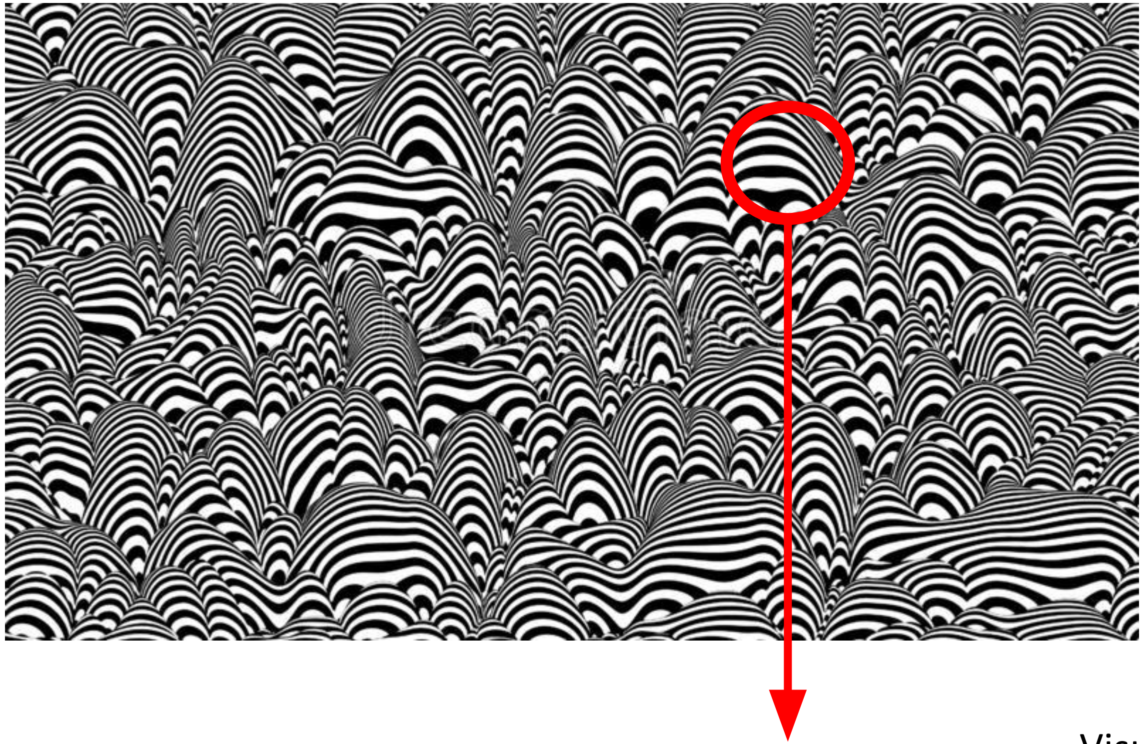
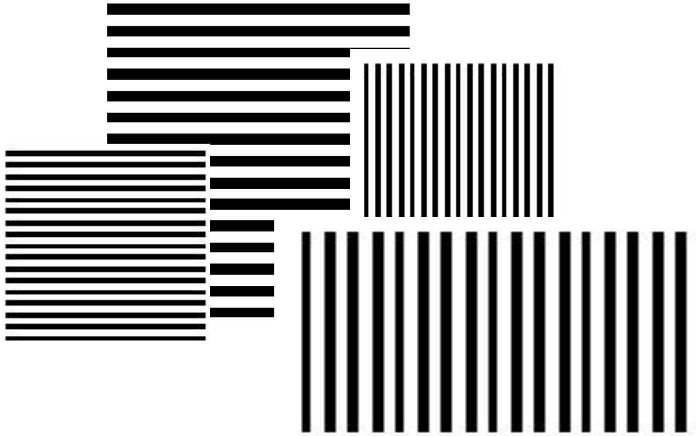
Image Processing - I

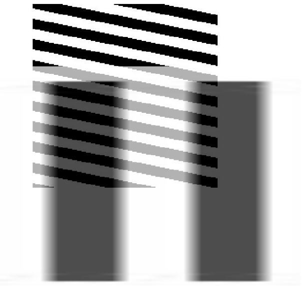
SAUMYA JETLEY [March 2022]

❑ Image Composition – Black and White Images



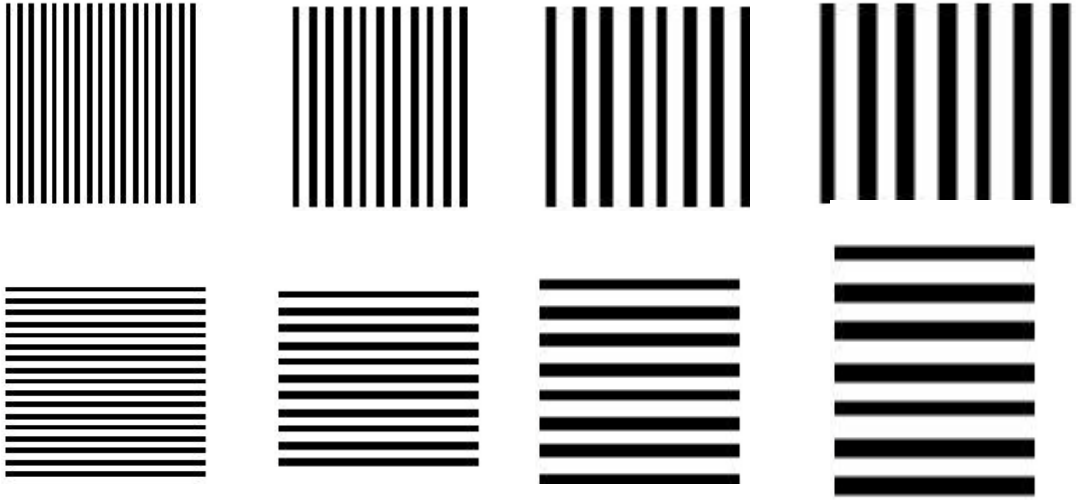
Images as juxtapositions of alternating black-n-white stripes of different frequencies.



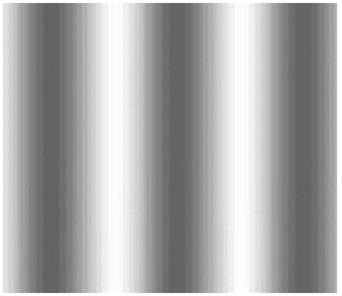


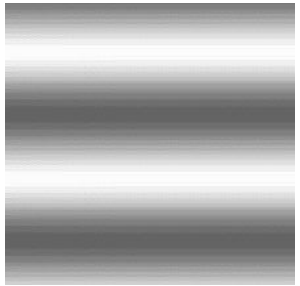
Visualise as:
White = see through
Black = Opaque

