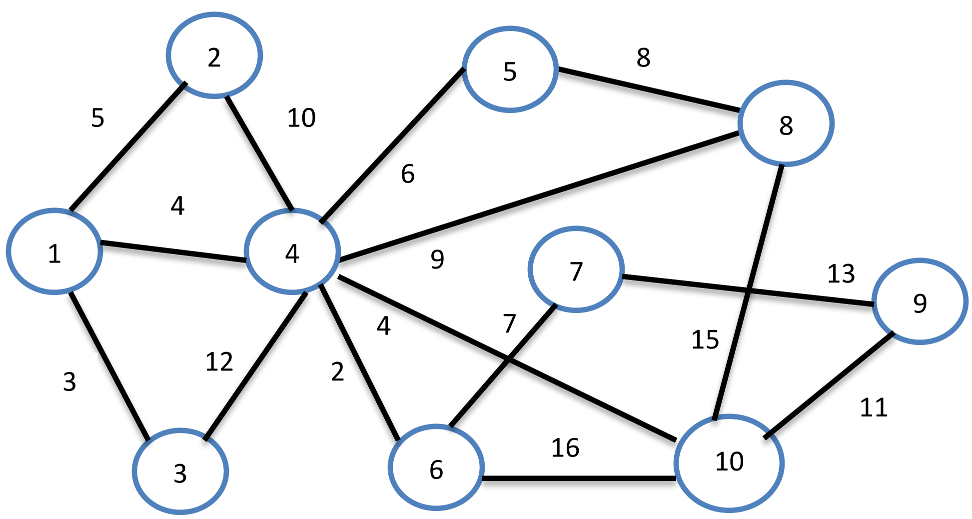
ASSIGNMENT 6 SOLUTION

Problem statement :

Implement Prims Algorithm to find Minimum Spanning Tree.
Your graph must have atleast 10 nodes and atleast 15 edges.
Submit screen shot of the program run.
It should print
a. The edges of the graph
b. Edges of the MST

Solution:
Consider the following graph as below:



The adjacency matrix for the above graph is :

	1	2	3	4	5	6	7	8	9	10
1	0	5	3	4	0	0	0	0	0	0
2	5	0	0	10	0	0	0	0	0	0
3	3	0	0	12	0	0	0	0	0	0
4	4	10	12	0	6	2	0	9	0	4
5	0	0	0	6	0	0	0	8	0	0
6	0	0	0	2	0	0	7	0	0	16
7	0	0	0	0	0	7	0	0	13	0
8	0	0	0	9	8	0	0	0	0	15
9	0	0	0	0	0	0	13	0	0	11
10	0	0	0	4	0	16	0	15	11	0

A spanning tree of a graph is a tree that has all the vertices of the graph connected by some edges. A graph can have one or more spanning trees. If the graph has N vertices, then the spanning tree will have N-1 edges.

A minimum spanning tree is a spanning tree that has minimum weight than all other spanning trees of the graph.

Prim's algorithm is a greedy algorithm that tries to find the minimum spanning tree of a graph. Prim's algorithm requires a graph, weight of the edges and also the root from which we want to find the spanning tree as inputs.

The algorithm is as below:

```
MST_PRIM(G, w, r)
Foreach u € V[G]
        Key[u] ← ∞ ; p[u] ← NULL;
Key[r] ←0 ; Q ←V[G];
While (Q ≠ NULL){
        U ←EXTRACT_MIN(Q)
        Foreach v € Adj[u] {
                If v€Q AND w(u,v)<key[v]
                        P[v]←u ; key[v] ←w(u,v)
        }
}
```

Let us begin the traversal from row 1.

1. The smallest distance in row 1 is graph[1][3]=3.

	1	2	3	4	5	6	7	8	9	10
1	0	5	3	4	0	0	0	0	0	0
2	5	0	0	10	0	0	0	0	0	0
3	3	0	0	12	0	0	0	0	0	0
4	4	10	12	0	6	2	0	9	0	4
5	0	0	0	6	0	0	0	8	0	0
6	0	0	0	2	0	0	7	0	0	16
7	0	0	0	0	0	7	0	0	13	0
8	0	0	0	9	8	0	0	0	0	15
9	0	0	0	0	0	0	13	0	0	11
10	0	0	0	4	0	16	0	15	11	0

2. Therefore, p[3]=1 and w(1,3)= 3. Marking nodes {1,3} as visited, we will consider rows 1 and 3 and find the minimum weighted edge. Graph[1][4]=4 is the minimum.

