# Locally Conservative Discontinuous Petrov-Galerkin Finite Elements for Fluid Problems

Truman Ellis, Leszek Demkowicz, and Jesse Chan

Institute for Computational Engineering and Sciences,
The University of Texas at Austin,
Austin, TX 78712

**Abstract**

## 1 Intoduction

Verteeg and Malalasekera, in *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*[3, p. 110-113] cite three characteristics that they consider essential to any numerical discretization of convection-diffusion type problems: conservativeness, boundedness, and transportiveness.

Perot[2] argues

> Accuracy, stability, and consistency are the mathematical concepts that are typically used to analyze numerical methods for partial differential equations (PDEs). These important tools quantify how well the mathematics of a PDE is represented, but they fail to say anything about how well the physics of the system is represented by a particular numerical method. In practice, physical fidelity of a numerical solution can be just as important (perhaps even more important to a physicist) as these more traditional mathematical concepts. A numerical solution that violates the underlying physics (destroying mass or entropy, for example) is in many respects just as flawed as an unstable solution.

The discontinuous Petrov-Galerkin finite element method has been described as least squares finite elements with a twist. The key difference is that least square methods seek to minimize the residual of the solution in some Hilbert space norm, while DPG seeks the minimization in a dual norm through the inverse Riesz map. Exact mass conservation has been an issue that has plagued least squares finite elements for a long time. Several approaches have been used to try to adress this. Chang and Nelson[1] developed the 'restricted LSFEM'[1] be augmenting the least squares equations with a Lagrange multiplier explicitly enforcing mass conservation element-wise. Our conservative formulation of DPG takes a similar approach and both methods share similar negative of transforming a minimization method to a saddle-point problem.

The discontinuous Petrov-Galerkin finite element method has shown a lot of promise for convection-diffusion type problems including robustness in the face of singularly perturbed problems.

## References

[1] C. L. Chang and John J. Nelson. Least-squares finite element method for the stokes problem with zero residual of mass conservation. *SIAM J. Num. Anal.*, 34:480–489, 1997.

[2] J. B. Perot. Discrete conservation properties of unstructured mesh schemes. *Annual Review of Fluid Mechanics*, 43:299–318, 2011.

[3] H.K. Versteeg and W. Malalasekera. *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*. Prentice Hall, 2007.