❑ Image Composition – Gray Scale

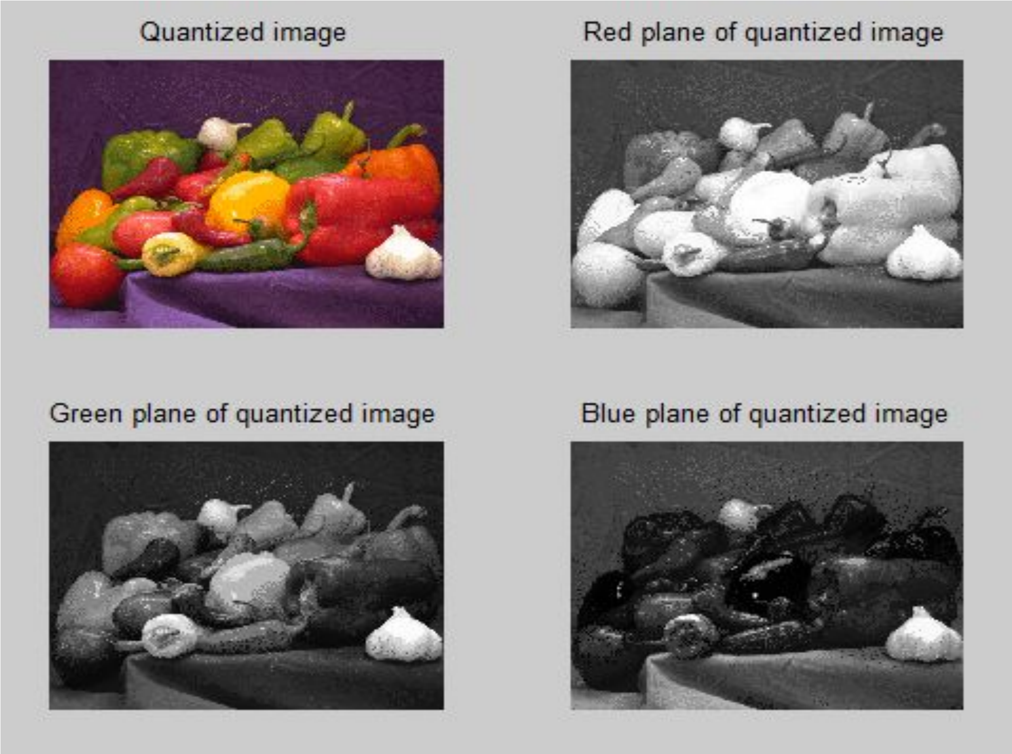


Vertical & Horizontal bands with their analog equivalent
[Sine and Cosine Waves]





❑ Image Composition – RGB images



❑ Fourier Transform

In 1D:

Courtesy of Lucas V. Barbosa via wikipedia

- **Fourier series**: Defines a periodic function, over a cycle's interval, as a weighted combination of harmonic sinusoids.

- **Fourier transform**: Generalises this analysis over 'unbounded' intervals; yields the weights of the harmonics.

Fourier Transform

Excavating the image to find these wave patterns !

In 2D:

$$F(k,l) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} f(i,j)\, e^{-\iota 2\pi \left(\frac{ki}{N} + \frac{lj}{N}\right)}$$

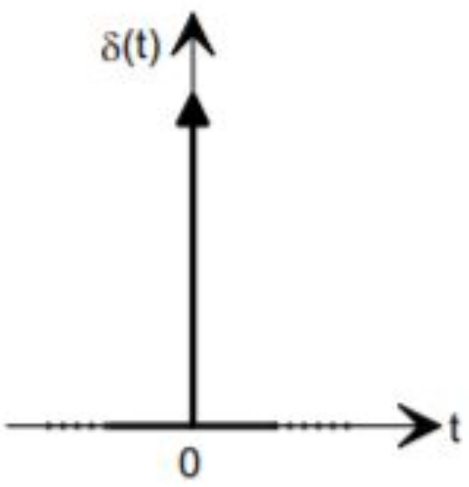
Euler's formula here

- Can be seen as an integral transform, where a function is mapped from original function space to a new function space using integration operation
- What else is an integral transform ? **Convolution!**

❑ Mathematical background !

1. The **dirac delta (impulse)** function,

- Non-physical, singularity function
- $$\delta(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ \text{undefined} & \text{at } x = 0 \end{cases}$$
- Such that : $$\int_{-\infty}^{\infty} \delta(x)dx = 1,$$



Courtesy of http://web.mit.edu/2.14/www/Handouts/Convolution.pdf

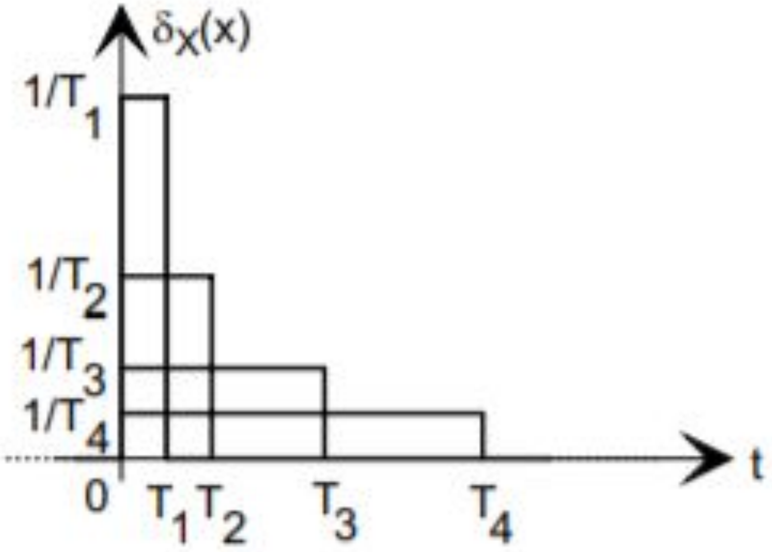
❑ Mathematical background !

2. Approximated using **Unit Pulses**,

- $$\delta_T(t) = \begin{cases} 0 & \text{for } t \le 0 \\ 1/T & 0 < t \le T \\ 0 & \text{for } t > 0. \end{cases}$$

- where : $$\delta(t) = \lim_{T \to 0} \delta_T(t).$$

- The impulse function is also approximated by the limiting forms of various other functions such as:
  o Triangular
  o Gaussian
  o Sinc (sin(x)/x)



Unit pulses of different durations; the magnitude adjusts to the duration to maintain unit area under the curve

Courtesy of http://web.mit.edu/2.14/www/Handouts/Convolution.pdf

❑ Mathematical background !

3. Property of **sifting,**

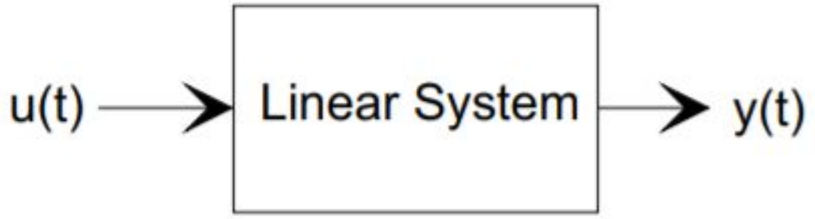
- The value of the integrand at the point of occurence

- $$\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a)$$

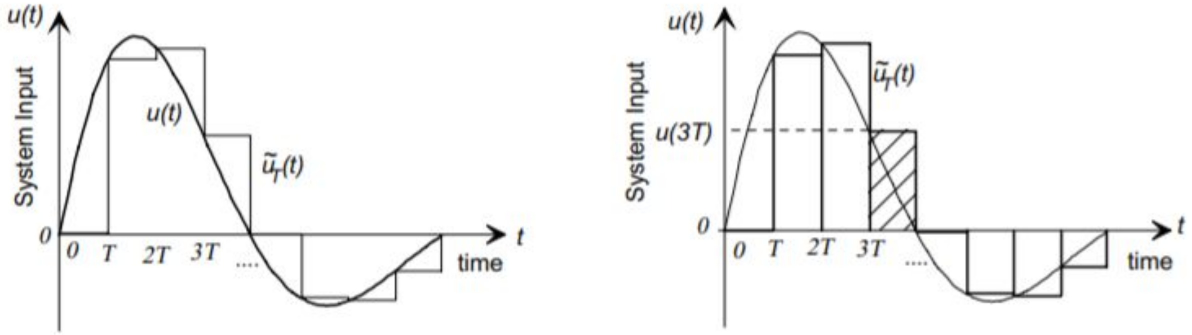
Courtesy of http://web.mit.edu/2.14/www/Handouts/Convolution.pdf

❑ Mathematical background !

