Tutorial: Functional Dependencies

1. Consider the schema $R = \{A, B, C, D, E, F\}$ with a set of functional dependencies $\Sigma = \{\{A, B, C\} \rightarrow \{E\}, \{B, D\} \rightarrow \{A\}, \{C, F\} \rightarrow \{B\}\}$.

   (a) Compute $\{C, D, F\}^+$

   > **Solution:** Using Algorithm #1, we can trace the iteration as follows.
   >
   > 1. Starting
   >
   > $$\Omega = \{\{A, B, C\} \rightarrow \{E\}, \{B, D\} \rightarrow \{A\}, \{C, F\} \rightarrow \{B\}\}$$
   > $$\Gamma = \{C, D, F\}$$
   >
   > 2. Using $\{C, F\} \rightarrow \{B\}$ $\qquad (\{C, F\} \subseteq \{C, D, F\})$
   >
   > $$\Omega = \{\{A, B, C\} \rightarrow \{E\}, \{B, D\} \rightarrow \{A\}\}$$
   > $$\Gamma = \{B, C, D, F\}$$
   >
   > 3. Using $\{B, D\} \rightarrow \{A\}$ $\qquad (\{B, D\} \subseteq \{B, C, D, F\})$
   >
   > $$\Omega = \{\{A, B, C\} \rightarrow \{E\}\}$$
   > $$\Gamma = \{A, B, C, D, F\}$$
   >
   > 4. Using $\{A, B, C\} \rightarrow \{E\}$ $\qquad (\{A, B, C\} \subseteq \{A, B, C, D, F\})$
   >
   > $$\Omega = \{\}$$
   > $$\Gamma = \{A, B, C, D, E, F\}$$
   >
   > 5. Answer: $\{A, B, C, D, E, F\}$

   (b) Find all the candidate keys of R.

   > **Solution:** We may enumerate all possible subsets of attributes of R and compute the closure. But this will be time-consuming. So there are some tricks you can do. Not all tricks work, so you are encouraged not to use shortcuts. However, we guarantee that the following trick works.
   >
   > 1. Start from smallest cardinality.
   > 2. Look at attributes not on the right hand side of any functional dependencies.
   >
   > Following trick #2, we can see that $\{C, D, F\}$ do not appear on any right hand side. So $\{C, D, F\}$

must be on every key. But since $\{C, D, F\}$ is already a key, all other superkeys must be a superset of $\{C, D, F\}$ as they must include $\{C, D, F\}$.
So the only key is $\{C, D, F\}$.

2. Consider the schema $R = \{A, B, C, D\}$ with a set of functional dependencies $\Sigma = \{\{A\} \to \{C, D\}, \{A, C\} \to \{D\}, \{A, D\} \to \{B\}, \{C\} \to \{D\}\}$.
   (a) Find the minimal cover for Σ.

   Solution: Using Algorithm #2, we can trace the execution as follows.

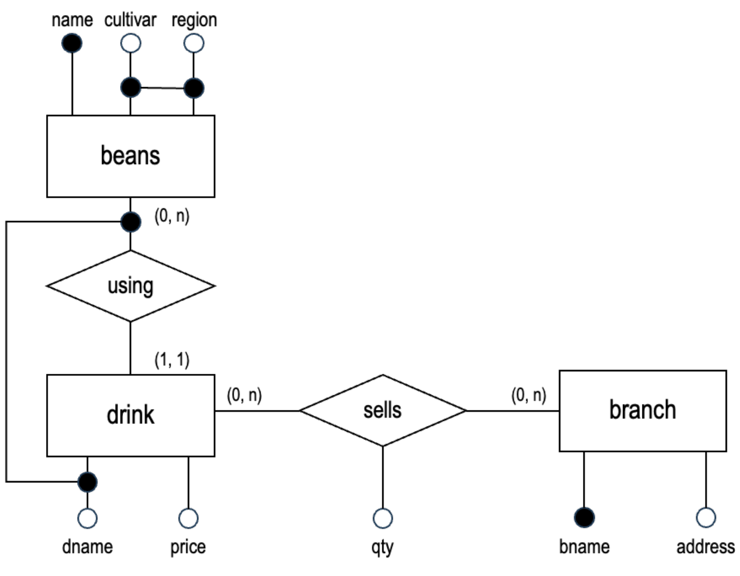
   1. $\Sigma = \{\{A\} \to \{C, D\}, \{A, C\} \to \{D\}, \{A, D\} \to \{B\}, \{C\} \to \{D\}\}$
   2. $\Sigma_1 = \{\{A\} \to \{C\}, \{A\} \to \{D\}, \{A, C\} \to \{D\}, \{A, D\} \to \{B\}, \{C\} \to \{D\}\}$
   3. $\Sigma_2 = \{\{A\} \to \{C\}, \{A\} \to \{D\}, \{A\} \to \{B\}, \{C\} \to \{D\}\}$
   4. $\Sigma_3 = \{\{A\} \to \{C\}, \{A\} \to \{B\}, \{C\} \to \{D\}\}$

