The LNM Institute of Information Technology

CSE6011 Mathematical Structures for Engineers

Midterm Exam

September 3^3, 2017 | Time: $2.3^2.5$ min | Total Marks: 2.3^3

This is an **open notes exam**. Only **handwritten notebooks are allowed**. No printed or Xerox materials allowed in the exam. Notebooks cannot be shared. All the best!

More importantly, **no doubt clearance during the exam!!**

1. Let $A = \{\phi\}$. Let $B = 2^{2^A}$. Answer the following questions: (6)
   (a) Is $\phi \in B$? $\phi \subseteq B$?
   (b) Is $\{\phi\} \in B$? $\{\phi\} \subseteq B$?
   (c) Is $\{\{\phi\}\} \in B$? $\{\{\phi\}\} \subseteq B$?
2. Say True or False for the following and justify your answers:
   (a) The set of all real numbers under the usual multiplication operation is not a group. (2)
   (b) If G is a group such that $a^2 = e$ for all $a \in G$, then G is abelian. (2)
   (c) The set of integers $\mathcal{Z}$ with the binary operation $\cdot$ defined as $a \cdot b = a + b + 1$ for $a, b \in \mathcal{Z}$ is a group. The identity element of this group is 1. (2)
   (d) For the same problem as above the inverse of a is $a - 2$. (2)
   (e) Every ring is a field. (2)
3. If G is a finite group, show that there exists a positive integer N such that $a^N = e$ for all $a \in G$. Here a is said to be of order N. (3)
4. Let a, b be two elements of a group G. Assume that a has order 5 and $a^3b = ba^3$. Prove that $ab = ba$. (3)
5. Prove that if N is a normal subgroup of G that contains a Sylow p-subgroup of G, then the number of Sylow p-subgroups of N is the same as that of G. (3)
6. Let G be a group of order 340. What is the nature of subgroups of G? Is there any relations between the subgroups? Prove that G has a normal and a cyclic subgroup of order 85. (4)
7. The LNMIIT has 2000 students enrolled for various UG programs in 6 departments (assume that the number of students in the departments are almost the same, total number of girl students is 20% and there are no siblings). A Quiz master wants to conduct a quiz competition to all the students by forming various groups. His plan is to conduct quiz for each group separately one by one. For forming groups he likes to partition the students set into groups of almost of same size but at the same time he also does not want to conduct the quiz many times (means he does not want more groups also). So he defines a relation on the set of all students in three ways: