Chapter 26 Direct-Current Circuits

Apostolos Delis

June 5, 2019

Contents

1 Resistors In Series and Parallel

- **Direct Current Circuits (DC)**: direction of the current does not change with time
- **Alternating Current Circuits (AC)**: the current oscillates back and fort
- For any combination of resistors we can always find a single resistor that could replace the combination and result in the same total current and potential difference
- The resistance of this single resistor is called the **equivalent resistance** of the combination.

$$V_{ab} = IR_{eq} \quad \text{or} \quad R_{eq} = \frac{V_{ab}}{I} \tag{1}$$

where V_{ab} is the potential difference between terminals a and b in the network and I is the current at point a or b

1.1 Resistors in Series

- When resistors are in series, the current I is the same in all of them. Applying $V = IR$ to each resistor, with R_1, R_2 and R_3, we have

$$V_{ax} = IR_1 \qquad V_{xy} = IR_2 \qquad V_{yb} = IR_3 \tag{2}$$

- The potential differences across each resistor don't need to be the same, . The potential difference V_{ab} across the entire combination is the sum of these individual potential differences

$$V_{ab} = V_{ax} + V_{xy} + V_{yb} = I(R_1 + R_2 + R_3) \tag{3}$$

and so:

$$\frac{V_{ab}}{I} = R_1 + R_2 + R_3 \tag{4}$$

- The Ration V_{ab}/I is the equivalent resistance R_{eq}. So

$$R_{eq} = R_1 + R_2 + R_3 \tag{5}$$

This of course can be generalized for any n number of resistors,

$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_n \tag{6}$$

1.2 Resistors in Parallel

- If the resistors are in parallel, the current through each resistor does not have to be the same. Suppose there are three resistors, with currents I_1, I_2, and I_3, then:

$$I_1 = \frac{V_{ab}}{R_1} \qquad I_2 = \frac{V_{ab}}{R_2} \qquad I_3 = \frac{V_{ab}}{R_3} \tag{7}$$

- Because charge neither accumulates at nor drains out of point a, the total current I must equal the sum of the three currents in the resistors:

$$I = I_1 + I_2 + I_3 = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \quad \text{or} \quad \frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{8}$$

- Then for any n number of resistors in parallel, we have that the equivalent resistance R_{eq} is:

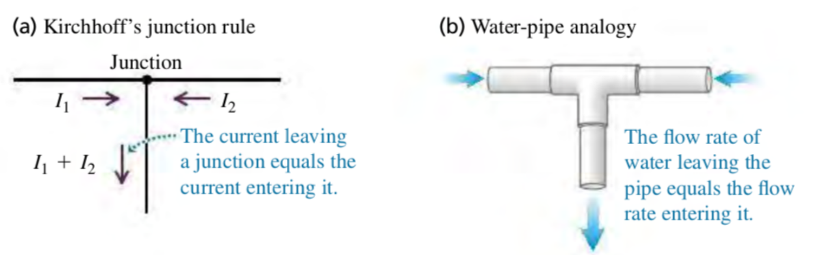
$$R_{eq} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{9}$$

2 Kirchhoff's Rules

- Many practical resistor networks cannot be reduced to simple series-parallel combinations
- A **Junction** in a circuit is a point where three or more conductors meet
- A **Loop** is any closed conducting path
- For Junctions, Kirchhoff's rule states that the sum of the current must equal 0

$$\sum I = 0 \tag{10}$$

26.7 Kirchhoff's junction rule states that as much current flows into a junction as flows out of it.



- For loops, Kirchhoff's rule states that the sum of the potential differences around any loop must equal 0

$$\sum V = 0 \tag{11}$$

- The loop rule is a statement that the electrostatic force is conservative.

3 Electrical Measuring Instruments

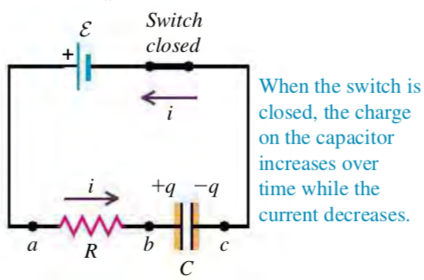
- **Ammeters**: current-measuring instruments that measure how much current is passing through them. An ideal Ammeter has zero resistance, but real ammeters always have as little resistance as possible.
- Ammeters can be added to a circuit to measure currents larger than its full scale by connecting a resistor in parallel, so that some of the current bypasses the meter coil. The parallel resistor is called the **shunt resistor** R_{sh}
- **Voltmeters**: are voltage-measuring instruments that measures the potential difference between two points. Real voltmeters always have finite resistance, but a voltmeter should have large enough resistance that connecting it in a circuit does not change the other currents appreciably.
- A voltmeter and an ammeter can be used together to measure resistance and power. The resistance R of a resistor equals the potential difference V_{ab} between its terminals divided by the current I, so $R = V_{ab}/I$, then power is $P = V_{ab}I$
- **Ohmmeters**: An alternative method for measuring resistance. An Ohmmeter consists of a meter, a resistor, and a source (often a flashlight battery) connected in series
- **Potentiometer**: is an instrument that can be used to measure the emf. It balances an unknown potential difference against an adjustable, measurable potential differencef a source without drawing any current from the source;

4 R-C Circuits

In the simple act of charging or discharging a capacitor we find a situation in which the currents, voltages, and powers do change with time. Many devices incorporate circuits in which a capacitor is alternately charged and discharged. These include flashing traffic lights, automobile turn signals, and electronic flash units.

