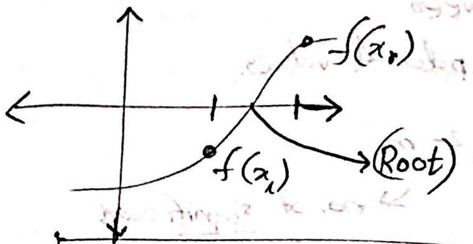
#1

Bisection Method of solving Non-Linear Equations

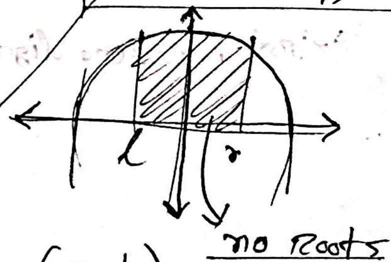
Methods (For non-linear eq) – i) Bisection
ii) Newton Raphson
iii) Secant

⊛ Bisection Method uses binary search.

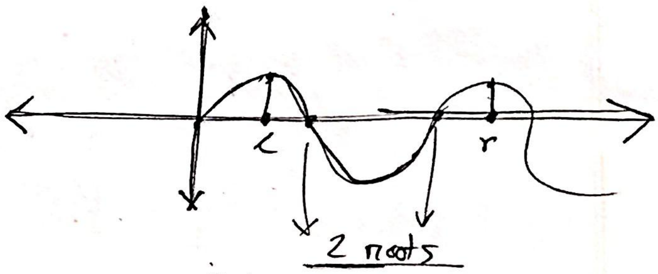
→ A continuous function has (at least one) root if $\boxed{f(x_l) \cdot f(x_r) < 0}$ (odd no. of roots)
(left) (right)



But if $\boxed{f(x_l) \cdot f(x_r) > 0}$ there can be roots

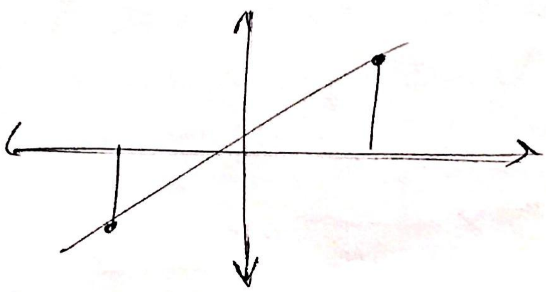


(even no. of roots)



$f(x_l) \cdot f(x_r) > 0$

→ Why use binary search?
Because points in a line are sorted.



(i) Randomly initialize l, r.
(ii) Calculate mid.
(iii) Compare $f(x_{mid})$
(iv) Return when $f(x_{mid}) = 0$.

→ (x_{mid}) converges to root.

When does the program end or return something?

(i) When $f(x_{mid})$ exactly equals to zero. (takes long time)

(ii) Having an approximate error:

$$|\epsilon_a| = \left| \frac{x_{mid}^{new} - x_{mid}^{old}}{x_{mid}^{new}} \right| \times 100$$

→ Difference between updated values.

(iii) $\epsilon_a = 0.5 \times 10^{2-m}$

no. of significant digits correct.

first non-zero digit.

#2

Chapter - 1.02
Measuring Errors

22.11.21
MONDAY

Error → True / Approximate or → Absolute / Relative

True Error = (True Value − Approx Value)

Ex− Linear Approximation

$$f'(x) = \frac{f(x+h)-f(x)}{h}$$

True Value → $f'(x) = \frac{d}{dx}[f(x)]$

Problem

→ Difficult to interpret the true error.

Relative Error

$$\frac{\text{True Error}}{\text{True Value}} = \frac{T.V - A.V.}{T.V}$$

Approximate Error (if we don't know true value)

$$\underbrace{\frac{x_m^{new} - x_m^{old}}{x_m^{new}} \times 100\%}_{\text{Relative Approx Error.}}$$

Maclaurin

~~McLaren~~ Series (to approximate the values of a function)

$$f(x) = f(0) + xf'(0) + x^2\frac{f''(0)}{2!} + x^3\frac{f'''(0)}{3!} \cdots\cdots$$

$$= \sum_{n=0}^{\infty} \frac{f^n(0)}{n!} \times x^n$$

Trailing Zeros

→ Non non-zero sign digit matched.

109
101

1 sig digit correct

1091
1092

3 sig digit correct.

2.90

has 3 significant digits
(~~digits~~ zeros coming after sig digit are significant).

কেন?

4.90000....
4.90000....

2 matches not infinite matches

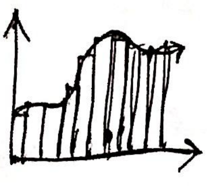
⊛ তাই Trailing zeroes match করি না unless we have a matching significant digit after that.

