Quiz-3   MATH-II, 16th APRIL 2014
Section: A   TIME: 15 MINUTES, MAXIMUM MARKS: 10

NAME:_____   ROLL No.:_____

1. Find and classify the PDE arise from the surface $(x-a)^2 + (y-b)^2 + z^2 = 1$. Find the singular integral of the resulting PDE.

   Sol. Given surface is

   $$F(x,y) = (x-a)^2 + (y-b)^2 + z^2 - 1 = 0 \tag{1}$$

   Differentiating equation (1) w.r.t. x and y respectively, we get

   $$(x-a) + zp = 0 \tag{2}$$
   $$(y-b) + zq = 0 \tag{3}$$

   Squaring equations (2), (3) and adding these two, we arrive at

   $$(x-a)^2 + (y-b)^2 + z^2p^2 + z^2q^2 + 2(x-a)zp + 2(y-b)zq = 0. \tag{4}$$

   Now using the equation (1)- (3), we get the non-linear PDE

   $$z^2(1 + p^2 + q^2) = 1.$$

   In order to get singular integral, we need to differentiate equation (1) w.r.t. a and b to get

   $$\frac{\partial F}{\partial a} = -2(x-a) = 0 \implies a = x;$$
   $$\frac{\partial F}{\partial b} = -2(y-b) = 0 \implies b = y;$$

   Substituting the value of a and b in equation (1), we get the singular solution as $z = \pm 1$.