The LNM Institute of Information Technology, Jaipur
Mid-semester Examination, Autumn Semester (2017-18)
Signals and Systems (ECE 219)

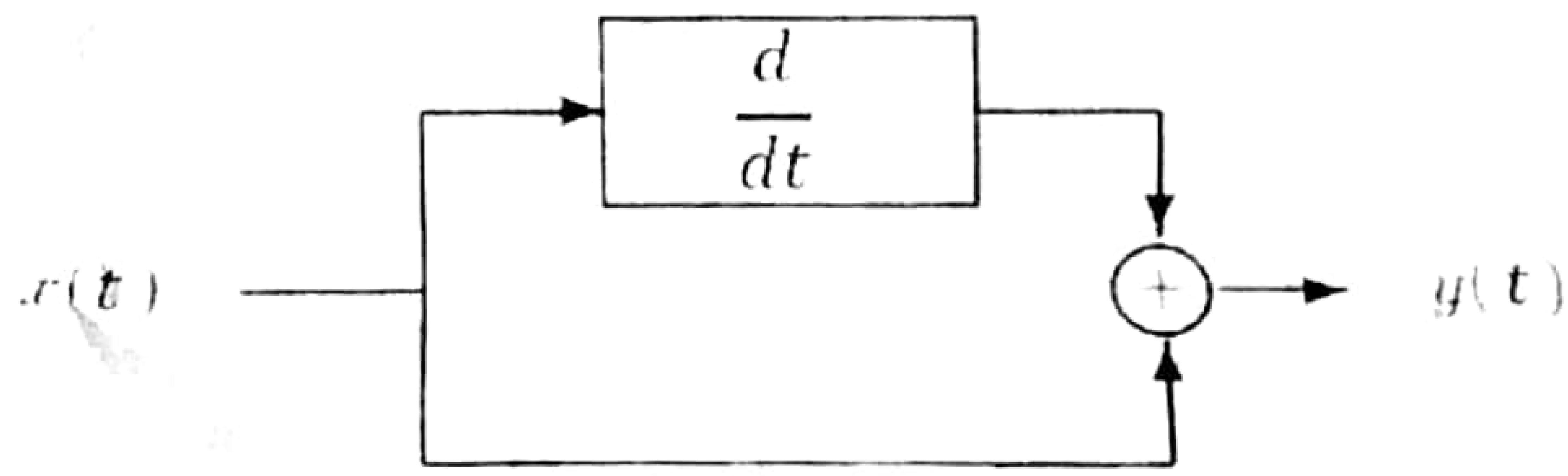
Time: 90 Min. M.M.: 25

Instructions to students: All questions are compulsory. Do the questions in order and all parts of the questions should be at the same place.

1. a) Plot the waveform of $x(t) = \frac{d}{dt}[-u(t+1) + r(t+1) - r(t-1) - u(t-1)]$.
   b) Find odd and even component of $x(t) = \begin{cases} Ae^{-at}, & t > 0 \\ 0, & t < 0 \end{cases}$.
   c) Find the autocorrelation function $R_{xx}(\tau)$ of $x(t) = A\sin(wt + \Phi)$. Utilizing $R_{xx}(\tau)$ expression, find the power of signal $x(t)$. [*Hint: use* $\tau = 0$ *in second part*]
   d) State the Parseval's theorem mathematically. [1+1+2+1]