4. **Convolution**,



A linear time invariant system, input and response

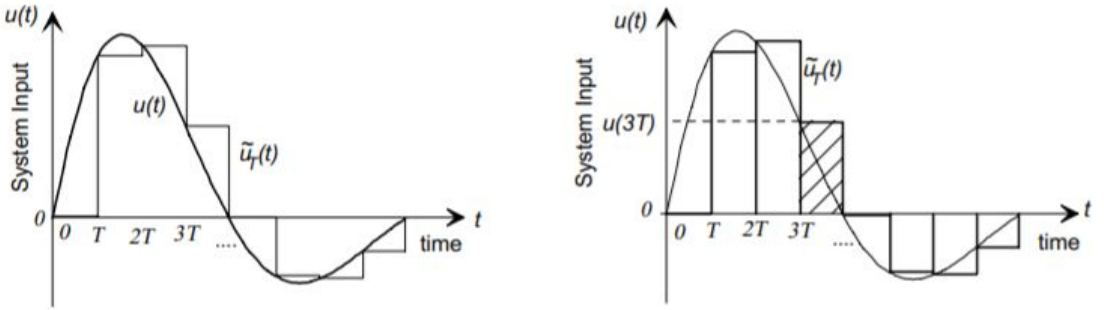


An arbitrary continuous input, and its staircase approximation

Courtesy of http://web.mit.edu/2.14/www/Handouts/Convolution.pdf

❑ Mathematical background !

4. **Convolution**,



An arbitrary continuous input, and its staircase approximation

The staircase approximation may be viewed as a sum of overlapping unit-pulses of different magnitudes:

$$\tilde{u}_T(t) = \sum_{n=-\infty}^{\infty} p_n(t)$$

where,

$$p_n(t) = \begin{cases} u(nT) & nT \leq t < (n+1)T \\ 0 & \text{otherwise} \end{cases}$$

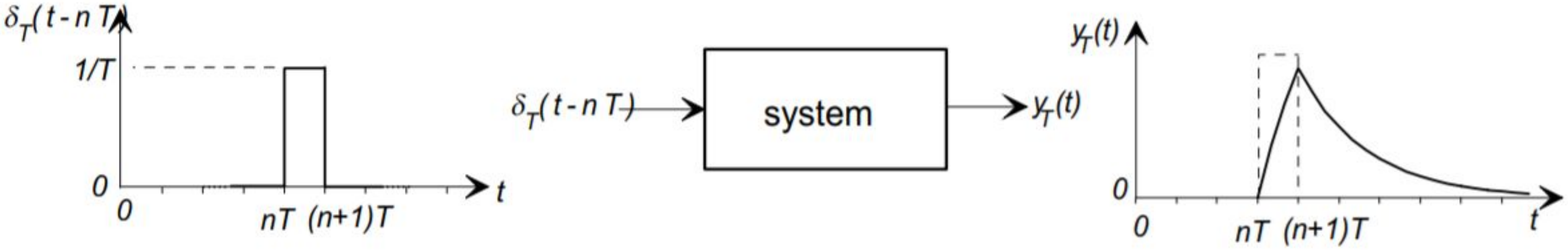
$$\tilde{u}_T(t) = \sum_{n=-\infty}^{\infty} u(nT)\delta_T(t - nT)T.$$

Courtesy of http://web.mit.edu/2.14/www/Handouts/Convolution.pdf

❑ Mathematical background !

4. **Convolution**,

assume that the system response to $\delta_T(t)$ is a known function and is designated $h_T(t)$



$$\tilde{u}_T(t) = \sum_{n=-\infty}^{\infty} u(nT)\delta_T(t - nT)T.$$

[system]

$$\tilde{y}_T(t) = \sum_{n=-\infty}^{\infty} u(nT)h_T(t - nT)T$$

Courtesy of http://web.mit.edu/2.14/www/Handouts/Convolution.pdf

- Mathematical background !

4. **Convolution**,

$$\tilde{y}_T(t) = \sum_{n=-\infty}^{\infty} u(nT)h_T(t-nT)T$$

Impulse response is 0 for t<0, so future components of the input do not contribute to the sum

$$\tilde{y}_T(t) = \sum_{n=-\infty}^{N} u(nT)h_T(t-nT)T$$

In the limit of T → 0

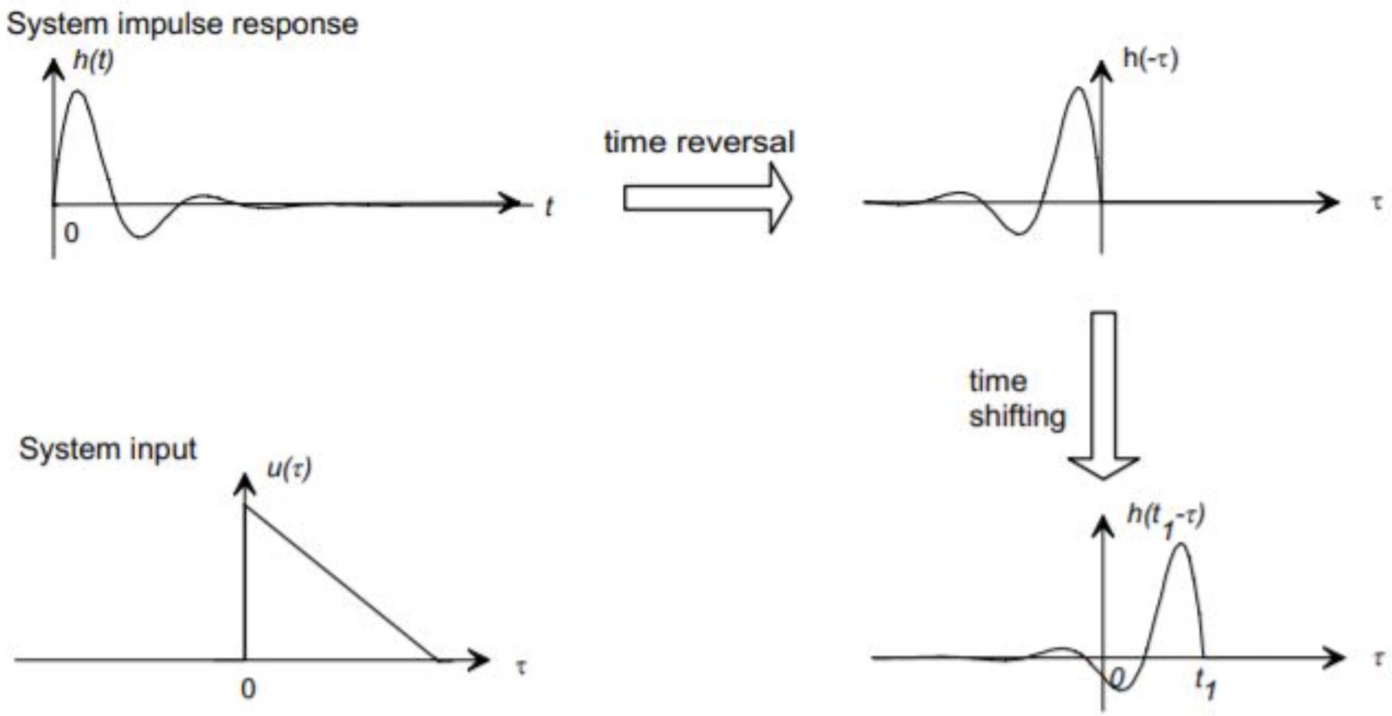
$$\begin{aligned} y(t) &= \lim_{T\to 0} \sum_{n=-\infty}^{N} u(nT)h_T(t-nT)T \\ &= \int_{-\infty}^{t} u(\tau)h(t-\tau)d\tau \end{aligned}$$

$$\boxed{y(t) = u(t) \otimes h(t) = \int_{-\infty}^{t} u(\tau)h(t-\tau)d\tau.}$$

Courtesy of http://web.mit.edu/2.14/www/Handouts/Convolution.pdf

❑ Mathematical background !

