

End Semester Exam

Part-I

MATH-III, 21ST NOV., 2016

TIME: 45 MINUTES, MAXIMUM MARKS:10

Name: _____

Roll No.: _____

Note: Each question carry 2 marks. Overwriting will be treated as a wrong answer. Use only the last page of main answer sheet for rough work and calculation. Write down the final answer, no marks for formula or incomplete solution.

1. The PDE which characterizes the surfaces

$$F\left(\frac{xy}{z}, \frac{x-y}{z}\right) = 0$$

is _____

Solution: $x^2p + y^2q = (x+y)z$ (Here $u = \frac{xy}{z}$, $v = \frac{x-y}{z}$, $P = \frac{\partial(u,v)}{\partial(y,z)} = -\frac{x^2}{z^3}$, $Q = \frac{\partial(u,v)}{\partial(z,x)} = -\frac{y^2}{z^3}$, $R = \frac{\partial(u,v)}{\partial(x,y)} = -\frac{x+y}{z^2}$)

2. (a) The classification (elliptic, parabolic, or hyperbolic) of the PDE

$$xu_{xx} + (x-y)u_{xy} - yu_{yy} = 0, \quad x > 0, \quad y > 0,$$

is _____ type.

(b) Tick Appropriately: The PDE $u_{xxx} + (u_{yy})^4 = 0$ is first/second/third/forth order and linear/semi-linear/quasi-linear/non-linear PDE.

Solution: (a) Hyperarabolic (Here $b^2 - 4ac = (x-y)^2 + 4xy = (x+y)^2$)

(b) Third order semi-linear PDE

3. Solution for following PDE

$$\begin{aligned} u_{tt} - u_{xx} &= 0, & 0 < x < \infty, \quad t > 0 \\ u(x, 0) &= 0, & u_t(x, 0) = \cos x, \quad 0 \leq x < \infty, \\ u(0, t) &= 0, & t \geq 0. \end{aligned}$$

is $\sin x \cos t$

4. Solution of the wave equation (Hint: Duhamel's principle):

$$\begin{aligned} u_{tt} &= u_{xx} + 2x, & -\infty < x < \infty, \quad t > 0, \\ u(x, 0) &= u_t(x, 0) = 0, & -\infty < x < \infty. \end{aligned}$$

is xt^2

5. Solution for the following Laplace equation:

$$\begin{aligned} u_{xx} + u_{yy} &= 0, & 0 < x, y < \pi, \\ u(x, 0) &= 2x, \quad 0 \leq x \leq \pi, & u(x, \pi) = 0, \quad 0 \leq x \leq \pi, \\ u(0, y) &= u(\pi, y) = 0, & 0 \leq y \leq \pi. \end{aligned}$$

is $u(x, y) = \sum_{n=1}^{\infty} \frac{(-1)^n 4 \sin nx \sinh[n(y - \pi)]}{n \sinh(n\pi)}$