SOLUTION.

Q.1. (a)

$$x(t) = 10 \sin 5t \cdot \cos 10t = 5\left[\sin 15t - \sin 5t\right].$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\, dt .$$

$$= \lim_{T\to\infty} \frac{25}{T}\int_{-T/2}^{T/2} \left(\sin^2 15t + \sin^2 5t + 2\sin 15t \cdot \sin 5t\right) dt$$

$$= \lim_{T\to\infty} \frac{25}{T}\left[t\right]_{-T/2}^{T/2}$$

$$= 25. \quad 1$$

(b)

$$\int_{-\infty}^{\infty} \delta(t-2)\, \underbrace{\sin(\pi t)}_{x(t)}\, dt = \int_{-\infty}^{\infty} \delta(t-2)\cdot x(t)\, dt .$$

$$= x(2) = \sin \pi t \Big|_{t=2}$$

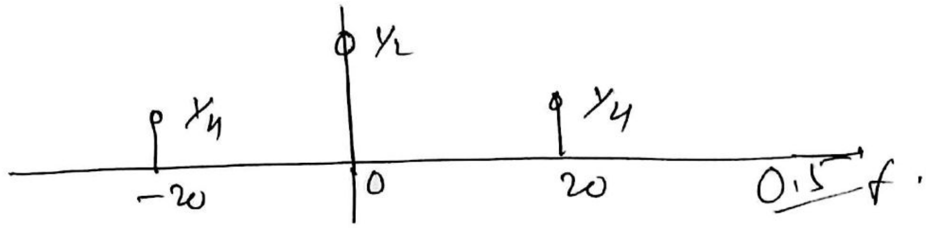
$$= 0. \quad 1$$

(c)

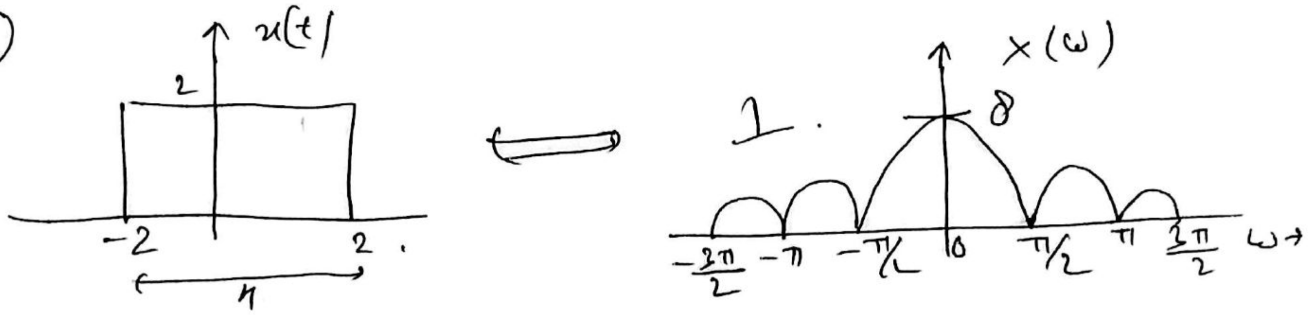
$$\left[\cos(\underbrace{20\pi t}_{A})\right]^2 = (\cos A)^2 = \frac{1}{2}\left[e^{JA} + e^{-JA}\right]^2$$

$$= \frac{1}{4}\left[e^{2JA} + e^{-2JA} + 2\right].$$

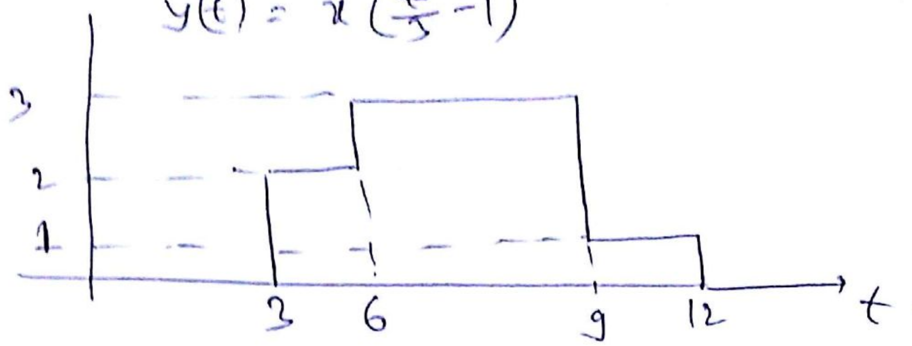
$$= \frac{1}{4}\left[e^{J2\pi(20)t} + e^{J2\pi(-20)t}\right] + \frac{1}{2} \quad \underline{0.5}$$



