The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-II
Assignment 6

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1. (*i*) If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is homogeneous, then $1/(Mx+Ny)$ is an integrating factor unless $Mx + Ny \equiv 0$, (*ii*) if the differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact but is of the form $f_1(xy)ydx + f_2(xy)xdy = 0$, then $1/(Mx - Ny)$ is an integrating factor unless $Mx - Ny \equiv 0$. Using it, solve the following differential equations:

   (a) $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$ (b) $x^2ydx - (x^3 - y^3)dy = 0$ (b) $y(1 + xy)dx + x(1 - xy)dy = 0$

2. Solve the following deferential equations:

   (a) $(x + 2y^3)\frac{dy}{dx} = y$ (b) $(1 + y^2) + (x - e^{-\tan^{-1} y})\frac{dy}{dx} = 0$

3. Reduce to linear deferential equations:

   (a) $x\frac{dy}{dx} + y\log y = xye^x$ (b) $\frac{dy}{dx} + \frac{1}{x}\sin 2y = x^2\cos^2 y$ (c) $(xy^2 + e^{-\frac{1}{x^3}})dx - x^2ydy = 0$

4. Find the orthogonal trajectories of the following families of curves:

   (a) $y = ax^2$ (b) $x^2 + y^2 = 2ax$.

5. Find the orthogonal trajectories of the parabolas $r = \frac{2c}{(1+\cos\theta)}$, where c is a parameter.

6. Prove that the orthogonal trajectories of $r^n \cos n\theta = c^n$ is $r^n \sin n\theta = c^n$.

7. Find the family of oblique trajectories which intersect the family of hyperbola $xy = c$ at an angle of 45°.
   Note: An oblique trajectory is a curve that intersect each member of a given family of curve at a constant angle $\alpha \neq 90°$.

8. Study the existence of solutions of the initial value problem

$$xy' = \frac{3}{x^3}, \qquad y(1) = -1$$

9. Study the existence of solutions of the initial value problem

$$y' = \sqrt{|y|}, \qquad y(0) = 0$$

10. Show that $xy' = 4y$, $y(0) = 1$ has no solution. Does this contradict existence theorem.

11. Find all initial conditions such that the initial value problem $(x^2 - 2x)y' = 2(x-1)y$, $y(x_0) = y_0$ has (a) no solution (b) Infinitely many solutions (c) Unique solution.

12. Find the solution of the initial value problem $y' = 2y - x$, $y(0) = 1$, using the Picard's iteration method. Compare with the exact solution.