

Electronics I  
Mid Term Exam

Date: 16<sup>th</sup> September 2016

Time: 90 Minutes

Max Marks. 30

Notes: Attempt any 5 questions. Each question carries 6 marks.

Start every solution on fresh page.

Highlight your answers by boxing or underlining them.

Assumptions made should be written clearly.

1: Using mesh analysis (loop analysis), calculate the values of loop current  $I_1$ ,  $I_2$ ,  $I_3$ ,  $I_4$  and Voltage  $V_o$  in Figure 1.

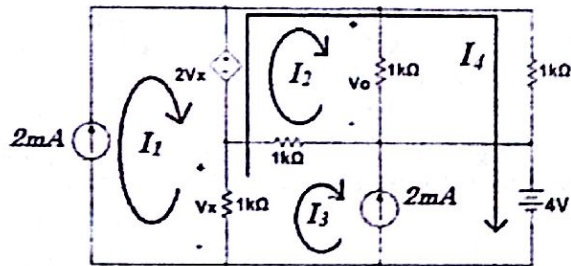


Figure 1

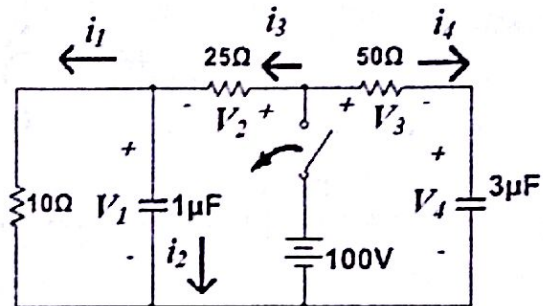


Figure 2

2: Assume that all capacitors were initially uncharged in Figure 2, and at time  $t = 0$  the switch was closed.

Find the voltages ( $V_1$ ,  $V_2$ ,  $V_3$  and  $V_4$ ) and currents ( $i_1$ ,  $i_2$ ,  $i_3$  and  $i_4$ ) indicated in the circuit:

- Immediately after the switch closes (i.e. at  $t = 0^+$  sec).
- After the switch has been closed for a long time (i.e.  $t \gg 5\tau$ ).

3: For the circuit shown in Figure 3, calculate total circuit impedance ' $Z$ ' and circuit current ' $I$ '. Also draw the voltage phasor diagram of circuit.

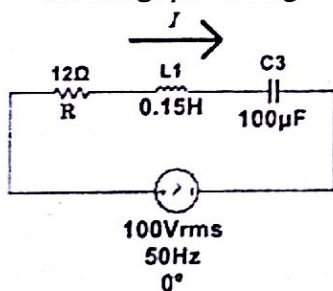


Figure 3

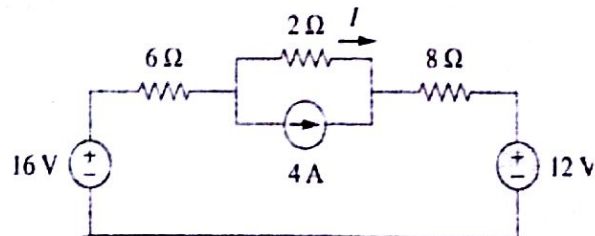


Figure 4

4: Determine the current flowing through  $2\Omega$  resistor in Figure 4, by building Thevenin's equivalent for the rest of the circuit.

5: Use Superposition theorem to solve for current  $i$  in the circuit shown in Figure 5.

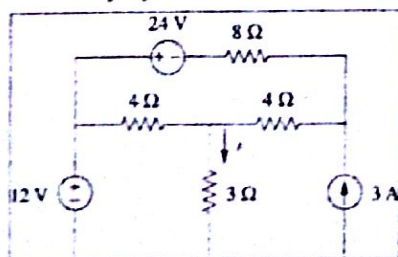


Figure 5

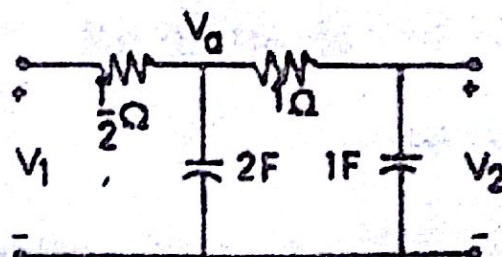


Figure 6

6: Consider the circuit given in Figure 6. Express the transfer function  $H(j\omega) = V_2(j\omega)/V_1(j\omega)$  in terms of  $R$  and  $C$ . Calculate and plot the amplitude Bode plot for the given values of the parameters.

7: Consider the circuit given in Figure 7. The switch opens at  $t = 0$ s. Find the voltage across capacitor and current in inductor as a function of time.

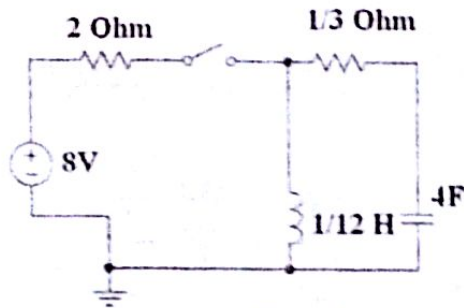


Figure 7.