(d)



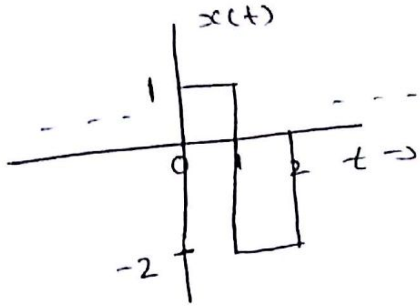
(e) $y(t) = x\left(\frac{t}{3} - 1\right)$



1.

(f)

$$x(t) = \begin{cases} 1 & 0 \le t \le 1 \\ -2 & 1 < t < 2 \end{cases} =$$



• $x(t)$ is periodic with $T = 2$

$$\therefore \quad x(t) = \cdots + 3u(t+2) - 3u(t+1) + 3u(t) - 3u(t-1) + 3u(t-2) \cdots \quad 1$$

$$\frac{dx(t)}{dt} = \cdots + 3\delta(t+2) - 3\delta(t+1) + 3\delta(t) - 3\delta(t-1) + 3\delta(t-2) \cdots$$

1

$$\delta_{T_0}(t) = \cdots + \delta(t+2) + \delta(t) + \delta(t-2) + \cdots$$

$$3\delta_{T_0}(t) = \cdots + 3\delta(t+2) + 3\delta(t) + 3\delta(t-2) + \cdots$$

$$3\delta_{T_0}(t-1) = \cdots + 3\delta(t+1) + 3\delta(t-1) + 3\delta(t-3) + \cdots \quad 1$$

$\therefore$ we can say that

$$\frac{dx(t)}{dt} = 3\delta_{T_0}(t) - 3\delta_{T_0}(t-1) \quad — \quad 1$$

Comparing it with $\frac{dx(t)}{dt} = G_1 \delta_{T_0}(t - t_1) + G_2 \delta_{T_0}(t - t_2)$

2

we get $\underline{G_1 = 3, \ t_1 = 0, \ G_2 = -3, \ t_2 = 1.}$

Q. 2.

$T = 1.$

$\omega_0 = \frac{2\pi}{T}$

$\omega_0 = 2\pi$

$$a_0 = \frac{1}{1}\int_0^1 t \cdot dt = \left[\frac{t^2}{2}\right]_0^1 = 1/2.$$ 1

$$a_n = \frac{2}{1}\int_0^1 t \cdot \cos(2\pi n t)\, dt$$

$$= 2\left[\frac{t}{2\pi n}\left\{\sin 2\pi n(1) - \sin 2\pi n(0)\right\}\right.$$

$$\left. - \int_0^1 \left\{1 \cdot \int \cos 2\pi n t\, dt\right\} dt\right]$$

$$= 0 + \frac{2}{(2\pi n)^2}\left[\cos 2\pi n - 1\right]$$

$$= \frac{2}{(2\pi n)^2}\left[1 - 1\right]$$

1.5

$$= 0$$

$$b_n = \frac{2}{1}\int_0^1 t \cdot \sin(2\pi n t)\, dt$$

$$= 2\left\{\left[t \cdot \left\{-\frac{\cos n 2\pi t}{2\pi n}\right\}\right]_0^1 - \int_0^1 \left\{1 \int \sin 2\pi n t\, dt\right\} dt\right.$$

$$= 2\left[\frac{\sin 2\pi n}{4 n^2 \pi^2} - \frac{\cos 2\pi n}{2\pi n}\right]$$

$$= 2\left(-\frac{1}{2n\pi}\right)$$

$$= -\frac{1}{n\pi}$$ 1.5

$$x(t) = 1/2 + \sum_{n=1}^{\infty}\left(-\frac{1}{n\pi}\right)\sin n\omega_0 t$$

1.

