State Space Approach to Adaptive Filters

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***Abstract*—State space approach (such as the Kalman filter) has been extensively used in feedback control systems. They are known to give sub-optimal results for linear and nonlinear systems with Gaussian and non-Gaussian noise. The stability of such algorithms has also been well established in the literature. The adaptive filtering problem can be rephrased as a state space problem and state space solutions can be used to address the solutions in place of adaptive filters. There is a guaranteed convergence for these algorithms under some conditions. Moreover, the step size in adaptive filters is replaced by a gain in the state space domain. This has an expression that is easily implementable. Hence, this project aims to introduce the concept of state space systems to analyse adaptive filters.**

***Index Terms*—State Space, Kalman Filter, Adaptive filters, LMS, NLMS, RLS.**

State space representation is widely used in control systems due to their steady state and stability properties. A popular state space solution is obtained using the well known Kalman filter. The space solution for the adaptive filter algorithm, particularly the least mean squares (LMS) and the recursive least squares (RLS) filters has been studied fairly in the literature. For instance, Lopes et. al. [1] proposed a Kalman filter based LMS. The Kalman filter in this context is explained in [2]. Some of the recent works in state space least mean squares involve fractional calculus [3] for removal of power line interference (PLI) in ECG signals [4]. However, not a lot of work has focused on using state space solutions in nonlinear systems.

I. Objective

1) Implement a state space solution for the adaptive filter algorithm, particularly the least mean squares (LMS) filter. The implementation is based on the methodology proposed by Malik et. al. [5], [6]. These papers propose a state space LMS (SS-LMS) as a generalisation of the LMS and NLMS filters. The algorithms is used in noise estimation and signal tracking, especially for sinusoidal systems.
2) Analyse a state space solution of an adaptive filter which minimises the squared sum of the relative misalignment in the filter coefficients. This is realised using an unscenteed Kalman filter (UKF). The model, convergence, noise statistics have been theoretically derived.

The state space representation for a time varying FIR model is shown in fig. 1. The state pace model for a discrete time system with time index k is given by:

$$\begin{aligned}\mathbf{x}[k+1] &= \mathbf{A}[k+1|k]\mathbf{x}[k] + \mathbf{B}[k]\mathbf{g}[k] \\ \mathbf{y}[k] &= \mathbf{C}[k]\mathbf{x}[k] + \nu[k]\end{aligned} \tag{1}$$

where $\mathbf{x}[k]$ represents the state of the system, $\mathbf{A}[k+1|k]$ is the state transition matrix relating the previous state to the current state, $\mathbf{B}$ relates the control input $\mathbf{g}[k]$ to the state, $\mathbf{y}[k]$ is the observed measurement, $\mathbf{C}$ is the matrix relating the measurement to the state and ν is the Gaussian noise in the measurement. For the case of adaptive filters, the filter coefficients of the adaptive FIR filter represent the system states. The transition matrix is the recursive relation between consecutive time filter coefficients. The equations are as follows:

$$\begin{aligned}\mathbf{w}[k+1] &= \mathbf{I}\mathbf{w}[k] + \mu\mathbf{u}[k]e[k] \\ d[k] &= \mathbf{u}^T[k]\mathbf{w}[k+1] + \nu\end{aligned} \tag{2}$$

where $e[k] = d[k] - \mathbf{u}^T[k]\mathbf{w}[k+1]$ is the estimation error, $d[k]$ is the unknown desired signal, $\hat{d}[k] = \mathbf{u}^T[k]\mathbf{w}[k+1]$ is the estimate of the desired signal. Here $e[k]$ can be considered the control input.

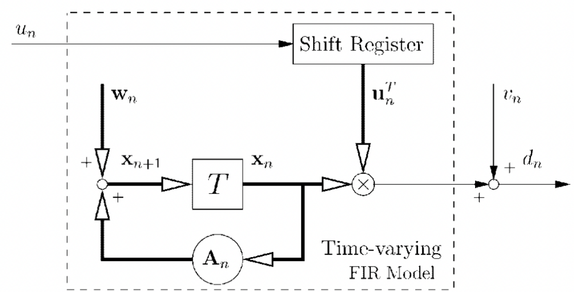

Fig. 1: State space representation of a time varying FIR model [2]

The timeline is as follows:

1) End of Oct - Code the algorithm in MATLAB and get the results for the first objective. Derive the UKF algorithm, convergence and error bounds.
2) End of Nov - Implement the UKF algorithm for similar applications as [5], [6]. Try to include more examples of noise removal.

References

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