# M3 Midsem

## Year 2020-21 – Y19 Batch

1st Oct 2020 | Max Marks : 30 | Duration : 90 Mins

In Online Mode Via Moodle

## Q1

Question 1
Not answered
Marked out of 1.00
Flag question

(a) Show that for any two complex numbers $z_1$ and $z_2$, $|z_1 + z_2| \geq ||z_1| - |z_2||$. [2]

(b) Show that the function $f(z) = \sqrt{|xy|}$ is not differentiable at the $(0, 0)$, but the CR-Equations are satisfied at that point. [3]

(c) Let a function $f$ be analytic everywhere in a domain $D$. Prove that if $f(z)$ is real valued for all $z$ in $D$, then $f(z)$ must be constant throughout $D$. [2]

## Q2

Question 2
Not answered
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(a) For any nonzero $z$, define $f(z) = ln\ r + e^{i\theta}$, where $z = re^{i\theta}$ and $\frac{\pi}{4} < \theta \leq \frac{9\pi}{4}$. Show that the function is discontinuous on the ray $\theta = \frac{\pi}{4}$. [3]

(b) Evaluate $(-1)^{\frac{1}{\pi}}$. [2]

(c) If $f(z) = zIm(z)$, then determine the set of points where $f'(z)$ exists, and find the value of $f'(z)$ at all such points. [3]

## Q3

Question 3
Not answered
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(a) Evaluate the following integral: $\int_C \exp(z^5)\, dz$, where $C$ is any simple closed contour in the counterclockwise direction. [2]

(b) Using generalised Cauchy integral formula, evaluate $\int_C \frac{\exp(3z)}{z^3} dz$, where $C$ is the positively oriented unit circle $|z| = 1$. [3]

(c) Prove / Disprove : $\sin 3z$ is bounded function in $\mathbb{C}$. [2]

## Q4

Question 4
Not answered
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(a) Assume that $f(z)$ is entire and $v(x,\ y) = \text{Im}\,[f(z)]$ has an upper bound $v_0$ in the $xy$ plane. Show that $v(x,\ y)$ must be constant throughout the plane. [3]

(b) Consider the function $f(z) = z^2 \sin\left(\frac{1}{z^2}\right)$.

(i) Find the Laurent series representation of the function $f(z)$ in the domain $0 < |z| < \infty$. Hence find the residue of the function $f$ at $z = 0$. [3]

(ii) Using the residue, evaluate the following integral $\int_C f(z)\, dz$, where $C$ is the circle $|z| = 3$ in the positive sense. [2]