Tutorial: Third Normal Form

1. Consider the schema $R = \{A, B, C, D\}$ with the set of functional dependencies $\Sigma = \{\{A\} \rightarrow \{B, C, D\}, \{C\} \rightarrow \{D\}\}$.

   (a) Is the decomposition into $\sigma = \{R_1(A, B, D), R_2(A, C)\}$ a dependency preserving decomposition?

   Solution: NO.
   The projection of Σ on R_1 (*we can denote this as* $\Sigma|_{R_1}$) is $\{\{A\} \rightarrow \{B, D\}\}$. The projection $\Sigma|_{R_2}$ is $\{\{A\} \rightarrow \{C\}\}$.
   The union of the projection is $\Sigma_\cup = \Sigma|_{R_1} \cup \Sigma|_{R_2} = \{\{A\} \rightarrow \{B, D\}, \{A\} \rightarrow \{C\}\}$. We need to check if it is equivalent to Σ.
   Note that by construction $\Sigma_\cup$ is logically entailed by Σ. So we only need to show that Σ is logically entailed by $\Sigma_\cup$.

   - Consider $\{\{A\} \rightarrow \{B, C, D\}\}$.
     - $\{A\}^+$ with respect to $\Sigma_\cup$ is $\{A, B, C, D\}$. $\{A\} \rightarrow \{B, C, D\}$ is logically entailed by $\Sigma_\cup$.
   - Consider $\{\{C\} \rightarrow \{D\}\}$.
     - $\{C\}^+$ with respect to $\Sigma_\cup$ is $\{C\}$. $\{C\} \rightarrow \{D\}$ is not logically entailed by $\Sigma_\cup$.

   Therefore, $\Sigma \not\equiv \Sigma_\cup$. The decomposition is not dependency preserving.

   (b) Is the decomposition into $\sigma = \{R_1(A, B, C), R_2(C, D)\}$ a dependency preserving decomposition?

   Solution: YES.
   The projection of Σ on R_1 (*we can denote this as* $\Sigma|_{R_1}$) is $\{\{A\} \rightarrow \{B, C\}\}$. The projection $\Sigma|_{R_2}$ is $\{\{C\} \rightarrow \{D\}\}$.
   The union of the projection is $\Sigma_\cup = \Sigma|_{R_1} \cup \Sigma|_{R_2} = \{\{A\} \rightarrow \{B, C\}, \{C\} \rightarrow \{D\}\}$. We need to check if it is equivalent to Σ.

   - Consider $\{\{A\} \rightarrow \{B, C, D\}\}$.
     - $\{A\}^+$ with respect to $\Sigma_\cup$ is $\{A, B, C, D\}$. $\{A\} \rightarrow \{B, C, D\}$ is logically entailed by $\Sigma_\cup$.
   - Consider $\{\{C\} \rightarrow \{D\}\}$.
     - $\{C\}^+$ with respect to $\Sigma_\cup$ is $\{C, D\}$. $\{C\} \rightarrow \{D\}$ is logically entailed by $\Sigma_\cup$.

   Therefore, $\Sigma \equiv \Sigma_\cup$. The decomposition is dependency preserving.

2. Consider the schema $R = \{A, B, C, D, E\}$ with a set of functional dependencies $\Sigma = \{\{A\} \rightarrow \{B, D, E\}, \{C, D\} \rightarrow \{A\}, \{E\} \rightarrow \{B, D\}\}$.

   (a) Is R in third normal form with respect to Σ?

   Solution: R is not in third normal form with respect to Σ. Consider the following functional dependency $\{E\} \rightarrow \{B\}$. This violates the third normal form property of R with respect to Σ because

   - $\{B\} \not\subseteq \{E\}$ so $\{E\} \rightarrow \{B\}$ is non-trivial.
   - $\{E\}^+ = \{B, D, E\}$ so $\{E\}$ is not the superkey of R.
   - The keys of R are $\{A, C\}$, $\{C, D\}$, and $\{C, E\}$. So B is not a prime attribute of R with respect to Σ.

   Therefore, R is not in third normal form with respect to Σ.

   (b) If R is not in third normal form with respect to Σ, find a third normal form decomposition of R with respect to Σ that are both dependency preserving and lossless join.

   Solution: To guarantee both properties, we need to use the 3NF synthesis algorithm. The first step is to find the minimal cover.

   - $\Sigma_1 = \{\{A\} \rightarrow \{B\}, \{A\} \rightarrow \{D\}, \{A\} \rightarrow \{E\}, \{C, D\} \rightarrow \{A\}, \{E\} \rightarrow \{B\}, \{E\} \rightarrow \{D\}\}$
   - $\Sigma_2 = \{\{A\} \rightarrow \{B\}, \{A\} \rightarrow \{D\}, \{A\} \rightarrow \{E\}, \{C, D\} \rightarrow \{A\}, \{E\} \rightarrow \{B\}, \{E\} \rightarrow \{D\}\}$
   - $\Sigma_3 = \{\{A\} \rightarrow \{E\}, \{C, D\} \rightarrow \{A\}, \{E\} \rightarrow \{B\}, \{E\} \rightarrow \{D\}\}$

   Then we find the canonical cover.

   - $\Sigma_4 = \{\{A\} \rightarrow \{E\}, \{C, D\} \rightarrow \{A\}, \{E\} \rightarrow \{B, D\}\}$

   Using canonical cover, we construct the table from each functional dependency.

   - $\delta = \{R_1(A, E), R_2(A, C, D), R_3(B, D, E)\}$

   Lastly, to ensure lossless join decomposition, we need to check if at least one of the key is already in one of the fragments. Here, either $\{A, C\}$ or $\{C, D\}$ is in R_2, so we do not need to construct a table for the key.

   (c) Is your decomposition in part (b) above in Boyce-Codd normal form with respect to Σ.

   Solution: R_2 is not in Boyce-Codd normal form because $\{A\} \rightarrow \{D\}$.

   - $\{D\} \not\subseteq \{A\}$ so $\{A\} \rightarrow \{D\}$ is non-trivial.
   - $\{A\}^+ = \{A, B, D, E\}$ so $\{A\}$ is not the superkey of R_2.

   However, do note that there is a possibility that by using 3NF synthesis algorithm, the decomposed schema is in Boyce-Codd normal form. In which case, we are lucky because this is dependency preserving Boyce-Codd normal form decomposition. Unfortunately, it is not always the case.

Solution: The general steps in solving the problem is to compute the candidate keys: $\{A, C\}$, $\{C, D\}$, and $\{C, E\}$. Then we can compute the prime attributes: $\{A, C, D, E\}$.

From here, we can easily check part (a). Consider $\{A\} \rightarrow \{B\}$. Since $\{B\} \not\subseteq \{A\}$, it is non-trivial. $\{A\}$ is not a superkey because it is not a superset of *any* key. Finally, B is not a prime attribute.

The second step is to compute the minimal cover. By first computing the key, you will likely reduce the overall amount of computation needed to check for lossless-join dependency preserving decomposition either in BCNF or 3NF.