Midterm-1

Discrete Mathematical Structures
B.Tech 1st Year, The LNMIIT Jaipur

(Attempt all questions)

Q-1: Answer the following questions with proper justifications: **2×5=10**

(a) Check if the following matrix is *unipotent*, *idempotent* or *nilpotent*:

$$\mathbf{A} = \begin{bmatrix} 45 & -22 & -19 \\ 33 & -16 & -14 \\ 69 & -34 & -29 \end{bmatrix}$$

(b) Let, $A = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7\}$ and $f : A \to A$, $g : A \to A$ be two mappings defined as follows:

$$f = \{(1,3),\ (2,3),\ (5,4),\ (3,1),\ (4,2),\ (7,5),\ (6,7)\}$$
$$g = \{(2,4),\ (5,7),\ (6,1),\ (3,2),\ (1,3),\ (7,5),\ (4,6)\}$$

Find $f \circ g$ and $g \circ f$. Check if any of f and g is *bijective*. If so, find its inverse.

(c) A relation R on $\mathbb{N}$ is defined as follows: aRb iff $|a - b| \geq 3$ for $a,\ b \in \mathbb{N}$. Check if R is *reflexive*, *irreflexive*, *symmetric*, *asymmetric*, *antisymmetric* or *transitive*. Then, find the relation R^{-1}.

(d) Compute the sum of the distinct numbers, obtained by all possible permutations of 39247, taken all at a time.

(e) Let, R be a relation on the set $A = \{a,\ b,\ c,\ d,\ e\}$ that has the following boolean matrix:

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Write down the relation R and check (with proper justification) whether it is an equivalence relation or not. If so, find the partition A/R.

Q-2: Solve the following homogeneous recurrence relation: **5**

$$a_n \ = \ 3\,(a_{n-1} \ + \ a_{n-2}) \ \text{ for all } n \geq 3;\ a_1 = 4,\ a_2 = -2$$

Q-3: Show that if 5 points are selected in a square whose sides are of length 1 inch, then at least two points must be no more than $\sqrt{2}$ inches apart. **5**

Q-4: Let, $A = \{a,\ b,\ c,\ d,\ e\}$ be a set and R be a relation on A, defined as:

$$R = \{(a,a)\,,\ (a,c)\,,\ (a,e)\,,\ (b,d)\,,\ (c,a)\,,\ (d,c)\,,\ (d,d)\,,\ (e,a)\,,\ (e,c)\}$$

Check whether R is *circular* or *transitive* with proper justification. Write down the relation R^2. Use the *Warshall's algorithm* to find the smallest transitive relation containing R. **2+3+5=10**