LNMIIT/B. Tech(CSE,CCE,ECE)/IC/2019-20/EVEN/MTH221/MT

THE LNM INSTITUTE OF INFORMATION TECHNOLOGY
DEPARTMENT OF MATHEMATICS
PROBABILITY AND STATISTICS: MTH221
MID SEMESTER EXAM
Date: 22/02/2020

Time: 90 Minutes | Maximum Marks: 30

NOTE: You should attempt all questions. Marks awarded are shown next to the question. **NO USE OF CALCULATORS.** Please make an index showing the question number and page number on the front page of your answer sheet in the following format, otherwise you may be penalized by the deduction of **4 marks.**

Question No.				
Page No.				

1. Let $A_n, n \geq 1$ be a sequence of events. If $A_1 \subset A_2 \subset \cdots$, then prove that $P\left(\bigcup_{k=1}^{\infty} A_k\right) = \lim_{k\to\infty} P(A_k)$. **[4 marks]**

2. A person wrote n letters to different persons, sealed them in n envelopes and wrote the n different addresses randomly one on each of them. Find the probability that at least one of the letters reaches its correct destination. **[5 marks]**

3. Can the following function be a cummulative distribution function (CDF)?
$$g(x) = \begin{cases} 0, & -\infty < x < 0, \\ \frac{1}{5}, & 0 \leq x < 1, \\ \frac{3}{5}, & 1 \leq x < 3, \\ 1, & 3 \leq x < \infty. \end{cases}$$
If yes, determine whether the corresponding random variable X is a discrete/absolutely continuous random variable and find it's PMF/PDF. **[4 marks]**

4. The amount of time in hours that a computer functions before breaking down is a continuous random variable with probability density function (PDF) given by **[4 marks]**
$$f_X(x) = \begin{cases} \lambda e^{-x/100}, & x \geq 0, \\ 0, & x < 0. \end{cases}$$
What is the probability that a computer will function between 50 and 150 hours before breaking down?

5. Let X be a random variable with $P(X=2) = \frac{1}{4}$ and its CDF is **[2+3 marks]**
$$F_X(x) = \begin{cases} 0, & x < -3 \\ \alpha(x+3), & -3 \leq x < 2, \\ \frac{3}{4}, & 2 \leq x < 4, \\ \beta x^2, & 4 \leq x < \frac{8}{\sqrt{3}}, \\ 1, & x \geq \frac{8}{\sqrt{3}}. \end{cases}$$
(a) Find α, β if 2 is the only jump point in CDF $F_X(x)$.
(b) Compute conditional probability $P(X < 3 | X \geq 2)$.

6. Suppose that the length of a phone call in minutes is an exponential random variable X (denote the length of call made by the person in the booth) with parameter $\lambda = \frac{1}{10}$. Find the conditional distribution $F_X\left(\frac{x}{M}\right)$ i.e $P(X \leq x | M)$ with conditioning the event $M = \{X \geq 20\}$. **[4 marks]**

7. Let X be a random variable with PDF $f_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$, for $-\infty < x < \infty$ and CDF $F_X(x)$. Let $Y = X^2$ be another random variable. Find the CDF of Y in terms of $F_X(x)$ and also find PDF of random variable Y. **[4 marks]**

End of paper