Problem 1

If you earned simple interest (without compounding), then the total growth in your account after 25 years would be 4% per year × 25 years = 100%. Therefore, your money would double. With compound interest, your money would grow faster than it would with simple interest and therefore would require less than 25 years to double.

Problem 2

Since we are assuming that it is currently 2019, 104 years have passed since 1915.

a.

$$\begin{aligned} \text{Future value}_{\text{Year (2019 – 1915)}} &= \text{Present value} \times (1 + r)^{(2019-1915)} \\ &= \$1{,}000 \times 1.05^{104} \\ &= \$159{,}840.60 \end{aligned}$$

b.

$$\begin{aligned} \text{Present value} &= \text{Future value} / (1 + r)^{t} \\ &= \$1{,}000{,}000 / (1.05)^{(2019-1915)} \\ &= \$6{,}256.23 \end{aligned}$$

Problem 3

$FV = PV \times (1 + r)^t$

a.

$FV = A\$100 \times (1.03)^{116} = A\$3{,}084.03$

b.

$FV = A\$100 \times (1.06)^{116} = A\$86{,}194.66$

Problem 4

$$\begin{aligned} FV &= PV \times (1 + r)^t \\ (1 + r)^t &= FV / PV \\ t \times \ln(1 + r) &= \ln(FV / PV) \\ t &= \ln(FV / PV) / \ln(1 + r) \end{aligned}$$

a.

$$\begin{aligned} t &= \ln(\$2{,}000 / \$500) / \ln 1.05 \\ &= 1.3863 / 0.0488 \\ &= 28.41 \text{ years} \end{aligned}$$

b.

$$\begin{aligned} t &= \ln(\$2{,}000 / \$500) / \ln 1.090 \\ &= 1.3863 / 0.0862 \\ &= 16.09 \text{ years} \end{aligned}$$

c.

$$\begin{aligned} t &= \ln(\$2{,}000 / \$500) / \ln 1.170 \\ &= 1.3863 / 0.1570 \\ &= 8.83 \text{ years} \end{aligned}$$

Problem 5

You earned compound interest of 6% for 8 years and 4% for 13 years. Your \$2,500 has grown to:

$\$2,500 \times (1.06)^8 \times (1.04)^{13} = \$6,634.69$

Problem 6

$PV = C((1 / r) - \{1 / [r(1 + r)^t]\})$

a-1.

$PV = \$825 \times ((1 / 0.03) - \{1 / [0.03(1.03)^{12}]\})$
$= \$8,212.05$

a-2.

$PV = \$625 \times ((1 / 0.03) - \{1 / [0.03(1.03)^{17}]\})$
$= \$8,228.82$

a-3.

You would prefer the \$625 a year for 17 years because that option has the higher present value when the interest rate is 3 percent.

b-1.

$PV = \$825 \times ((1 / 0.12) - \{1 / [0.12(1.12)^{12}]\})$
$= \$5,110.36$

b-2.

$PV = \$625 \times ((1 / 0.12) - \{1 / [0.12(1.12)^{17}]\})$
$= \$4,449.77$

b-3.

You would prefer the \$825 a year for 12 years because that option has the higher present value when the interest rate is 12 percent.

When the interest rate is low, as in part (a), the longer (i.e., 17-year) but smaller annuity is more valuable because the impact of discounting on the present value of future payments is less significant.

Problem 7

a.

$PV = C((1 / r) - \{1 / [r(1 + r)^t]\})$
$= \$7,000 \times ((1 / 0.06) - \{1 / [0.06(1.06)^5]\})$
$= \$29,486.55$

b.

Since the present value of the lease is less than the purchase price, it is cheaper to lease.

c.

$PV_{AD} = PV_{OA} \times (1 + r)$
$= \$29,486.55 \times 1.06$
$= \$31,255.74$

Note that "AD" stands for annuity due and "OA" stands for ordinary annuity.

d.

Since the purchase price is less than the present value of the lease, it is cheaper to buy.

Problem 8

First, you need to compute the monthly mortgage payment. Because the payments are monthly, the interest rate and the number of time periods must also be expressed in terms of months. It is assumed that annuity payments occur at the end of each time period unless a problem indicates otherwise.

$$PV = C((1 / r) - \{1 / [r(1 + r)^t]\})$$
$$\$210{,}000 = C \times [[1 / (0.12 / 12)] - (1 / \{(0.12 / 12)[1 + (0.12 / 12)]^{(25 \times 12)}\})]$$
$$C = \$2{,}211.77$$

Second, compute the loan balance at the end of 16 years. Use the monthly loan payment rounded to 2 decimal places and adjust the number of months to equal the number of payments remaining.

$$PV = C((1 / r) - \{1 / [r(1 + r)^t]\})$$
$$= \$2{,}211.77 \times [[1 / (0.12 / 12)] - (1 / \{(0.12 / 12)[1 + (0.12 / 12)]^{[(25 - 16) \times 12]}\})]$$
$$= \$145{,}662.28$$

Problem 9

a.

$$PV = FV / (1 + r)^t$$
$$= \$100 / 1.083^3$$
$$= \$78.73$$

b.

$$\text{Real value} = \text{Nominal value} / (1 + \text{Inflation rate})^t$$
$$= \$100 / 1.033^3$$
$$= \$90.72$$

c.

$$\text{Real interest rate} = [(1 + \text{Nominal rate}) / (1 + \text{Inflation rate})] - 1$$
$$= (1.083 / 1.033) - 1$$
$$= 0.0484, \text{ or } 4.84\%$$

d.

Discount the real value using the real interest rate:

$$PV = FV / (1 + r)^t$$
$$= \$90.72 / 1.0484^3$$
$$= \$78.73$$

Problem 10

First, determine the amount of savings required on the date of retirement:

$$\begin{aligned} PV &= C((1 / r) - \{1 / [r(1 + r)^t]\}) \\ &= \$35{,}000 \times ((1 / 0.06) - \{1 / [0.06(1.06)^{15}]\}) \\ &= \$339{,}928.71 \end{aligned}$$

Now compute the annual savings needed to accumulate the needed retirement savings:

$$\begin{aligned} FV &= C \times \{[(1 + r)^t - 1] / r\} \\ \$339{,}928.71 &= C \times [(1.06^{30} - 1) / 0.06] \\ C &= \$4{,}299.73 \end{aligned}$$