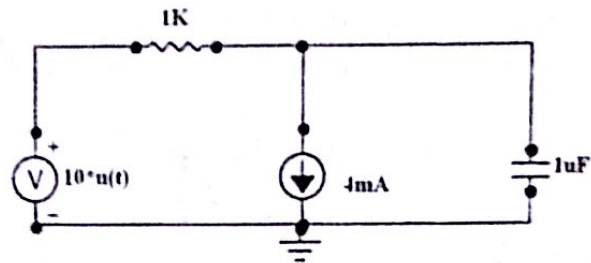
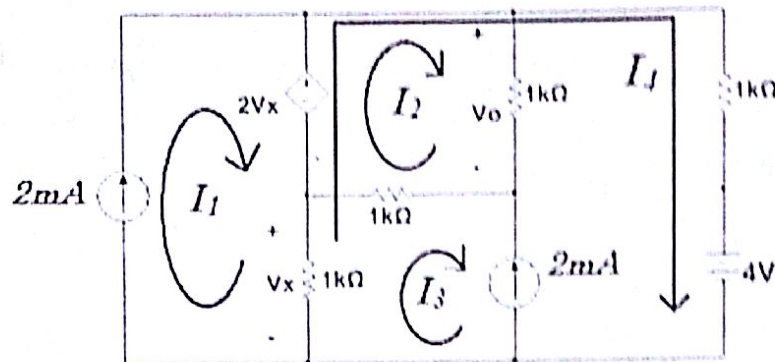


Figure 8

8: Find the voltage across capacitor, in Figure 8, as a function of time by using Laplace transform approach.

Q1. Using mesh analysis (loop analysis), calculate the values of current  $I_1$ ,  $I_2$ ,  $I_3$ ,  $I_4$  and Voltage  $V_o$ .



Sol 1:

Above network has 4 loops, and therefore four linearly independent equations are required to determine the loop currents. From the circuit provided, we can directly estimate the value of current  $I_1$  and  $I_3$  as:

$$I_1 = 2mA \quad \text{and} \quad [2] \quad (1)$$

$$I_3 = -2mA \quad (2)$$

Applying KVL in loop 2 we get:

$$-2V_x + 10^3 I_2 + (I_2 - I_3)10^3 = 0 \quad (3)$$

Similarly, applying KVL in loop 3 we get:

$$[(I_4 + I_3 - I_1) \times 1k] - 2V_x + (1k \times I_4) + 4 = 0 \quad (4)$$

From loop 3 we can estimate  $V_x$ :

$$V_x = 1k \times (I_1 - I_2 - I_4) \quad (5)$$

Substituting equation (1), (2) and (5) into equation (4) gives:

$$2 \times 10^3 I_2 + 2 \times 10^3 I_4 = 6$$

$$4 \times 10^3 I_4 = 8$$

Hence

$$I_4 = 2mA \quad [2] \quad (6)$$



Substituting equation (2) and (5) in (3) yields:

$$I_2 = 1mA$$

[1]

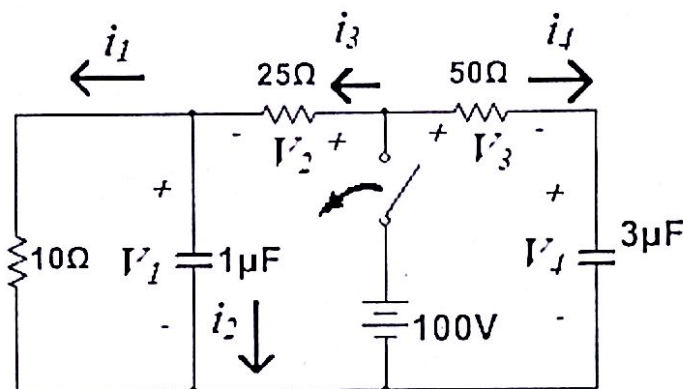
Therefore from loop 2 we can estimate  $V_0$ :

$$V_0 = I_2 \times 1k = 10^{-3} \times 10^3 = 1V$$

[21]

Q4. Assuming all the capacitors were initially uncharged, and at time  $t = 0$  the switch was closed. Find the voltages ( $V_1$ ,  $V_2$ ,  $V_3$  and  $V_4$ ) and currents ( $i_1$ ,  $i_2$ ,  $i_3$  and  $i_4$ ) indicated in the circuit for:

- Immediately after the switch closes (i.e. at  $t = 0^+$  sec).
- After the switch is closed for a long time (i.e.  $t \gg 5\tau$ ).



Ans-4

**Sol: Step 1**

At time  $t = 0^+$  Secs, capacitors has 0V across them because the capacitor voltages cannot change abruptly.

$$\text{Therefore, } V_1(0^+) = 0V \quad \text{and } V_4(0^+) = 0V$$

Along with that, capacitor acts as short circuit at time  $t=0^+$ , which results that 100V voltage source is across the resistances  $25\Omega$  and  $50\Omega$ .

$$\text{Therefore, } V_2(0^+) = V_3(0^+) = 100V$$

Three initial currents can be found from the voltages calculated above.

$$i_1(0^+) = \frac{0}{10} = 0A$$

$$i_3(0^+) = \frac{100}{25} = 4A$$

$$i_4(0^+) = \frac{100}{50} = 2A$$

[2]

Remaining initial currents can be found by applying KCL at the node at the top of  $1\mu F$

$$i_2(0^+) = i_3(0^+) - i_1(0^+) = 4 - 0 = 4A \quad [1]$$

**Step 2:**

After the switch is closed for a long time (i.e.,  $t \gg 5\tau$ ). The Capacitor voltages become constant and so the capacitor acts as open circuit.

