THE LNM INSTITUTE OF INFORMATION TECHNOLOGY
JAIPUR, RAJASTHAN

# End Semester Exam

MATH-III,, 21ST NOV, 2016

**Part-I**

TIME: 45 MINUTES, MAXIMUM MARKS:10

**Name:**________________________ **Roll No.:**________________________

**Note:** Each question carry 2 marks. Overwriting will be treated as a wrong answer. Use only the last page of main answer sheet for rough work and calculation. Write down the final answer, no marks for formula or incomplete solution.

1. The PDE which characterizes the surfaces

$$F(xyz, x+y+z) = 0,$$

is ________________________

Solution: $x(y-z)p + y(z-x)q = (x-y)z$ (Here $u = xyz,\ v = x+y+z,\ P = \frac{\partial(u,v)}{\partial(y,z)} = x(z-y),\ Q = \frac{\partial(u,v)}{\partial(z,x)} = y(x-z),\ R = \frac{\partial(u,v)}{\partial(x,y)} = z(y-x)$)

2. $(a)$ The classification (elliptic, parabolic, or hyperbolic) of PDE

$$xu_{xx} + (x-y)u_{xy} - yu_{yy} = 0,\ x > 0,\ y < 0,$$

is ________________________ type.

$(b)$ Tick Appropriately: The PDE $x^3u_{xxxx} + xy^3u_{yy} = 0$ is first/second/third/forth order and linear/semi-linear/quasi-linear/non-linear PDE.

Solution: (a) Hyperbolic if $x+y \neq 0$ and parabolic for $x+y = 0$ (Here $b^2 - 4ac = (x-y)^2 + 4xy = (x+y)^2$)
(b) Forth order linear PDE

3. Solution for following PDE

$$\begin{aligned} u_{tt} - u_{xx} &= 0, && 0 < x < \infty,\ t > 0 \\ u(x,0) &= \cos x, && u_t(x,0) = 0, 0 \leq x < \infty, \\ u(0,t) &= 0, && t \geq 0. \end{aligned}$$

is $u(x,t) = \begin{cases} \cos x \cos t & x \geq t \\ -\sin x \sin t & x \leq t \end{cases}$

4. Solution of the wave equation (Hint: Duhamel's principle):

$$u_{tt} = u_{xx} + 2t, \qquad -\infty < x < \infty,\ t > 0,$$

$$u(x,0) = u_t(x,0) = 0, \quad -\infty < x < \infty.$$

is $\frac{t^3}{3}$

5. Solution for the following Laplace equation:

$$\begin{aligned} u_{xx} + u_{yy} &= 0, && 0 < x, y < 2\pi, \\ u(x,0) = 2x,\ 0 \leq x &\leq 2\pi, && u(x,2\pi) = 0,\ 0 \leq x \leq 2\pi, \\ u(0,y) = u(2\pi,y) &= 0, && 0 \leq y \leq 2\pi. \end{aligned}$$

is $u(x,y) = \sum_{n=1}^{\infty} \frac{(-1)^n 8 \sin(\frac{nx}{2}) \sinh[\frac{n(y-2\pi)}{2}]}{n \sinh(n\pi)}$.