



PECOS

Predictive Engineering and Computational Sciences

Camellia

A Discontinuous Petrov-Galerkin Toolbox Using Trilinos

Nate Roberts

The University of Texas at Austin

October 13, 2011



Acknowledgments

Collaborators:

- Leszek Demkowicz (UT/PECOS)
- Denis Ridzal (Sandia/CSRI)
- Pavel Bochev (Sandia/CSRI)
- Jesse Chan (UT/PECOS)

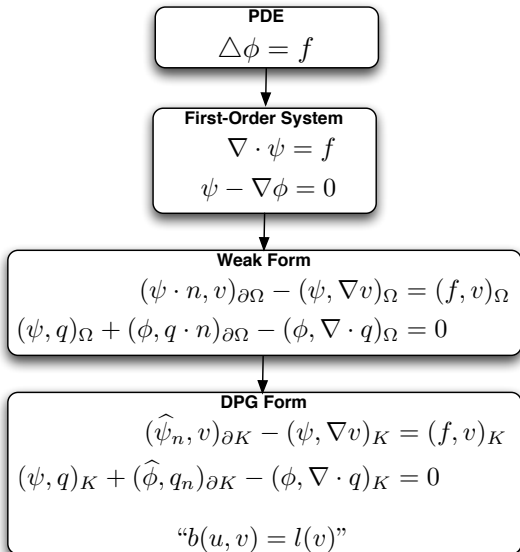
Funding:

- Sandia (summer internships)
- PECOS (GRA funding)
- ICES (CSEM Fellowship)

Outline

- ① DPG Intro
- ② Previous Work
- ③ Goals for Camellia
- ④ Verification Approach
- ⑤ Convergence Studies
- ⑥ Current and Future Work

From Strong-Form PDE to DPG Form



Solving with DPG

Continuous Test Space

DPG Form

$$b(u_h, v) = l(v)$$

Optimal Test Functions

For each $u \in U_h$, find
 $v_u \in V : (v_u, w)_V = b(u, w) \forall w \in V$

Discrete Test Space

DPG Form

$$b(u_h, v_h) = l(v_h)$$

Optimal Test Functions

For each $u \in U_h$, find
 $v_u \in V_{p+\Delta p} : (v_u, w)_V = b(u, w)$
 $\forall w \in V_{p+\Delta p}$

Stiffness Matrix

$$K_{ij} = b(e_i, v_{e_j}) = (v_{e_i}, v_{e_j})_V = (v_{e_j}, v_{e_i})_V = b(e_j, v_{e_i}) = K_{ji}$$

Optimality

$$\|u - u_h\|_E \leq \|u - w_h\|_E \\ \forall w_h \in U_h$$

$$\left(\|u\|_E = \sup_{\|v\|_V=1} b(u, v) = \|v_u\|_V \right)$$

Previous Work (Summer 2010)

Applied DPG to the Stokes problem, implementing atop Leszek Demkowicz's hpDPG code:

- Uniform quad mesh
- Manufactured solution following Cockburn et. al.:

$$u_1 = -e^x(y \cos y + \sin y)$$

$$u_2 = e^x y \sin y$$

$$p = 2\mu e^x \sin y$$

- Convergence studies with several inner products, including some results that compete well with Cockburn et. al.

Goals for Camellia

Immediate Goals

- Mesh
 - ▶ Uniform Quads (2D)
- Linear PDE support
- Arbitrary inner product
- Serial computation
- Built atop Trilinos

Ultimate Goals

- Mesh
 - ▶ Quads & triangles
 - ▶ hp-adaptivity
 - ▶ 3D?
- MPI
- Nonlinear PDE support

Verification Approach

Unit Tests

- Aspire to a test-first methodology.
- Test individual features using e.g. simple PDEs, and building up from there.
- 7095 lines of code for unit tests (compared to 10,888 for core code).

Convergence Studies

For Poisson and two Stokes formulations (VVP and VSP), run

- h-convergence studies (triangles and quads),
- “perverse” p-refinement pattern on 16×16 mesh, and
- “hybrid” mesh of quads and triangles.
- 2708 lines of code for convergence studies.

