Algorithm To Decide Whether CFL Is Finite

Theory of Automata & Computation

Context Free Language-

Before you go through this article, make sure that you have gone through the previous article on Context Free Language.

We have discussed-

- Context free language is generated using a context free grammar.
- Each context free language is accepted by a Pushdown automaton.

In this article, we will discuss a decision algorithm of CFL.

Algorithm To Decide Whether CFL Is Finite Or Not-

For a given CFG, there exists an algorithm to decide whether its language is finite or not.

Step-01:

Reduce the given grammar completely by-

- Eliminating ∈ productions
- Eliminating unit productions
- Eliminating useless productions

Also Watch- How To Reduce Grammar?

Step-02:

- Draw a directed graph whose nodes are variables of the given grammar.

- There exists an edge from node A to node B if there exists a production of the form $A \rightarrow \alpha B \beta$.

Now, following 2 cases are possible-

Case-01:

- Directed graph contains a cycle.
- In this case, language of the given grammar is infinite.

Case-02:

- Directed graph does not contain any cycle.
- In this case, language of the given grammar is finite.

Also Read- Algorithm To Decide Whether CFL Is Empty

PRACTICE PROBLEMS BASED ON DECIDING WHETHER CFL IS FINITE-

Problem-01:

Check whether language of the following grammar is finite or not-

$$S \rightarrow AB / a$$

$$A \rightarrow BC / b$$

$$B \rightarrow CC / c$$

Solution-

Step-01:

The given grammar is already completely reduced.

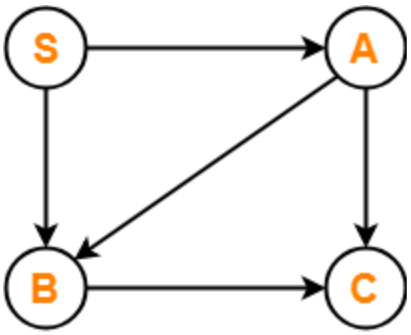
Step-02:

We will draw a directed graph whose nodes will be S , A , B , C.

Now,

- Due to the production S → AB, directed graph will have edges S → A and S → B.
- Due to the production A → BC, directed graph will have edges A → B and A → C.
- Due to the production B → CC, directed graph will have edge B → C.

The required directed graph is-



Directed Graph

Clearly,

- The directed graph does not contain any cycle.
- Therefore, language of the given grammar is finite.

Problem-02:

Check whether language of the following grammar is finite or not-

S → XS / b

$$X \rightarrow YZ$$

$$Y \rightarrow ab$$

$$Z \rightarrow XY$$

Solution-

Step-01:

The given grammar is already completely reduced.

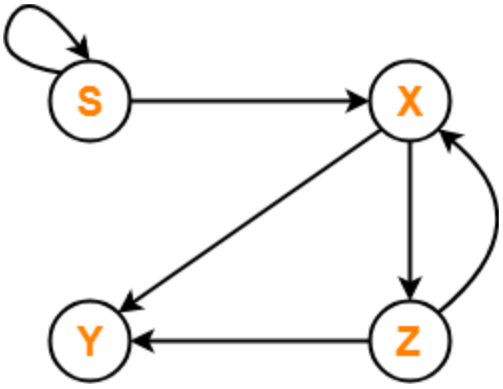
Step-02:

We will draw a directed graph whose nodes will be S , X , Y , Z.

Now,

- Due to the production S → XS / b, directed graph will have edges S → X and S → S.
- Due to the production X → YZ, directed graph will have edges X → Y and X Z.
- Due to the production Z → XY, directed graph will have edges Z → X and Z Y.

The required directed graph is-



Clearly,

The directed graph contain cycles.

Therefore, language of the given grammar is infinite.