MA1522 ASSIGNMENT 1 SOLUTIONS

- Do NOT upload this assignment problem set to any website.
- The assignment carries a total number of 30 marks. The marks for each question or part are as indicated.

(1) [1 mark for each question] Which of the following statements are true? Which are false? You do not need to justify your answers.

Summarized Answer: $\boxed{\text{F, F, F, F, T, F, F, T, T, F}}$

(a) Let $\mathbf{A}$ and $\mathbf{B}$ be two matrices of the same size. If $\mathbf{A}+\mathbf{B}=\mathbf{0}$, then either $\mathbf{A}=\mathbf{0}$ or $\mathbf{B}=\mathbf{0}$. Here, $\mathbf{0}$ is the zero matrix of the same size as $\mathbf{A}$ and $\mathbf{B}$.

Answer: $\boxed{\text{False}}$. For example, let $\mathbf{A}=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$. Then, both $\mathbf{A}$ and $-\mathbf{A}$ are nonzero, but $\mathbf{A}+(-\mathbf{A})=\mathbf{0}$.

(b) Let $\mathbf{A}$ and $\mathbf{B}$ be two square matrices of the same size. If $\mathbf{A}\cdot\mathbf{B}=\mathbf{0}$, then either $\mathbf{A}=\mathbf{0}$ or $\mathbf{B}=\mathbf{0}$. Here, $\mathbf{0}$ is the zero matrix of the same size as $\mathbf{A}$ and $\mathbf{B}$.

Answer: $\boxed{\text{False}}$. For example, let $\mathbf{A}=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$. Then $\mathbf{A}\neq\mathbf{0}$, but $\mathbf{A}\cdot\mathbf{A}=\mathbf{0}$.

(c) Let $\mathbf{A}$ be a 3×3 matrix,

$$\mathbf{x}=\begin{pmatrix}x\\ y\\ z\end{pmatrix};\quad \mathbf{b}_1=\begin{pmatrix}1\\ 2\\ 3\end{pmatrix}\quad\text{and}\quad \mathbf{b}_2=\begin{pmatrix}4\\ 5\\ 6\end{pmatrix}.$$

It is possible that $\mathbf{Ax}=\mathbf{b}_1$ has a unique solution and $\mathbf{Ax}=\mathbf{b}_2$ has infinitely many solutions.

Answer: $\boxed{\text{False}}$. If $\mathbf{Ax}=\mathbf{b}_1$ has a unique solution, then the REF of $\mathbf{A}$ has no zero rows, hence $\mathbf{A}$ is invertible. Therefore, $\mathbf{Ax}=\mathbf{b}_2$ also has a unique solution. Contradiction.

(d) Let $\mathbf{A},\mathbf{x},\mathbf{b}_1$ and $\mathbf{b}_2$ be as in (c). It is possible that $\mathbf{Ax}=\mathbf{b}_1$ has a unique solution and $\mathbf{Ax}=\mathbf{b}_2$ has no solutions. **Answer**:

$\boxed{\text{False}}$. Same reason as in (c).

(e) Let $\mathbf{A}, \mathbf{x}, \mathbf{b}_1$ and $\mathbf{b}_2$ be as in (c). It is possible that $\mathbf{A}\mathbf{x} = \mathbf{b}_1$ has infinitely many solutions and $\mathbf{A}\mathbf{x} = \mathbf{b}_2$ has no solutions.

Answer: $\boxed{\text{True}}$. For example, let

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}, \quad \mathbf{b}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{b}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

(f) For any $m \times n$ matrix $\mathbf{A}$, the products $\mathbf{A} \cdot \mathbf{A}^T$ and $\mathbf{A}^T \cdot \mathbf{A}$ are always defined and $\mathbf{A} \cdot \mathbf{A}^T = \mathbf{A}^T \cdot \mathbf{A}$.

