Suppose you have a set of embedded points $\{\mathbf{x}_i\} \subset \mathbb{R}^n$ representing your high-dimensional object — say, a hypercube.

You define a **privileged basis** $\{\hat{b}_k\}$ and construct **privileged bivectors** $\hat{b}_k \wedge \hat{b}_l$ spanning 2D planes within $\mathbb{R}^n$
.

You then define, for each plane (k, l), a one-parameter family of **rotation matrices** $R_{kl}(\theta) \in \mathrm{SO}(n)$, where $\theta \in [0, 2\pi)$ sweeps the angle of rotation in that 2D plane.

The **spotlight vector** at angle θ is then:

$$\hat{v}_{kl}(\theta) = R_{kl}(\theta)\hat{b}_k$$

The **spotlight resonance signal** $f_{kl}(\theta)$ is defined as:

$$f_{kl}(\theta) = \frac{1}{N}\sum_{i=1}^{N} 1\left(\langle \hat{v}_{kl}(\theta), \hat{x}_i \rangle \geq \epsilon\right)$$

where:

- $1(\cdot)$ is the indicator function (1 if the condition is true, 0 otherwise),
- $\epsilon = \cos(\varphi)$ defines the spotlight aperture,
- $\hat{x}_i = \frac{\mathbf{x}_i}{\|\mathbf{x}_i\|}$ are unit-normalized embedded points.

Now, the **angular Fourier transform** of $f_{kl}(\theta)$ gives you the hidden symmetry structure:

$$\tilde{f}_{kl}(m) = \frac{1}{2\pi}\int_0^{2\pi} f_{kl}(\theta)e^{-im\theta}d\theta$$

where $m \in \mathbb{Z}$ is the angular frequency.

If the system is isotropic, $\tilde{f}_{kl}(m) \approx 0$ for all $m \neq 0$.

If there is hidden axis alignment (symmetry breaking), you get large magnitudes $|\tilde{f}_{kl}(m)|$ at certain frequencies, telling you about the angular structure.

Or to package it *even funkier*, you could write:

$$\text{Spotlight Resonance Spectrum:} \quad \tilde{f}(m) = \mathcal{F}_\theta \left[\frac{1}{|B|} \sum_{(k,l) \in B} f_{kl}(\theta) \right]$$

where $\mathcal{F}_\theta$ is the angular Fourier transform over θ, and B is the set of all privileged bivector planes.

You could visualize it like this:

- $f_{kl}(\theta)$ is a wiggly function in θ space.
- Its Fourier spectrum $\tilde{f}_{kl}(m)$ shows sharp peaks if the structure aligns with privileged axes.
- The full set $\{\tilde{f}_{kl}(m)\}$ reconstructs the "angular fingerprint" of hidden symmetries.

When I said the spotlight resonance signal $f_{kl}(\theta)$ is a "wiggly function," I meant:
it's a **function of rotation angle** θ — **not** of position in space, and **not** of time.
It's literally: "as you rotate your spotlight *around a privileged plane*, how much stuff is inside the spotlight cone?"

So:

- **Input** to f_{kl} is **angle** $\theta \in [0, 2\pi)$.
- **Output** is **fraction** of vectors (samples) that are close to the spotlight direction.

If you plotted $f_{kl}(\theta)$ vs θ, you'd see a curve that goes up and down — "wiggling" — because at some angles the spotlight hits a dense part of the embedded data, and at others it misses.

That's what I meant by "wiggly function" — a **function over the circle** S^1.
More precisely:
$f_{kl} : S^1 \to [0, 1]$.

You're literally sweeping around a circle of angles, recording a number at each angle, and getting a signal.
That's why it's natural to Fourier transform it: because it's a periodic function on a circle.