

Theorem 1 (ISHA Guarantee). *Fix $\delta \in (0, 1)$. Under Assumptions 1 and 2, for any $\epsilon > 0$ define*

$$\begin{aligned} z_{n,\epsilon} &:= \frac{\log(2 \log_2(n)/\delta)}{\nu_0(\mu_* + \epsilon)} \max \left\{ 1, 64R \log(4n \log_2(n)/\delta) \sup_{x \geq \mu_* + \epsilon} (x - \mu_*)^{-2} \nu_0(x) \right\} \\ &\leq \log(2 \log_2(n)/\delta) \max \left\{ \frac{1}{\nu_0(\mu_* + \epsilon)}, \right. \\ &\quad \left. 64R \log(4n \log_2(n)/\delta) \left(\epsilon^{-2} + \frac{1}{\nu_0(\mu_* + \epsilon)} \int_{x=\mu_* + \epsilon}^{\infty} \frac{1}{(x - \mu_*)^2} d\nu_0(x) \right) \right\}. \end{aligned}$$

If ISHA is run with n arms and $T = \lceil n \log_2(n) \rceil$ total pulls where $n \geq z_{n,\epsilon}$ then with probability at least $1 - \delta$ the single arm returned is no greater than $\mu_ + \epsilon$.*

Theorem 2 (ISHA Guarantee, α version). *Fix $\delta \in (0, 1)$. Under Assumptions 1 and 2, for any $\epsilon > 0$ define $\alpha = \nu_0(\mu_* + \epsilon)$ and*

$$\begin{aligned} z_{n,\epsilon} &:= \frac{1}{\alpha} \log \left(\frac{2 \lg(n)}{\delta} \right) \max \left\{ 1, 64R \log \left(\frac{4n \lg(n)}{\delta} \right) \sup_{x \geq \mu_* + \epsilon} (x - \mu_*)^{-2} \nu_0(x) \right\} \\ &\leq \log \left(\frac{2 \lg(n)}{\delta} \right) \max \left\{ \frac{1}{\alpha}, \right. \\ &\quad \left. 64R \log \left(\frac{4n \lg(n)}{\delta} \right) \left(\frac{1}{\epsilon^2} + \frac{1}{\alpha} \int_{x=\mu_* + \epsilon}^{\infty} \frac{1}{(x - \mu_*)^2} d\nu_0(x) \right) \right\}. \end{aligned}$$

If ISHA is run with n arms and $T = \lceil n \log_2(n) \rceil$ total pulls where $n \geq z_{n,\epsilon}$ then with probability at least $1 - \delta$ the single arm returned is no greater than $\mu_ + \epsilon$.*

Number of arms required satisfies both of:

$$n \geq \frac{1}{\alpha} \log \left(\frac{2 \lg(n)}{\delta} \right) \tag{1}$$

$$n \geq 64R \log \left(\frac{2 \lg(n)}{\delta} \right) \log \left(\frac{4T}{\delta} \right) \left(\frac{1}{\epsilon^2} + \frac{1}{\alpha} \int_{x=\mu_* + \epsilon}^{\infty} \frac{1}{(x - \mu_*)^2} d\nu_0(x) \right) \tag{2}$$