

PDE

$$\Delta \phi = f$$



First-Order System

$$\begin{aligned}\nabla \cdot \psi &= f \\ \psi - \nabla \phi &= 0\end{aligned}$$



Integration by Parts

$$\begin{aligned}(\psi \cdot n, v)_{\Gamma_h} - (\psi, \nabla v)_{\Omega_h} &= (f, v)_{\Omega_h} \\ (\psi, q)_{\Omega_h} + (\phi, q \cdot n)_{\Gamma_h} - (\phi, \nabla \cdot q)_{\Omega_h} &= 0\end{aligned}$$



Ultraweak (DPG) Variational Formulation

$$\begin{aligned}(\hat{\psi}_n, v)_{\Gamma_h} - (\psi, \nabla v)_{\Omega_h} \\ + (\psi, q)_{\Omega_h} + (\hat{\phi}, q_n)_{\Gamma_h} - (\phi, \nabla \cdot q)_{\Omega_h} &= (f, v)_{\Omega_h}\end{aligned}$$

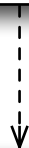
$$b((\phi, \psi, \hat{\phi}, \hat{\psi}_n), (v, q)) = (f, v)_{\Omega_h}$$

$$“b(u, v) = l(v)”$$

$$Au = f$$



$$(u, A^*v)_{\Omega_h} + \langle u, v \rangle_{\Gamma_h} = l(v)$$



$$(u, A^*v)_{\Omega_h} + \langle \hat{u}, v \rangle_{\Gamma_h} = l(v)$$

$$b((u, \hat{u}), v) = l(v)$$

$$“b(u, v) = l(v)”$$