The LNM Institute of Information Technology, Jaipur
Second Mid Sem Examination 2011
Mathematics II

Date: 14th Mar 2011 \qquad Full Mark 40 \qquad Duration: 1 hour

1. (i) Find the eigen values and eigen vectors of the matrix (4)

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

(ii) Find a polynomial solution $y(x)$ of (5)

$$(1 - x^2)y'' - 2xy' + 20y = 0$$

such that $y(1) = 10$.

2. (i) Find the radius of convergence of the following series (2)

$$1 + (x + 1) + 2(x + 1)^2 + ... + n(x + 1)^n + ...$$

centered around $x_0 = -1$.

(ii) Show that the equation $xy'' + y' - y = 0$ has only one Frobenius series solution. Find the first three non-zero terms of this series. (4)

(iii) Prove that between each pair of consecutive positive zeros of $J_p(x)$, there is exactly one zero of $J_{p+1}(x)$ and vice versa. (5)

3. (i) Let $\Phi(x)$ be a non-trivial solution of

$$y'' + \left[k + 2\sin\left(x + \frac{\pi}{4}\right)\right]y = 0, \qquad k > 3.$$

Show that $\Phi(x)$ has at least 5 zeros in the interval $[0, 5\pi]$. (6)

(ii) Find the eigen values and corresponding eigen functions of the boundary value problem

$$y'' + \lambda y = 0$$

with boundary conditions $y(0) = 0, y'(\pi) = 0$. (4)

4. (i) The current $I(t)$ in a circuit involving resistance, conductance and capacitance is described by the initial value problem

$$\frac{d^2 I}{dt^2} + 2\frac{dI}{dt} - 3I = 5u(t)$$
$$I(0) = 8, \frac{dI}{dt}(0) = 0,$$

where $u(t)$ is the unit step function. Using Laplace transform find the current as a function of time. (5)

(ii) Using Laplace transform find the solution of the integral equation

$$y(t) + \int_0^t y(\tau)d\tau = f(t)$$

with

$$f(t) = \begin{cases} 1 & a < t < b \\ 0 & \text{elsewhere.} \end{cases}$$

(5)

End of paper