

The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-II
Assignment 7

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16UCS086

1. Reduce the following second order differential equations to system of first order differential equations and hence solve.

(i) $xy'' + y' = y'^2$ (ii) $yy'' - y'^2 = 0$ (iii) $yy'' + y'^2 + 1 = 0$ (iv) $y'' - 2y' \coth x = 0$

2. Find general solution of the following differential equations given a known solution y_1 :

(i) $x(1-x)y'' + 2(1-2x)y' - 2y = 0$ $y_1 = \frac{1}{x}$
(ii) $(1-x^2)y'' - 2xy' + 2y = 0$ $y_1 = x$

3. Verify that $\sin x/\sqrt{x}$ is a solution of $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0$ over any interval on the positive x -axis and hence find its general solution.

Note: This ODE is the special case of Bessels equation corresponding to $p = 1/2$.

4. Solve the following differential equations:

(i) $y'' - 4y' + 3y = 0$ (ii) $y'' + 2y' + (\omega^2 + 1)y = 0$, ω is real.

5. Solve the following initial value problems:

(i) $y'' + 4y' + 4y = 0$ $y(0) = 1, y'(0) = -1$
(ii) $y'' - 2y' - 3y = 0$ $y(0) = 1, y'(0) = 3$

6. The equation

$$x^2 \frac{d^2y}{dx^2} + ax \frac{dy}{dx} + by = 0.$$

where a, b are constants, is called the Euler-Cauchy equation. Show that under the transformation $x = e^t$ for the independent variable, the above reduces to

$$\frac{d^2y}{dt^2} + (a-1) \frac{dy}{dt} + by = 0,$$

which is an equation with constant coefficients. Hence solve:

(i) $x^2y'' + 2xy' - 12y = 0$ (ii) $x^2y'' + xy' + y = 0$ (iii) $x^2y'' - xy' + y = 0$

7. By using the method of variation of parameters, find the general solution of:

(i) $y'' + 4y = 2\cos^2 x + 10e^x$ (ii) $y'' + y = x \sin x$
(iii) $y'' + y = 1 + \sin x$ (iv) $xy'' - y' = x^2(3+x)e^x$

Supplementary problems from "Advanced Engg. Maths. by E. Kreyszig (8th Edn.)

- (a) Page 75 - 76, Q.10,17,27,28 (b) Page 80, Q.13,17,19,20
(c) Page 96, Q.2,6,7 (d) Page 100, Q.12,16
(e) Page 104, Q.16 (f) Page 131 - 132, Q.4,11,18,20
(g) Page 137, Q.14,16,20