Q3 (a)

$$x(t) = t e^{-|t|}$$

$$e^{-|t|} = \begin{cases} e^{t} & , t<0 \\ e^{-t} & , t>0 \end{cases} = e^{t}u(-t) + e^{-t}u(t) \quad (1)$$

$$\text{F.T. of } e^{-|t|} = \text{F.T.}\left(e^{t}u(-t)\right) + \text{F.T.}\left(e^{-t}u(t)\right)$$

$$= \frac{1}{1-j\omega} + \frac{1}{1+j\omega}$$

$$= \frac{2}{1+\omega^2}$$

$$\therefore \quad e^{-|t|} \longleftrightarrow \frac{2}{1+\omega^2} \quad — (1) \quad (2)$$

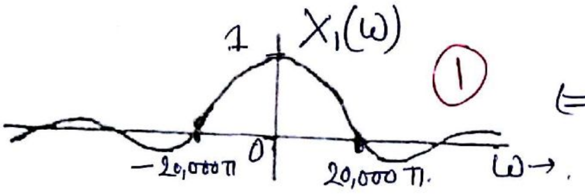
• Using the differentiation property in frequency domain in (1)

$$t e^{-|t|} \longleftrightarrow j \frac{d}{d\omega}\left(\frac{2}{1+\omega^2}\right) = -\frac{4j\omega}{(1+\omega^2)^2}$$

$$\therefore \quad t e^{-|t|} \longleftrightarrow \frac{-4j\omega}{(1+\omega^2)^2} \quad (2)$$

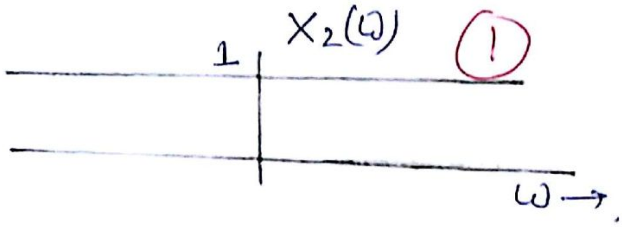
Q.3 (b)

(i)



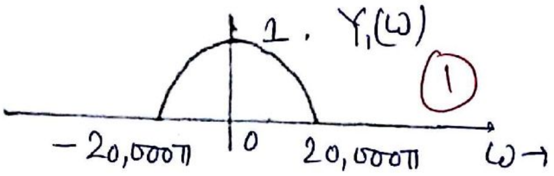
(1)

$\Longleftrightarrow$



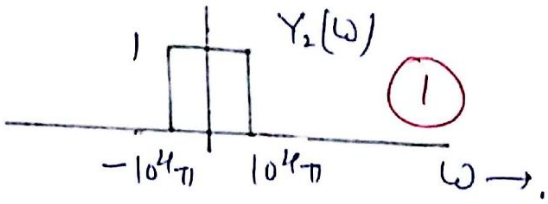
(1)

(ii)



(1)

$\Longleftrightarrow$



(1)

(iii) $\because$ $y_1(t) \cdot y_2(t) \Longleftrightarrow Y_1(\omega) * Y_2(\omega)$

B.W. of $Y_1(\omega) = 20{,}000\pi = 10$ KHz.

B.W. of $Y_2(\omega) = 10^4\pi = 5$ KHz.

B.W of $Y(\omega) = 15$ KHz. (1)

Q. (3) (c)

$$x[n] = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} X(\Omega)\, e^{jn\Omega}\, d\Omega$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \Delta\left(\frac{\Omega}{\pi}\right) e^{jn\Omega}\, d\Omega \quad (2)$$

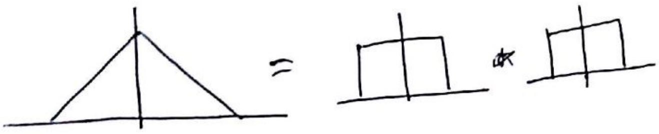
By Property.

$\because \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \Delta\left(\frac{\omega}{\pi}\right) e^{j\omega t}\, d\omega = \frac{1}{4}\operatorname{sinc}^2\left(\frac{\pi t}{4}\right)$

$\because \Delta\left(\frac{\omega}{\pi}\right) \Leftrightarrow \frac{1}{4}\operatorname{sinc}^2\left(\frac{\pi t}{4}\right)$

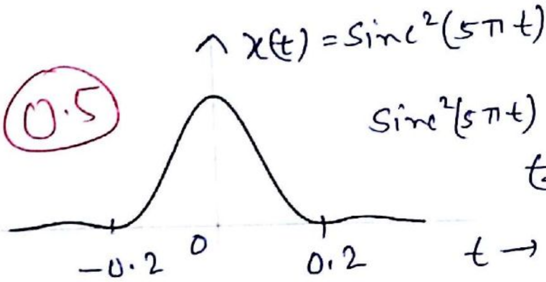
Put $\omega = \Omega$ & $t = n$.

