46	7.45	10.32

Since there are 5(5-1)/2 = 10 confidence intervals, each must be at level 99 percent to achieve an overall confidence level of at least 90 percent. The following are the results for the paired-t and Welch approaches (a statistically significant difference is marked by a *:

Paired-*t* confidence interval for $\mu_{i_2} - \mu_{i_1}$ **(**$i_2 > i_i$ **):**

	$i_2 = 2$	$i_2 = 3$	$i_2 = 4$	$i_2 = 5$
$i_1 = 1$	$-28.37 \pm 5.89^*$	$-23.65 \pm 5.94^*$	$-11.15 \pm 4.83^*$	-2.64 ± 7.91
$i_1 = 2$		$4.72 \pm 3.68^*$	$17.21 \pm 4.73^*$	$25.72 \pm 4.52^*$
$i_1 = 3$			$12.49 \pm 2.09^*$	$21.00 \pm 4.85^*$
$i_1 = 4$				$8.51 \pm 5.71^*$

Welch confidence interval for $\mu_{i_2} - \mu_{i_1}$ ($i_2 > i_i$):

	$i_2 = 2$	$i_2 = 3$	$i_2 = 4$	$i_2 = 5$
$i_1 = 1$	$-28.37 \pm 6.07^*$	$-23.65 \pm 5.95^*$	$-11.15 \pm 5.97^*$	-2.64 ± 6.11
$i_1 = 2$		$4.72 \pm 3.42^*$	$17.21 \pm 3.80^*$	$25.72 \pm 4.11^*$
$i_1 = 3$			$12.49 \pm 3.08^*$	$21.00 \pm 3.49^*$
$i_1 = 4$				$8.51 \pm 3.86^*$

Both approaches were able to distinguish between the two configurations in a pair in the same 9 out of 10 cases, and compared to the magnitude of the output values (about 70 to 110), these half-lengths appear small enough to be useful. We might add that we tried the above analysis based on 5 (rather than 10) replications of each configuration, and ended up with intervals that seemed to be too wide to be of much use.

10.9. Reducing the mean inspection time by 10 percent decreases the utilization factor of the inspector, which in turn decreases the average delay in queue at the inspector.

10.10. In Example 10.7 the confidence interval is larger, since the confidence level is 97.5 percent as compared to 90 percent in Example 10.3.

10.11. $\mathrm{Var}(X_{ij} - X_{lj}) = \mathrm{Var}(X_{ij}) + \mathrm{Var}(X_{lj}) - 2\mathrm{Cov}(X_{ij}, X_{lj})$

$$= 2\psi_i + \tau^2 + 2\psi_l + \tau^2 - 2(\psi_i + \psi_l)$$

$$= 2\tau^2$$

10.12. $E(S^2) = 2\tau^2 E\{\chi^2_{(k-1)(n_0-1)}/[(k-1)(n_0-1)]\}$

$$= 2\tau^2 \frac{E[\chi^2_{(k-1)(n_0-1)}]}{(k-1)(n_0-1)}$$

$$= 2\tau^2 \frac{(k-1)(n_0-1)}{(k-1)(n_0-1)}$$

$$= 2\tau^2$$