LNMIIT
The LNM Institute of
Information Technology

THE LNM INSTITUTE OF INFORMATION TECHNOLOGY
DEPARTMENT OF MATHEMATICS
MATHEMATICS-1 & MTH102
MID TERM EXAM

Time: 90 Minutes   Date: 03/10/2019   Maximum Marks: 30

Note: You should attempt all questions. **There are total eight questions.** Marks awarded are shown next to the question. Please make an index showing the question number and page number on the front page of your answer sheet in the following format. Without proper justification of proof, answer will not be considered. **Calculator is not allowed.**

Question No.				
Page No.				

1. Test the convergence and absolute convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n - \log n}$. [4]

2. Test the convergence/divergence of the following sequence: [3]

$$x_1 = 1, \quad x_{n+1} = \frac{1}{2 + x_n} \quad \text{for } n \geq 1.$$

3. Suppose we know that $f(x)$ is continuous and differentiable on the interval $[-7, 0]$ such that $f(-7) = -3$ and $f'(x) \leq 2$ for all $x \in [-7, 0]$. Using Mean Value Theorem (MVT), find the largest possible value for $f(0)$. [4]

4. Let $f(x) = x \sin(\frac{a}{bx})$ if $x \neq 0$ and $f(x) = 0$ if $x = 0$, where a and b are non-zero fixed real numbers. Prove that $f(x)$ is continuous, but not differentiable at $x = 0$. [4]

5. Let $f : [0, 2] \to \mathbb{R}$ such that [4]

$$f(x) = \begin{cases} 3, & \text{if } 0 \leq x < 1 \\ 2, & \text{if } x = 1 \\ 1, & 1 < x \leq 2 \end{cases}$$

For each $\epsilon > 0$, find a partition P of $[0, 2]$ such that $U(P, f) - L(P, f) < \epsilon$.

6. Consider $a_n := \sum_{i=1}^{n} \frac{i}{n^2 + i^2}$ for $n \in \mathbb{N}$. Find $\lim_{n \to \infty} a_n$. [3]

7. Let $f(x) := (x^2 - 1)/(x + 2)$ for $x \in \mathbb{R}$ with $x \neq -2$. Determine the intervals, where f is convex and concave. Moreover, determine all the points of local maxima, local minima and point of inflection. [5]

8. Find the greatest and the least values of $f : [0, 2] \to \mathbb{R}$ where $f(x) = 4x^3 - 8x^2 + 5x$. [3]

End of paper