$$x[n] = \frac{1}{4}\operatorname{sinc}^2\left(\frac{\pi n}{4}\right) \quad (2)$$



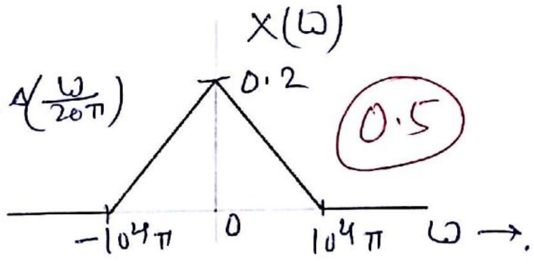
Q (4) (a)

(i) - (0.5)



$\operatorname{sinc}^2(5\pi t) \Longrightarrow 0.2\,\Delta\left(\frac{\omega}{20\pi}\right)$

$\Longleftrightarrow$



(0.5)

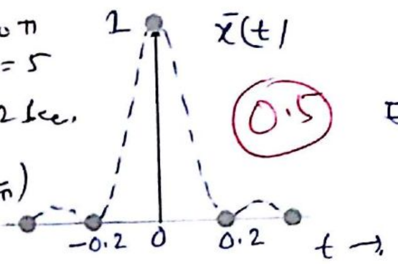
(ii) -

$\omega_s = 2\pi B = 10\pi$

$\Longrightarrow f_s = 5$

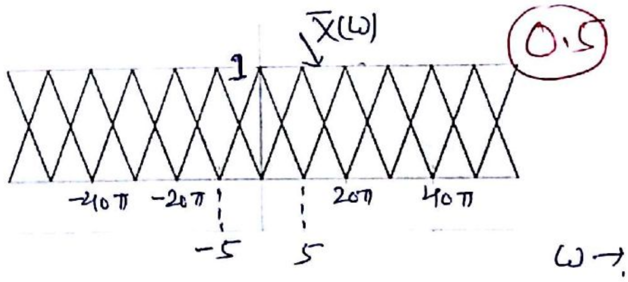
$T = \frac{1}{f_s} = 0.2$ sec.

$\frac{1}{T}X(\omega) = \frac{0.2}{T}\Delta\left(\frac{\omega}{20\pi}\right) = \Delta\left(\frac{\omega}{20\pi}\right)$

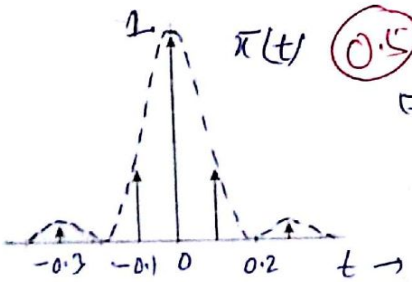


(0.5)

$\Longrightarrow$

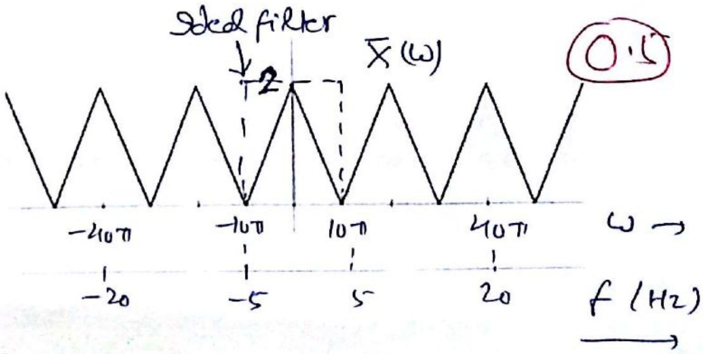


(0.5)



(0.5)

