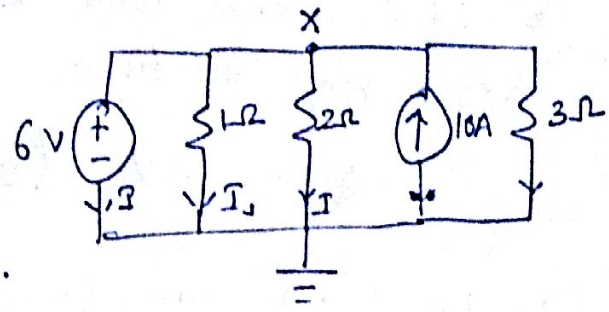
E-I 2015-16

Mid Term Solution.

1

1) This is a very straight forward question and requires a basic knowledge of Concepts.



V at Node X = 6V

∵ there is a voltage source.

$\therefore I_{1\Omega} = \frac{6}{1} = 6A$

$I_{2\Omega} = \frac{6}{2} = 3A$

$I_{3\Omega} = \frac{6}{3} = 2A$

By KCL $I_{6V} = 10 - (6+3+2) = -1A$

$\therefore P_{1\Omega} = I_{1\Omega}^2 R = 36W$

$P_{2\Omega} = I_{2\Omega}^2 R = 18W$

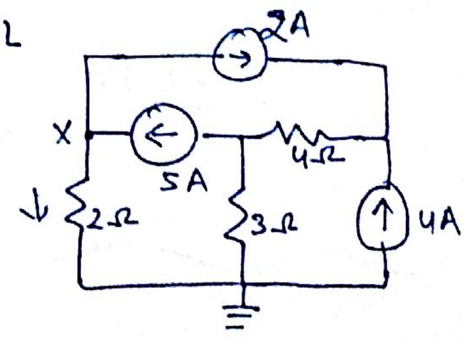
$P_{3\Omega} = I_{3\Omega}^2 R = 12W$

$P_{10A} = V.I = -6 \times 10 = -60W$

$P_{6V} = V.I = 6 \times -1 = -6W$

Total power = $36 + 12 + 18 - 60 - 6 = 0$ ✓

2) Our aim is to find $P_{2\Omega}$ using KCL and not to solve the complete circuit.



KCL at X.

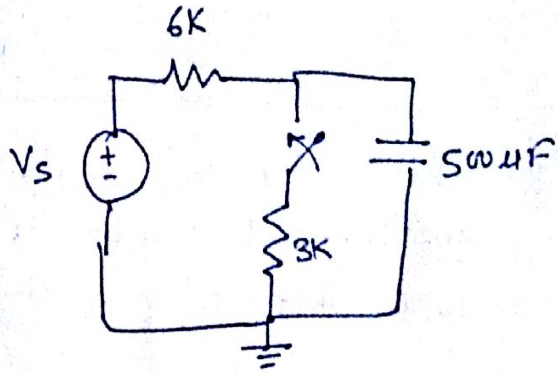
$I = \cancel{2+}\ 5 - 2 = 3A$

$\therefore P_{2\Omega} = 3^2 \times 2 = 18W$

Thats it No more time wasting.

3) → for $t<0$

The capacitor has been connected to 12 V source with 6k resistor for a very long time.



$\therefore V_c = 12V$ (for $t<0$)

→ for $t>0$ and $t<10$

$V_s = 24\,V$ and thus V_c would increase and reach 24 V eventually.

$$V_c(t) = V_c(\infty) + [V_c(0) - V_c(\infty)]e^{-t/\tau}$$

$$\tau = RC = 6\times10^3 \times 500\times10^{-6} = 3$$

$$\therefore V_c(t) = 24 - 12e^{-t/3}$$

→ For $t>10$

$$V_c(10) = 24 - 12e^{-10/3}$$
$$\underline{23.54V}$$

Since the switch is closed now and thus V_c is parallel to V_{3k} and both should be equal.

$\therefore V_c$ would discharge to 8V with $\tau = 3k \times 500\mu F = 1.5$

$$\therefore V_c(t) = 8 + (23.54-8)\exp\left(\frac{t-10}{1.5}\right)$$

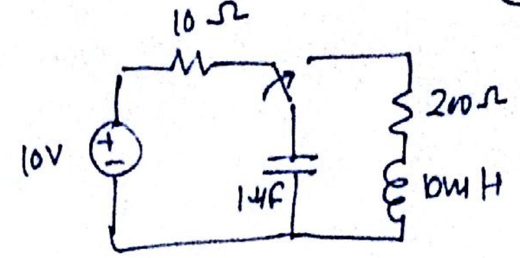
$$\cancel{8 + 15.54e^{\frac{t-10}{1.5}}}\ V$$

$$8 + 15.54e^{-\frac{(t-10)}{1.5}}\ V$$

Be Careful about $t-10$

4)

4) Before $t=0$

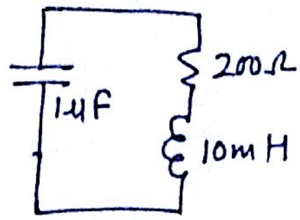


$i_L(0) = 0$

$v_c(0) = 10V$

After $t=0$ The circuit becomes a source free ckt

By KVL



$$\frac{1}{C}\int i\,dt + L\frac{di}{dt} + Ri = 0$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

$$\rho = \frac{R}{2}\sqrt{\frac{C}{L}} \quad , \quad \omega_n = \frac{1}{\sqrt{LC}}$$

$$= 1 \qquad \omega_n = \frac{1}{\sqrt{10^{-2}\times 10^{-6}}} = 100$$

assuming e^{st} = solution.

$$\therefore \Rightarrow s^2 + 2\rho\omega_n s + \omega_n^2 = 0$$

$$S = -100$$

Solution is $Ae^{st} + Bt\,e^{st}$

$$= Ae^{-100t} + Bt\,e^{-100t}$$

at $t=0$

$$i(0) = 0 = Ae^{-100\times 0} + B\cdot 0\cdot e^{-100\cdot 0}$$

$$\Rightarrow \underline{A = 0}$$

$$V_c = -(V_R + V_L)$$

$$= -\left[Ri + \frac{L\,di}{dt}\right] =$$

at $t=0$ $\quad 10 = -\left[R\,i(0) + L\frac{d\,i(0)}{dt}\right]$

$$i(t) = Ae^{-100t} + tBe^{-100t}$$

$$10 = -0 + B\left[-100A - 100B\right]$$

$$= B = \frac{1}{10}$$

$$\therefore i(t) = 0 \cdot e^{-100t} + \frac{1}{10} t e^{-100t}$$

I might have made slight mistake in this last step.
so plese cross check this step.

5) Currently this topic is not covered in any of the sections.

Best Wishes.