Question 1

a. With simple interest, you earn 4% of $1,000, or $40 each year. There is no interest on interest. After 10 years, you earn total interest of $400, and your account accumulates to $1,400.

$$\begin{aligned} FV_{\text{Simple Interest}} &= PV + PV \times (r \times t) \\ &= \$1{,}000 + \$1{,}000 \times (.04 \times 10) \\ &= \$1{,}400 \end{aligned}$$

b.
$$\begin{aligned} FV &= PV \times (1 + r)^t \\ &= \$1{,}000 \times 1.04^{10} \\ &= \$1{,}480.24 \end{aligned}$$

With compound interest, each year you earn interest on the principal AND the interest accumulated in all prior years. In this case, the compound interest amounts to:

Compound interest = $1,480.24 – 1,400 = $80.24

Question 2

a. The present value of the ultimate sales price is $PV = FV / (1 + r)^t$ = $4 million/$(1.08)^5$ = $2.722 million.

b. The present value of the sales price is less than the purchase price of the property, so this would not be an attractive opportunity.

c.
$$\begin{aligned} PV &= \text{Per-year rent} \times ((1 / r) - \{1 / [r(1 + r)^t]\}) + \text{Sales price} / (1 + r)^t \\ &= \$200{,}000 \times ((1 / .08) - \{1 / [.08\,(1.08)^5]\}) + \$4{,}000{,}000 / 1.08^5 \\ &= \$3{,}520{,}874.80, \text{ or } \$3.521 \text{ million} \end{aligned}$$

The investment is attractive now because the present value of the future cash flows exceeds the current purchase price of the property.

Question 3

a. This is an annuity problem; use trial and error (You can also use the RATE function in Excel) to solve for r in the following equation:

$$\$600 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{240}}\right] = \$80{,}000 \Rightarrow r = 0.548\%$$

b. $EAR = (1 + 0.00548)^{12} - 1 = 0.0678 = 6.78\%$

c. Compute the payment by solving for C in the following equation:

$$C \times \left[\frac{1}{0.005} - \frac{1}{0.005 \times (1.005)^{240}}\right] = \$80{,}000 \Rightarrow C = \text{PMT} = \$573.14$$

Question 4

a. You borrow \$1,000 and repay the loan by making 12 monthly payments of \$100. Solve for r in the following equation:

$$\$100 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{12}}\right] = \$1{,}000 \Rightarrow r = 2.923\% \text{ per month}$$

[Using a financial calculator, enter PV = (–)1,000, FV = 0, n = 12, PMT = 100; compute r = 2.923%. You can also use the RATE function in Excel]

Therefore, the APR is 2.923% × 12 = 35.076%.

b. The effective annual rate is $(1.02923)^{12} - 1 = 0.41302 = 41.302\%$.

If you borrowed \$1,000 today and paid back \$1,200 one year from today, the true rate would be 20%. You should have known that the true rate must be greater than 20% because the twelve \$100 payments are made before the end of the year, thus increasing the true rate above 20%.

Question 5

a. Assume the retirees receive their first cash flow at the end of their first year of retirement.

$$\text{The real interest rate} = \frac{1.08}{1.05} - 1 = .0286 \text{ or } 2.86\%$$

The present value of their retirement savings at retirement must be

$$\text{PV} = \$30,000 \times \left[\frac{1}{0.0286} - \frac{1}{0.0286 \times (1.0286)^{25}}\right] = \$530,638$$

The real annual savings must be

$$C \times \left[\frac{1.0286^{50} - 1}{0.0286}\right] = \$530{,}638 \Rightarrow C = PMT = \$4{,}908.08$$

b. If the *real* amount saved is \$4,908.08 and prices rise at 5% per year, then the amount saved at the end of 1 year, in nominal terms, will be:

\$4,908.08 × 1.05 = \$5,147.52

c. If the *real* amount saved is \$4,908.08 and prices rise at 5% per year, then the amount saved at the end of 50 year, in nominal terms, will be:

$\$4{,}908.08 \times (1.05)^{50} = \$55{,}712.78$

d. The first expenditure occurs at the end of the first year of retirement, which is 51 years from today. If the real amount spent is \$30,000 and prices rise at 5% per year, then the amount spent in that year will be::

$\$30{,}000 \times (1.05)^{51} = \$361{,}223.09$

e. If the *real* amount spent is \$30,000 and prices rise at 5% per year, then the amount spent in their last year of retirement, will be:

$\$30{,}000 \times (1.05)^{75} = \$1{,}164{,}980.58$