Answer: $\boxed{\text{False}}$. If $m \neq n$, then $\mathbf{A} \cdot \mathbf{A}^T$ is an $m \times m$ matrix, whereas $\mathbf{A}^T \cdot \mathbf{A}$ is an $n \times n$ matrix. For a concrete example, let

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \text{then } \mathbf{A} \cdot \mathbf{A}^T = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}, \quad \mathbf{A} \cdot \mathbf{A}^T = (5),$$

which are not equal. Note that the first part of the statement is true.

(g) The product of two elementary matrices is an elementary matrix.

Answer. $\boxed{\text{False}}$. For example, let

$$\mathbf{E}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{E}_2 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}.$$

Then

$$\mathbf{E_1} \cdot \mathbf{E_2} = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$$

is not an elementary matrix.

(h) If $\mathbf{x}$ and $\mathbf{y}$ are nonzero $n \times 1$ vectors and $\mathbf{A} = \mathbf{x}\mathbf{y}^T$, then the row echelon form of $\mathbf{A}$ will have exactly one nonzero row.

Answer: $\boxed{\text{True}}$. Let $\mathbf{x} = (x_1, \ldots, x_n)^T$ and $\mathbf{y} = (y_1, \ldots, y_n)^T$. Then

$$\mathbf{A} = \mathbf{x}\mathbf{y}^T = \begin{pmatrix} x_1\mathbf{y}^T \\ x_2\mathbf{y}^T \\ \vdots \\ x_n\mathbf{y}^T \end{pmatrix}.$$

Since $\mathbf{x}$ and $\mathbf{y}$ are nonzero, say $x_i \neq 0$ and $y_j \neq 0$, the i-th row of $\mathbf{A}$ is nonzero. By interchanging row 1 with row i if necessary, we may assume that $x_1 \neq 0$ and hence row 1 of $\mathbf{A}$ is nonzero. By performing $R_j - \frac{x_j}{x_1}R_1$ for $1 < j \leq n$, all rows become zero except row 1. The statement is justified.

(i) A homogeneous linear system is always consistent.

Answer. $\boxed{\text{True}}$. See Theorem on slide 45 in Chapter 2, or simply observe that the vector $\mathbf{0}$ is always a solution.

(j) Every diagonal matrix is LU-factorizable.

Answer. $\boxed{\text{False}}$. The diagonal matrix $\mathbf{D}$ is in upper triangular form, but may not be in REF. For example,

$$\mathbf{D} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is not LU-factorizable under our definition.
However, there are textbooks and internet resources like Wikipedia, which only require that $\mathbf{U}$ is upper triangular, in that case, the statement is true. Because of this, we treat the answer TRUE also as correct.

(2) (i) [2 marks] Given a cubic curve with equation

$$y = a + bx + cx^2 + dx^3,$$

where a, b, c, d are real constants, that passes through the points $(0, 10), (1, 7), (3, -11)$ and $(4, -14)$, find the values of a, b, c, d. You do not need to show the row reductions. However, you should write down REF or RREF of some matrix, if you have used them.

Answer. By substituting $(x, y) = (1, 10), (1, 7), (3, -11)$ and $(4, -14)$ into the equation of the cubic curve, we obtain the following linear system:

$$\begin{cases} a & & & & & & & = & 10 \\ a & + & b & + & c & + & d & = & 7 \\ a & + & 3b & + & 9c & + & 27d & = & -11 \\ a & + & 4b & + & 16c & + & 64d & = & -14. \end{cases}$$

Using Gaussian elimination, we have

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 10 \\ 1 & 1 & 1 & 1 & 7 \\ 1 & 3 & 9 & 27 & -11 \\ 1 & 4 & 16 & 64 & -14 \end{array}\right)$$

$$\xrightarrow{R_2-R_1, R_3-R_1, R_4-R_1} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 10 \\ 0 & 1 & 1 & 1 & -3 \\ 0 & 3 & 9 & 27 & -21 \\ 0 & 4 & 16 & 64 & -24 \end{array}\right)$$

$$\xrightarrow{R_3-3R_2, R_4-4R_2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 10 \\ 0 & 1 & 1 & 1 & -3 \\ 0 & 0 & 6 & 24 & -12 \\ 0 & 0 & 12 & 60 & -12 \end{array}\right)$$

$$\xrightarrow{R_4-2R_3} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 10 \\ 0 & 1 & 1 & 1 & -3 \\ 0 & 0 & 6 & 24 & -12 \\ 0 & 0 & 0 & 12 & 12 \end{array}\right)$$