$$\text{Therefore, } i_2(\infty) = i_4(\infty) = 0A$$

With  $1\mu F$  capacitor acting as open circuit, the  $10\Omega$  and  $25\Omega$  resistors are in series across  $100V$  voltage source.

$$\text{Therefore, } i_1(\infty) = i_3(\infty) = \frac{100}{35} = 2.86A$$

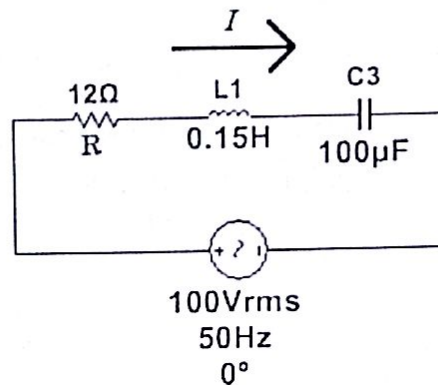
From the resistance and calculated currents, we can calculate the voltages:

$$\begin{aligned} V_1(\infty) &= 10 \times 2.86 = 28.6V \\ V_2(\infty) &= 25 \times 2.86 = 71.4V \\ V_3(\infty) &= 0 \times 50 = 0V \end{aligned} \quad [2]$$

Now, from the right hand mesh

$$V_4(\infty) = 100 - V_3(\infty) = 100 - 0 = 100V \quad [1]$$

Q5. For the circuit shown below, calculate total circuit impedance 'Z' and circuit current 'I'. Also draw the voltage phasor diagram of circuit.



Am - 5

**Sol:** Voltage Source is provided in RMS, therefore student can either solve the circuit with RMS values or they can convert the Voltage to Voltage peak to peak ( $V_{pp}$ )

$$V_{pp} = V_{RMS} \times 2.83 = 100 \times 2.83 = 283V$$

Calculating the reactance of inductor and capacitor:

$$X_L = 2\pi fL = 2 \times 3.141 \times 50 \times 0.15 = 47.13\Omega$$

and

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.141 \times 50 \times 100 \times 10^{-6}} = 31.83\Omega$$

Total circuit impedance is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{12^2 + (47.13 - 31.83)^2}$$

$$Z = \sqrt{144 + 234} = 19.4\Omega \quad [2]$$

Current I in the circuit is given by:

$$I = \frac{V_s}{Z} = \frac{283}{19.4} = 14.59A \quad [1]$$

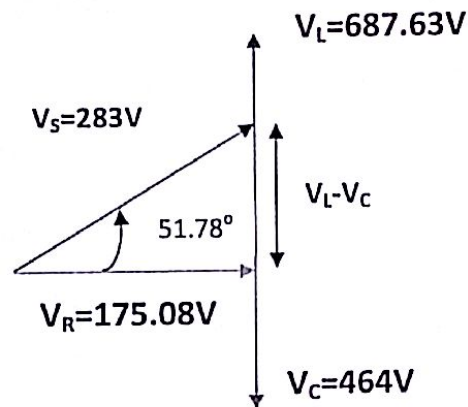
$$V_R = I \times R = 14.59 \times 12 = 175.08V$$

$$V_L = I \times X_L = 14.59 \times 47.13 = 687.63V$$

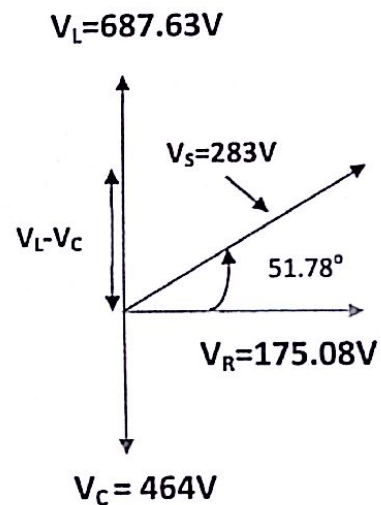
$$V_C = I \times X_C = 14.59 \times 31.83 = 464.39V$$

[1]

To plot the phasor diagram we take the current through the whole circuit as the reference, because current  $I$  is same through all components.



OR



[2]

Phase angle between the Source voltage and Current is given by:

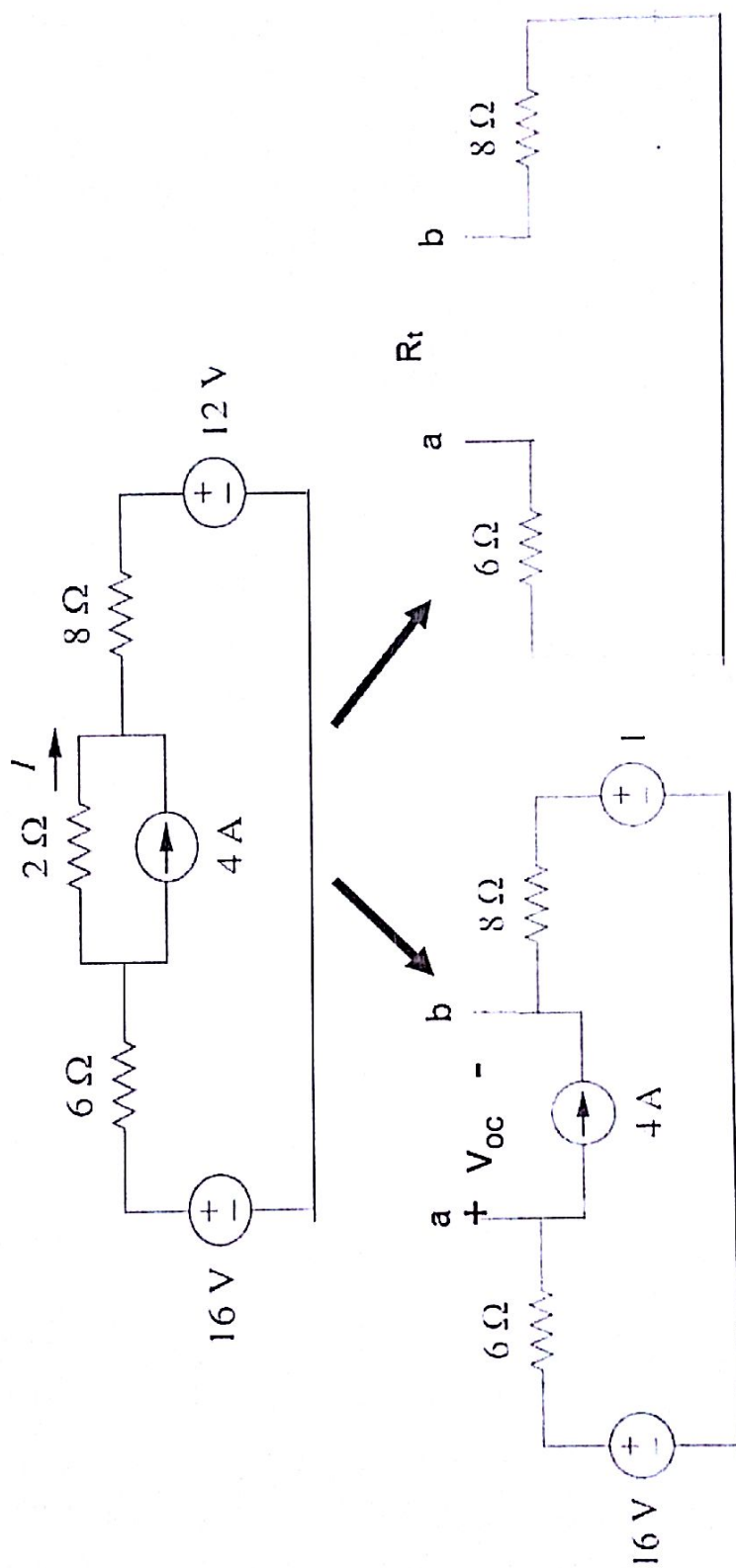
$$\cos \phi = \frac{R}{Z} = \frac{12}{19.4} = 0.618$$

or

$$\phi = \cos^{-1}(0.618) = 51.78$$



Q. 4 Determine current in  $2\Omega$  resistor by building Thevenin's equivalent for the rest of the circuit obtained by excluding it.



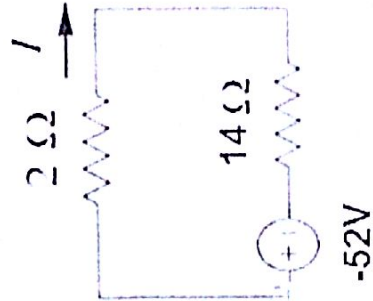
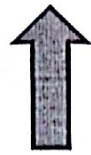
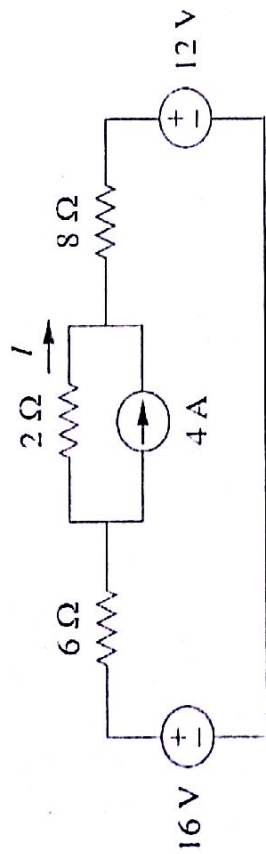
$$-16 + 4 \times 6 + v_{oc} + 4 \times 8 + 12 = 0$$

$$R_t = 14\Omega$$

[2]

