

# Chapter 21 Electric Charge And Electric Field

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## 1 Electric Charge

Electrostatics: the interactions between electric charges that are at rest

### 1.1 Electric Charge and The Structure of Matter

- The structure of atoms can be described in terms of three particles: the negatively charged electron, the positively charged proton, and the uncharged neutron
- The protons and neutrons in an atom make up a small, very dense core called the nucleus
- Note that the masses of the proton and neutron are nearly equal and are roughly 2000 times the mass of the electron.
- The negative charge of the electron has exactly the same magnitude as the positive charge of the proton.
- The number of protons or electrons in a neutral atom of an element is called the atomic number of the element.

- A negative ion is an atom that has gained one or more electrons. This gain or loss of electrons is called ionization.

## 1.2 Electric Charge is Conserved

- Principle of Conservation of Charge: The algebraic sum of all the electric charges in any closed system is constant.
- The second important principle is: The magnitude of charge of the electron or proton is a natural unit of charge.
- Every observable amount of electric charge is always an integer multiple of this basic unit. We say that charge is quantized. Charge can't be divided more than one electron or proton.

## 2 Conductors, Insulators, And Induced Charges

- Conductors permit the easy movement of charge through them, while insulators do not
- Most metals are good conductors, while most nonmetals are insulators
- Some materials called semiconductors are intermediate in their properties between good conductors and good insulators.

### 2.1 Charging By Induction

- We can charge a metal ball using a copper wire and an electrically charged plastic rod, however this requires giving off some excess electrons
- Charging by induction lets the plastic rod give another body a charge of opposite sign without losing any of its own charge

### 2.2 Electric Forces On Uncharged Objects

- A charged body can exert forces even on objects that are not charged themselves
- Polarization: all the positive charge in an object goes to one side, all the negative goes to the other

## 3 Coulomb's Law

- The magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them
- The magnitude  $F$  of the force of two point charges  $q_1$  and  $q_2$  that are a distance  $r$  from each other can be expressed as

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2} \quad (1)$$

- Thus, this force is similar to the force of gravity between two objects

### 3.1 Fundamental Electric Constants

- The SI electric units include most of the familiar units such as the volt, the ampere, the ohm, and the watt.
- The SI unit of electric charge is called one coulomb (1 C), in coulombs, we have that

$$k = \frac{1}{4\pi\epsilon_0} = 8.987551787 \times 10^9 N \cdot m^2/C^2 \quad (2)$$

- This value will often just get approximated to  $9.0 \times 10^9 N \cdot m^2/C^2$

## 3.2 Superposition of Forces

Experiments show that when two charges exert forces simultaneously on a third charge, the total force acting on that charge is the vector sum of the forces that the two charges would exert individually. This phenomena is known as the superposition of forces

# 4 Electric Field and Electric Forces

## 4.1 Electric Field

- An electric field is a force similar to gravity in that it does not need to physically contact an object and can simply act across empty space
- The electric force on a charged body is exerted by the electric field created by other charged bodies.
- The electric field at a certain point is equal to the electric force per unit charge experienced by a charge at that point:

$$\vec{E} = \frac{\vec{F}_0}{q_0} \quad (3)$$

- So in this equation,  $\vec{F}_0$  is the Electric force at the test charge  $q_0$ , while  $\vec{E}$  is in terms of electric force per unit charge

## 4.2 Electric Field of a Point Charge

- We call the location of the charge the source point, and we call the point P where we are determining the field the field point.
- It also often useful to introduce  $\hat{r}$ , a unit vector that points along the line from the source point to the field point
- Take the displacement vector  $\vec{r}$  and divide by the distance  $|r|$ , so then  $\hat{r} = \vec{r}/|r|$
- The magnitude  $E$  of the electric field at  $P$  is:

$$E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2} \quad (4)$$

- Using the unit vector  $\hat{r}$ , we can write a vector equation that gives both magnitude and direction of the electric field

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \quad (5)$$

# 5 Electric-Field Calculations

## 5.1 The Superposition of Electric Fields

- The total force  $\vec{F}_0$  that the charge distribution exerts on  $q_0$  is the vector sum of the total forces

$$\vec{F}_0 = \vec{F}_1 + \vec{F}_2 + \dots = q_0\vec{E}_1 + q_0\vec{E}_2 + \dots \quad (6)$$