$\Longrightarrow$



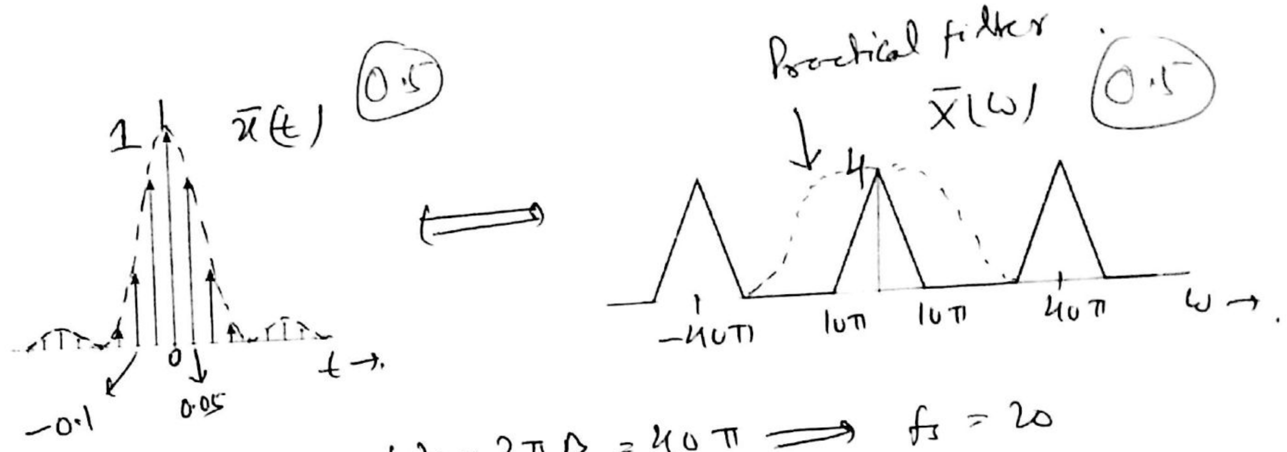
(0.5)

$\omega_s = 2\pi B = 20\pi \Longrightarrow f_s = 10$

$T = \frac{1}{f_s} = 0.1$ sec.

$$\frac{1}{T}X(\omega) = \frac{0.2}{T}\Delta\left(\frac{\omega}{20\pi}\right) = 0.2\,\Delta\left(\frac{\omega}{20\pi}\right)$$

(iii)



$$\omega_s = 2\pi B = 40\pi \Longrightarrow f_s = 20$$

$$T = \frac{1}{f_s} = 0.05 \text{ sec.}$$

$$\frac{1}{T} X(\omega) = \frac{0.2}{T} \Delta\left(\frac{\omega}{20\pi}\right) = 4\Delta\left(\frac{\omega}{20\pi}\right)$$

(iii) -
(i) 5 ~~Hz~~ $\rightarrow$ Aliasing
(ii) 10 Hz $\rightarrow$ Perfect reconstruction. 0.5
(iii) 20 Hz $\rightarrow$ Perfect reconstruction. 0.5

Q4 (b) The fourier transform $X(f)$ of the given signal $x(t)$ is

$$X(f) = \frac{1}{2}\left[\delta(f-100) + \delta(f+100)\right] + \left[\delta(f-160) + \delta(f+160)\right]$$

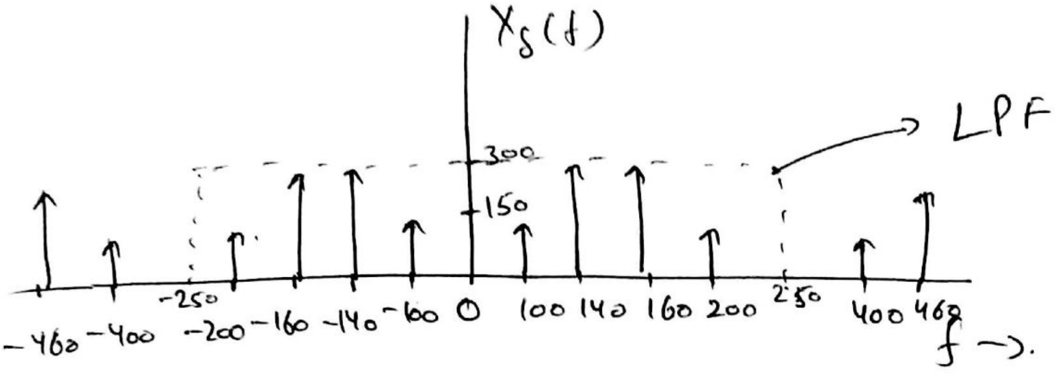
1

- The sampling frequency is $f_s = 300$ Hz. The fourier transform of sampled signal is given as

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

$$= 300 \sum_{n=-\infty}^{\infty} X(f - 300n).$$

$$X_\delta(f) = \cdots + 300X(f+300) + 300X(f) + 300X(f-300) + \cdots \quad (2)$$