Convergence Study: Stokes VSP

Stokes VSP, u_1 (triangular mesh)						
Mesh Size	$k = 1$	rate	$k = 2$	rate	$k = 3$	rate
1×1	9.4e-1	-	2.6e-1	-	5.0e-2	-
2×2	3.1e-1	1.62	4.3e-2	2.63	4.0e-3	3.66
4×4	8.0e-2	1.94	5.8e-3	2.88	2.6e-4	3.95
8×8	2.0e-2	1.99	7.4e-4	2.97	1.6e-5	3.99
16×16	5.1e-3	2.00	9.3e-4	2.99	1.0e-6	4.00
32×32	1.3e-3	2.00	1.2e-5	3.00	6.4e-8	4.00

Table: L^2 error and h -convergence rates for u_1 , $k = 1, 2, 3$. We observe optimal convergence.

Variable Polynomial Orders Study

For this test, we used a 16×16 mesh and the following polynomial order pattern (repeated 4 times):

4	4	4	4	1	1	1	1	2	2	2	2	3	3	3	3
3	3	3	3	4	4	4	4	1	1	1	1	2	2	2	2
2	2	2	2	3	3	3	3	4	4	4	4	1	1	1	1
1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4

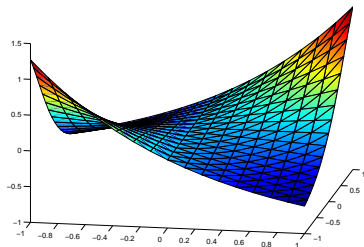
Variable Polynomial Orders Study, Poisson Results

Triangles					
ϕ		ψ_1		ψ_2	
$k = 1$	mixed k	$k = 1$	mixed k	$k = 1$	mixed k
2.0e-3	9.1e-4	3.4e-3	1.7e-3	2.4e-3	1.1e-3

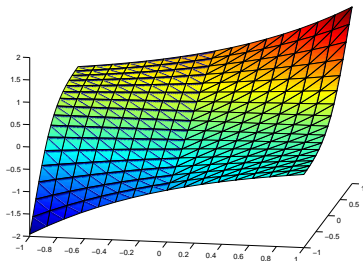
Quads					
ϕ		ψ_1		ψ_2	
$k = 1$	mixed k	$k = 1$	mixed k	$k = 1$	mixed k
1.0e-3	3.7e-4	2.3e-3	6.6e-4	2.9e-3	1.2e-3

Poisson Manufactured Solution, “Hybrid” Cubic Mesh

ϕ

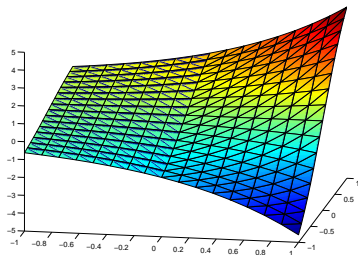


ψ_1

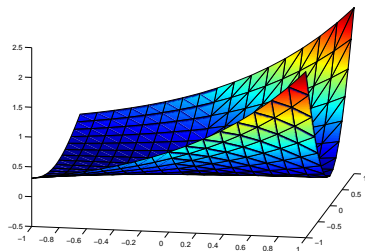


Stokes Manufactured Solution, “Hybrid” Cubic Mesh

Pressure



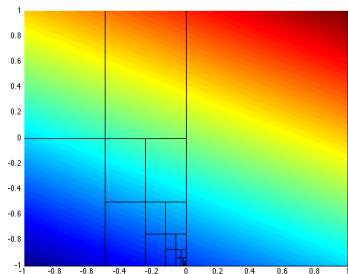
Velocity (y component)



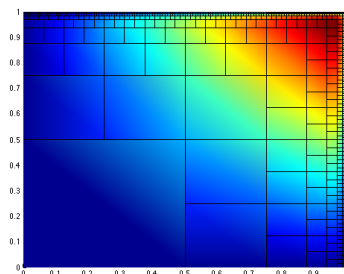
Current and Future Work

- h -refinements: now working, but not extensively tested
- adaptivity: coming soon
- Nonlinear PDE support
- MPI
- 3D?

Linear Poisson



Convection-Diffusion



Thank you!

Questions?