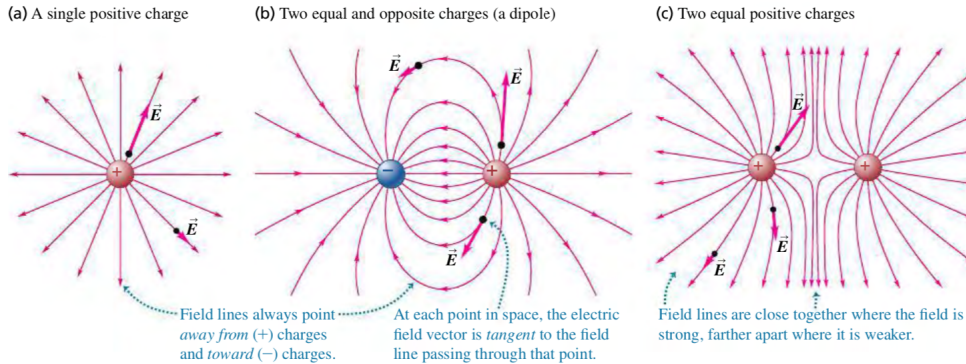
- The combined effect of all the charges in the distribution is described by the total electric field  $\vec{E}$  at point  $P$

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \vec{E}_1 + \vec{E}_2 + \dots \quad (7)$$

- The total electric field at P is the vector sum of the fields at P due to each point charge in the charge distribution, this is the principle of superposition of electric fields.
- When charge is distributed along a line, over a surface, or through a volume, we also want the charge density of the medium

## 6 Electric Field Lines

- An electric field line is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric-field vector at that point.
- Electric field lines show the direction of  $\vec{E}$  at each point, and their spacing gives a general idea of the magnitude of  $\vec{E}$  at each point
- Electric field lines for three different charge distributions. In general, the magnitude of  $\vec{E}$  is different at different points along a given field line



## 7 Electric Dipoles

An electric dipole is a pair of point charges with equal magnitude and opposite sign separated by a distance  $d$

### 7.1 Force and Torque on an Electric Dipole

- To begin, place an electric dipole in a uniform external electric field  $\vec{E}$ . Both forces  $\vec{F}_+$ ,  $\vec{F}_-$  on the two charges have magnitude  $qE$
- The magnitude of the net torque is twice the magnitude of the individual torques

$$\tau = (qE)(d \sin \phi) \quad (8)$$

- Where  $d \sin \phi$  is the perpendicular distance between the lines of action of the two forces
- The product of charge  $q$  and distance  $d$  is the magnitude of a quantity known as the electric dipole moment, denoted  $p = qd$
- In terms of vector torque on the electric dipole, we can write the vector form:

$$\vec{\tau} = \vec{p} \times \vec{E} \quad (9)$$

- Torque is thus greatest when  $\vec{p}$  and  $\vec{E}$  are perpendicular and is zero when they are parallel or antiparallel

### 7.2 Potential Energy Of An Electric Dipole

- When a dipole changes direction in an electric field, the electric-field torque does work on it
- The work  $dW$  done by a torque  $\tau$  during an infinitesimal displacement  $d\phi$  is given by  $dW = \tau d\phi$  Because torque is in the direction of decreasing  $\phi$ , we must have that  $\tau = -pE \sin \phi$  and

$$dW = \tau d\phi = -pE \sin \phi d\phi \quad (10)$$

- So the total work done from  $\phi_1$  to  $\phi_2$  is given by

$$W = \int_{\phi_1}^{\phi_2} (-pE \sin \phi) d\phi = pE(\cos \phi_2 - \cos \phi_1) \quad (11)$$

- Work is the negative change of potential energy, so work is  $W = U_1 - U_2$ , so then we have the equation of potential energy:

$$U(\phi) = -pE \cos \phi \quad (12)$$

- In vector form, we recognize the scalar product  $\vec{\mathbf{p}} \cdot \vec{\mathbf{E}} = pE \cos \phi$ , so:

$$U = -\vec{\mathbf{p}} \cdot \vec{\mathbf{E}} \quad (13)$$