The LNM Institute of Information Technology
Jaipur, Rajsthan

Math-II (2014-15), Quiz-I: Section-A

Name: Roll No:

Time: 15 Minutes Maximum Marks: 10

Q1. Consider the following linear system of equations

$$\begin{aligned} x - y + 2z &= 1 \\ 2x + 2z &= 1 \\ x - 3y + 4z &= 2. \end{aligned}$$

Is this system consistent ? Explain the reason. If so, describe explicitly all solutions. [05 Marks]

Sol. The coefficient matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 2 \\ 1 & -3 & 4 \end{bmatrix}$

The row reduced echelon form of A is given by $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$. So row rank of A is 2.

Row reduced echelon form of the augmented matrix $[A\ b] = \begin{bmatrix} 1 & -1 & 2 & 1 \\ 2 & 0 & 2 & 1 \\ 1 & -3 & 4 & 2 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 1 & 1/2 \\ 0 & 1 & -1 & -1/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. So the row rank of $[A\ b]$ is 2.

Since, row rank of A = row rank of $[A\ b] = 2 <$ number of unknowns (4), the given system has many solutions and the system is consistent.

First and second columns of row reduced matrix are leading columns and hence x, y are basic variables and z is free variable. So by taking $z = s \in \mathbb{R}$, solutions of the system are given by the following set:

$$\{(1/2 - s, s - 1/2, s)^t : s \in \mathbb{R}\}$$

Q2. Find the null space and a basis for the null space of $\begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & 2 & 4 & 2 \\ 3 & 1 & 4 & 1 \end{bmatrix}$. [05 Marks]

Sol. Null space $N(A)$ of $A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & 2 & 4 & 2 \\ 3 & 1 & 4 & 1 \end{bmatrix}$ is defined as

$$N(A) = \{\mathbf{x} \in \mathbb{R}^4 : A\mathbf{x} = \mathbf{0}\}$$

The row reduced echelon form of A is $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

First and second columns of row reduced matrix are leading columns and hence x_1, x_2 are basic variables and x_3, x_4 are free variable. By choosing arbitrary values for x_3 and x_4, we get the solution space (Null space of A) as

$$N(A) = \{(-s, -s-t, s, t)^t \in \mathbb{R}^4 \colon s, t \in \mathbb{R}\}$$

Since $N(A)$ can be written as $\begin{bmatrix} -s \\ -s-t \\ s \\ t \end{bmatrix} = s \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$, the column vectors

$$\{(-1, -1, 1, 0)^t, (0, -1, 0, 1)^t\}$$

spans $N(A)$ and form a basis for $N(A)$.