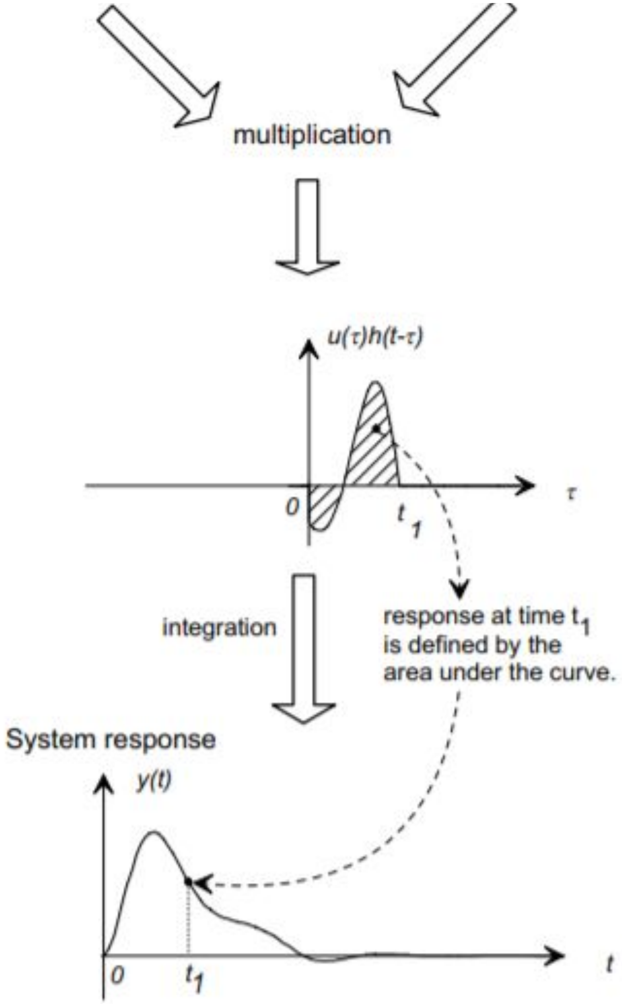
4. **Convolution** - Example visualisation



Courtesy of http://web.mit.edu/2.14/www/Handouts/Convolution.pdf

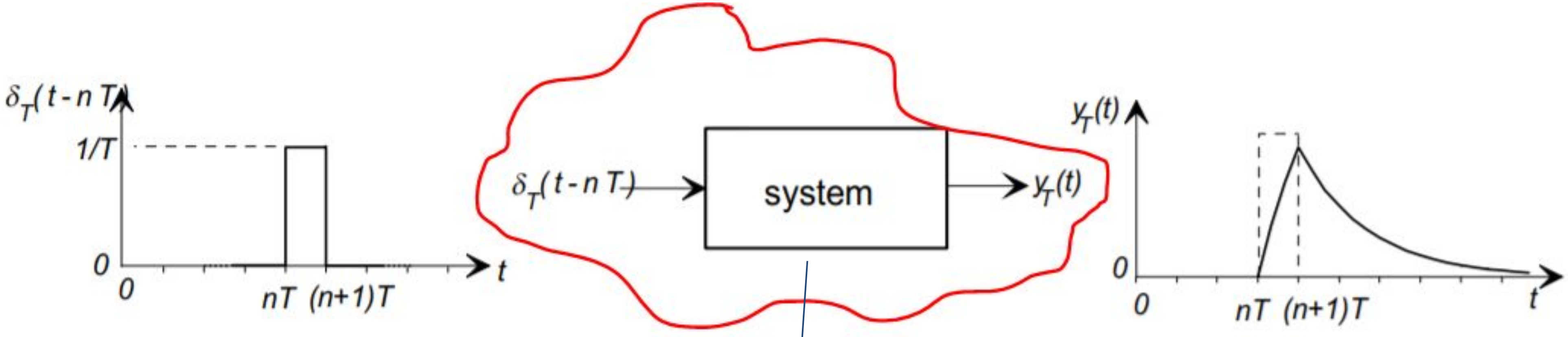
❑ Mathematical background !

4. **Convolution** - Example visualisation



Courtesy of http://web.mit.edu/2.14/www/Handouts/Convolution.pdf

- Fourier Transform and Convolution: Making connections !