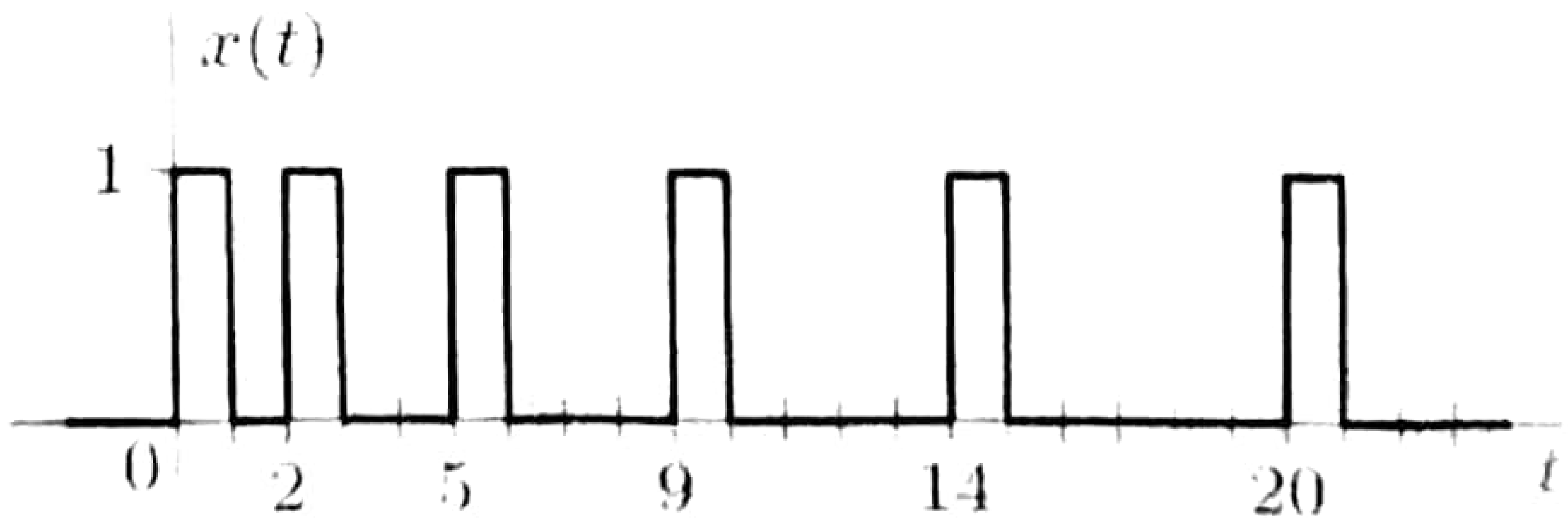
2. Consider two systems are connected in cascade. One have the impulse response $h_1(t) = e^{-t}u(-t)$ and the other system is shown in Fig. 1. Calculate the overall impulse response of the cascaded system. [5]

Fig. 1



3. A binary signal $x(t)$ with $x(t) = 0$, $t < 0$ is shown in Fig. 2 For positive time, $x(t)$ toggles between one and zero as follows: one for 1 second, zero for 1 second, one for 1 second, zero for 2 seconds, one for 1 second, zero for 3 seconds, and so forth. That is, the "on" time is always one second but the "off" time increases by one second between each toggle. Determine the energy and power of $x(t)$.
[2 + 3]

Fig. 2



4. Find the convolution sum between two sequences x[n] and g[n], shown in Fig. 3. Also, plot the convolved sequence. [5]

Fig. 3

5. Plot an impulse train of amplitude 1 and period T_0 with proper labelling. Find the exponential Fourier series of impulse train and plot its frequency response. [5]

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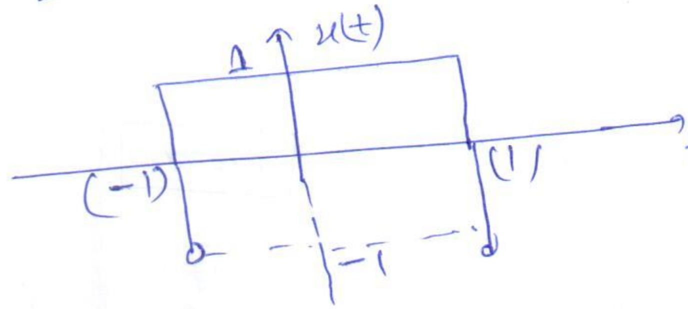
$w = 2\pi f$

Mid Sem. Exam. 2017. (Solution)

Q.1.