   (b) Find the canonical cover for Σ.

   Solution: From the answer to the previous part, we can obtain the following:

   1. $\Sigma_4 = \{\{A\} \to \{B, C\}, \{C\} \to \{D\}\}$

3. Consider the following entity-relationship diagram regarding coffee shops. This involves the branch, the drink, and the coffee bean.



For simplicity, you may use the following characters to represent the attributes.

Attribute	Character
`name`	A
`cultivar`	B
`region`	C
`dname`	D

Attribute	Character
`price`	E
`qty`	F
`bname`	G
`address`	H

(a) Find the attribute closure of $\{$`name`, `dname`$\}$. In other words, find $\{$`name`, `dname`$\}^+$.

Solution: Since $\{$`name`$\}$ is one of the candidate keys (*the other being* $\{$`cultivar`, `region`$\}$) of the entity set `beans`, we know that $\{$`name`$\} \to \{$`cultivar`, `region`$\}$.

Since {`name`, `dname`} is the primary key of the weak entity set `drink` (*merged with* `using`), we know that {`name`, `dname`} → {`price`}.
Other functional dependencies are listed below for completeness.

- {`bname`} → {`address`}
- {`cultivar`, `region`} → {`name`}
- {`cultivar`, `region`, `dname`} → {`price`}
- {`name`, `dname`, `bname`} → {`qty`}
- {`cultivar`, `region`, `dname`, `bname`} → {`qty`}

The rest can actually be logically entailed from here. So starting from {`name`, `dname`}, following Algorithm #1 introduced in the lecture, we can perform the following iteration:

1. Starting

$$\begin{aligned}\Omega = \{&\{\texttt{name}\} \rightarrow \{\texttt{cultivar}, \texttt{region}\}, \{\texttt{name}, \texttt{dname}\} \rightarrow \{\texttt{price}\}, \{\texttt{bname}\} \rightarrow \{\texttt{address}\},\\ &\{\texttt{cultivar}, \texttt{region}\} \rightarrow \{\texttt{name}\}, \{\texttt{name}, \texttt{dname}, \texttt{bname}\} \rightarrow \{\texttt{qty}\},\\ &\{\texttt{cultivar}, \texttt{region}, \texttt{dname}\} \rightarrow \{\texttt{price}\},\\ &\{\texttt{cultivar}, \texttt{region}, \texttt{dname}, \texttt{bname}\} \rightarrow \{\texttt{qty}\}\}\\ \Gamma = \{&\texttt{name}, \texttt{dname}\}\end{aligned}$$

2. Using $\{\texttt{name}\} \rightarrow \{\texttt{cultivar}, \texttt{region}\}$ $\quad(\{\texttt{name}\} \subseteq \Gamma)$

$$\begin{aligned}\Omega = \{&\{\texttt{name}, \texttt{dname}\} \rightarrow \{\texttt{price}\}, \{\texttt{bname}\} \rightarrow \{\texttt{address}\},\\ &\{\texttt{cultivar}, \texttt{region}\} \rightarrow \{\texttt{name}\}, \{\texttt{name}, \texttt{dname}, \texttt{bname}\} \rightarrow \{\texttt{qty}\},\\ &\{\texttt{cultivar}, \texttt{region}, \texttt{dname}\} \rightarrow \{\texttt{price}\},\\ &\{\texttt{cultivar}, \texttt{region}, \texttt{dname}, \texttt{bname}\} \rightarrow \{\texttt{qty}\}\}\\ \Gamma = \{&\texttt{name}, \texttt{dname}, \texttt{cultivar}, \texttt{region}\}\end{aligned}$$