$$v_{oc} = -52\text{ V} \quad [2]$$

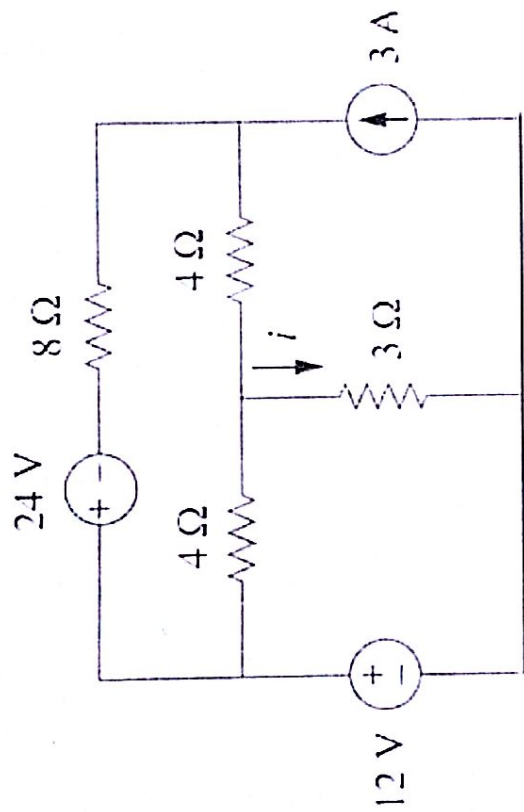




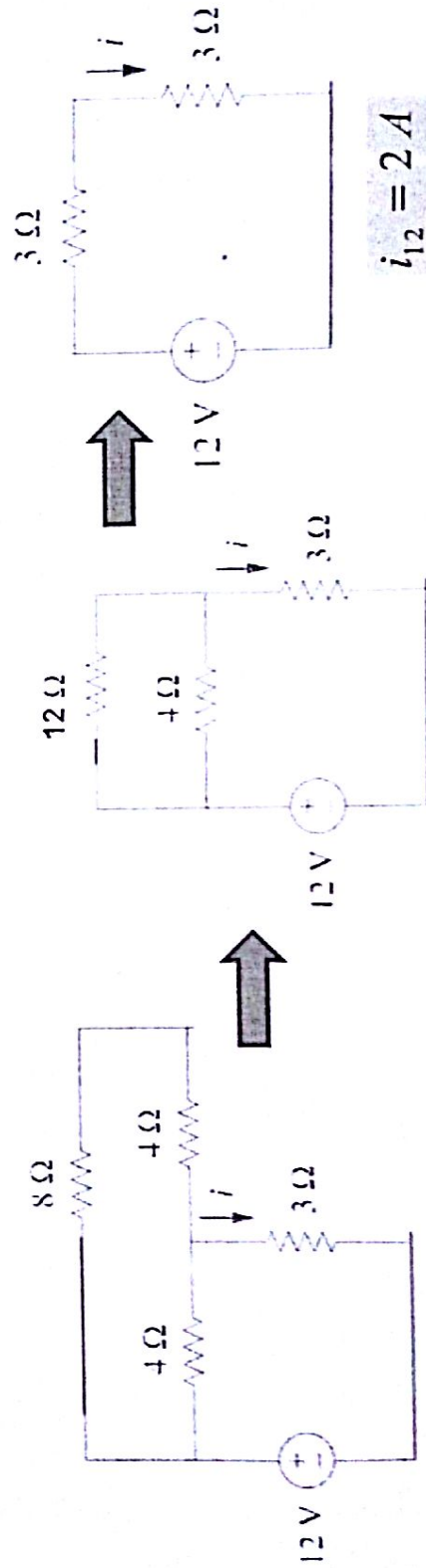
$$(2 + 14)I - (-52) = 0$$

$$I = 3.25 \text{ A} \quad [2]$$

Q. 5 Use Superposition theorem to solve for current  $i$  in the following circuit:

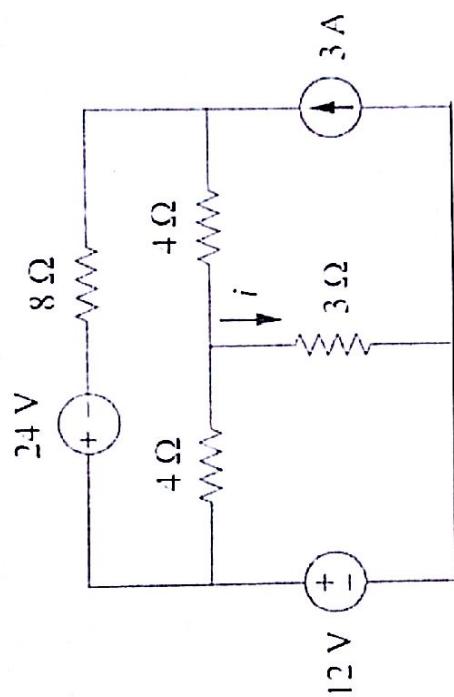


Find current due to 12V source only

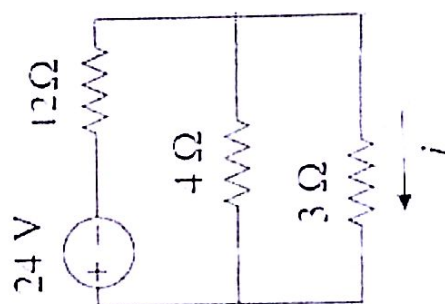
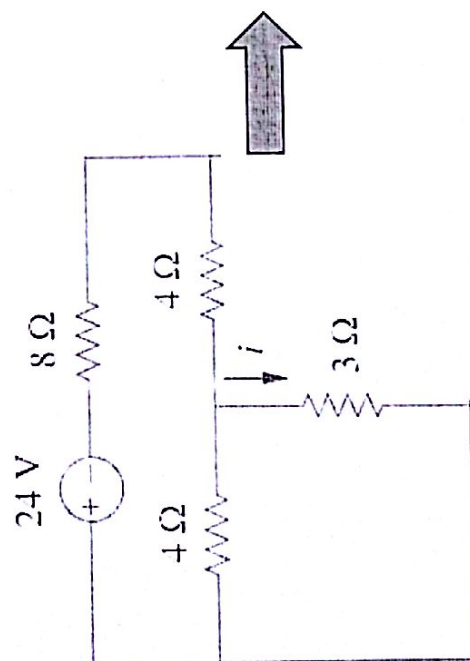


$$i_{12} = 2A$$

[2]