(a)

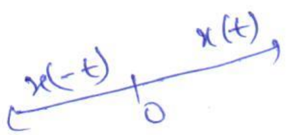
$$x(t) = [u(t+1) - u(t-1)] - \delta(t+1) - \delta(t-1)$$ 0.5



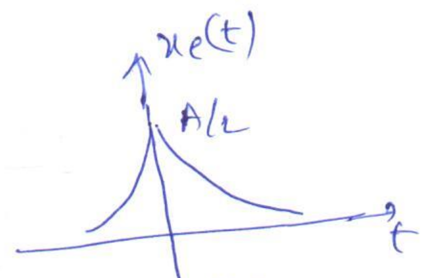
0.5

(b)

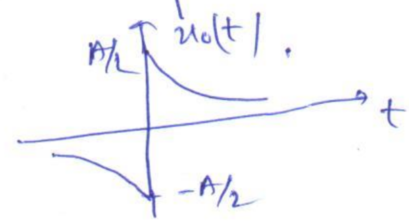
$$x(t) = \begin{cases} Ae^{-\alpha t} & t>0 \\ 0 & t<0 \end{cases}$$



$$x(-t) = \begin{cases} Ae^{\alpha t} & t<0 \\ 0 & t>0 \end{cases}$$



0.5 $$x_e(t) = \begin{cases} \tfrac{1}{2} Ae^{-\alpha t} & t>0 \\ \tfrac{1}{2} Ae^{\alpha t} & t<0 \end{cases}$$



0.5 $$x_o(t) = \begin{cases} \tfrac{A}{2} e^{-\alpha t} & t>0 \\ -\tfrac{A}{2} e^{\alpha t} & t<0 \end{cases}$$

(c)

$$R_{xx}(\tau) = \frac{A^2}{2} \cos(\omega\tau)$$ 1

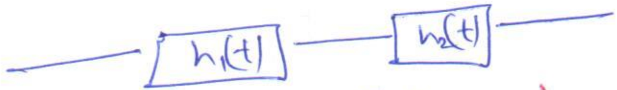
at $\tau = 0$ $\quad P_x = R_{xx}(0) = \frac{A^2}{2}$ 1

(d)

$$P_x = D_0^2 + 2\sum_{n=1}^{\infty} |D_n|^2$$ 1

Q.2.

$$h_2(t) = \delta(t) + \frac{d}{dt}[\delta(t)]$$ 1



1

$$h(t) = h_1(t) * h_2(t) = e^{-t}u(-t) * \delta(t) + e^{-t}u(-t) * \frac{d}{dt}[\delta(t)].$$

$= e^{-t}.u(-t) * \delta(t) + \delta(t) * \frac{d}{dt}[e^{-t}. u(t)]$ 1

$= e^{-t}.u(-t) + \delta(t)[-e^{-t} u(-t) + e^{-t}\delta(-t)]$

$= e^{-t}u(-t) - e^{-t}u(-t) - e^{-t}\delta(-t)$

$= -e^{-t}\delta(-t)$

$= -\delta(t)$

2

Q.3. ∞ No. of Pulses $\therefore$ $\boxed{E = \infty}$ 2.

for (1)	for (0)
1	1
1	2
1	3
1	4
⋮	⋮
N	N
(AP)	(AP)

