Solutions to Problems in Chapter 13 of
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13.1. The following behavior often occurs:

- The sheep population (SP) gets large (> 450) because more green grass is available.
- The wolf population (WP) then gets large (> 400) because there are a lot of sheep to eat, which in turn causes the sheep population to get small (< 20).
- The WP plummets toward 0 because sheep are few and far between – less than 1 percent of the patches contain sheep.
- Once the WP is exterminated, the SP gets very large (> 900) because there is plenty of green grass to eat.
- Eventually, the SP and the (green-grass)/4 population reach an "equilibrium" at levels of approximately 750 and 190, respectfully.

13.2. $\bar{X}_{50}(100) = 74.69, \bar{X}_{75}(100) = 77.00, S^2_{50}(100) = 230.94, S^2_{75}(100) = 238.75$

$\hat{f} = 197.95,\ t_{197.95,0.975} = 1.972$ (from Excel)

A 95 percent confidence interval for $\mu_{50} - \mu_{75}$ is -2.31 ± 4.27 or [-6.58,1.96]. Since the confidence interval contains 0, the observed difference between μ_{50} and μ_{75}, that is, $\bar{X}_{50}(100) - \bar{X}_{75}(100)$, is not statistically significant at level 0.05. It is surprising that increasing the initial number of wolves from 50 to 75 has virtually no impact on the final number of wolves.

13.3. There is no value of Δt that guarantees that the states of the cells (i.e., whether they are green or red) are updated when they should be.

13.4. Consider reversing the roles of the blue and red forces. Thus, $b(0) = 300$ and $\alpha = 0.4$ for the blue force, and $r(0) = 125$ and $\beta = 0.1$ for the red force. Since

$$b(0) / r(0) = 2.4 > 2 = \sqrt{\alpha / \beta}$$

the "blue force" (actually the red force) will win the battle. If we use the given formulas for t_{end} and $b(t_{\text{end}})$, then we get that the battle ends at 5.99 hours and that the "blue force" has 165.83 units at this time.

13.5. The *bottom* part of the main "rectangle" for the system layout will work if component 2 works *and* at least one of components 3 and 4 work. Thus, this part of the system will fail at time $S = \min[X_2, \max(X_3, X_4)]$. The main rectangle of the system layout will work if the top part or the bottom part of the rectangle works, and thus the time to failure of this part of the system is $T = \max\{X_1, S\}$. It follows that the whole system will fail at time $Y = \min(T, X_5)$.

13.6. Let $\bar{X}_i(25000)$ be the average time to failure across $n = 25000$ replications if component i is replaced by two such components in parallel, for $i = 1,2,5$. Also, let $S_i^2(25000)$ be the corresponding sample variance. Then we obtained the following results:

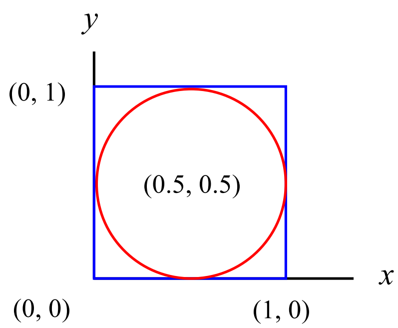
$$\bar{X}_1(25000) = 4.982, \bar{X}_2(25000) = 4.554, \text{ and } \bar{X}_5(25000) = 5.991$$
$$S_1^2(25000) = 18.975, S_2^2(25000) = 15.133, S_5^2(25000) = 17.564$$

Thus, it appears that adding a second component 5 in parallel will increase the expected time to failure the most. (Recall for the original system that the average time to failure was 4.33.) Using the Welch approach and the Bonferroni inequality, we obtained the following 98.67 percent confidence intervals for the differences in means:

Difference in means	**98.67 percent confidence interval**
$\mu_1 - \mu_2$	0.428 ± 0.088
$\mu_1 - \mu_5$	-1.009 ± 0.091
$\mu_2 - \mu_5$	-1.437 ± 0.087

Note that none of the confidence intervals contain 0, so it appears that the means in each pair are different.

13.7. Consider the following square and circle:



Generate a random point (X_i, Y_i) in the square (for $i = 1, 2, \ldots, 25{,}000$) by letting $X_i \sim \mathrm{U}(0,1)$, $Y_i \sim \mathrm{U}(0,1)$, and X_i, Y_i independent. If

$$(X_i - 0.5)^2 + (Y_i - 0.5)^2 < (0.5)^2$$

then the point is in the circle and set $Z_i = 1$. Otherwise, the point is not in the circle and set $Z_i = 0$. Then a point estimate for π is

$$\hat{\pi} = 4 \times \frac{\sum_{i=1}^{25000} Z_i}{25000} = 3.128$$

and a 95 percent confidence interval for π (see Sec. 9.4.1) is $[3.108, 3.149]$, which contains $\pi = 3.14159$.