	1	2	3	4	5	6	7	8	9	10
1	0	5	3	4	0	0	0	0	0	0
2	5	0	0	10	0	0	0	0	0	0
3	3	0	0	12	0	0	0	0	0	0
4	4	10	12	0	6	2	0	9	0	4
5	0	0	0	6	0	0	0	8	0	0
6	0	0	0	2	0	0	7	0	0	16
7	0	0	0	0	0	7	0	0	13	0
8	0	0	0	9	8	0	0	0	0	15
9	0	0	0	0	0	0	13	0	0	11
10	0	0	0	4	0	16	0	15	11	0

3. Therefore, p[4]=1 and w(1,4)= 1. Marking nodes {1,4,3} as visited, we will consider rows 1,3 and 4 and find the minimum weighted edge. Graph[4][6]=4 is the minimum

	1	2	3	4	5	6	7	8	9	10
1	0	5	3	4	0	0	0	0	0	0
2	5	0	0	10	0	0	0	0	0	0
3	3	0	0	12	0	0	0	0	0	0
4	4	10	12	0	6	2	0	9	0	4
5	0	0	0	6	0	0	0	8	0	0
6	0	0	0	2	0	0	7	0	0	16
7	0	0	0	0	0	7	0	0	13	0
8	0	0	0	9	8	0	0	0	0	15
9	0	0	0	0	0	0	13	0	0	11
10	0	0	0	4	0	16	0	15	11	0

4. Therefore, p[6]=2 and w(4,6)= 2. Marking nodes {1,4,3,6} as visited, we will consider rows 1,3,4 and 6 and find the minimum weighted edge. Graph[4][10]=4 is the minimum

	1	2	3	4	5	6	7	8	9	10
1	0	5	3	4	0	0	0	0	0	0
2	5	0	0	10	0	0	0	0	0	0
3	3	0	0	12	0	0	0	0	0	0
4	4	10	12	0	6	2	0	9	0	4
5	0	0	0	6	0	0	0	8	0	0
6	0	0	0	2	0	0	7	0	0	16
7	0	0	0	0	0	7	0	0	13	0
8	0	0	0	9	8	0	0	0	0	15

9	0	0	0	0	0	0	13	0	0	11
10	0	0	0	4	0	16	0	15	11	0

5. Therefore, p[10]=4 and w(4,10)= 4. Marking nodes {1,4,3,6,10} as visited, we will consider rows 1,3,4,6 and 10 and find the minimum weighted edge. Graph[1][2]=5 is the minimum

	1	2	3	4	5	6	7	8	9	10
1	0	5	3	4	0	0	0	0	0	0
2	5	0	0	10	0	0	0	0	0	0
3	3	0	0	12	0	0	0	0	0	0
4	4	10	12	0	6	2	0	9	0	4
5	0	0	0	6	0	0	0	8	0	0
6	0	0	0	2	0	0	7	0	0	16
7	0	0	0	0	0	7	0	0	13	0
8	0	0	0	9	8	0	0	0	0	15
9	0	0	0	0	0	0	13	0	0	11
10	0	0	0	4	0	16	0	15	11	0

6. Therefore, p[2]=1 and w(1,2)= 5. Marking nodes {1,4,3,6,10,2} as visited, we will consider rows 1,3,4,6,10 and 2 and find the minimum weighted edge. Graph[6][7]=7 is the minimum

	1	2	3	4	5	6	7	8	9	10
1	0	5	3	4	0	0	0	0	0	0
2	5	0	0	10	0	0	0	0	0	0
3	3	0	0	12	0	0	0	0	0	0
4	4	10	12	0	6	2	0	9	0	4
5	0	0	0	6	0	0	0	8	0	0
6	0	0	0	2	0	0	7	0	0	16
7	0	0	0	0	0	7	0	0	13	0
8	0	0	0	9	8	0	0	0	0	15
9	0	0	0	0	0	0	13	0	0	11
10	0	0	0	4	0	16	0	15	11	0

