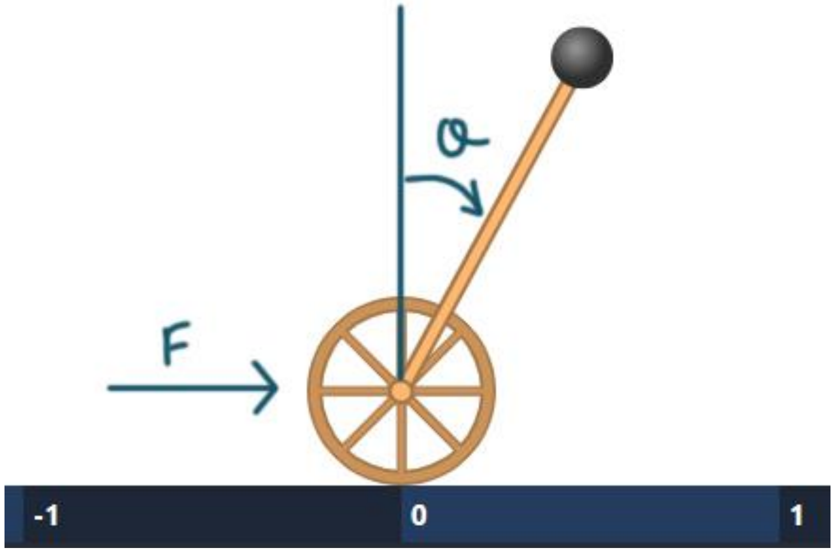

$$x_m = x + l sin\theta$$

$$y_m = l cos\theta$$

$$\dot{x}_m = \dot{x} + l\dot{\theta} cos\theta$$

$$\dot{y}_m = -l\dot{\theta} sin\theta$$

$$V = mgy_m = mgl cos\theta$$

$$T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}I\left(\frac{\dot{x}}{R}\right)^2 + \frac{1}{2}m(\dot{x}_m^2 + \dot{y}_m^2)$$

$$T = \frac{1}{2}(2M)\dot{x}^2 + \frac{1}{2}m(\dot{x}^2 + l^2\dot{\theta}^2\cos^2\theta + 2\dot{x}l\dot{\theta}cos\theta + l^2\dot{\theta}^2\sin^2\theta)$$

$$T = \frac{1}{2}(2M)\dot{x}^2 + \frac{1}{2}m(\dot{x}^2 + l^2\dot{\theta}^2 + 2\dot{x}l\dot{\theta}cos\theta)$$

$$T = \frac{1}{2}(2M + m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 + m\dot{x}l\dot{\theta}cos\theta$$

$$L = T - V$$

$$\frac{d}{dt}\frac{\delta L}{\delta \dot{q}_i} - \frac{\delta L}{\delta q_i} = Q_i$$

$$L = \frac{1}{2}(2M+m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 + m\dot{x}l\dot{\theta}cos\theta - mglcos\theta$$

$$\frac{\delta L}{\delta \dot{x}} = (2M+m)\dot{x} + ml\dot{\theta}cos\theta$$

$$\frac{\delta L}{\delta x} = 0$$

$$\frac{\delta L}{\delta \dot{\theta}} = ml^2\dot{\theta} + m\dot{x}lcos\theta$$

$$\frac{\delta L}{\delta \theta} = -m\dot{x}l\dot{\theta}sin\theta + mglsin\theta$$

$$Q_x = \frac{d}{dt}\Big((2M+m)2\dot{x} + ml\dot{\theta}cos\theta\Big) + m\dot{x}l\dot{\theta}sin\theta - mglsin\theta$$

$$where\ Q_x = F - \mu(M+m)g\dot{x}$$

$$Q_x = (2M+m)\ddot{x} + ml\big(\ddot{\theta}cos\theta - \dot{\theta}^2 sin\theta\big) \qquad 1$$

$$0 = l\ddot{\theta} + \big(\ddot{x}cos\theta - \cancel{\dot{x}\dot{\theta}sin\theta}\big) - gsin\theta + \cancel{\dot{x}\dot{\theta}sin\theta}$$

$$0 = l\ddot{\theta} + \ddot{x}cos\theta - gsin\theta \qquad 2$$

$$\ddot{\theta} = \frac{gsin\theta - \ddot{x}cos\theta}{l}$$

$$F - \mu(M+m)g\dot{x} + ml\dot{\theta}^2 sin\theta - ml\ddot{\theta}cos\theta = (2M+m)\ddot{x}$$

$$F - \mu(M+m)g\dot{x} + ml\dot{\theta}^2 sin\theta - mcos\theta(gsin\theta - \ddot{x}cos\theta) = (2M+m)\ddot{x}$$

$$\frac{F - \mu(M+m)g\dot{x} + ml\dot{\theta}^2 sin\theta - mgsin\theta cos\theta}{(2M+m-\ mcos^2\theta)} = \ddot{x}$$

And linearized system;

$$x_1 = x, \qquad x_2 = \dot{x}, \qquad x_3 = \theta, \qquad x_4 = \dot{\theta}$$

$$cos\theta \cong 1, \qquad sin\theta \cong \theta, \qquad \dot{\theta}^2 \cong 0$$

$$\frac{F-\mu(M+m)gx_2-mgx_3}{(2M)}=\ddot{x}$$

$$\ddot{\theta}=\frac{gx_3}{l}-\frac{F-\mu(M+m)gx_2-mgx_3}{(2Ml)}$$

$$\ddot{\theta}=-\frac{F}{2Ml}+\frac{\mu(M+m)g}{2Ml}x_2+\frac{2Mg+mg}{2Ml}x_3$$

Linear system In state space form

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\theta}\\ \ddot{\theta}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -\frac{\mu(M+M)g}{2M} & -\frac{mg}{2M} & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{\mu(M+m)g}{2Ml} & \frac{2Mg+mg}{2Ml} & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \theta\\ \dot{\theta}\end{bmatrix}+\begin{bmatrix}0\\ \frac{1}{2M}\\ 0\\ \frac{-1}{2Ml}\end{bmatrix}[F]$$