MATH 3: End-Semester Examination: Part-A
(To be returned after 45 mins..)

R.No.: **Section:** **Name:**

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Instructions:

- Attempt all questions. Use the main sheet for rough work. Only the answers should be written on this sheet.
- ***Answers will be rejected if there is any overwriting or cutting***. No partial credits. Each question carries 4 marks.
- **Calculations (Rough work) should be clearly demonstrated in the main sheet.**

Fill in the Blanks

1. Determine the subset of points in C for which $f(z) = 2iz\bar{z}$ is analytic. $\underline{\ \phi\ }$

Ans Let $z = x + iy$. Then, $f(z) = 2iz\bar{z} = 2i(x+iy)\overline{(x+iy)} = 2i(x+iy)(x-iy) = 2i(x2+y2)$ To check if a function $f(z)$ is analytic, we apply Cauchy-Riemann equations for $f(z) = u(x,y) + iv(x,y)$, i.e.,
$u_x = v_y$ and $u_y = -v_x$ However, we have $u = 0$ and $v = 2x^2 + 2y^2$, so $u_x = 0 \neq 2y = v_y$. Obviously, they do not satisfy the C-R equations. Hence, $f(z)$ is not analytic at any point.

2. The analytic function $f(z) = \sin z$ is conformal except at $z = \{\frac{(2k+1)\pi}{2} : k = 0, 1, 2, 3...\}$

Ans $f(z) = \sin z$ is analytic on C and $f'(z) = \cos z \neq 0$ except the points $z = \frac{(2k+1)\pi}{2}$ for $k = 0, 1, 2, 3....$ Thus , function $f(z) = \sinh z$ is conformal except at $\frac{(2k+1)\pi}{2}$.

3. For the function $\dfrac{1}{Z^2 - 3z + 2}$, find out all possible regions of Taylor's and Laurent series expansions about the point $z = -1$ ________________

Ans The function is not analytic at the points z= 1 and z = 2. The distance between the point $z = -1$ and $z = 1$ is 2, and between the point $z = -1$ and $z = 2$ is 3. Thus, we consider the regions, (i) $|z+1| < 2$ (ii) $2 < |z+1| < 3$ (iii) $|z+1| > 3$.
In the region, $|z+1| < 2$ the function is analytic, hence, we obtain Tayler series expansion. In other regions $2 < |z+1| < 3$ and $|z+1| > 3$, we obtain Laurent series expansions.

4. Solution for following PDE

$$\begin{aligned} u_{tt} - 4u_{xx} &= 0, \quad 0 < x < 20,\ t > 0 \\ u(x,0) &= \sin\frac{\pi x}{2} + 2\sin 2\pi x, u_t(x,0) = 0, \quad 0 \leq x \leq 20, \\ u(0,t) &= u(20,t) = 0, \quad t \geq 0 \end{aligned}$$

is $\underline{u(x,t) = \cos \pi t \sin \frac{\pi x}{2} + 2\cos 4\pi t \sin 2\pi x}$

5. Solution of the wave equation (Hint: Duhamel's principle):

$$u_{tt} = u_{xx} + t, \qquad -\infty < x < \infty,\ t > 0,$$

$$u(x,0) = u_t(x,0) = 0, \quad -\infty < x < \infty$$

is $\underline{t^3/6}$

6. The following Laplace equation

$$\triangle u = 0\ \forall\ x \in (0,1), 0 < y < \infty,\ u(x,0) = 0\ \forall\ x \in (0,1)$$

has unique solution. TRUE/FALSE (give justification)<u>FALSE as domain is not bounded</u>.