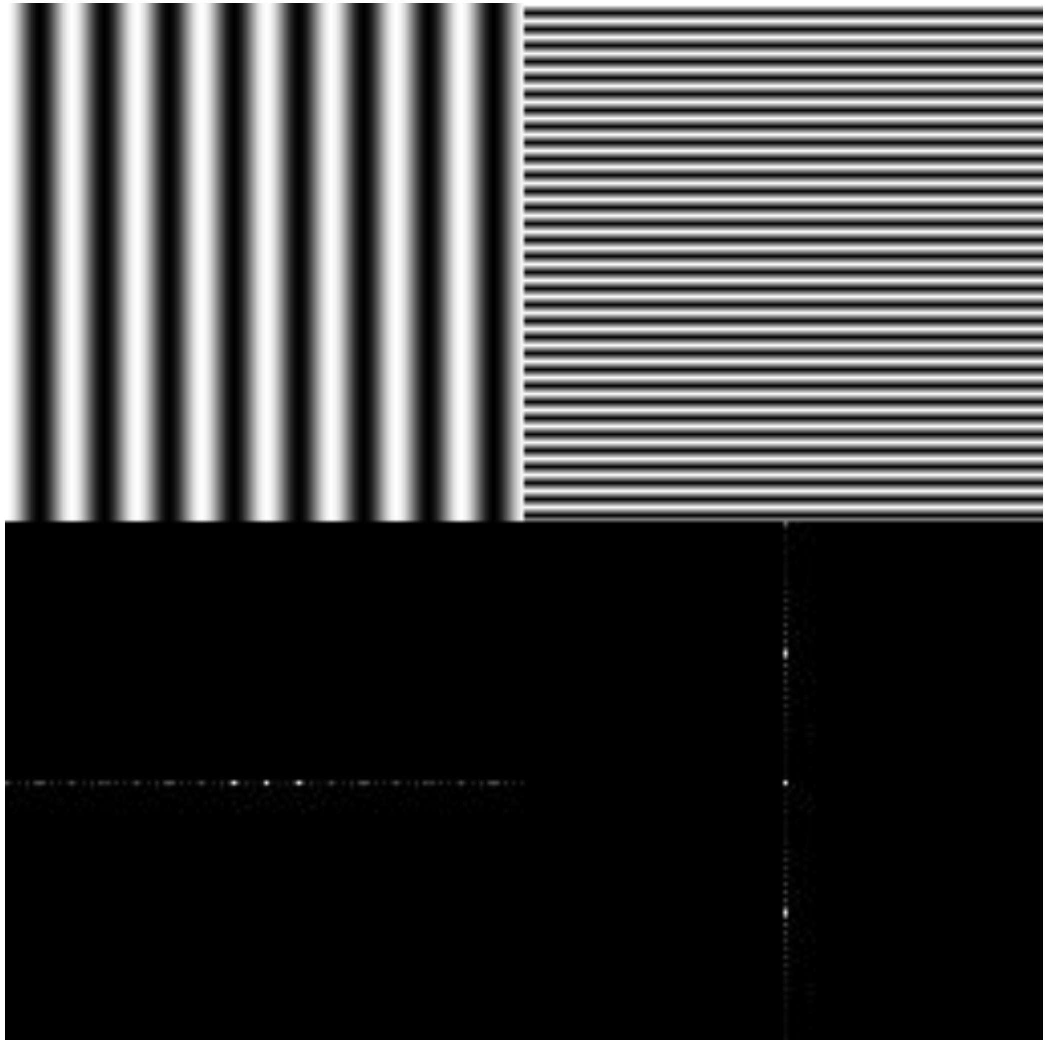


$$X_{1/T}(f) = \mathcal{F}\left\{\sum_{n=-\infty}^{\infty} x[n]\cdot\delta(t-nT)\right\}.$$

$$\left\{\sum_{n=-\infty}^{\infty} x[n]\cdot e^{-i2\pi fTn}\right\}.$$

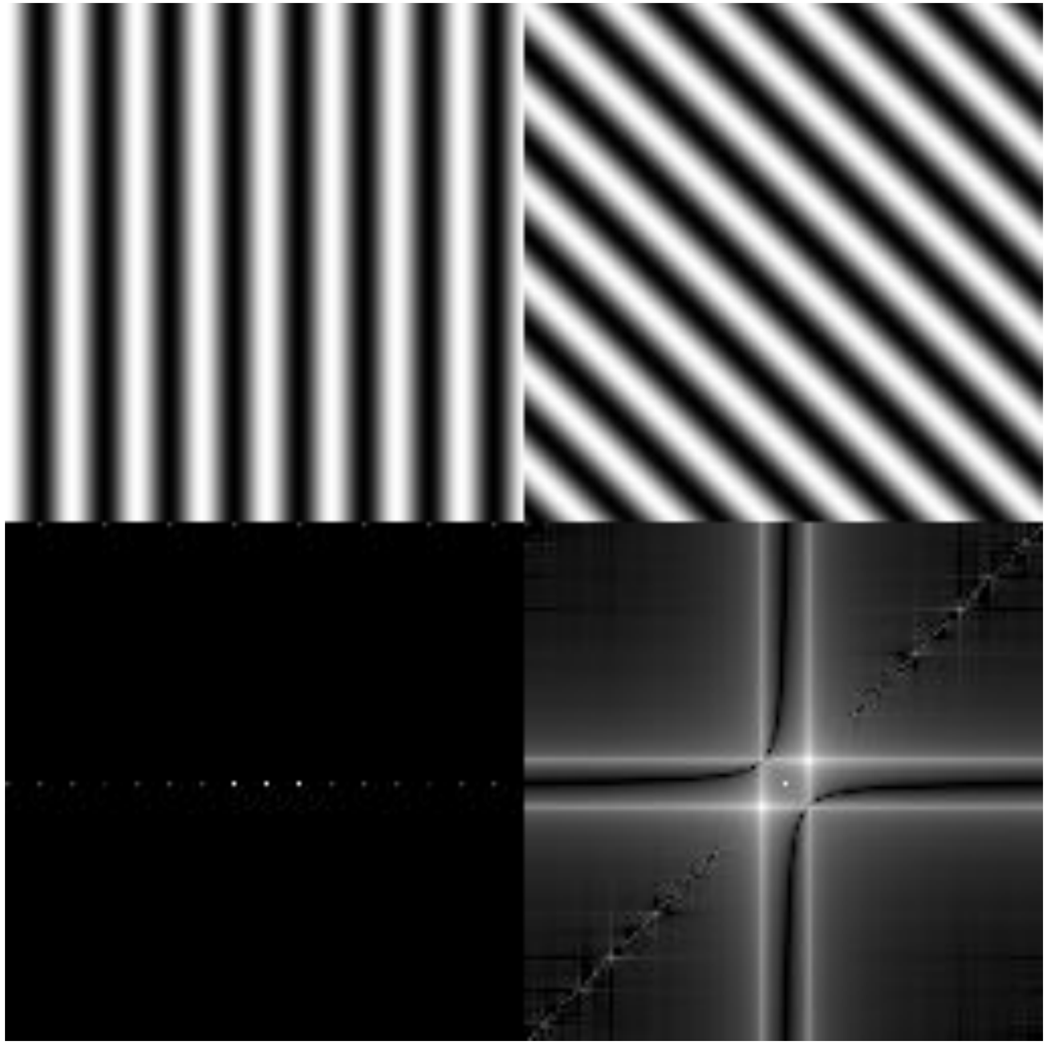
$$\boxed{F(k,l) = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} f(i,j)\; e^{-\imath 2\pi\left(\frac{ki}{N}+\frac{lj}{N}\right)}}$$

- Now we can EXTRACT the frequency information !!



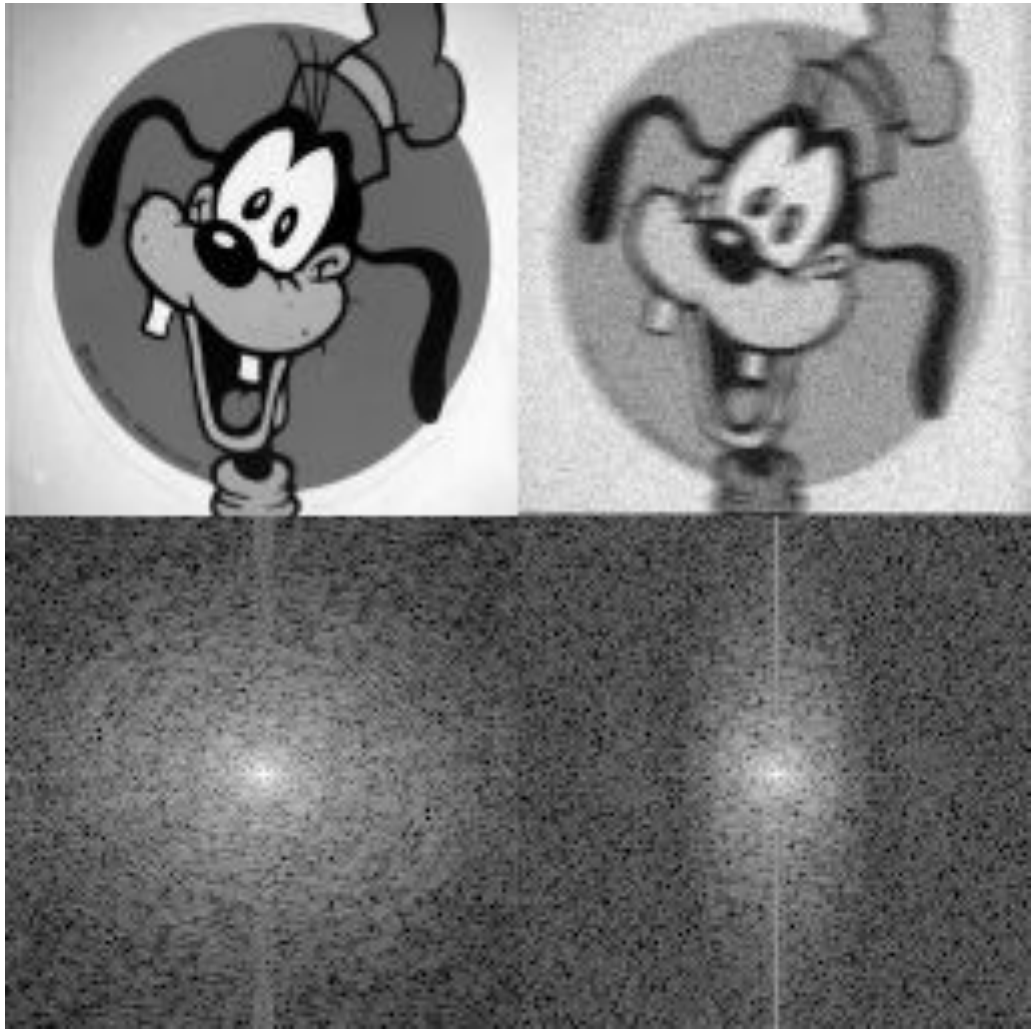
Courtesy of https://www.cs.auckland.ac.nz/courses/compsci773s1c/lectures/ImageProcessing-html/fourier.html

