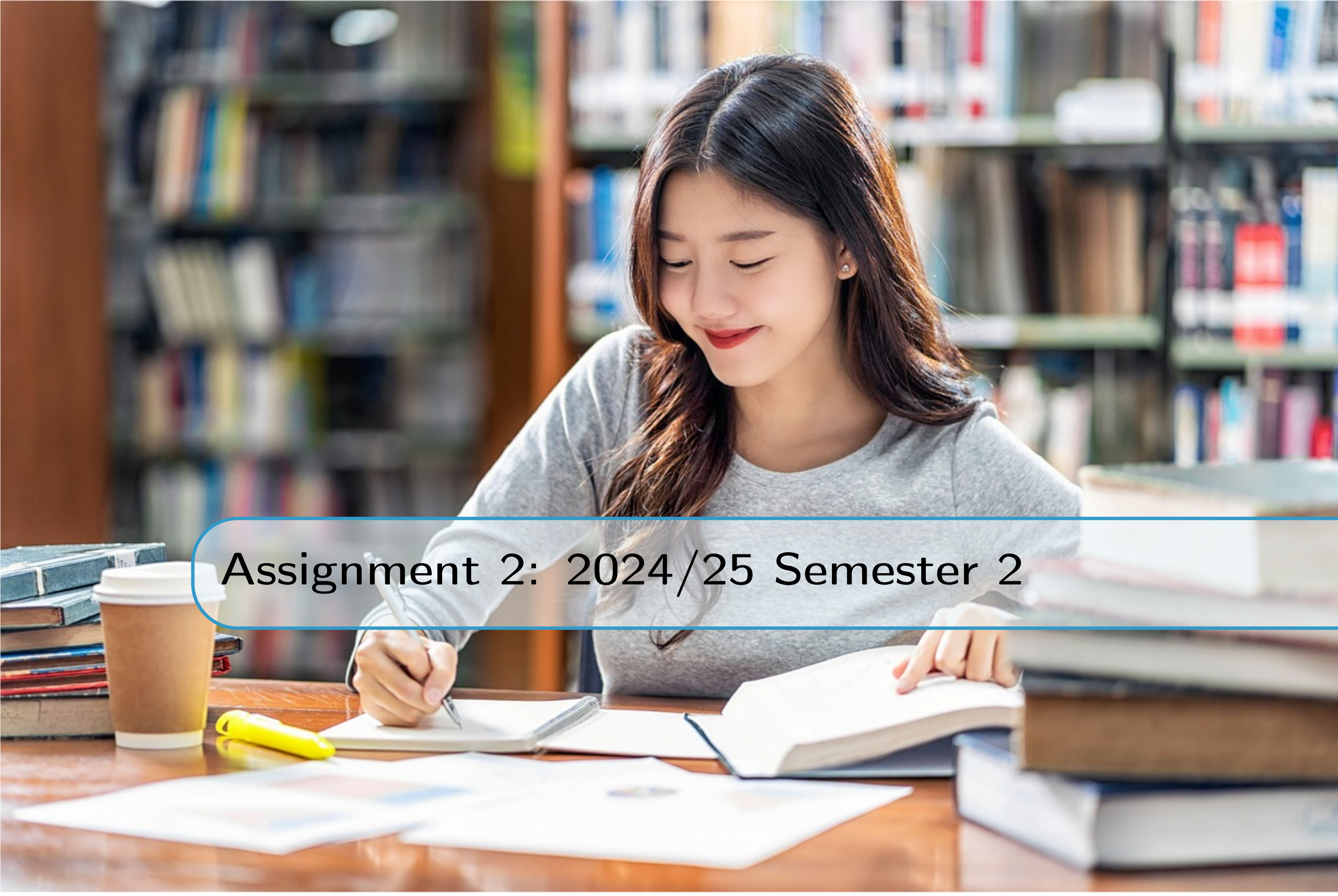

Assignment 2: 2024/25 Semester 2

In this assignment, we will be using the following relation as well as set of functional dependencies.

$$R = \{A, B, C, D, E, F\}$$

$$\Sigma = \{\ \{A,B\} \rightarrow \{B,E\}, \{B,D\} \rightarrow \{B\}, \{B,F\} \rightarrow \{A,C\}, \{A,E\} \rightarrow \{C,F\}, \{D\} \rightarrow \{A,C\}, \{E\} \rightarrow \{D,F\}, \{D,E,F\} \rightarrow \{D,F\}\ \}$$

Notes: As usual, we will compute the candidate keys and one minimal cover. We assume you know how to do this.

- **Keys:** $\{\ \{A,B\}, \{B,D\}, \{B,E\}, \{B,F\}\ \}$
- **Min Cover:** $\{\ \{A,B\} \rightarrow \{E\}, \{B,F\} \rightarrow \{A\}, \{D\} \rightarrow \{A\}, \{D\} \rightarrow \{C\}, \{E\} \rightarrow \{D\}, \{E\} \rightarrow \{F\}\ \}$

Question 1 and 2 are intended to show you that we can first preprocess Σ to not include *irrelevant* data. This can be done by removing *trivial* functional dependencies as well as splitting *non-trivial* functional dependencies (*Q1*) so that the resulting functional dependencies is completely non-trivial. If you prefer, you can refer to this process as *non-trivializing* the functional dependencies.