$P = \lim_{N \to \infty} \left(\frac{2N}{N^2 + 3N} \right)$

$\boxed{P = 0}$ 3

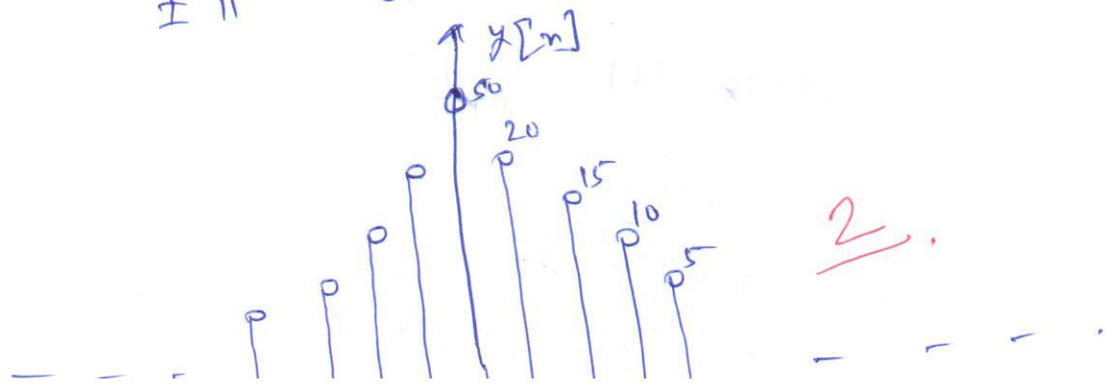
Q.4. $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, g[n-k]$.

$y[n] = 0 \quad 5 \le |n| \le 19 \;\&\; |n| \ge 15$

n	$y[n]$
0	$5\times5 + 5\times5 = 50$
±1	$5\times4 + 0 = 20$
±2	$5\times3 + 0 = 15$
±3	$5\times2 + 0 = 10$
±4	$5\times1 + 0 = 5$
±5	0
⋮	⋮
±10	$0\times0 + 5\times1 = 5$
±11	$0\times0 + 5\times4 = 20$

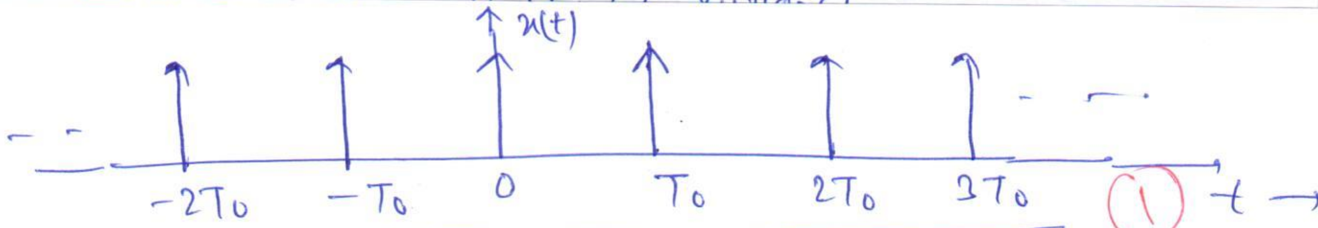
3

n	$y[n]$
±12	$0\times0 + 5\times3 = 15$
±13	$0\times0 + 5\times2 = 10$
±14	$0\times0 + 5\times1 = 5$
±15	0
⋮	
±18	0.



2.

Q.5



1

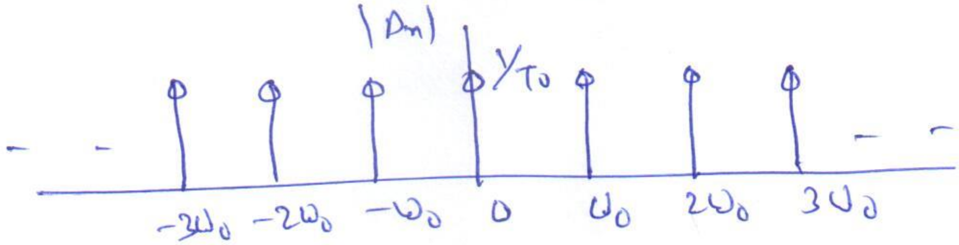
$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)}$$

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T_0}$$

$$\Rightarrow \quad D_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{-jn\omega_0 t}\, dt.$$

$$\boxed{D_n = 1/T_0}$$ 1

$$x(t) = \frac{1}{T_0}\sum_{n=-\infty}^{\infty} e^{jn\omega_0 t}.$$ 1



2.