CARLETON UNIVERSITY
Department of Mechanical & Aerospace Engineering

MAAE 3202 Mechanics of Solids II

Experiment D Castigliano's Theorem

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1. OBJECTIVES

To measure the deflections of curved bars in bending and to compare these results with those predicted by Castigliano's theorem.

2. THEORY

Energy methods are frequently used to analyze the deflections of beams and structures. Of the many available methods, the application of Castigliano's theorem is one of the most widely used. The following is a statement of the theorem:

"If the strain energy of a linearly elastic structure is expressed in terms of the system of external loads, the partial derivative of the strain energy with respect to a concentrated external load is the deflection of the structure at the point of application and in the direction of the load."

Mathematically, the theorem may then be stated as:

If $$U = U(P_i)$$

$$\Delta_i = \frac{\partial U}{\partial P_i}$$

where
- U is the strain energy of the elastic structure;
- P_i is the i-th discrete applied force or moment;
- Δ_i is the deflection (translational if P_i is a force, rotational if P_i is moment) in the same direction and at the point where the load is applied.

If the deflection is required either at a point where there is no unique point load or in a direction not aligned with the applied load, a dummy or fictitious load may be introduced at the desired point acting in the proper direction. The deflection is obtained by first differentiating U with respect to the dummy load and then taking the limit as the dummy load approaches zero.

3. APPARATUS

(a) Curved steel bars
(b) DIDACTEC Deflection of Curved Bars Apparatus, as shown in Figure 1
(c) Weights

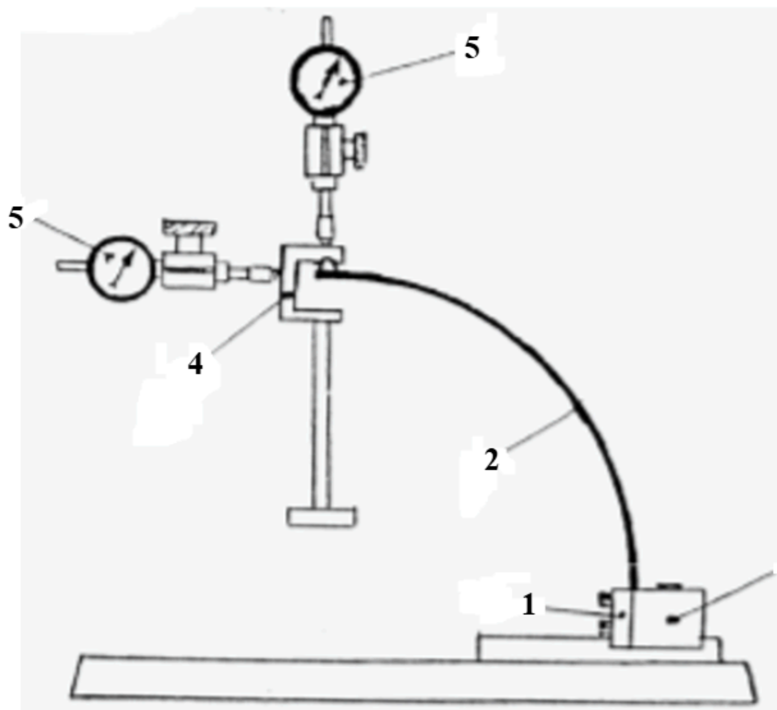


Figure 1 DIDACTEC Deflection of Curved Bars Apparatus

4. PROCEDURE

The items indicated in the procedure below are with reference to Figure 1.

(a) Release clamp (1) and place specimen (2) in position. Clamp the specimen. Release block (3) and re-position if necessary to suit selected specimen. Lock in required position.
(b) Place special load hanger (4) on the specimen.
(c) Position dial gauge supports so that the dial gauges (5) make contact with the load hanger (4) as shown. Set the dial gauges to zero.

(d) Add weights to the load hanger (4) incrementally, and for each load determine the deflections as measured by the gauges.