7. Therefore, p[7]=6 and w(6,7)= 7. Marking nodes {1,4,3,6,10,2,7} as visited, we will consider rows 1,3,4,6,10,2 and 7 and find the minimum weighted edge. Graph[4][5]=6 is the minimum

	1	2	3	4	5	6	7	8	9	10
1	0	5	3	4	0	0	0	0	0	0
2	5	0	0	10	0	0	0	0	0	0

3	3	0	0	12	0	0	0	0	0	0
4	4	10	12	0	6	2	0	9	0	4
5	0	0	0	6	0	0	0	8	0	0
6	0	0	0	2	0	0	7	0	0	16
7	0	0	0	0	0	7	0	0	13	0
8	0	0	0	9	8	0	0	0	0	15
9	0	0	0	0	0	0	13	0	0	11
10	0	0	0	4	0	16	0	15	11	0

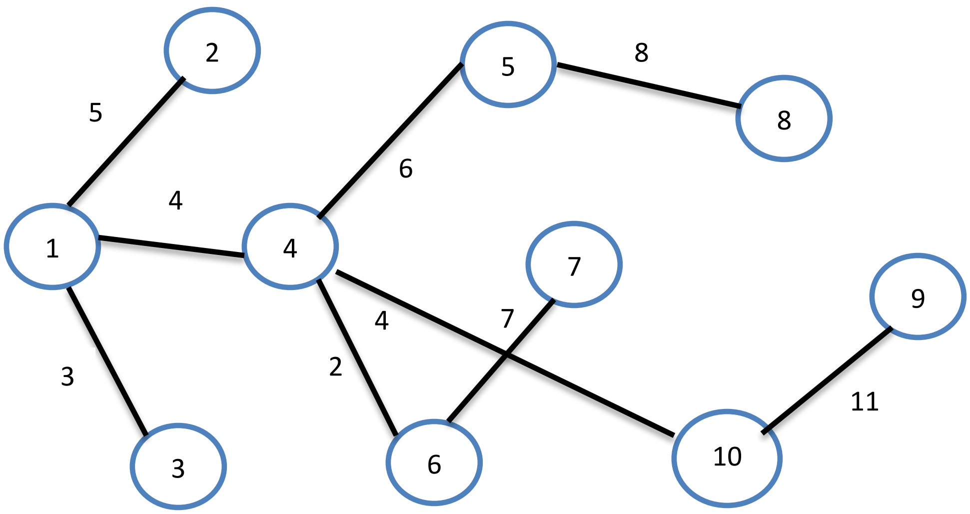
8. Therefore, p[5]=4 and w(5,4)= 6. Marking nodes {1,4,3,6,10,2,7,5} as visited, we will consider rows 1,3,4,6,10,2,7 and 5 and find the minimum weighted edge. Graph[8][5]=8 is the minimum

	1	2	3	4	5	6	7	8	9	10
1	0	5	3	4	0	0	0	0	0	0
2	5	0	0	10	0	0	0	0	0	0
3	3	0	0	12	0	0	0	0	0	0
4	4	10	12	0	6	2	0	9	0	4
5	0	0	0	6	0	0	0	8	0	0
6	0	0	0	2	0	0	7	0	0	16
7	0	0	0	0	0	7	0	0	13	0
8	0	0	0	9	8	0	0	0	0	15
9	0	0	0	0	0	0	13	0	0	11
10	0	0	0	4	0	16	0	15	11	0

9. Therefore, p[8]=5 and w(5,8)= 8. Marking nodes {1,4,3,6,10,2,7,5,8} as visited, we will consider rows 1,3,4,6,10,2,7,5 and 8 and find the minimum weighted edge. Graph[9][10]=11 is the minimum

	1	2	3	4	5	6	7	8	9	10
1	0	5	3	4	0	0	0	0	0	0
2	5	0	0	10	0	0	0	0	0	0
3	3	0	0	12	0	0	0	0	0	0
4	4	10	12	0	6	2	0	9	0	4
5	0	0	0	6	0	0	0	8	0	0
6	0	0	0	2	0	0	7	0	0	16
7	0	0	0	0	0	7	0	0	13	0
8	0	0	0	9	8	0	0	0	0	15
9	0	0	0	0	0	0	13	0	0	11
10	0	0	0	4	0	16	0	15	11	0

Since all the nodes have been visited, the minimum spanning tree is as shown below,



The total weight of the minimum spanning tree is : 50