Chapter 23 Electric Potential

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1 Electric Potential Energy

- When a force $\vec{\mathbf{F}}$ acts on a particle that moves from point a to point b, the work done $W_{a\to b}$ by the force is given by the line integral:
$$W_{a\to b} = \int_a^b \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}} = \int_a^b F\cos\phi dl \tag{1}$$
where $d\vec{\mathbf{l}}$ is an infinitesimal displacement along the particle's path and ϕ is the angle between $\vec{\mathbf{F}}$ and $d\vec{\mathbf{l}}$ at each point along the path

- If the force $\vec{\mathbf{F}}$ is conservative, the work done by $\vec{\mathbf{F}}$ can always be expressed in terms of **potential energy** U. Then we have that
$$W_{a\to b} = U_a - U_b = -(U_a - U_b) = -\Delta U \tag{2}$$

- The work energy theorem states that the change in kinetic energy ΔK during the displacement equals the total work done on the particle. Then total mechanical energy is conserved with $K_a + U_a = K_b + U_b$

1.1 Electric Potential Energy in a Uniform Field

- Suppose a pair of parallel metal plates setup a uniform downward electric field with magnitude E. The field exerts a downward force with magnitude $F = q_0 E$ on a positive test charge q_0.

- The charge moves down a distance d, from point a to b, so the total work done can be calculated as follows:
$$W_{a\to b} = Fd = q_0 Ed \tag{3}$$

- Whether the test charge is positive or negative, when it moves with the field and decreases when it moves against the field. U increases if the test charge q_0 moves in the direction opposite of the electric force $\vec{\mathbf{F}} = q_0\vec{\mathbf{E}}$, but increases when q_0 moves in the same direction as the electric force $\vec{\mathbf{F}}$

1.2 Electric Potential Energy of Two Point Charges

- It is usefull to calculate the work done on a test charge q_0 moving in the electric field caused by a single, stationary point charge q
- Consider the radial component along the radial line, the force on q_0 is given by Coulomb's law,

$$F_r = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2} \tag{4}$$

- The force is not constant during the displacement, so we must integrate to calculate $W_{a\to b}$ done on q_0 by this force:

$$W_{a\to b} = \int_{r_a}^{r_b} F_r dr = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2} dr = \frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) \tag{5}$$

- The work done by the electric force for this path depends on only the endpoints, so the work done on q_0 by $\vec{\mathbf{E}}$ depends only on r_a and r_b, not on the details of the path.
- This means that if q_0 returns to the initial starting point, the work done is 0, this means that the force on q_0 is conservative
- Electric potential energy is then defined as:

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r} \tag{6}$$

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1.3 Electric Potential Energy with Several Point Charges

- Suppose the electric field $\vec{\mathbf{E}}$ in which the charge q_0 moves is caused by several point charges $q_1, q_2, q_3, ...$ at distances $r_1, r_2, ...$ from q_0. The total electric field at each point is the vector sum of the fields

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + ...\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \tag{7}$$

- For every electric field due to a static charge distribution, the force exerted by that field is conservative

2 Electric Potential

- Potential V at any point in the electric field as the potential energy per unit charge, or as the potential energy U per unit charge associated with q_0

$$V = \frac{U}{q_0} \tag{8}$$

- Then we can relate this to work from a to b

$$\frac{W_{a\to b}}{q_0} = -\frac{\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = -(V_b - V_a) = V_a - V_b \tag{9}$$

- This equation states that the potential (in V) of a with respect to b, equals the work (in J) done by the electric force when a charge moves from a to b

2.1 Calculating Electric Potential

- To find the potential V due to a single point charge, q, we have

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \tag{10}$$

- Similarly, when you have a collection of point charges:

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \tag{11}$$

- If we have a continuous distribution of charge along a line, over a surface, or throught a volume, we divide the charge into elements dq and then integrate:

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \tag{12}$$

2.2 Finding Electric Potential from Electric Field

- When given a collection of point charges, and the electric field can be found easily, it is easier to determine V from $\vec{\mathbf{E}}$. The force $\vec{\mathbf{F}}$ on the test charge q_0 can be written as $\vec{\mathbf{F}} = q_0\vec{\mathbf{E}}$ so the work done by the electric force is:

$$W_{a\to b} = \int_a^b \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}} = \int_a^b q_0\vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} \tag{13}$$

- If we divide this by q_0 and compare the result, we find that:

$$V_a - V_b = \int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \int_a^b E\cos\phi dl \tag{14}$$

  Where E is the electric field magnitude and ϕ is the angle between $\vec{\mathbf{E}}$ and $d\vec{\mathbf{l}}$

- Again the value $V_a - V_b$ is independent of the path taken from a to b, just as the value of $W_{a\to b}$ is independent of path