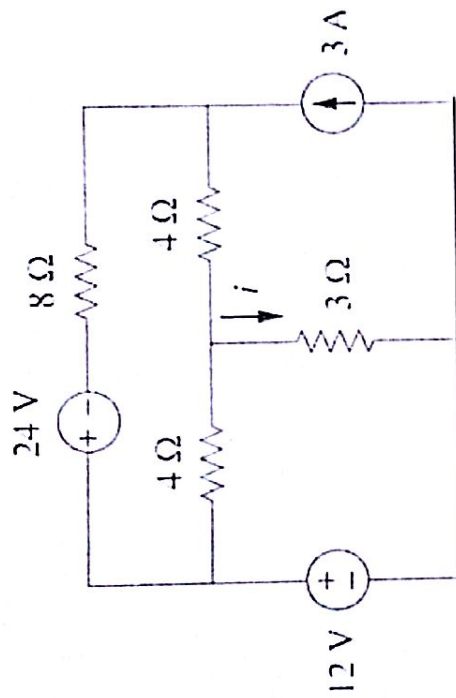


Find current due to 24V source only

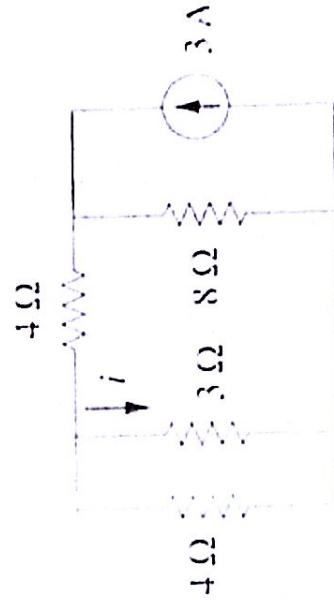
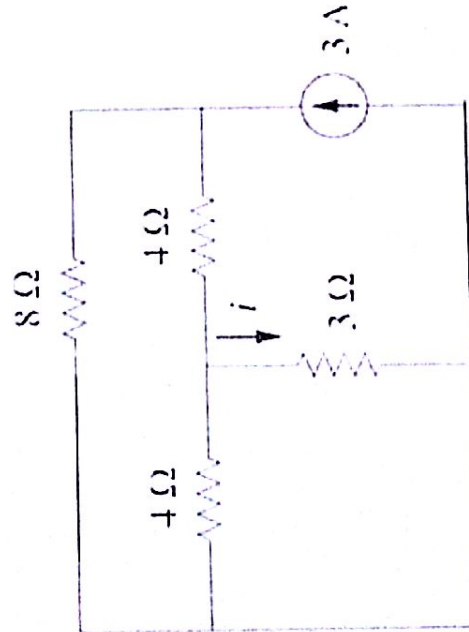


$$i_{24} = -1A$$

[2]



Find current due to 3A source only



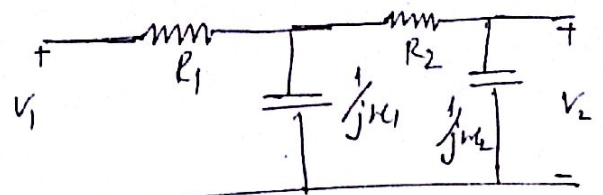
$$i_3 = 1A$$

[2]

$$\text{Net current} = i_{12} + i_{24} + i_3 = 2A$$



6



$$Z_{eq} = R_1 + \left( \frac{1}{j\omega C_1} \right) \parallel \left( R_2 + \frac{1}{j\omega C_2} \right) = R_1 + \frac{\frac{1}{j\omega C_1} \times (R_2 + \frac{1}{j\omega C_2})}{\frac{1}{j\omega C_1} + R_2 + \frac{1}{j\omega C_2}}$$

$$i = \frac{V_1}{Z_{eq}}$$

Current through output capacitor,  $i_c = i \times \frac{1/j\omega C_1}{R_2 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}}$

$$\therefore V_2 = i_c \times \frac{1}{j\omega C_2} = i \times \frac{1/j\omega C_1}{R_2 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} \times \frac{1}{j\omega C_2}$$

$$= \frac{V_1}{Z_{eq}} \times \frac{1}{j\omega C_1} \times \frac{1}{j\omega C_2} \times \frac{1}{R_2 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}}$$

[2]

$$\therefore H(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)} = \frac{1}{Z_{eq}} \times \frac{1}{j\omega C_1} \times \frac{1}{j\omega C_2} \times \frac{1}{R_2 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}}$$

$$= \frac{(\frac{1}{j\omega C_1} + R_2 + \frac{1}{j\omega C_2})}{R_1 (\frac{1}{j\omega C_1} + R_2 + \frac{1}{j\omega C_2}) + \frac{1}{j\omega C_1} (R_2 + \frac{1}{j\omega C_2})} \times \frac{1}{j\omega C_1} \times \frac{1}{j\omega C_2}$$

$$= \frac{1}{j\omega C_1 \times j\omega C_2 \left( \frac{R_1}{j\omega C_1} + R_2 + \frac{R_1}{j\omega C_2} \right) + j\omega C_2 (R_2 + \frac{1}{j\omega C_1})}$$

$$= \frac{1}{j\omega C_2 R_1 + R_1 R_2 (-\omega^2 C_1 C_2) + j\omega C_1 R_1 + j\omega C_2 R_2 + 1}$$

$$= \frac{1}{(1 - \omega^2 C_1 C_2 R_1 R_2) + j\omega (C_1 R_1 + C_2 R_2 + C_2 R_1)}$$

