Continuous Test Space

DPG Form

$$b(u_h, v) = l(v)$$

Optimal Test Functions

For each $u \in U_h$, find
$v_u \in V : (v_u, w)_V = b(u, w) \forall w \in V$

Discrete Test Space

DPG Form

$$b(u_h, v_h) = l(v_h)$$

Optimal Test Functions

For each $u \in U_h$, find
$v_u \in V_{p+\triangle p} : (v_u, w)_V = b(u, w)$
$\forall w \in V_{p+\triangle p}$

Stiffness Matrix

$$K_{ij} = b(e_i, v_{e_j}) = (v_{e_i}, v_{e_j})_V = (v_{e_j}, v_{e_i})_V = b(e_j, v_{e_i}) = K_{ji}$$

Optimality

$$||u - u_h||_E \leq ||u - w_h||_E \quad \forall w_h \in U_h$$

$$\left(||u||_E = \sup_{||v||_V = 1} b(u, v) = ||v_u||_V \right)$$