The LNM Institute of Information Technology
Jaipur, Rajsthan

Math-II (2014-15), Quiz-2: Section-A

Name: Roll No:

Time: 15 Minutes Maximum Marks: 10

Q1. if $\{u_1, u_2, \cdots, u_n\}$ is an orthonormal basis of an inner product space V, then show that for any $v \in V$ can be written as $v = \sum_{i=1}^{n} < v, u_i > u_i$. [5]

Sol. Since $\{u_1, u_2, \cdots, u_n\}$ is an orthonormal basis of the inner product space V, any $v \in V$ can be written as $v = \sum_{i=1}^{n} \alpha_i u_i$.

Now by taking inner product with u_j for $j = 1, \cdots, n$, we have

$$< v, u_j >=< \sum_{i=1}^{n} \alpha_i u_i, u_j >= \sum_{i=1}^{n} \alpha_i < u_i, u_j >= \alpha_j < u_j, u_j >= \alpha_j.$$

Substituting the value of α_i, we obtain $v = \sum_{i=1}^{n} < v, u_i > u_i$.

Q2 Prove that similar matrices have the same eigen values. [5]

Sol. Two matrices A and B are similar if there exists an invertible matrix P such that $B = P^{-1}AP$.

$B - \lambda I = P^{-1}AP - \lambda P^{-1}P = P^{-1}(AP - \lambda P) = P^{-1}(A - \lambda I)P$.

By taking determinant, we get

$\det(B - \lambda I) = \det(P^{-1}) \det(A - \lambda I) \det(P) = \det(A - \lambda I)$

Therefore, the characteristic polynomial of A and B are same and the similar matrices have the same eigen values.