

The LNM Institute of Information Technology, Jaipur
Mathematics-II
Mid Term
February 21, 2018

Duration: 90 mins.

Max.Marks: 30

Name: _____ Roll No.: _____

1. (a) For which values of g, h and k the following system of linear equations is consistent?

$$x - 4y + 7z = g$$

$$3y - 5z = h$$

$$-2x + 5y - 9z = k$$

[3]

- (b) Prove that any square matrix A can be written as the sum of a symmetric matrix and a skew-symmetric matrix. [2]

2. (a) Let $V = C[0, 1]$. Prove or disprove: $S = \{f \in V : f(\frac{1}{2}) = 0\}$ is a subspace of V . [2]

- (b) Let V be a vector space over a field F . Let U and W be two subspaces of V . Prove that $U \cap W$ is also a subspace of V . [3]

3. (a) What is the span of $\{1\}$ in the vector space \mathbb{C} over the field \mathbb{C} ? What is the span of $\{1\}$ in the vector space \mathbb{C} over the field \mathbb{R} . [2]

- (b) Write the span of $\{(1, 0, 0), (0, -1, 0)\}$ in \mathbb{R}^3 . Explain geometrically. [2]

4. (a) Show that $B = \{(1, 2, 0), (1, 3, 2), (0, 1, 3)\}$ forms a basis for \mathbb{R}^3 . [3]

- (b) Determine the coordinate vector of $v = (2, 7, -4)$ with respect to the ordered basis B given in Part (a). [3]

5. (a) Suppose $\{u_1, u_2\}$ is an orthogonal set in an inner product space V . Show that $\|u_1 + u_2\|^2 = \|u_1\|^2 + \|u_2\|^2$. [2]

- (b) Use Gram-Schmidt process to transform the basis $\{(1, 0, 1), (1, 0, -1), (0, 3, 4)\}$ for \mathbb{R}^3 into an orthogonal basis with the standard inner product. [3]

6. (a) Find all value/s of $k \in \mathbb{R}$ for which $A = \begin{pmatrix} 1 & k & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is diagonalizable. [3]

- (b) Let A be a square matrix. Prove that A and A^T have same set of eigen values. [2]