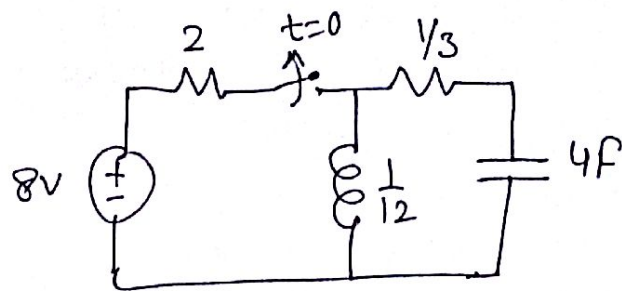
$$\left. \begin{array}{l} R_1 = \frac{1}{2}, R_2 = 1 \\ C_1 = 2, C_2 = 1 \end{array} \right\}$$

$$\therefore H(j\omega) = \frac{1}{(1 - \omega^2 \cdot 2 \cdot \frac{1}{2}) + j\omega (1 + 1 + \frac{1}{2})} = \frac{1}{1 - \omega^2 + \frac{5}{2} j\omega}$$

[2]

$$\therefore H(j\omega) = \frac{2}{-2(1 + j\frac{\omega}{2})(1 + j\frac{\omega}{2})} = \frac{1}{(1 + j\frac{\omega}{2})(1 + j\frac{\omega}{2})} \Big|_{\omega_1 = \frac{1}{2}, \omega_2 = 2} = \frac{2}{(2 + j\omega)(1 + 2j\omega)}$$

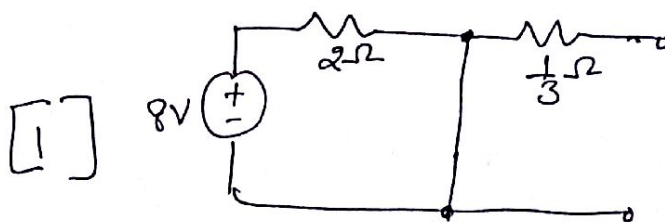
Q.7)



Before  $t=0$  the equivalent ckt is ( $t=0^-$ )

$$\therefore i_L(t) = \frac{8}{2} = 4A$$

$$V_C(t=0^-) = 0V$$



After  $t=0$  the equivalent ckt is

By KVL

$$\frac{1}{12} \frac{di}{dt} + \frac{1}{C} \int i dt + \frac{1}{3} i = 0$$

$$\frac{1}{12} \frac{d^2 i}{dt^2} + \frac{i}{4} + \frac{1}{3} \frac{di}{dt} = 0$$

$$\Rightarrow \frac{d^2 i}{dt^2} + 4 \frac{di}{dt} + 3i = 0$$

assuming  $i = e^{st}$  is solution.

$$(s^2 + 4s + 3) e^{st} = 0$$

$$\therefore s^2 + 4s + 3 = 0$$

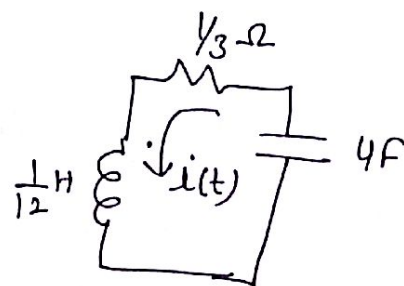
$$s_1 = -3, s_2 = -1$$

$$\therefore i(t) = A e^{-3t} + B e^{-t}$$

at  $t=0$

$$i(0) = A e^{-3 \times 0} + B e^{-0}$$

$$4 = A + B$$



[1]

(1)

(11)

[1]

(111)

(IV)

Also

$$V_R + V_L + V_C = 0$$

$$V_C = -V_R - V_L$$

at  $t=0$

$$V_C(0^+) = -V_R(0^+) - V_L(0^+)$$

$$V_C(0^+) = R \cdot i(0^+) - L \frac{di(0^+)}{dt}$$

$$0 = -\frac{1}{3} \times 4 - \frac{1}{12} \left( \frac{d}{dt} (A e^{-3t} + B e^{-t}) \right) \Big|_{t=0}$$

$$0 = -\frac{4}{3} - \frac{1}{12} (-3A - B)$$

$$-\frac{4}{3} + \frac{4}{3} = \frac{1}{4} A + \frac{B}{12}$$

$$16 = 3A + B \quad \text{--- (V)}$$

From (IV) and (V)

$$\begin{matrix} A = 6 \\ B = -2 \end{matrix} \quad \begin{bmatrix} 1 \end{bmatrix}$$

$$\therefore \boxed{i_L(t) = 6e^{-3t} - 2e^{-t}} \quad \begin{bmatrix} 1 \end{bmatrix}$$

$$V_C(t) = \frac{1}{C} \int i dt = \frac{1}{12} \int_0^{\infty} (6e^{-3t} - 2e^{-t}) dt$$

