**Mid Term Exam Question Paper**

Subject: **Design and Analysis of Algorithms (CSE 325)**

Max Marks = **50** Time = **1.5 hours**

Note: Answer all parts of the question at one place only. Non-conformity will lead to answers being not corrected or deduction of marks. Also write your answers to the point in order to attempt all questions in time.

Q1. Fill in the blanks [5 marks]

a. The upper bound for number of iterations in Gale-Shapley algorithm for stable matching between n men and n women is ____________.

b. An algorithm is "efficient" if ____________________.

c. No efficient algorithm is known for _____________ class of problems

d. Bucket sort can be used when the numbers to be sorted are ________________.

e. The 3rd step (before re-arranging) in counting sort computes ____________ in auxiliary array.

Q2. Answer the following questions very concisely. [3 x 3 = 9 marks]

a) Two algorithms **A** and **B** have their running times $T(n) = 5T(n/4) + n\log n$ and $T(n) = aT(n/8) + n\log n$ respectively. Find the maximum value of '**a**' such that B algorithm is faster than A?

b) To sort an array of integers with range from 1 to $n^c$ using radix sort in O(n) time, find the value of base in which the numbers can be represented?

c) For quick sort, let there be a procedure (with O(n) time) for finding a good pivot element which splits into two, each of which contains atleast one-third of elements. Write the recurrence for the worst case T(n) and its asymptotic bound.

Q3. **Attempt any three** of the following questions. [3 x 6 = 18 marks]

a) Solve the recurrence $T(n) = 2T(n/2) + n\log_2 n$ using either substitution or recurrence tree method. (Hint: Take or substitute $n=2^m$)

b) Derive the asymptotically tight bound for the BUILD MAX HEAP algorithm (discussed in class) which is used to re-arrange any given array into a max heap array. Note: you won't get credit if your answer is not asymptotically tight bound.

c) Let f(n) and g(n) be asymptotically nonnegative functions. Using the basic definition of theta notation, prove that $max(f(n), g(n)) = \Theta(f(n) + g(n))$

d) How would you modify Strassen's algorithm to multiply *n x n* matrices in which *n* is not an exact power of 2 so that the algorithm still has a running time of Theta(n^lg(7))?(Hint: Think of extending the idea of block matrix multiplication)

Q4. Although merge sort runs in $\Theta(\log n)$ worst-case time and insertion sort runs in $\Theta(n^2)$ worst-case time, the constant factors in insertion sort can make it faster in practice for small problem sizes on many machines. Thus, it makes sense to coarsen the leaves of the recursion by using insertion sort within merge sort when subproblems become sufficiently small. Consider a modification to merge sort in which n/k sublists of length k are sorted using insertion sort and then merged using the standard merging mechanism, where k is a value to be determined. [10 marks]

a. What is the worst case time complexity for insertion sort to sort the n/k sublists, each of length k. [2 mark]

b. Write the recurrence relation for the modified algorithm in worst case and solve it to determine the asymptotic running time. [5 mark]

c. What is the largest value of k as a function of n for which the modified algorithm has the same running time as standard merge sort, in terms of Θ-notation? [2 mark]

d. How should we choose k in practice? [1 mark]

Q5. Given an array of N integers (which can be positive or negative, representing a person's emotional index on each day). Let the happiness in an interval be defined as the sum of the values in that interval. Design an O(N log N) algorithm to find the happiest interval of the person and its happiness value. For example, if the given array is {-2, -5, 6, -2, -3, $\boxed{1, 5, -6}$}, then the happiest interval value is 7 (see highlighted elements). Write the pseudo code of the algorithm and justify its correctness and time complexity. [8 marks].