Chapter 25 Current, Resistance, and Electromotive Force

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1 Current

- A current is any motion of charge from one region to another
- In electrostatic situations, the electric field is zero everywhere within the conductor, and there is no current. However, this does not mean that all charges within the conductor are at rest.
- Consider what happens if a constant, steady electric field $\vec{\mathbf{E}}$ is established inside a conductor. A particle inside the conducting material is then subjectued to steady force $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$
- If the charged particle were moving in a vacuum, this steady force would cause a steady acceleration in the direction of $\vec{\mathbf{F}}$, and after a time, the charged particle would be moving in that direction at high speed
- A charged particle moving through a conductor however undergoes frequent collisions with massive, nearly stationary ions of the material. Such collisions cause the particle's direction of motion to undergo random change, with a net effect of the electric field $\vec{\mathbf{E}}$ is that three is a very slow net motion or drift of the moving charged particles as a group in the direction of the electric force $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$

1.1 The Direction of Current Flow

- The electric field $\vec{\mathbf{E}}$ does work on the moving charges, resulting in kinetic energy transferrred to the material of the conductor by means of collisions with the ions
- A conventional current is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.
- In a metallic conductor, the moving charges are electrons, but the current still points in the direction positive charges would flow.
- If the net charge dQ flows through an area in a time dt, the current I through the area is:

$$I = \frac{dQ}{dt} \tag{1}$$

1.2 Current, Drift Velocity, and Current

- We can express current in terms of the drift velocity of the moving charges.
- Suppose there are n moving charged particles per unit volume, then n is the concentration of particles, now suppose they all have a drift velocity with magnitude v_d
- In a time interval dt, each particle moves a distance $v_d dt$
- The volume of the cylinder is $Av_d dt$ and the number of particles is $nAv_d dt$. If each particle has a charge q, the charge dQ flows out of the end of the cylinder during the time dt is:

$$dQ = q(nAv_d dt) = nqv_d A dt \tag{2}$$

- The current is then defined as:

$$I = \frac{dQ}{dt} = nqv_d A \tag{3}$$

- The current per unit cross-sectional area is called the **current density** J

$$J = \frac{I}{A} = nqv_d \tag{4}$$

- So to summarize, I is the current through an area, $\frac{dQ}{dt}$ is the rage at which charge flows though area, n is concentration of moving charged particles, q is charge per particle, v_d is drift speed, and A is cross-sectional area

2 Resistivity

- The current density $\vec{\mathbf{J}}$ in a conductor depends on the electric field $\vec{\mathbf{E}}$ and on the properties of the material
- For some materials, especially metals, at a given temperature, $\vec{\mathbf{J}}$ is n:nohearly directly proportional to $\vec{\mathbf{E}}$, and the ratio of the magnitudes of E and J is nearly constant. This relationship is known as Ohm's Law
- We define the **resistivity** ρ in terms of the magnitude of the electric field divided by the magnitude of the current density caused by that field:

$$\rho = \frac{E}{J} \tag{5}$$

- The reciprocal of resistivity is **conductivity**. With units $(\Omega \cdot m)^{-1}$
- A material that obey's Ohm's law reasonably well is called an ohmic conductor or a linear conductor, for such materials, ρ is a constant that does not depend on the value of E

2.1 Resistivity and Temperature

- The resistivity of a metallic conductor nearly always increases with increasing temperature. As temperature increases, the ions in the conductor vibrate with greater amplitude, making it more likely that a moving electron will collide with an ion
- Over a small temperature range (up to $100C^\circ$), the resistivity of a metal can be represented approximately by the equation:

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)] \tag{6}$$

where ρ_0 is the resistivity at a reference temperature T_0 and α is the temperature coefficient of resistivity

3 Resistance

- For a conductor of resistivity ρ, the current density $\vec{\mathbf{J}}$ at a point where the electric field is $\vec{\mathbf{E}}$ is given by:
$$\vec{\mathbf{E}} = \rho\vec{\mathbf{J}} \tag{7}$$
- Suppose our conductor is a wire with unifrom cross-sectional area A and length L. Let V be the potential difference between the higher-potential and lower-potential ends of the conductor, so that V is positive
- The direction of the current is always from the higher-potential end to the lower-potential end, and this is because the current in a conductor flows in the direction of $\vec{\mathbf{E}}$ and $\vec{\mathbf{E}}$ points in the direction of decreasing electric potential
- We can also relate the value of the current I to the potential difference between the ends of the conductor. If the magnitudes of the current density $\vec{\mathbf{J}}$ and the electric field $\vec{\mathbf{E}}$ are unform throughout the conductor, the total current $I = JA$ and the potential difference is $V = EL$, which leads to:
$$\frac{V}{L} = \frac{\rho I}{A} \quad \text{or} \quad V = \frac{\rho L}{A} I \tag{8}$$
- The ratio of V to I for a particular conductor is called its **resistance** R
$$R = \frac{V}{I} = \frac{\rho L}{A} \tag{9}$$
where ρ is the resistivity of the conductor material, L is the length of the conductor, and A is the cross-sectional area
- If ρ is constant, then so is R, this leads to the equation often called Ohm's Law:
$$V = IR \tag{10}$$
Where V is voltage between ends of a conductor, I is the current in the conductor, and R is the resistance in the conductor
- Note that because resistivity of ta material varies with temperature, the resistance of a specific conductor also varies with temperature. For small temperature ranges (similar to ρ), we have that:
$$R(T) = R_0[1 + \alpha(T - T_0)] \tag{11}$$

