The LNM Institute of Information Technology
Department of Electronics and Communication Engineering
Signals and Systems (ECE217)
Exam Type: End Term Exam

Time: 180 minutes **Date:** 06/12/2019 **Max. Marks:** 50

Instruction: ***1. Attempt the questions sequentially.***
2. Please check there must be five questions and two printed pages.

1) a) State whether the following system is linear, causal, time-invariant and stable?
$y(n) = 2x(n+1) + [x(n-1)]^2$ [4]

b) Determine the power and energy for the following signal:

i) $x(t) = rect\left(\frac{t}{T_0}\right)\cos(w_0 t)$ [3]

ii) $x(t)$ as given in Fig.1 [3]

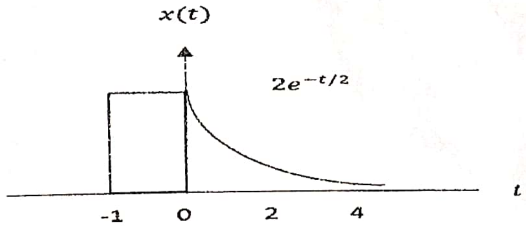

Fig. 1

2) a) Given the periodic train of delta function with period T_0

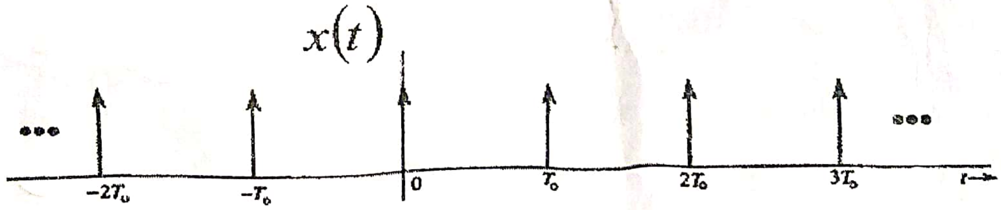

Fig. 2

i) Write the time domain expression of the signal $x(t)$. [1]
ii) Find the exponential Fourier series coefficient. [2]
iii) Write the exponential Fourier series expression of the signal $x(t)$. [2]

b) Find the z-transform of $x[n] = |n|(0.125)^{|n|}$. [2]

c) Draw the region of convergence of $Y(z)$, if $y[n] = x_1[n+3] * x_2[-n+1]$, where
$x_1[n] = \left(\frac{1}{2}\right)^n u[n]$ and $x_2[n] = \left(\frac{1}{3}\right)^n u[n]$. [3]

3) a) Find the Fourier Transform of: (i) $x(t) = \frac{3}{2+t^2}$, (ii) $x[n] = 2^n cos\left(\frac{\pi n}{4}\right) u[-n]$. [4]

b) Find the inverse Fourier Transform of: (i) $X(\omega) = e^{-\pi\omega^2}$, (ii) $X(e^{j\omega}) = \frac{3}{(1-e^{-1-j\omega})^3}$. [4]

c) Given that $x(n)$ has Fourier transform $X(e^{j\omega})$, express the Fourier Transform of the following signals in the terms of $X(e^{j\omega})$:

i) $x_1(n) = x(1-n) + x(-1-n)$, ii) $x_2(n) = (n-1)^2 x(n)$ [2]

4) a) A causal LTI system has the frequency response $H(\omega)$ shown in Fig. 3. For each of the input signals given below, determine the filtered output signal $y(t)$.

i) $x(t) = e^{jt}$, **ii)** $x(t) = \sin(\omega_o t)\, u(t)$, **iii)** $X(\omega) = \frac{1}{j\omega(6+j\omega)}$, **iv)** $X(\omega) = \frac{1}{2+j\omega}$ [4]

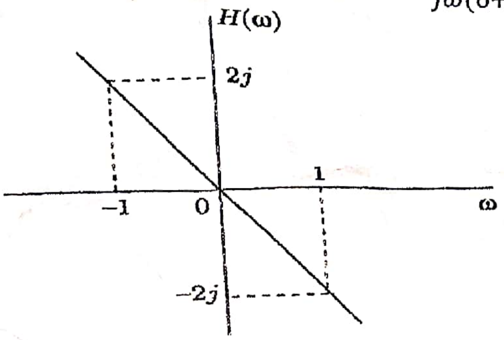

Fig. 3

b) What is Sampling Theorem? Explain Ideal Sampling, Natural Sampling and Flat Top Sampling. [4]

c) Let a signal $y(t) = x_1(t) * x_2(t)$ is sampled using impulse-train sampling, where $X_1(\omega) = 0$ for $|\omega| > 1000\pi$ and $X_2(\omega) = 0$ for $|\omega| > 2000\pi$. What is the allowable range of values of sampling period T of $y(t)$ so as to avoid aliasing? [2]

5) a) A causal LTI system S with impulse response $h(t)$ is related through the following differential equation with $x(t)$ as input and $y(t)$ as output:

$$\frac{d^3y(t)}{dt^3} + (1+\alpha)\frac{d^2y(t)}{dt^2} + \alpha(1+\alpha)\frac{dy(t)}{dt} + \alpha^2 y(t) = x(t).$$

For what values of α, S is guaranteed to be stable? [2]

b) Find the inverse Laplace Transform of $H(s) = \frac{4s^2+15s+8}{(s+2)^2(s-1)}$, assuming that:

i) $h(t)$ is Causal, **ii)** Fourier Transform of $h(t)$ exists, i.e. $h(t)$ is absolutely integrable. [2]

c) Consider the system S, characterized by the given differential equation

$$\frac{d^3y(t)}{dt^3} + 6\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 6y(t) = x(t)$$

i) Determine the Zero State Response of this system for the input $x(t) = e^{-4t}u(t)$.

ii) Determine the Zero Input Response of the system for $t > 0^-$ given that, $y(0^-) = 1$, $\dot{y}(0^-) = -1$, $\ddot{y}(0^-) = 1$.

iii) Determine the total response of the system when the input is $x(t) = e^{-4t}u(t)$ and the initial conditions are the same as those specified in part (ii). [3]

d) Determine the inverse Z-Transform of $X(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}}$, if

i) ROC $|z| > 1$, **ii)** ROC $|z| < 0.5$, **iii)** ROC $0.5 < |z| < 1$ [3]