Chapter 24 Capacitance and Dielectrics

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1 Capacitors and Capacitance

- A capacitor is a device that stores electric potential energy and electric charge
- Any two conductors seperated by an insulator form a capacitor, in most applications, capacitors start with zero net charge and electrons are transferred from one conductor to another
- The two conductors have charges with equal magnitude and opposite sign, and the net charge as a whole remains zero
- When a capacitor is said to have a charge Q, it means that one end will have a charge of $+Q$ and one side will have a charge $-Q$
- The electric field at any point in the region between the conductors is proportional to the magnitude Q of the charge on each conductor. It follows that the potential difference V_{ab} between conductors is also proportional to Q
- The ration of the charge to potential difference is known as **capacitance** C of the capacitor:

$$C = \frac{Q}{V_{ab}} \tag{1}$$

1.1 Calculating Capacitance: Capacitors in Vacuum

- We can calculate the capacitance C of a given capacitor by finding the potential difference V_{ab} between conductors for a given charge Q, for now consider the capacitors in a vaccum, so the empty space that seperates the conductors of the capacitor
- The simplest capacitor is two parallel conducting plates with area A, called a **parallel-plate capacitor**
- By using Gauss's law, we found that $E = \sigma/\epsilon_0$, where σ is the magnitude of the surface charge density of each plate. so alternatively, we have that $\sigma = Q/A$, so the field magnitude can be expressed as:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \tag{2}$$

- The field is uniform and the distance between the plates is d, so the potential difference between the two plates is:

$$V_{ab} = Ed = \frac{1}{\epsilon_0}\frac{Qd}{A} \tag{3}$$

- For any capacitor in vaccum, the capacitance C only depends on the shapes, dimensions, and seperation of conductors that make up the capacitor

2 Capacitors in Series and Parallel

Capacitors can be combined in many ways to achieve the results needed, the simplest ways to combine capacitors is to put them in series or in parallel.

2.1 Capacitors in Series

- Two capacitors are connected in series by conducting wires between points a and b. When a constant positive potential difference V_{ab} is applied between points a and b, the capacitors become charged
- When you have n capacitors, the equivalent capacitance can be found from the reciprocals of the sums of all the capacitors

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n} \tag{4}$$

2.2 Capacitors in Parallel

- When capacitors are connected in parallel, there can be much higher total capacitance, since the total amount of capacitor area is increased
- When you have n capacitors, the equivalent capacitance is a sum of all the individual capacitors' capacitance:

$$C_{eq} = C_1 + C_2 + C_3 + ... + C_n \tag{5}$$

3 24.3, Energy Storage in Capacitors and Electric-Field Energy

- We can calculate the potential energy U of a charged capacitor by calculating the work W required to charge it. When a capacitor is charged, the final charge is Q and the potential difference is V

$$V = \frac{Q}{C} \tag{6}$$

- Let v and q be the charge and potential difference, then $q = q/C$, then the work dW required to transfer an additional element of charge dq is

$$dW = vdq = \frac{qdq}{C} \tag{7}$$

- The total work W to increase the capacitor charge q from zero to Q is

$$W = \int_0^W dW = \frac{1}{C}\int_0^Q qdq = \frac{Q^2}{2C} \tag{8}$$

- We can define the potential energy of an uncharged capacitor to be zero, then W is equal to the potential energy U of the charged capacitor. The final charge is $Q = CV$ so U can be expressed as:

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV \tag{9}$$