3.1 Electromotive Force

- **Electromotive force** is the influence that makes current flow from lower to higher potential
- Every complete circuit with a steady current must include a source of emf such as batteries, electric generators, solar cells, etc
- The SI unit of emf is the same as that for potential, the volt ($1V = 1J/C$). A typical flashlight battery has an emf of $1.5V$, so the battery does $1.5J$ of work on every coulomb of charge that passes through it.
- An example of an ideal source of emf that maintains the potential differenve between the conductors a and b, called terminals, has associated with this potential difference an electric field $\vec{\mathbf{E}}$ in the region around the terminals
- A charge q within the source experiences an electric force $\vec{\mathbf{F}}_e = q\vec{\mathbf{E}}$, but the source also provides an additional influence, which is the nonelectrostatic force $\vec{\mathbf{F}}_n$, which pushes charge from b to a in an uphill direction against the electric force $\vec{\mathbf{F}}_e$
- If a positive charge q is moved from b to a inside the source, the nonelectrostatic force $\vec{\mathbf{F}}_n$ does a positive amount of work $W_n = q\mathcal{E}$ on the charge, and is opposite to the electrostatic force $\vec{\mathbf{F}}_e$, so the potential energy increases by an qV_{ab}
- By combining previous equations, we can see that the potential difference between the ends of the wire is given by:
$$\mathcal{E} = V_{ab} = IR \tag{12}$$

3.2 Internal Resistance

- Real sources of emf in a circuit don't behave in exactly the way described since the potential difference across a real source in a circuit is not equal to the emf, since the charge moving through the material of a real source encounters resistance.
- This **Internal Resistance** r of the source begaves according to Ohm's Law, r is constant and independent of the current I.
- As the current moves through r, it experiences an associated drop in potential equal to Ir, so the potential difference V_{ab} between the terminals is:

$$V_{ab} = \mathbf{E} - Ir \tag{13}$$

- The potential V_{ab} is called **terminal voltage** and is less than the emf $\mathcal{E}$ because of the term Ir representing the potential drop across the internal resistance r
- The increase in potential energy qV_{ab} as a charge q moves from b to a within the source is less than the work $q\mathcal{E}$ done by the nonelectrostatic force $\vec{\mathbf{F}}_n$, since some potential energy is lost in traversing the internal resistance

4 Energy and Power in Electric Circuits

- As the amount of charge q passes through the circuit element, there is a change in potential energy equal to qV_{ab}
- If the potential at a is lower than at b, then V_{ab} is negative and there is a net transfer of energy out the circuit, so qV_{ab} can denote the quantity of energy that is either delivered to a circuit element or exracted from that element
- If the current though the element is I, then in a time interval dt, an amount of charge $dQ = Idt$ passes through the element. The potential energy change for this amount of charge is $V_{ab}dQ = V_{ab}Idt$
- Dividing this expression by dt, we obtain the rate at which energy is transferrred, also known as power, denoted as:

$$P = V_{ab}I \tag{14}$$

- If the circuit element is a resistor, the potential difference is $V_{ab} = IR$ The power is thus:

$$P = V_{ab}I = I^2R = \frac{V_{ab}^2}{R} \tag{15}$$

  where R is the resistance of the resistor and I is the current in the resistor

- For the power output of a source, we have that if a is a higher potential than point b, then $V_a > V_b$ and V_{ab} is positive, then energy is being delivered to the external circuit at a rate of:

$$P = V_{ab}I \tag{16}$$

  where

$$V_{ab} = \mathcal{E} - Ir \tag{17}$$

  so

$$P = V_{ab}I = \mathcal{E}I - I^2r \tag{18}$$

- For power input into source, we have that the curent I in the circuit is opposite to what it was for power output, so we then have that:

$$V_{ab} = \mathcal{E} + Ir \tag{19}$$

  and than we see that for power:

$$P = V_{ab}I = \mathcal{E}I + I^2r \tag{20}$$