The corresponding system is

$$\left\{\begin{array}{ccccccccc} a & & & & & & & = & 10 \\ & & b & + & c & + & d & = & -3 \\ & & & & 6c & + & 24d & = & -12 \\ & & & & & & 12d & = & 12. \end{array}\right.$$

Using backward substitution, we have $\boxed{a = 10, b = 2, c = -6, d = 1}$. In other words, the cubic equation is

$$y = 10 + 2x - 6x^2 + x^3.$$

(ii) [2 marks] Can you find real constants a, b, c such that the quadratic curve with equation

$$y = a + bx + cx^2,$$

passes through the same four points $(0, 10), (1, 7), (3, -11)$ and $(4, -14)$? Why?

Answer. $\boxed{\text{It is not possible}}$. As in part (i), we have the corresponding linear system:

$$\left\{\begin{array}{ccccccc} a & & & & & = & 10 \\ a & + & b & + & c & = & 7 \\ a & + & 3b & + & 9c & = & -11 \\ a & + & 4b & + & 16c & = & -14. \end{array}\right.$$

Using a similar Gaussian elimination and in part (i), we have

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 1 & 1 & 1 & 7 \\ 1 & 3 & 9 & -11 \\ 1 & 4 & 16 & -14 \end{array}\right)$$

$$\xrightarrow{R_2-R_1, R_3-R_1, R_4-R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & 1 & -3 \\ 0 & 3 & 9 & -21 \\ 0 & 4 & 16 & -24 \end{array}\right)$$

$$\xrightarrow{R_3-3R_2, R_4-4R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 6 & -12 \\ 0 & 0 & 12 & -12 \end{array}\right)$$

$$\xrightarrow{R_4-2R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 6 & -12 \\ 0 & 0 & 0 & 12 \end{array}\right)$$

The last row tells us the system is inconsistent.

(3) [4 marks] Consider the following linear system:

$$\begin{cases} ax + y + z = a^3 \\ x + ay + z = 1 \\ x + y + az = a, \end{cases}$$

where a is a real constant. Use Gaussian elimination method to find the conditions on a such that

(i) the system has no solution;
(ii) the system has a unique solution;
(iii) the system has infinitely many solutions.

You do not need to find the solutions in cases (ii) and (iii). Please show your row reductions. Finding answers using any other method will not fetch any marks.

Answer. Consider the augmented matrix and apply Gaussian elimination.

$$\left(\begin{array}{ccc|c} a & 1 & 1 & a^3 \\ 1 & a & 1 & 1 \\ 1 & 1 & a & a \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & a & 1 & 1 \\ a & 1 & 1 & a^3 \\ 1 & 1 & a & a \end{array}\right) \xrightarrow[R_3-R_1]{R_2-aR_1} \left(\begin{array}{ccc|c} 1 & a & 1 & 1 \\ 0 & 1-a^2 & 1-a & a^3-a \\ 0 & 1-a & a-1 & a-1 \end{array}\right)$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & a & 1 & 1 \\ 0 & 1-a & a-1 & a-1 \\ 0 & 1-a^2 & 1-a & a^3-a \end{array}\right)$$

$$\xrightarrow{R_3-(1+a)R_2} \left(\begin{array}{ccc|c} 1 & a & 1 & 1 \\ 0 & 1-a & a-1 & a-1 \\ 0 & 0 & (1-a)(2+a) & (a-1)^2(1+a) \end{array}\right).$$

(i) If $\boxed{a = -2}$, the matrix is reduced to $\left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 3 & -3 & -3 \\ 0 & 0 & 0 & -9 \end{array}\right)$, and the linear system has no solution.

