Equivalence of Two Finite Automata:

- Two finite automata over ∑ are equivalent if they accept the same set of strings over ∑

- When the two finite automata are not equivalent there is some string w over ∑ satisfying the following

  One automaton reaches a final state on application of 'w' whereas the other automaton reaches a non-final state.

- Comparison method is used to test the equivalence of two finite automata over ∑

Comparison Method:

Let M and M' are two finite automata over ∑ we construct comparison table consisting of n+1 columns where n is the number of input symbols

The first column consists of pair of vertices of the form (q,q') where q

∈ M and q' M'. if(q,q') appear in some row of the first column then corresponding entry in the a-column(a ∈∑) is (q_a,q'_a), where q_a and $q'_a$are reachable from q,q' respectively on application of a.

The comparison table is constructed by starting with the pair of initial vertices q_{in}, q'_{in} of M,M' in the first column.

The first elements in the subsequent columns are (q_a,q'_a), where q_a and $q'_a$are reachable from q_{in}, q'_{in}respectively.

We repeat the construction by considering the pairs in the second and subsequent columns which are not in the first column

The row wise construction is repeated.
There are two cases:

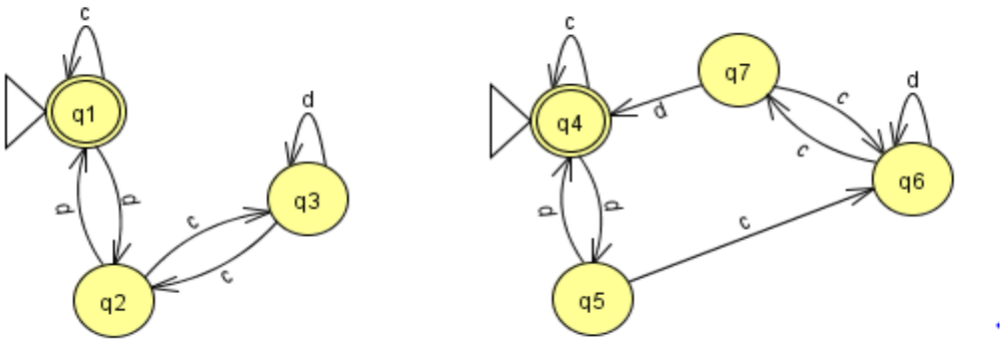
Case 1: if we reach a pair (q,q') such that q is a final state of M, and q' is a non final state of M' or vice versa, we terminate the construction and conclude that M and M' are not equivalent.

Case 2: Here the construction is terminated when no new element appears in the second and subsequent columns which are not in the first column.

In this case we conclude that M and M' equivalent.

Example:

Test whether the following DFA's are equal or not



Comp

arison Method:

(q,q')	(q_c,q'_c)	(q_d,q'_d)
(q_1,q_4)	(q_1,q_4)	(q_2,q_5)
(q_2,q_5)	(q_3,q_6)	(q_1,q_4)
(q_3,q_6)	(q_2,q_7)	(q_3,q_6)
(q_2,q_7)	(q_3,q_6)	(q_1,q_4)

The initial state in M and M’ are q_1 and q_4 respectively. Hence the first element of the first column in the comparison table must be (q_1,q_4). The first q_1 andq_4 are c-reachable from the respective initial states.

As we do not get a pair (q,q’) where q is a final state and q’ is a non final state at every row, we proceed until all the elements in the second and third columns are also in the first column

Therefore M and M’ are equivalent