- Now we can EXTRACT the frequency information !!



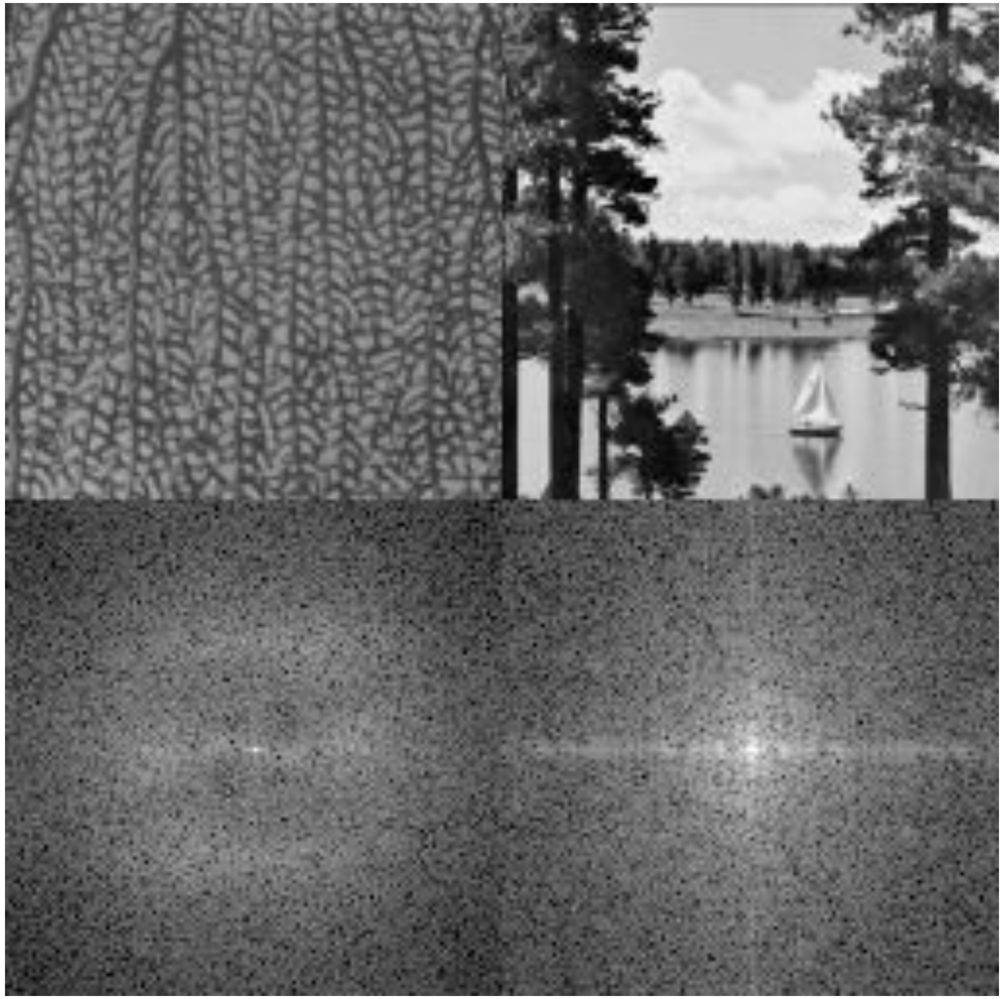
Courtesy of https://www.cs.auckland.ac.nz/courses/compsci773s1c/lectures/ImageProcessing-html/fourier.html

- Now we can EXTRACT the frequency information !!



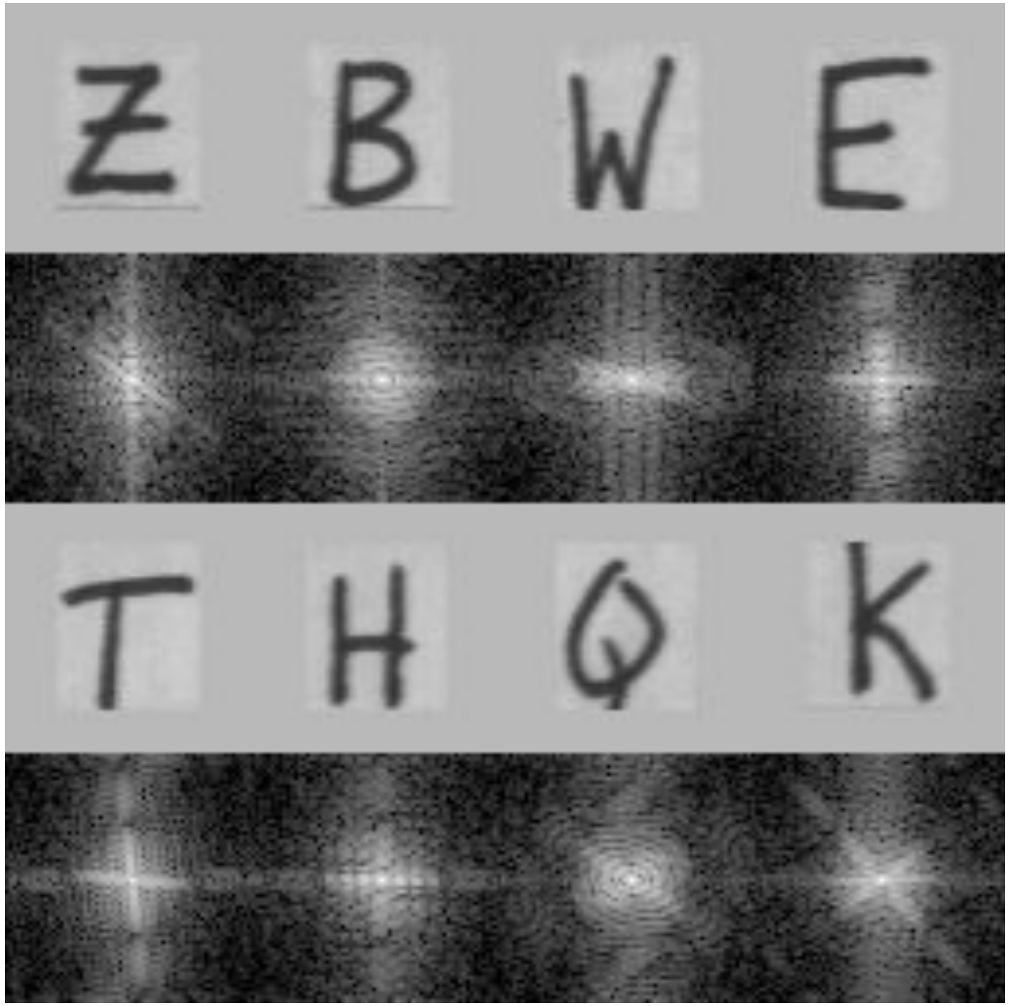
Courtesy of https://www.cs.auckland.ac.nz/courses/compsci773s1c/lectures/ImageProcessing-html/fourier.html

- Now we can EXTRACT the frequency information !!



