

I is given by:

$$I = P_1 \cdot I + P_2 \cdot (\cos 2i_1 \cdot Q - \sin 2i_1 \cdot U)$$

since $I = 1$, this means

$$I = P_1 + P_2 \cdot (\cos 2i_1 \cdot Q - \sin 2i_1 \cdot U) \equiv P_1 + P_2 \cdot \gamma$$

where

$$\gamma = \cos 2i_1 \cdot Q - \sin 2i_1 \cdot U$$

P_1 and P_2 are functions of (μ, ν) where $\mu = \cos \theta$. So for a given (μ, ν) , P_1 and P_2 are constants. In this case, I is maximized when $P_2 \cdot \gamma$ is maximized. If $P_2 < 0$, this occurs when γ is minimized, and if $P_2 > 0$, this occurs when γ is maximized. So the question is, what is the range of values that γ can take?

The variables inside γ are correlated: $\sin 2i_1 = \sqrt{1 - \cos^2 2i_1}$ and $|U| < \sqrt{1 - Q^2}$.

I wrote a small program that chooses a random $\cos 2i_1$ between -1 and 1, and calculates the corresponding $\sin 2i_1$. Then a random Q is sampled between -1 and 1, and U is randomly sampled between $-\sqrt{1 - Q^2}$ and $\sqrt{1 - Q^2}$:

```
program gamma_range

  implicit none
  integer,parameter :: p = 8
  real(p) :: cosx,sinx,q,u
  real(p) :: gamma,gamma_min=0._p,gamma_max=0._p
  integer :: iter

  do iter=1,100000000
    call random_number(cosx) ; cosx = cosx * 2._p - 1._p
    sinx = sqrt(1._p-cosx*cosx)
    call random_number(q) ; q = q*2._p-1._p
    call random_number(u) ; u = (u*2._p-1._p) * sqrt(1._p-q*q)
    gamma = cosx*q-sinx*u
    if(gamma > gamma_max) gamma_max = gamma
    if(gamma < gamma_min) gamma_min = gamma
  end do

  print *, 'Min = ', gamma_min
  print *, 'Max = ', gamma_max

end program gamma_range
```

Which produces the following results

```
$ ./a.out
Min = -0.999999580188680
Max = 0.999992504333863
```

Therefore it seems that $\gamma \in [-1 : 1]$. If $P_2 < 0$, the maximum value of $P_2 \cdot \gamma$ is $-P_2$, and if $P_2 > 0$, the maximum value of $P_2 \cdot \gamma$ is P_2 . So the maximum value of $P_2 \cdot \gamma$ is $|P_2|$. This means that the maximum value of I is

$$I = P_1 + |P_2|$$

Since $P_1 + |P_2|$ is still a function of μ , this means that at a given frequency, the ‘ceiling’ value used in the rejection criterion is the maximum value of $P_1 + |P_2|$ at that frequency. It should therefore be possible to pre-compute the maximum $P_1 + |P_2|$ at each frequency. For the scattering of a photon of a frequency ν , the maximum I for the rejection criterion could then be found from simple interpolation.