(ii) If $\boxed{a \neq 1}$ and $\boxed{a \neq -2}$, then linear the system has a unique solution.

(iii) If $\boxed{a = 1}$, the matrix is reduced to $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$, and the linear system has infinitely many solutions (with 2 arbitrary parameters).

(4) [2 marks] Let $\mathbf{A}$ and $\mathbf{B}$ be 3×3 matrices such that

$$\mathbf{A}^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \mathbf{A}.$$

Find $\mathbf{B}^{-1}$ and explain how you derive your answer.

Answer. Let $\mathbf{E}$ denote the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, which is an elementary matrix with $\mathbf{E}^{-1} = \mathbf{E}$. Thus

$$\mathbf{B}^{-1} = \mathbf{A}^{-1} \cdot \mathbf{E}^{-1} = \mathbf{A}^{-1} \cdot \mathbf{E}.$$

So

$$\mathbf{B}^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 3 & 2 \\ 4 & 6 & 5 \\ 7 & 10 & 8 \end{pmatrix}}.$$

(5) [2 marks] Let $\mathbf{A}$ be a 2025×2025 matrix such that $a_{ij} = 0$ for $i < j$ and $a_{ij} = 1$ for $i \geq j$, in other words, $\mathbf{A}$ is a lower triangular matrix whose entries below (and including) diagonal are all equal to 1. Find $\mathbf{A}^{-1}$ and explain how you derive your answer.

Answer. We can work out the answer for $n \times n$ matrix $\mathbf{A}$. Let $\mathbf{B} = (b_{ij})_{n \times n}$, where

$$b_{ij} = \begin{cases} 1, & \text{i=j;} \\ -1, & \text{i=j+1;} \\ 0, & \text{otherwise.} \end{cases}$$

In other words, the only nonzero entries in $\mathbf{B}$ are on the diagonal, which have value 1; and on subdiagonal (the line just below diagonal),which have value -1. $\mathbf{A}^{-1}$ looks like

$$\boxed{\begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & \dots & 0 \\ 0 & -1 & 1 & \dots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & & -1 & 1 \end{pmatrix}}$$

One can verify that $\mathbf{AB} = \mathbf{BA} = \mathbf{I_n}$. In case you want to see the mathematical detail, we have the (i,j)-th entry of $\mathbf{AB}$ is

$$\begin{aligned} \sum_{k=1}^{n} a_{ik}b_{kj} &= a_{ij}b_{jj} + a_{i(j+1)}b_{(j+1)j} \quad \text{(because } b \text{ only has two nonzero terms)} \\ &= a_{ij} - a_{i(j+1)} \quad \text{because } b_{jj} = 1 \text{ and } b_{(j+1)j} = -1 \\ &= \begin{cases} 1 - 1 = 0, & \text{if } j \le i - 1; \\ 1 - 0 = 1, & \text{if } j = i; \\ 0 - 0 = 0, & \text{if } j > i. \end{cases} \end{aligned}$$

Thus, the only nonzero entries of $\mathbf{AB}$ is on the diagonal and they all equal to 1, in other words, $\mathbf{AB} = \mathbf{I_n}$. $\mathbf{BA} = \mathbf{I_n}$ can be verified similarly. Note: You don't need to give a formal proof. Typical explanations can be (1) Observe patterns using matrices of smaller size or (2) Perform row operations on $(\mathbf{A}|\mathbf{I_n})$ to get $(\mathbf{I_n}|\mathbf{A^{-1}})$ and observe patterns from there.

(6) [2 marks] Let $\mathbf{A}$ be a square matrix such that

$$\mathbf{A^2} - \mathbf{3A} - \mathbf{I} = \mathbf{0}.$$

Find $\mathbf{A}^{-1}$ and explain how you derive your answer.