5. REQUIREMENTS

(a) Plot graphs of the measured vertical deflection *v*, and horizontal deflection *u*, versus the vertical end load *W*.

(b) Calculate the theoretical deflection of the curved bar tested using Castigliano's theorem for each load condition. Compare them with the experimentally determined values. Discuss the reasons for any discrepancies between the two sets of results.

(c) Discuss the significance of Castigliano's theorem. What are the limitations of this method? Why is a dummy load required?

<u>General case</u>

Assuming uniform cross-section of the curved bar in Figure 2, then the horizontal and vertical deflections of the loaded end can be calculated, as detailed in the course notes.

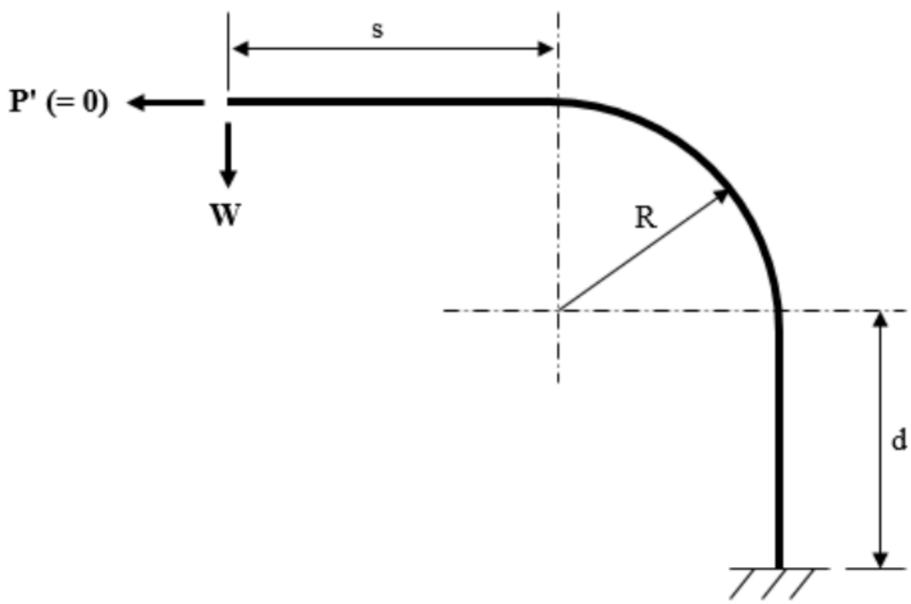


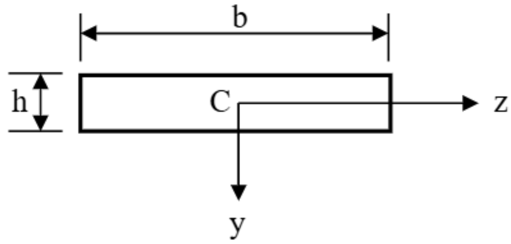
Figure 2 A curved bar under bending

$$\Delta_W = \frac{Ws^3}{3EI} + \frac{WR}{EI}\left(\frac{\pi s^2}{2} + \frac{\pi R^2}{4} + 2sR\right) + \frac{Wd}{EI}(s^2 + 2sR + R^2)$$

$$\Delta_{P\prime} = \frac{WR^2}{EI}\left[s\left(\frac{\pi}{2} - 1\right) + \frac{R}{2}\right] + \frac{Wd}{EI}\left(sR + R^2 + \frac{sd}{2} + \frac{Rd}{2}\right)$$

Specimens

Material: Steel with E = 207 GPa and cross-section dimensions b = 25.4 mm and h = 3.2 mm



Specimen (Number)	Dimension s (mm)	Dimension d (mm)	Dimension R (mm)
1	75	75	75
2	0	0	150
3	0	75	75
4	150	150	0

Hanger: 0.16 kg