4.1 Charging a Capacitor

- A circuit such that has a resistor and a capacitor in series is called an R-C circuit



- Suppose the battery has a constant emf of $\mathcal{E}$ and zero internal resistance ($r = 0$). Now suppose the capacitor of the circuit is initially charged at $t = 0$. Then the switch is closed, completing the circuit.
- Because the capacitor is initially uncharged, the potential difference v_{bc} across it is zero at $t = 0$. By Kirchhoff's loop law, the voltage V_{ab} across the resistor R is equal to the battery $\mathcal{E}$
- The initial current through the resistor I_0 is given by Ohm's law: $I_0 = \frac{v_{ab}}{R} = \frac{\mathcal{E}}{R}$.
- As the capcitor charges, the voltage v_{bc} increases and the potential difference v_{ab} across the resistor decreases, corresponding to a decrease in current. The sum of these two voltages is constant and equalt to $\mathcal{E}$
- Let q represent the charge on the capacitor and i the current of the circuit at time t after the switch has been closed. The instantaneous potential differences v_{ab} and v_{bc} are:

$$v_{ab} = iR \qquad\qquad v_{bc} = \frac{q}{C} \tag{12}$$

- After Kirchhoff's loop rule, we havr that:

$$\mathcal{E} - iR - \frac{q}{C} = 0 \tag{13}$$

- The potential drops by an amount iR as we travel from a to b and by $\frac{q}{C}$ as we travel from b to c. So we find that:

$$i = \frac{\mathcal{E}}{R} - \frac{q}{RC} \tag{14}$$

- At time $t = 0$, when the switch is first closed, the capcitor is uncharged so $q = 0$. So using this we find that $I_0 = \mathcal{E}/R$. As the charge q increases, the term q/RC becomes larger and the capcitor charge approaches its final value, which we call Q_f

- The current decreases and eventually becomes zero. When $i = 0$, we have:

$$\frac{\mathcal{E}}{R} = \frac{Q_f}{RC} \qquad\qquad Q_f = C\mathcal{E} \tag{15}$$

- We can derive general expressions for charge q and current i as functions of time. Since i equals the rate at which positive charge arrives at the positive plate of the capacitor, $i = dq/dt$, so

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} = -\frac{1}{RC}(q - C\mathcal{E}) \tag{16}$$

  Which can be rearranged as:

$$\frac{dq}{q - C\mathcal{E}} = -\frac{dt}{RC} \tag{17}$$

  integrating both sides we have that:

$$\int_0^q \frac{dq'}{q' - C\mathcal{E}} = -\int_0^t \frac{dt'}{RC} \tag{18}$$

  Which becomes:

$$\ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC} \tag{19}$$

  Exponentiating both sides and simplifying:

$$\frac{q - C\mathcal{E}}{-C\mathcal{E}} = e^{-t/RC} \tag{20}$$

- So for an R-C Circuit, the charging capacitor charge is:

$$q = C\mathcal{E}(1 - e^{-t/RC}) = Q_f(1 - e^{-t/RC}) \tag{21}$$

- The instantaneous current i is just the time derivative of q

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC} \tag{22}$$

4.2 Time Constant

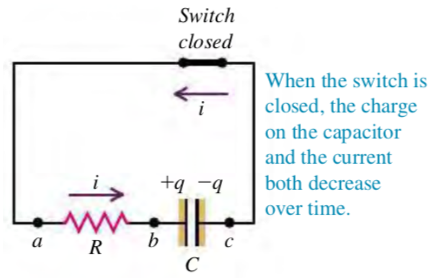
- After a time equal to RC, the current in the R-C circuit has decreased to $1/e$ of its initial value. The capcitor now has reached $(1 - 1/e) = 0.632)$ of its final value $Q_f = C\mathcal{E}$. The product RC is therefor a measure of how quickly the capacitor charges. This value RC is known as the time constant or relaxiont time of the circuit, τ

$$\tau = RC \tag{23}$$

- When τ is small, the capcitor charges quickly, while larger values of τ cause for longer charging.

4.3 Discharging a Capacitor

- Suppose after the capacitor has aqcuired a charge Q_0, we remove the battery from R-C circuit and connect points a and c to an open switch. When the switch closes, The capacitor then discharges through the resistor, and its charge eventually decreases to zero



- Kirchhoff's loop rule when $\mathcal{E} = 0$ is:

$$i = \frac{dq}{dt} = -\frac{q}{RC} \tag{24}$$

- The current i is now negative, this is because positive charge q is leaving the left capacitor plate, so the current is in the direction opposite to that in the diagram.

- To find q as a function of time, integrate:

$$\int_{Q_0}^{q} \frac{dq'}{q'} = -\frac{1}{RC}\int_0^t dt' \tag{25}$$

Which becomes:

$$\ln \frac{q}{Q_0} = -\frac{t}{RC} \tag{26}$$

- So the capacitor charge in a discharging circuit can be modeled with the following function:

$$q = Q_0 e^{-t/RC} \tag{27}$$

- The instantaneous current i is the derivative of this with respect to time

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC} \tag{28}$$

- While the capacitor is charging, the instantaneous rate at which the battery delivers energy to the circuit is $P = \mathcal{E}i$

- The total energy supplied by the battery during charging of the capcitor equals the battery emf $\mathcal{E}$ multiplied by the total charge Q_f