3. Using $\{\texttt{cultivar}, \texttt{region}\} \rightarrow \{\texttt{name}\}$ $\quad(\{\texttt{cultivar}, \texttt{region}\} \subseteq \Gamma)$

$$\begin{aligned}\Omega = \{&\{\texttt{name}, \texttt{dname}\} \rightarrow \{\texttt{price}\}, \{\texttt{bname}\} \rightarrow \{\texttt{address}\},\\ &\{\texttt{name}, \texttt{dname}, \texttt{bname}\} \rightarrow \{\texttt{qty}\}\}\\ \Gamma = \{&\texttt{name}, \texttt{dname}, \texttt{cultivar}, \texttt{region}\}\end{aligned}$$

4. Using $\{\texttt{name}, \texttt{dname}\} \rightarrow \{\texttt{price}\}$ $\quad(\{\texttt{name}, \texttt{dname}\} \subseteq \Gamma)$

$$\begin{aligned}\Omega = \{&\{\texttt{bname}\} \rightarrow \{\texttt{address}\}, \{\texttt{name}, \texttt{dname}, \texttt{bname}\} \rightarrow \{\texttt{qty}\},\\ &\{\texttt{cultivar}, \texttt{region}, \texttt{dname}\} \rightarrow \{\texttt{price}\},\\ &\{\texttt{cultivar}, \texttt{region}, \texttt{dname}, \texttt{bname}\} \rightarrow \{\texttt{qty}\}\}\\ \Gamma = \{&\texttt{name}, \texttt{dname}, \texttt{cultivar}, \texttt{region}, \texttt{price}\}\end{aligned}$$

5. Using $\{\texttt{cultivar}, \texttt{region}, \texttt{dname}\} \rightarrow \{\texttt{price}\}$ $\quad(\{\texttt{cultivar}, \texttt{region}, \texttt{dname}\} \subseteq \Gamma)$

$$\begin{aligned}\Omega = \{&\{\texttt{bname}\} \rightarrow \{\texttt{address}\}, \{\texttt{name}, \texttt{dname}, \texttt{bname}\} \rightarrow \{\texttt{qty}\},\\ &\{\texttt{cultivar}, \texttt{region}, \texttt{dname}, \texttt{bname}\} \rightarrow \{\texttt{qty}\}\}\\ \Gamma = \{&\texttt{name}, \texttt{dname}, \texttt{cultivar}, \texttt{region}, \texttt{price}\}\end{aligned}$$

6. Answer: $\{\texttt{name}, \texttt{dname}, \texttt{cultivar}, \texttt{region}, \texttt{price}\}$

(b) Let Σ^+ be the closure of the set of functional dependencies that holds according to the entity-relationship diagram. We assume that we are using a single table but with the functional dependencies enforced (*e.g., via triggers?*). Find the canonical cover of Σ^+.

Solution: We already know the set of functional dependencies that holds according to entity-relationship diagram based on the previous question.

- {`name`} → {`cultivar`, `region`}
- {`name`, `dname`} → {`price`}
- {`bname`} → {`address`}
- {`cultivar`, `region`} → {`name`}
- {`cultivar`, `region`, `dname`} → {`price`}
- {`name`, `dname`, `bname`} → {`qty`}
- {`cultivar`, `region`, `dname`, `bname`} → {`qty`}

By running Algorithm #3, you should obtain the following canonical cover.

- {`name`} → {`cultivar`, `region`}
- {`name`, `dname`} → {`price`}
- {`bname`} → {`address`}
- {`cultivar`, `region`} → {`name`}
- {`name`, `dname`, `bname`} → {`qty`}

Notice how these corresponds to the primary keys (or unique and not null constraint) of the tables constructed from schema translation.