Answer. From the given equation, we get

$$\mathbf{A}(\mathbf{A} - \mathbf{3I}) = \mathbf{I}.$$

Thus $\mathbf{A}$ is invertible and $\boxed{\mathbf{A}^{-1} = \mathbf{A} - \mathbf{3I}}$.

(7) [2 marks] Let

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}.$$

Find $\mathbf{A}^{2025}$ and explain how you derive your answer.

Answer. Let

$$\mathbf{B} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \quad \mathbf{0}_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Then $\mathbf{A}$ has a block form as a diagonal matrix

$$\mathbf{A} = \begin{pmatrix} \mathbf{B} & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{C} \end{pmatrix}.$$

Hence,

$$\mathbf{A}^{2025} = \begin{pmatrix} \mathbf{B}^{2025} & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{C}^{2025} \end{pmatrix}.$$

By observation of patterns or better using mathematical induction, we have

$$\mathbf{B}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix} \quad \mathbf{C}^n = \begin{pmatrix} 2^{n-1} & -2^{n-1} \\ -2^{n-1} & 2^{n-1} \end{pmatrix}.$$

Thus

$$\boxed{\mathbf{A} = \begin{pmatrix} 1 & 4050 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2^{2024} & -2^{2024} \\ 0 & 0 & -2^{2024} & 2^{2024} \end{pmatrix}}.$$

(8) It is given that

$$\mathbf{A} \xrightarrow{R_2+3R_1} \xrightarrow{R3-R1} \xrightarrow{R4+R2} \mathbf{U} = \begin{pmatrix} 1 & 2 & 2 & 4 \\ 0 & 17 & 2 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 14 \end{pmatrix}.$$

(i) [2 marks] Find a 4×4 matrix $\mathbf{L}$ such that $\mathbf{A} = \mathbf{LU}$. Explain how you derive your answer.

Answer. Following the algorithm on lecture notes, we have

$$\boxed{\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}}.$$

One can also do it step by step. The elementary matrices and their inverses corresponding to the three row operations are:

$$\mathbf{E_1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathbf{E_1}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$\mathbf{E_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathbf{E_2}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$\mathbf{E_3} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \quad \mathbf{E_3}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}.$$

Since $\mathbf{E_3 E_2 E_1 A} = \mathbf{U}$, we have $\mathbf{A} = \mathbf{E_1^{-1} E_2^{-1} E_3^{-1} U}$. Thus,

$$\mathbf{L} = \mathbf{E_1^{-1} E_2^{-1} E_3^{-1}}$$

which gives us the same result.

(ii) [2 marks] Use your answer in (i) to solve

$$\mathbf{Ax} = \begin{pmatrix} 2 \\ 3 \\ -1 \\ 5 \end{pmatrix}.$$

Show how you derive your answer. Solving the system using any other method will not fetch any marks.

Answer. Let $\mathbf{b}$ denote the vector on the right hand side. We first solve the system of equations $\mathbf{Ly} = \mathbf{b}$, namely,

$$\begin{cases} y_1 & & & & = 2 \\ -3y_1 + y_2 & & & & = 3 \\ y_1 & & + y_3 & & = -1 \\ & -y_2 & & + y_4 & = 5. \end{cases}$$

Working from the top to bottom, we can easily get $y_1 = 2, y_2 = 9, y_3 = -3, y_4 = 14$. Next, we get $\mathbf{x}$ by solving the system $\mathbf{Ux} = \mathbf{y}$, that is,

$$\begin{cases} x_1 + 2x_2 + 2x_3 + 4x_4 = 2 \\ \quad 17x_2 + 2x_3 + 4x_4 = 9 \\ \quad\quad x_3 + 3x_4 = -3 \\ \quad\quad\quad 14x_4 = 14. \end{cases}$$

Using backward substitution, we have $\boxed{x_1 = 8, x_2 = 1, x_3 = -6, x_4 = 1}$.

END OF ASSIGNMENT ONE