PDE

$$\triangle \phi = f$$

First-Order System

$$\nabla \cdot \psi = f$$
$$\psi - \nabla \phi = 0$$

Integration by Parts

$$(\psi \cdot n, v)_{\Gamma_h} - (\psi, \nabla v)_{\Omega_h} = (f, v)_{\Omega_h}$$
$$(\psi, q)_{\Omega_h} + (\phi, q \cdot n)_{\Gamma_h} - (\phi, \nabla \cdot q)_{\Omega_h} = 0$$

Ultraweak (DPG) Variational Formulation

$$(\widehat{\psi}_n, v)_{\Gamma_h} - (\psi, \nabla v)_{\Omega_h}$$
$$+(\psi, q)_{\Omega_h} + (\widehat{\phi}, q_n)_{\Gamma_h} - (\phi, \nabla \cdot q)_{\Omega_h} = (f, v)_{\Omega_h}$$

$$b((\phi, \psi, \widehat{\phi}, \widehat{\psi}_n), (v, q)) = (f, v)_{\Omega_h}$$

"$b(u, v) = l(v)$"

$$Au = f$$

$$(u, A^* v)_{\Omega_h} + \langle u, v \rangle_{\Gamma_h} = l(v)$$

$$(u, A^* v)_{\Omega_h} + \langle \widehat{u}, v \rangle_{\Gamma_h} = l(v)$$

$$b((u, \widehat{u}), v) = l(v)$$

"$b(u, v) = l(v)$"