

**ANTENNA ENGINEERING**  
**Mid Term Examination, 26<sup>th</sup> February 2019**

**Max. Marks: 30**

**Duration: 90 minutes**

NOTE: ALL QUESTIONS CARRY EQUAL MARKS. Attempt all questions. Do not answer the same question more than once. Start the answer to a new question on a new page. You are allowed to bring in one A4 size paper sheet (with anything written on it, that you think may help you in exam).

1. Consider a Hertzian dipole antenna carrying current  $I \cos(\omega t)$  Amperes where  $I$  is the peak value of current in Amperes,  $\omega$  is the angular frequency of the signal in radians/second, and  $t$  is time in seconds. Allow the antenna to be situated at the origin of a spherical coordinate system. Also allow the antenna to be aligned with the  $z$ -axis of the coordinate system used. Rigorously derive the mathematical expressions for the radiated field components in near-field and in far-field. Show that the near-field's contribution to total radiated power is nil.
2. Building on the results obtained in 1, rigorously derive the formula for the radiation resistance of an electrically-short monopole antenna.
3. Consider a thin 3-meter long copper wire. The wire is hung horizontally at a distance of 10-meter from the surface of the earth and is used as a center-fed dipole antenna at an operating frequency = 50 MHz. Calculate the approximate numerical value of the radiated electric-field strength at a point P situated 100-m away from the surface of the earth along an imaginary vertical straight line passing through the antenna center. Assume that the peak value of antenna current is 100 mA. Neglect earth's electromagnetic effects on the behavior of the antenna.
4. For the antenna considered in 3, calculate the effective length, the effective area, the radiation resistance, and the gain.
5. A. Consider two isotropic antennas situated distance  $d$  apart where  $d$  is measured in Km. If the frequency of operation is  $f$  where  $f$  is measured in MHz, show that the path loss (FSPL) between these antennas is approximately given by the equation  $FSPL \text{ (dB)} = 20 \log(d) + 20 \log(f) + 32.44$ .  
B. Calculate the gain of a 30-feet diameter parabolic dish antenna at 4 GHz operating frequency. Assume that the aperture efficiency is 60%. Note that 1 foot = 12 inch, 1 inch = 2.54 cm, and 1 meter = 100 cm.
6. A dipole antenna of length 1.5 cm and diameter 2 mm is made of copper wire (conductivity 57 MS/m) and is operating at 2 GHz frequency. Sketch the radiation pattern and calculate the directivity. Also calculate the input impedance, the radiation resistance, and the radiation efficiency.



## APPENDIX B VECTOR ANALYSIS

### Coordinate Transformations

Rectangular to cylindrical:

	$\hat{x}$	$\hat{y}$	$\hat{z}$
$\hat{r}$	$\cos \phi$	$\sin \phi$	0
$\hat{\phi}$	$-\sin \phi$	$\cos \phi$	0
$\hat{z}$	0	0	1

Rectangular to spherical:

	$\hat{r}$	$\hat{\theta}$	$\hat{\phi}$
$\hat{r}$	$\sin \theta \cos \phi$	$\sin \theta \sin \phi$	$\cos \theta$
$\hat{\theta}$	$\cos \theta \cos \phi$	$\cos \theta \sin \phi$	$-\sin \theta$
$\hat{\phi}$	$-\sin \phi$	$\cos \phi$	0

Cylindrical to spherical:

	$\hat{r}$	$\hat{\theta}$	$\hat{\phi}$
$\hat{r}$	$\sin \theta$	0	$\cos \theta$
$\hat{\theta}$	$\cos \theta$	0	$-\sin \theta$
$\hat{\phi}$	0	1	0

These tables can be used to transform unit vectors as well as vector components; e.g.,

$$\hat{r} = \hat{x} \cos \phi + \hat{y} \sin \phi$$

$$A_r = A_x \cos \phi + A_y \sin \phi$$

### Vector Differential Operators

Rectangular coordinates:

$$\nabla f = \hat{x} \frac{\partial f}{\partial x} + \hat{y} \frac{\partial f}{\partial y} + \hat{z} \frac{\partial f}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \hat{x} \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{y} \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{z} \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 \vec{A} = \hat{x} \nabla^2 A_x + \hat{y} \nabla^2 A_y + \hat{z} \nabla^2 A_z$$

Cylindrical coordinates:

$$\nabla f = \hat{r} \frac{\partial f}{\partial \rho} + \hat{\phi} \frac{1}{\rho} \frac{\partial f}{\partial \phi} + \hat{z} \frac{\partial f}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \hat{r} \left( \frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{\phi} \left( \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) + \hat{z} \left[ \frac{\partial (\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi} \right]$$

$$\nabla^2 f = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 \vec{A} = \nabla (\nabla \cdot \vec{A}) - \nabla \times \nabla \times \vec{A}$$

Spherical coordinates:

$$\nabla f = \hat{r} \frac{\partial f}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial f}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (A_\phi \sin \theta) - \frac{\partial A_\phi}{\partial \phi} \right] + \frac{\partial}{\partial r} \left[ \frac{1}{\sin \theta} \frac{\partial A_\theta}{\partial \phi} - \frac{\partial (r A_\phi)}{\partial r} \right]$$

$$+ \frac{\partial}{\partial r} \left[ \frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right]$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$$

$$\nabla^2 \vec{A} = \nabla (\nabla \cdot \vec{A}) - \nabla \times \nabla \times \vec{A}$$