The LNM Institute of Information Technology
Jaipur, Rajasthan

Quiz-3

MATH-II, 16^{th} April 2014

Section: A

Time: 15 minutes, Maximum Marks: 10

Name:________________________ Roll No.:________________________

1. Find the integral surface of the first order PDE $xp - yq = z$ containing the curve $\Gamma : x_0 = s^2, y_0 = s+1, z_0 = s$. (Here $z = z(x,y), p = z_x, q = z_y$)

Sol. The characteristic curve is given by

$$\frac{dx}{x} = -\frac{dy}{y} = \frac{dz}{z}$$

From first and second term of the identity we get $u = xy = c_1$.

By taking first and third term we get $v = \frac{x}{z} = c_2$.

These are two independent solutions of the characteristic curve. So the general solutions is given by $F(u,v) = 0$, or $F(xy, \frac{x}{z}) = 0$, or $xy = G(\frac{x}{z})$, where F and G are arbitrary functions.

We seek an integral surface containg the initial curve $\Gamma : x_0 = s^2, y_0 = s+1, z_0 = s$.

Imposing this condition on the general solution we find

$$s^2(s+1) = G(\frac{s^2}{s}) = G(s).$$

Hence, $xy = G(\frac{x}{z})$ implies the desired integral surface as $yz^3 = x(x+z)$.