The LNM Institute of Information Technology, Jaipur
Session: 2017-18, EVEN Semester, Quiz 2
Subject: Control System Engineering (Core)
Date: 28th March 2018, Full Marks: 10, Time: 30 Minutes

Name:

Roll. No.:

1. Open loop transfer function of a control system is given by:

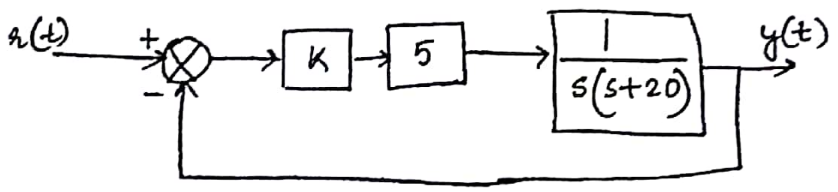
$$G(s)H(s) = \frac{k}{s(s+1)^2(s+2)}$$

Draw the root locus and determine the following:
   i) breakaway points,
   ii) angle of departure from complex poles,
   iii) value of k for the system to be marginally stabile.

2. Construct the Routh's table and comment about the stability of the system with given characteristic equation.

$$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$$

3. For the system shown below, determine the following specifications when $K = 60$.
   i) Peak overshoot,
   ii) Settling time.



[5+3+2 = 10]

Ans 2.

Routh's table

s^6	1	8	20	16
s^5	2	12	16	
s^4	2	12	16	
s^3	$\cancel{0}$ 8	$\cancel{0}$ 24		
s^2	6	16		
s^1	8/3			
s^0	16			

2 M

Auxiliary equation

$A(s) = 2s^4 + 12s^2 + 16$

$\frac{dA(s)}{ds} = 8s^3 + 24s$

Row of zero occurs once and 1st column of the table has same sign, the system is MARGINALLY STABLE.

1 M

Ans 3.

$$\frac{Y(s)}{R(s)} = \frac{\frac{5k}{s(s+20)}}{1+\frac{5k}{s(s+20)}}$$

$$= \frac{5k}{s(s+20)+5k}$$

$$= \frac{5K}{s^2+20s+5K} \quad \text{——— (i)}$$

Comparing with generalized 2nd order equation

$$TF = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \quad \text{——— (ii)}$$

from (i) & (ii),

$$\omega_n^2 = 5K \quad \text{and} \quad 2\xi\omega_n = 20$$

Given $K = 60$,

$\therefore$ $\omega_n = 17.32$ and $\xi = 0.58$

Now,

i) Peak overshoot (Mp) : $e^{-\pi\xi/\sqrt{1-\xi^2}}$ ——— (1M)

$\Rightarrow M_p = 0.1068$

$= 10.68\%$

ii) Settling time (t_s) : $\frac{4}{\xi\omega_n} = 0.4s$ (2% tolerance)

$\frac{3}{\xi\omega_n} = 0.3s$ (5% tolerance)

(1M)

Ans 1 :   Given,

$$G(s)H(s) = \frac{K}{s(s+1)^2(s+2)}$$

Step (a)   zeros $\rightarrow$ none

poles $\rightarrow$ 4 i.e. at $s = 0, -1, -1, -2$.

All 4 poles will terminate at ∞.

Step (b)

Angle of asymptotes $\theta = \frac{(2q+1)180^\circ}{p-z}$ $q = 0, 1, 2, 3$

$p - z = 4$

$$= \frac{(2q+1)\,180^\circ}{4}$$

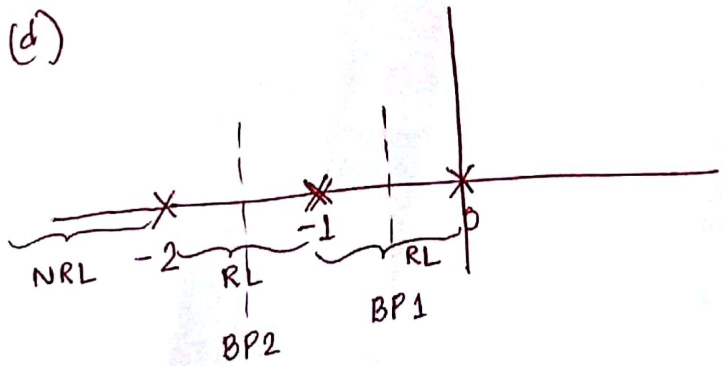
$$= 45^\circ, 135^\circ, 225^\circ, 315^\circ$$ 1M

Step (c)

centroid $\sigma = \frac{\Sigma(\text{real poles}) - \Sigma(\text{real zeros})}{p-z}$

$$= \frac{-4-0}{4} = -1$$

Step (d)



There exist 2 Break-away points

One between 0 & −1

another −1 & −2

RL : Root Locus region

NRL : Not Root-Locus region

Step (e) $G(s)H(s) = -1$

or $1 + G(s)H(s) = 0$

$\Rightarrow s(s+1)^2(s+2) + K = 0$

$\Rightarrow K = -s(s+1)^2(s+2)$.

Now : $\dfrac{dK}{ds} = 0$

$\Rightarrow \dfrac{d}{ds}\left[-s(s+1)^2(s+2)\right] = 0$ —— (1M)

$\Rightarrow s = -1, -1.707, -0.292$
$\quad\quad\;\times \quad\quad\checkmark \quad\quad \checkmark$

Break-away points = -1.707 & -0.292

Step (f) To find intersection pt. on imaginary axis.

$1 + G(s)H(s) = 0$ (1M)

$\Rightarrow s^4 + 4s^3 + 5s^2 + 2s + K = 0$

s^4	1	5	K
s^3	4	2	
s^2	18/4	K	
s^1	$\dfrac{9-4K}{4.5}$		
s^0	K		

for the system to be marginally stable

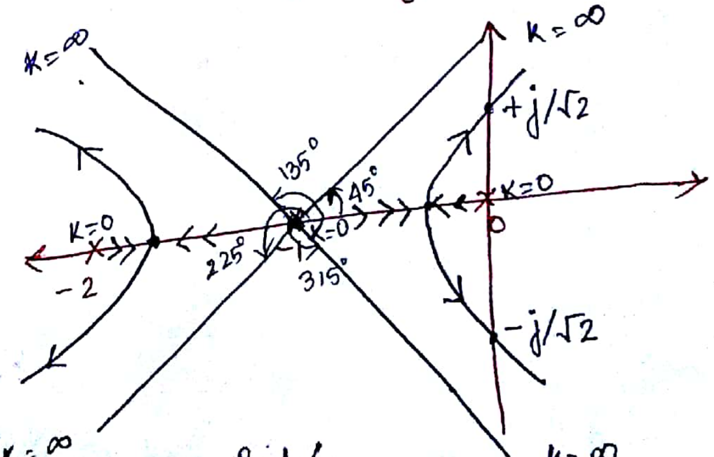
$\dfrac{9-4K_{max}}{4.5} = 0$

$\Rightarrow \boxed{K_{max} = 2.25}$ (1M)

$s = \pm j/\sqrt{2}$

Step (g) Angle of departure

(1M) —— As there are no true complex poles, so no angle of departure



Root Locus