Q1-Q3 Non-Trivial ONLY and Completely Non-Trivial.

Trivial

- $\{B,D\} \rightarrow \{B\}$
- $\{D,E,F\} \rightarrow \{D,F\}$

Non-Trivial ONLY

- $\{A,B\} \rightarrow \{B,E\}$

This can be split into:

- $\{A,B\} \rightarrow \{B\}$ (*trivial*)
- $\{A,B\} \rightarrow \{E\}$

Completely Non-Trivial

- $\{A,E\} \rightarrow \{C,F\}$
- $\{B,F\} \rightarrow \{A,C\}$
- $\{D\} \rightarrow \{A,C\}$
- $\{E\} \rightarrow \{D,F\}$

Q4 Attribute Closure.

$\{C, E\}^+ = \{A, C, D, E, F\}$

Q5-Q7 Superkeys, Keys, and Prime Attributes.

Keys: From here, the superset of these are *superkeys*. The union of these are the *prime attributes*.

$\{A, B\}$, $\{B, D\}$, $\{B, E\}$, $\{B, F\}$

Q8 Lossless-Join Decomposition.

The idea is to find the intersection of the union, then find the closure. If the closure is a superset of one of the set, then the decomposition is lossless. This gives us the following lossless-join decomposition.

$\{\{B, C, D, E\}, \{A, D, E, F\}\}$, $\{\{C, D, E, F\}, \{A, B, E, F\}\}$, $\{\{A, B, E, F\}, \{A, B, C, D\}\}$

We will illustrate with the first example.

- Common attributes in $\{\{B, C, D, E\}, \{A, D, E, F\}\}$ are $\{D, E\}$.
- $\{D, E\}^+ = \{A, C, D, E, F\}$.
- $\{A, C, D, E, F\} \supseteq \{A, D, E, F\}$

Q9 BCNF Violation.

As you have learnt about 3NF, we also put a comparison with 3NF.

BCNF Check: Fail if any $X \rightarrow \{A\}$ pass all check

1. $A \notin X$
2. $X^+ \neq R$

3NF Check: Fail if any $X \rightarrow \{A\}$ pass all check

1. $A \notin X$
2. $X^+ \neq R$
3. A is not prime attribute.

Usign the check, we get the following answer.

BCNF	3NF
– $\{A, E\} \rightarrow \{C\}$	– $\{A, E\} \rightarrow \{C\}$
– $\{A, E\} \rightarrow \{F\}$	*F is prime attribute*
– $\{D\} \rightarrow \{A\}$	*A is prime attribute*
– $\{D\} \rightarrow \{C\}$	– $\{D\} \rightarrow \{C\}$
– $\{E\} \rightarrow \{D\}$	*D is prime attribute*
– $\{E\} \rightarrow \{F\}$	*F is prime attribute*

Q10 BCNF Decomposition.

One possible step.

- Consider $\{D\} \rightarrow \{A\}$. It is a violation w.r.t. R and Σ (*step omitted*).
  Compute $D^+ = A, C, D$.
  Split R into the following.
  * $R_1 = \{A, C, D\}$ with $\Sigma|_{R_1} = \{\ \{D\} \rightarrow \{A, C\}\ \}$
    R_1 with $\Sigma|_{R_1}$ is in **BCNF**.
  * $R_2 = \{B, D, E, F\}$ with $\Sigma|_{R_2} = \{\ \{B, D\} \rightarrow \{F\}, \{B, F\} \rightarrow \{E\}, \{E\} \rightarrow \{D, F\}\ \}$
    Consider $\{E\} \rightarrow \{D, F\}$. It is a violation w.r.t. R_2 and $\Sigma|_{R_2}$ (*step omitted*).
    Compute $E^+ = E, D, F$.
    Split R_2 into the following.
    · $R_3 = \{D, E, F\}$ with $\Sigma|_{R_3} = \{\ \{E\} \rightarrow \{D, F\}\ \}$
      R_3 with $\Sigma|_{R_3}$ is in **BCNF**.
    · $R_4 = \{B, E\}$ with $\Sigma|_{R_4} = \emptyset$
      R_4 with $\Sigma|_{R_4}$ is in **BCNF**.

Therefore, one possible answer is $\delta = \{ \{A, C, D\}, \{D, E, F\}, \{B, E\} \}$. This is minimal as we cannot remove any options such that the resulting decomposition is still satisfying the required properties of:

- Each fragment is in BCNF with respect to their projection.
- The decomposition is a **valid** decomposition (*i.e.,* does not lose any attribute).
- The decomposition is a **lossless-join** decomposition.