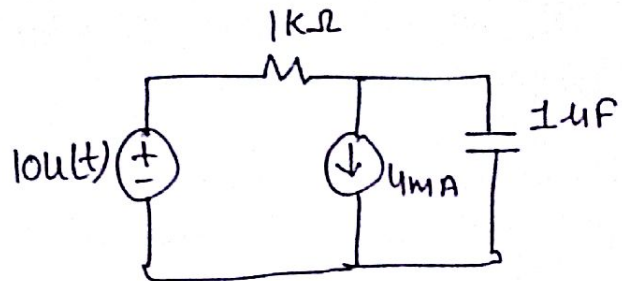
$$= \frac{1}{12} \left[ \frac{6}{-3} e^{-3t} - \frac{2}{-1} e^{-t} + V_C(0^+) \right]$$

$$\boxed{V_C(t) = -\frac{1}{6} e^{-3t} + \frac{1}{6} e^{-t}} \quad \begin{bmatrix} 1 \end{bmatrix}$$



Q.8)

KVL in time domain  
~~laplace ckt~~



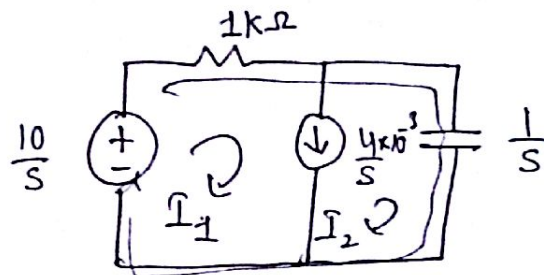
In loop 1.

$$10^3 \times i_1(t) + v_c(t) = 10u(t) \quad (i)$$

$$i_1(t) - i_2(t) = 4 \times 10^{-3} \quad (ii)$$

$$i_2 = C \frac{dv_c}{dt}$$

$$i_2 = 10^{-6} \times \frac{dv_c}{dt} \quad (iii)$$



→ In Laplace domain

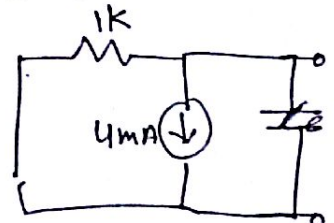
$$1000 I_1(s) + V_c(s) = \frac{10}{s}$$

$$I_1(s) - I_2(s) = \frac{4 \times 10^{-3}}{s} \quad [i]$$

$$10^{-6} [s V_c(s) - v_c(0)] = I_2(s)$$

→ To get  $v_c(0)$  the ckt before  $t=0$  is

$$\therefore v_c(0^-) = -4V$$



$$\therefore s V_c(s) + 4 = 10^6 \times I_2(s)$$

$$= 10^6 \left( I_1(s) - \frac{4 \times 10^{-3}}{s} \right)$$

$$= 10^3 \left( 1000 I_1(s) - \frac{4}{s} \right)$$

$$= 10^3 \left( \frac{10}{s} - v_c(s) - \frac{4}{s} \right)$$

$$s V_c(s) + 4 = \frac{6000}{s} - 1000 V_c(s)$$



$$s^2 V_c(s) + 4s = 6000 - 1000 \cdot s V_c(s)$$

$$s^2 V_c(s) + 1000 \cdot s V_c(s) = -4s + 6000$$

$$V_c(s) [s^2 + 1000s] = -4s + 6000$$

$$V_c(s) = \frac{-4s + 6000}{s(s+1000)} \quad [2]$$

$$= \frac{-4}{s+1000} + \frac{6000}{s(s+1000)} \quad [1]$$

taking inverse Laplace transform.

$$V_c(t) = -4e^{-1000t} + 6(1 - e^{-1000t}) \quad [1]$$

$$V_c(t) = 6 - 10e^{-1000t} \quad \checkmark$$


---

1)  $I_1 = 2\text{mA}, I_2 = 1\text{mA}, I_3 = -2\text{mA}, I_4 = 2\text{mA}$   
 $V_0 = 1\text{V}$

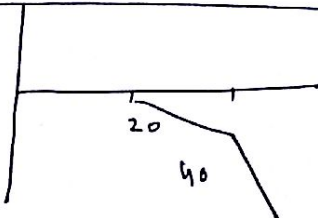
2) a)  $V_1 = 0, V_2 = 100, V_3 = 100, V_4 = 0$   
 $I_1 = 0, I_2 = 4\text{A}, I_3 = 4\text{A}, I_4 = 2\text{A}$

b)  $V_1 = 28.6, V_2 = 71.4, V_3 = 0, V_4 = 100$   
 $I_1 = 2.86\text{A}, I_2 = 0, I_3 = 2.86\text{A}, I_4 = 0\text{A}$

3)  $V_{pp} = 283\text{V}, I = 14.59\text{A}$   
 $Z = 19.4\Omega$   
 $V_R = 175.08\text{V}, V_L = 687.63\text{V}, V_C = 464.39\text{V}$   
 $\phi = 51.78$

4)  $V_{oc} = -52\text{V}, R_{TH} = 14\Omega, I = 3.25\text{A}$

5)  $I = 2\text{A}, I = -1\text{A}, I = 1\text{A} \therefore \text{Total} = 2\text{A}$

6)   $\frac{2}{(2+j\omega)(1+2j\omega)}$

7)  $S_1 = -3, S_2 = -1, A = 3$   
 $I_L(t) = 6e^{-3t} - 2e^{-t} \quad \left| \quad V_C(t) = -\frac{1}{6}e^{-3t} + \frac{1}{6}e^{-t} \right.$

8)  $V_C(s) = \frac{-4s + 6000}{s(s+1000)} = \frac{-4}{s+1000} + \frac{6000}{s(s+1000)}$   
 $\therefore V_C(t) = -4e^{-1000t} + 6(1 - e^{-1000t})$   
 $= 6 - 10e^{-1000t}$