# Group 1

**Problem 2.7.**

*Use the Well Ordering Principle to prove that any integer greater than or equal to 8 can be represented as the sum of nonnegative integer multiples of 3 and 5.*

Let for all integer , exists such that

By contradiction. Assume that is false. Then, some integers serve as counterexamples to it. The collection of counterexamples is:

Assuming is a nonempty set of integers. Then, by the Well Ordering Principle, has a minimum element, which we can call . And is the smallest counterexample among the integers.

Since is the smallest counterexample, we know will not equal to for any integer .

Then, We can check are holds.So must be greater than 10 to be a counterexample. But then, is greater than or equal to 8 and less than , so is hold.

So. exists such that

which means this does hold for , after all. Thus, is in empty, and we are done.

**Problem 2.15.**  
*We'll use the Well Ordering Principle to prove that for every positive integer , the sum of the first odd numbers is , that is,*

*for all .*

By contradiction. Assume that this theorem is false. Then, some positive integers serve as counterexamples to it. The collection of counterexamples is:

Assuming is a nonempty set of postive integers. Then, by the Well Ordering Principle, has a minimum element, which we can call . And is the smallest counterexample among the postive integers.

Since is the smallest counterexample, we know the theorem is false for but true for all postive integers . But the theorem is true for , so .Thus, . This means is a positive integer. That is,

But then, adding to both sides, we get

which means this theorem does hold for , after all, This is a contradiction, and we are done.