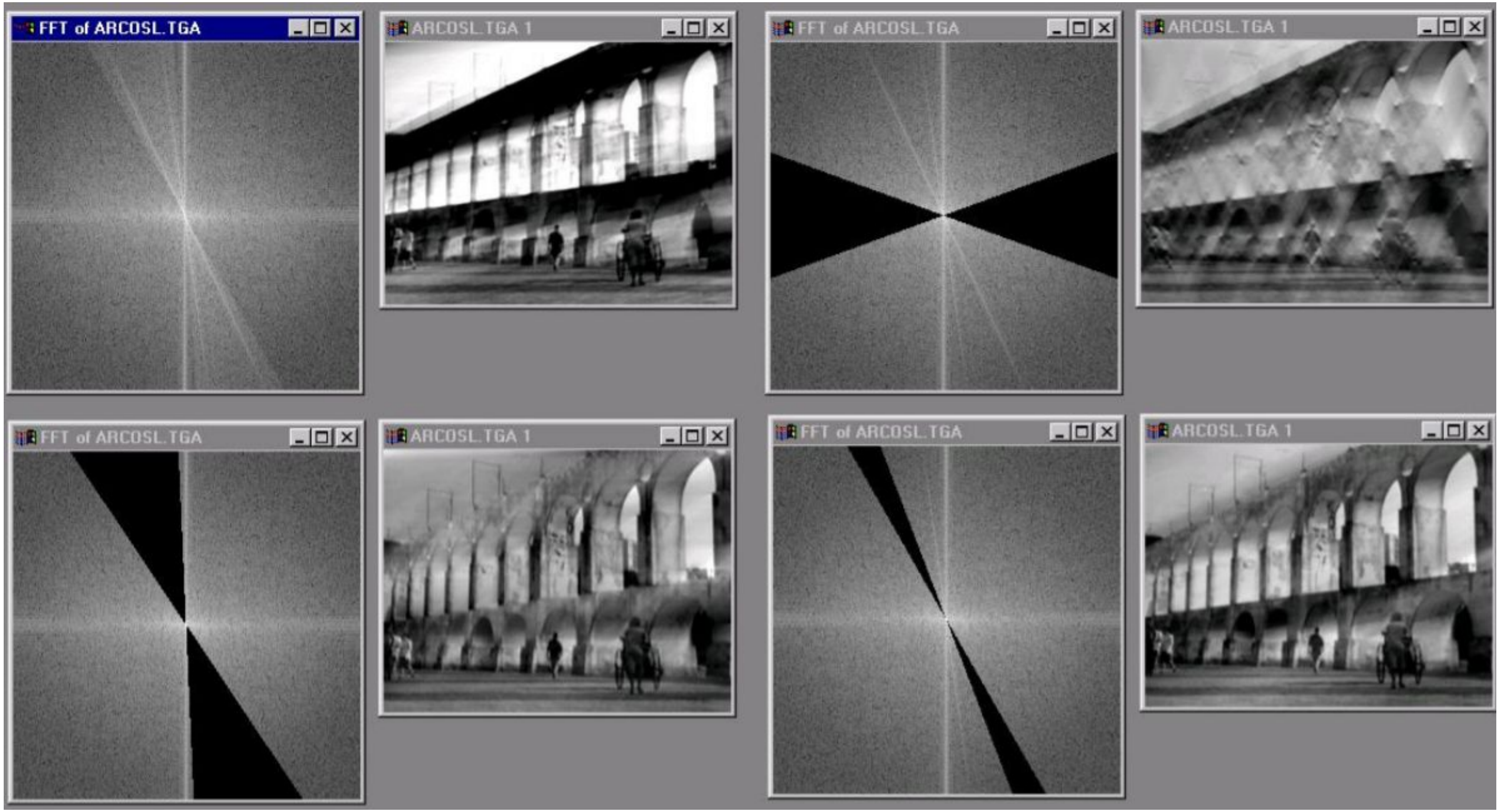
Courtesy of https://www.cs.auckland.ac.nz/courses/compsci773s1c/lectures/ImageProcessing-html/fourier.html

- Now we can EXTRACT the frequency information !!



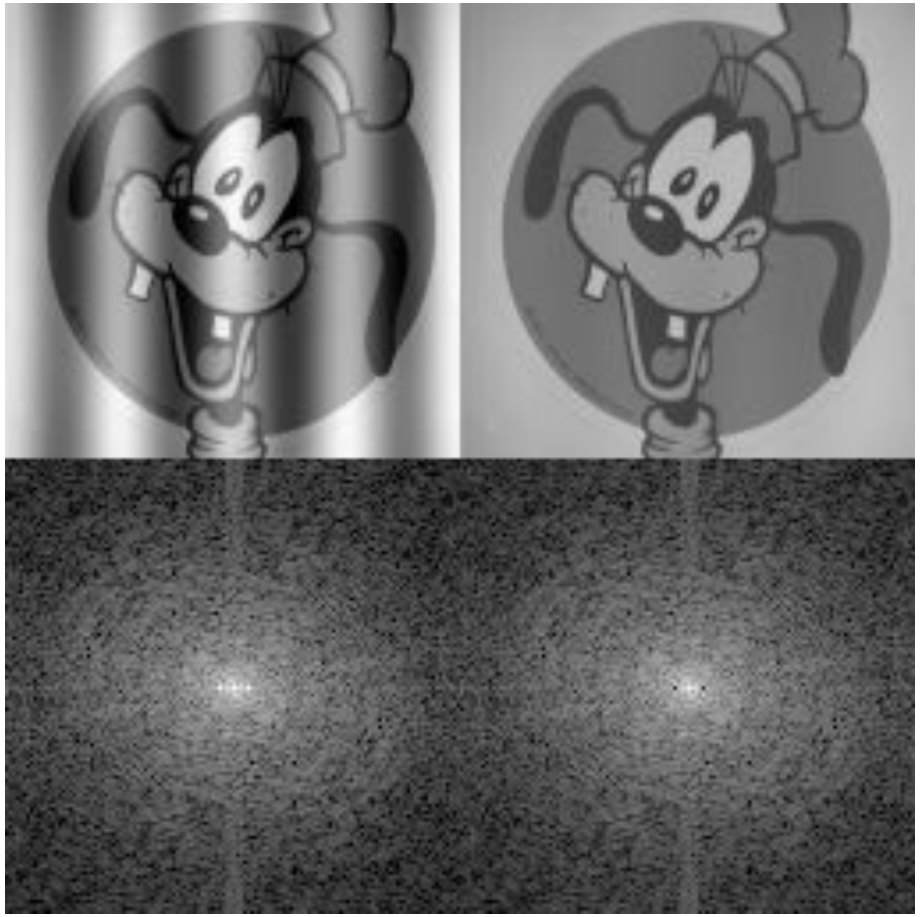
Courtesy of https://www.cs.auckland.ac.nz/courses/compsci773s1c/lectures/ImageProcessing-html/fourier.html