Types of error

a) Round off
2.322 → 2.3 (removed digits)

b) Truncation

i) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} - - - - - -$
→ $1 + x + \frac{x^2}{2!}$ (removed terms)



ii) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \approx \frac{f(x+h) - f(x)}{h}$ [Removed limit]

iii) Approximation of Integrals

#3

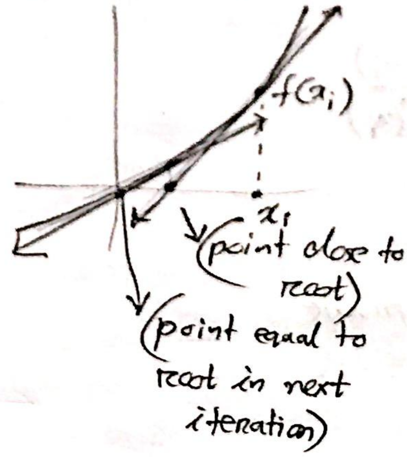
Newtonian - Rapson

29.11.21
Monday

Bracket Method → has 2 bounds

Newton-Rapson is not bracket method.

It has a tangent that will intersect the x-axis closer to the root.



$$f'(x_i) = \tan\theta$$
$$= \frac{f(x_i) - 0}{x_i - x_{i+1}}$$
$$\Rightarrow x_i - x_{i+1} = f(x_i) \Big/ f'(x_i)$$
$$\Rightarrow \boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

→ Repeat until threshold error.

Ex- $x^3 - 0.165x^2 + 3.993x \times 10^{-4}$ $\quad 0 \le x \le 0.11$

We don't start with 0 or 0.11 as these are max and min points and result in derivative is 0.

Let's choose $x = 0.05$

i) Calculate, $x_{i+1} = \ldots\ldots$

ii) Calculate error

If error less, than 5%, stop.
else, go back to step (i)
(depends on question)

Advantage → onek effiecient, 3ta iteration e bhalo value dei.

Disadvantage → i) Functions with parrell points (to x-axis) perra dei. Differentiable হতে হবে।

ii) Inflaction points (maxima, minima) ঘোরা দেয়।

($f''(x_i) > 0$) becomes ($f''(x_i) < 0$)

Proof ~~using~~ of Newton-Rapson:

using Taylor Series (first terms)

$$f(x_{i+1}) = f(x_i) + f(x_i) + f'(x_i)(x_{i+1} - x_i)$$

$$\Rightarrow x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Computational Problem

→ finding the derivate

Solution

→ Secant Method

→ How do we get inflaction points?

$x_i = 5$

$x_{i+1} = 1.54$

$x_{i+1} = -55$

$\}$ inflaction point.

#4

Secant Method

03-Dec-21
Friday

(*) Derivatives are expensive.

Sometimes, calculating one derivative is equivalent to 1000 iterations of Bisection Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \leftarrow \text{(Newton-Rapson)}$$

But, in $f'(x_i)$ it was $f'(x_i) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

If we remove $h \to 0$, then,

$$f'(x_i) = \frac{f(x+h)-f(x)}{h}$$

where ~~b~~ h is small but not ~~inde~~ infidecimally small.

The points in a secant triangle forms two similar triangles.

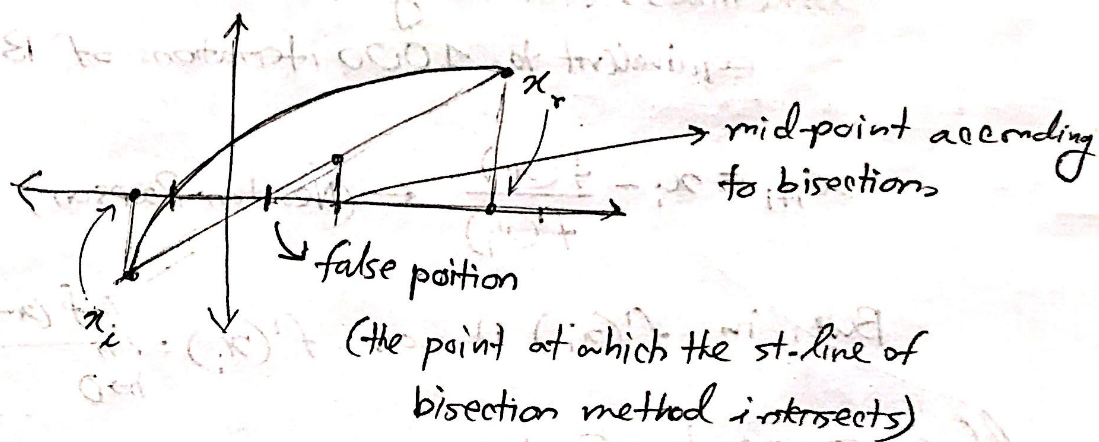
Secant (*)

$$\boxed{x_{i+1} = x_i - f(x_i) \times \frac{x_i - x_{i-1}}{f(x_i) - f(x_{i+1})}}$$

$$f'(x_i) \approx \frac{f(x_i) - f(x_{i+1})}{x_i - x_{i-1}}$$

False Position

→ Bracketting method



আবার Similiar Triangle কাহিনী করে Formula আসে।
তাই formula দেখতে Secant-এর মতো।