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The LNM Institute of Information Technology, Jaipur
Department of Mathematics
Mathematics-III MTH213
Mid Term

Duration: 90 mins. September 26, 2019 Max.Marks: 30

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NOTE: You should attempt all questions. Your writing should be legible and neat. Marks awarded are shown next to the question. **Start a new question on a new page and answer all its parts in the same place.** Please make an index showing the question number and page number on the front page of your answer sheet in the following format.

Question No.				
Page No.				

1. (a) Prove that for any complex number $z \neq 1$, we have [3]

$$1 + z + z^2 + \cdots + z^n = \frac{1 - z^{n+1}}{1 - z},$$

and then use it to derive *Lagrange's trigonometric identity:*

$$1 + \cos\theta + \cos 2\theta + \cdots + \cos n\theta = \frac{1}{2} + \frac{\sin[(2n+1)\theta/2]}{2\sin(\theta/2)}, \qquad 0 < \theta < 2\pi.$$

(b) Let $f : \mathbb{C} \longrightarrow \mathbb{C}$ be an analytic function such that $|f(z)| \geq 1$ for all $z \in \mathbb{C}$. Show that f is constant. [3]

2. (a) Show that the function $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2} & z \neq 0 \\ 0 & z = 0 \end{cases}$ satisfies the CR equations at $(0, 0)$ but is not differentiable there. [3]

(b) Suppose f is an analytic function on a domain D. Show that if $|f|^2$ is harmonic, then f is constant on D. [3]

3. (a) Show that $|\cos z|^2 = \cos^2 x + \sinh^2 y$. Conclude that the cosine function is unbounded in $\mathbb{C}$. [3]

(b) Let $f(z) = \frac{1}{z^2}$. Evaluate $\int_C f(z)dz$ where C is any simple closed contour in counterclockwise direction not passing through 0. [3]

4. (a) Let C_R denote the upper half circle $|z| = R$ $(R > 2)$, taken in the counterclockwise direction. Find an upperbound of

$$\left|\int_{C_R} \frac{2z^2 - 1}{z^4 + 5z^2 + 4} dz\right|.$$

Hence show that $\lim_{R\to\infty} \int_{C_R} \frac{2z^2 - 1}{z^4 + 5z^2 + 4} dz = 0$. [3]

(b) Find all the possible Laurent series expansion of $f(z) = \frac{2z}{z^2 - 9}$ about $z = 3$ and using that find the residue of $f(z)$ at $z = 3$. [3]

5. (a) Evaluate $\int_c z^2 e^{\frac{5}{z}}$ where C is any simple closed contour in counterclockwise direction with $z = 0$ inside it. [2]

(b) Using contour integral, evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2}$. [4]