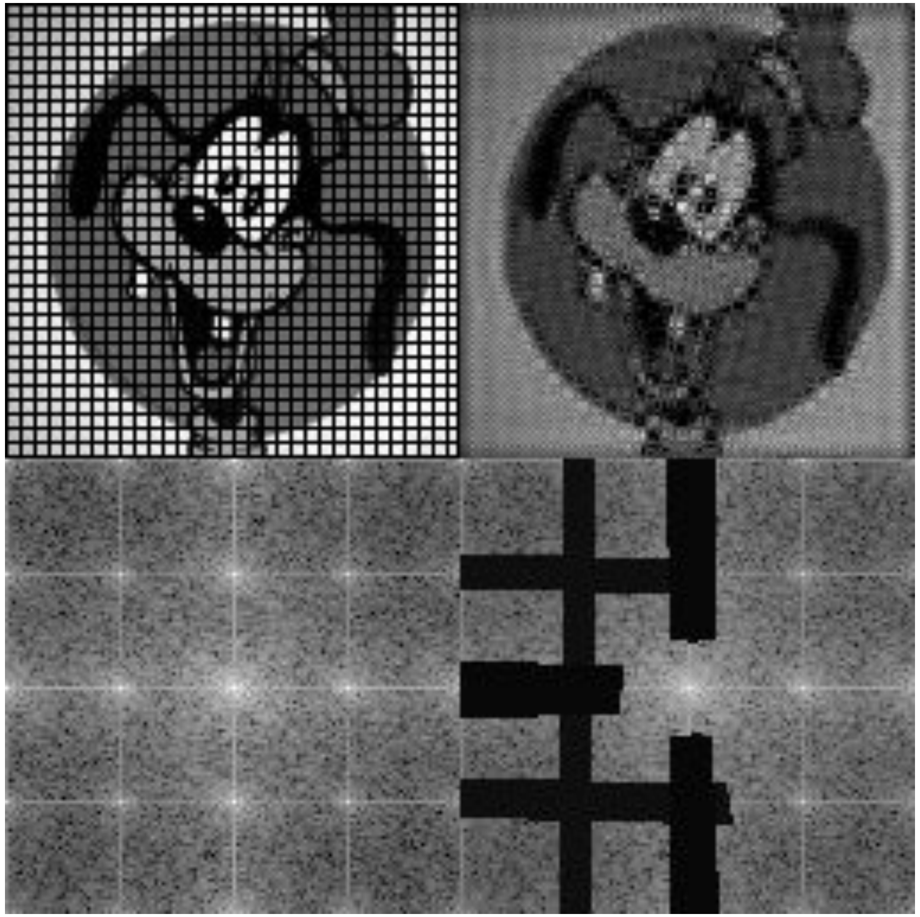
❑ Now we can MANIPULATE the frequency information !!



Courtesy of https://cs.brown.edu/courses/cs143/2017_Fall/lectures_Fall2017/06_Fall2017_ThinkingInFrequencyPart.pdf

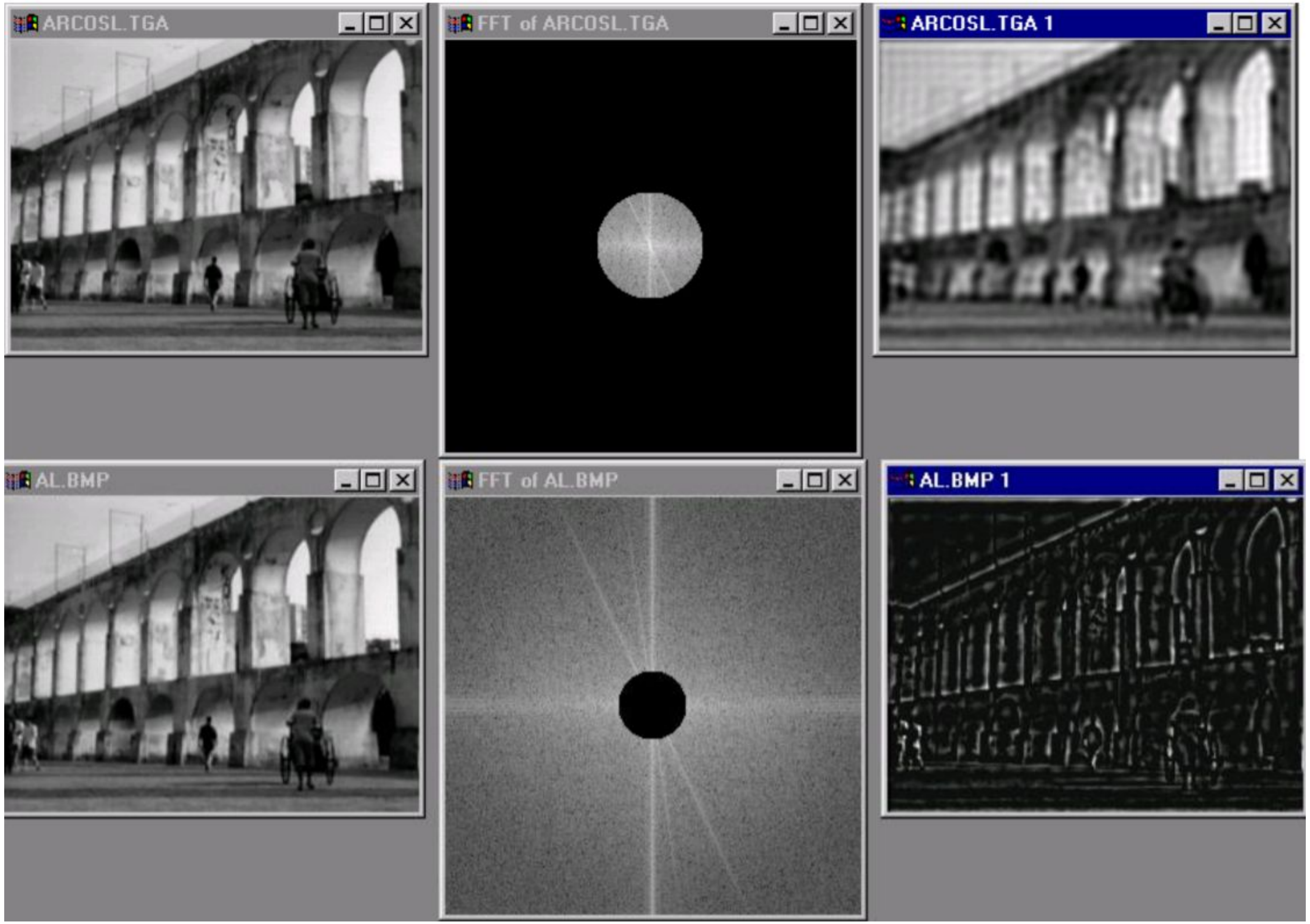
- Now we can MANIPULATE the frequency information !!





Courtesy of https://www.cs.auckland.ac.nz/courses/compsci773s1c/lectures/ImageProcessing-html/fourier.html

- High and low pass filtering



Courtesy of https://cs.brown.edu/courses/cs143/2017_Fall/lectures_Fall2017/06_Fall2017_ThinkingInFrequencyPart.pdf

- Content review

1. Space-frequency duality
2. Fourier Transform
3. LTI and ‘convolution’
4. Extracting and manipulating frequency information

Thank you!