Dust sublimation radius

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The factor $W(r)$ describes the probability that a photon, emitted at r in a random direction, is intercepted by the star, which has radius $r_\star$ at the wavelength considered. The factor 1-W(r) is the probability that a photon emitted at r escapes. $W(r)$ is the fraction of the solid angle that is covered by the star, so

$$W(r) = \frac{1}{2}\left(1 - \sqrt{1 - (r_\star/r)^2}\right)$$

This factor is called the *geometrical dilution factor*. Photons emitted just above $r_\star$ have a 50% chance of being re-absorbed by the star.

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The radiative equilibrium condition for determining the temperature T_d of a grain is

$$\int_0^\infty \kappa_\nu \, B_\nu(T_d) \, d\nu = \int_0^\infty \kappa_\nu \, J_\nu \, d\nu$$

The left hand side of this equation is the radiative cooling of a grain because the product $\kappa_\nu \, B_\nu(T_d)$ is the thermal emissivity at ν of a grain at temperature T_d. The right hand side is the radiative heating owing to the opacity κ_ν and the ambient radiation field. The quantity J_ν is the monochromatic mean intensity of the radiation field incident on the grain, and it is the angle average mean of the intensity i_ν. For the optically thin case in which the intensity incident on the grain is direct star light with a uniform intensity $I_\nu = F_\nu$ the mean intensity is given by

$$J_\nu = W(r) F_\nu$$

The result is that

$$\sigma \, T_d^4 \, \kappa_{\rm plank}(T_d) = \frac{L_\star}{4\pi r^2} \, W(r) \, \kappa_{\rm plank,atmos}$$

i.e.

$$T_d^4\,\kappa_{\rm plank}(T_d) = \frac{L_\star}{4\pi r_\star^2\sigma}\,W(r)\,\kappa_{\rm plank,atmos}$$

Note that for a blackbody spectrum, the fraction on the RHS reduces to $T_\star^4$. Re-arranging for $W(r)$ gives

$$W(r) = \frac{T_d^4}{L_\star/4\pi r_\star^2\sigma}\frac{\kappa_{\rm plank}(T_d)}{\kappa_{\rm plank,atmos}}$$

Expanding gives

$$\frac{1}{2}\left(1-\sqrt{1-(r_\star/r)^2}\right) = \frac{T_d^4}{L_\star/4\pi r_\star^2\sigma}\frac{\kappa_{\rm plank}(T_d)}{\kappa_{\rm plank,atmos}}$$

$$1-\sqrt{1-(r_\star/r)^2} = 2\,T_d^4\left(\frac{4\pi r_\star^2\sigma}{L_\star}\right)\frac{\kappa_{\rm plank}(T_d)}{\kappa_{\rm plank,atmos}}$$

$$\sqrt{1-(r_\star/r)^2} = 1-2\,T_d^4\left(\frac{4\pi r_\star^2\sigma}{L_\star}\right)\frac{\kappa_{\rm plank}(T_d)}{\kappa_{\rm plank,atmos}}$$

$$1-(r_\star/r)^2 = \left[1-2\,T_d^4\left(\frac{4\pi r_\star^2\sigma}{L_\star}\right)\frac{\kappa_{\rm plank}(T_d)}{\kappa_{\rm plank,atmos}}\right]^2$$

$$(r_\star/r)^2 = 1-\left[1-2\,T_d^4\left(\frac{4\pi r_\star^2\sigma}{L_\star}\right)\frac{\kappa_{\rm plank}(T_d)}{\kappa_{\rm plank,atmos}}\right]^2$$

$$r_\star/r = \left\{1-\left[1-2\,T_d^4\left(\frac{4\pi r_\star^2\sigma}{L_\star}\right)\frac{\kappa_{\rm plank}(T_d)}{\kappa_{\rm plank,atmos}}\right]^2\right\}^{1/2}$$

$$r = r_\star\left\{1-\left[1-2\,T_d^4\left(\frac{4\pi r_\star^2\sigma}{L_\star}\right)\frac{\kappa_{\rm plank}(T_d)}{\kappa_{\rm plank,atmos}}\right]^2\right\}^{-1/2}$$

In some cases, the term inside the brackets is $\ll 1$ in which case to maintain floating point accuracy we should carry out the binomial expansion. Writing

$$x \equiv T_d^4\left(\frac{4\pi r_\star^2\sigma}{L_\star}\right)\frac{\kappa_{\rm plank}(T_d)}{\kappa_{\rm plank,atmos}}$$

we have

$$r = r_\star \left\{ 1 - [1 - 2x]^2 \right\}^{-1/2}$$

For $x \ll 1$ we can write

$$r \approx r_\star \left\{ 1 - [1 - 4x] \right\}^{-1/2} = \frac{r_\star}{2\sqrt{x}}$$