- The Sampled Signal is passed through a LPF with Cutoff freq. 250 Hz. The frequency Component that will appear in the output will be 100, 140, 160 and 200 Hz.

$$\therefore Y(f) = \frac{1}{2}[\delta(f-100) + \delta(f+100)] + [\delta(f-140) + \delta(f+140)]$$
$$+ [\delta(f-160) + \delta(f+160)]$$
$$+ \frac{1}{2}[\delta(f-200) + \delta(f+200)]$$

$$\therefore y(t) = \cos(200\pi t) + 2\cos(280\pi t) + 2\cos(320\pi t) + \cos(400\pi t) \quad (2)$$

Q5 (a) $x(t) = \underbrace{e^{-t}u(t)}_{x_1(t)} + \underbrace{e^{-2t}u(-t)}_{x_2(t)}$

$$X_1(s) = \frac{1}{s+1} \qquad Re\ s > -1 \qquad 1$$

$$X_2(s) = \frac{-1}{s+2} \qquad Re\ s < -2 \qquad 1$$

$y_1(t)$ & $y_2(t)$ sys. response to $x_1(t)$ & $x_2(t)$

$$Y_1(s) = X_1(s)\cdot H(s)$$

$\xrightarrow{I/P} \boxed{H(s)} \xrightarrow{O/P}$

$$= \frac{1}{(s+1)(s+5)}$$

$$= \frac{1/4}{s+1} - \frac{1/4}{s+5}$$

$$\therefore y_1(t) = \frac{1}{4}\left(e^{-t} - e^{-5t}\right)u(t). \qquad 1$$

$$Y_2(s) = X_2(s)\cdot H(s) = \frac{-1}{(s+2)(s+5)} \qquad -5<s<-2$$

$$= \frac{-1/3}{s+2} + \frac{1/3}{s+5}.$$

$$y_2(t) = 1/3\left[e^{-2t}u(-t) + e^{-5t}u(t)\right] \qquad 1$$

$$y(t) = y_1(t) + y_2(t).$$

$$= \frac{1}{3}e^{-2t}u(-t) + \left(\frac{1}{4}e^{-t} + \frac{1}{12}e^{-5t}\right)u(t$$

1

Q5 (b) (i) $x[n] = \left(\frac{1}{3}\right)^n u[n] + (2)^n u[-n-1]$

$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-2z^{-1}} \quad ; \frac{1}{3} < |z| < 2$$

$$= \frac{-\frac{5}{3}z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-2z^{-1}\right)}.$$

- output $y[n] = 5\left(\frac{1}{3}\right)^n u[n] - 5\left(\frac{2}{3}\right)^n u[n]$

$$Y(z) = \frac{5}{1-\frac{1}{3}z^{-1}} - \frac{5}{1-\frac{2}{3}z^{-1}} \quad ; \quad |z| > \frac{2}{3}$$

$$= \frac{-\frac{5}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-\frac{2}{3}z^{-1})}$$

- $$H(z) = \frac{Y(z)}{X(z)} = \frac{1-2z^{-1}}{1-\frac{2}{3}z^{-1}} \quad , \quad |z| > \frac{2}{3}$$

2

(ii) $h[n] = ?$

$$H(z) = \frac{1-2z^{-1}}{1-\frac{2}{3}z^{-1}} \quad , \quad |z| > \frac{2}{3}$$

$$= \frac{1}{1-\frac{2}{3}z^{-1}} - \frac{2z^{-1}}{1-\frac{2}{3}z^{-1}}$$

Taking inverse z-Transform.

$$h[n] = \left(\frac{2}{3}\right)^n u[n] - 2\left(\frac{2}{3}\right)^{n-1} u[n-1]$$

$$h[n] = \left(\frac{2}{3}\right)^n [u[n] - 3u[n-1]]$$

2

(iii) Difference Equation relating $x[n]$ & $y[n]$.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-2z^{-1}}{1-\frac{2}{3}z^{-1}}$$

$$Y(z)\left[1-\frac{2}{3}z^{-1}\right] = X(z)\left[1-2z^{-1}\right]$$

$$Y(z) - \frac{2}{3}z^{-1}Y(z) = X(z) - 2z^{-1}X(z)$$

- taking inverse Z-T/f we get

$$y(n) - \frac{2}{3}y(n-1) = x(n) - 2x(n-1) \quad (1)$$

(Difference Eqⁿ)