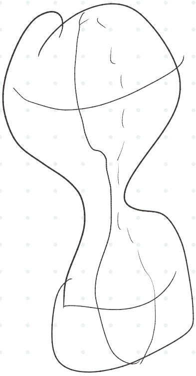


Constant power peanut

GW power:

$$|h_+|^2 + |h_\times|^2$$

Old solution (face-on):

$$h_+ = \frac{8M}{r} R^2 \Omega^2 \cos 2\Omega t$$

$$h_\times = -\frac{8M}{r} R^2 \Omega^2 \sin 2\Omega t$$

New solution (edge on):

$$h_+ = \frac{4M}{r} R^2 \Omega^2 \cos(2\Omega t)$$

$$h_\times = 0$$

$\Rightarrow$ Face-on power is 8x edge on!

How do orbits evolve when emitting GWs?

- GWs carry energy and angular momentum

$\Rightarrow$ orbits shrink!

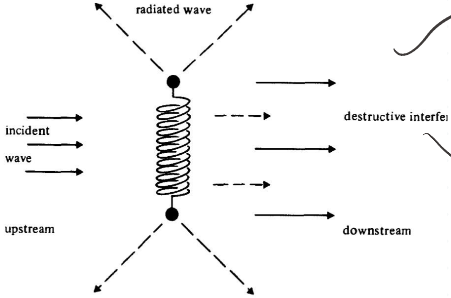
$$\boxed{\Omega^2 = \frac{GM_{tot}}{R^3}}$$

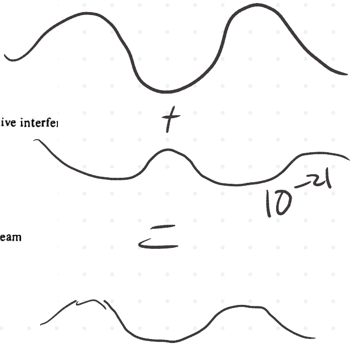
$$2\Omega\dot{\Omega} = -3\frac{GM}{R^4}\dot{R} \Rightarrow \dot{R} = -\frac{2}{3} R \frac{\dot{\Omega}}{\Omega}$$

One cool way to calculate energy loss:

arbitrarily, since we know from § 9.3 what waves they produce.

Schutz §9.4





Energy in spring is lost by GW, destructive interference downstream

$$\boxed{\frac{dE}{dt} = \frac{c^3 r^2}{32\pi G} \int d\Omega \, \langle \dot{h}_{ij}^{\mathrm{TT}} \dot{h}_{ij}^{\mathrm{TT}} \rangle \, .}$$

This mostly comes from $\boxed{\frac{d}{dt} T^{00}}$, we won't go into the details.

Power in GWs: $\langle P \rangle = \frac{1}{5} \langle \dddot{I}_{jk} \dddot{I}^{jk} \rangle$

$= - \langle \dot{E} \rangle$

$$h_{em} = \frac{2}{r} \ddot{I}_{em}(t-r)$$

$$\langle \dot{E} \rangle = -\frac{32}{5} \left(\frac{m_1 m_2}{m_1 + m_2} \right)^2 R^4 \Omega^6$$

$$= -\frac{32}{5} \frac{m_1^2 m_2^2}{R^5} (m_1 + m_2) \quad (1)$$

$$E = -\frac{m_1 m_2}{2R}$$

$$\frac{d}{dt}E \Rightarrow \dot{E} = \frac{m_1 m_2}{2R^2}\dot{R} \qquad (2)$$

$$(1) + (2) \Rightarrow \dot{R} = \frac{-64}{5}\,\frac{m_1 m_2 (m_1 + m_2)}{R^3}$$

<u>Reduced mass</u>: $\mu \equiv \dfrac{m_1 m_2}{m_1 + m_2}$

<u>Total mass</u>: $M \equiv m_1 + m_2$

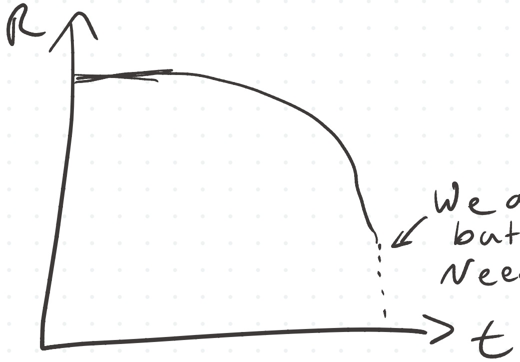
$$\dot{R} = -\frac{64}{5}\,\frac{\mu M^2}{R^3}$$

$$\int dt\, R^3 \cdot \frac{dR}{\cancel{dt}} = \int_t^{t_c} \frac{-64}{5}\mu M^2\, dt$$

$$\int_R^0 R^3\, dR = \frac{-64}{5}\mu M^2 (t_c - t)$$

$$-\frac{R^4}{4} = \frac{-64}{5}\mu M^2 (t_c - t)$$

$$\Rightarrow R(t) = \left(\frac{256}{5}\mu M^2\right)^{1/4} (t_c - t)^{1/4}$$



We assumed $v \ll c$, but $v \rightarrow c$ here. Need more terms.

"Post-Newtonian" expansion

Gravitational Wave frequency

$$\Omega_{orb} = \underbrace{2\pi f_{orb}}_{= \pi f_{GW}} = \frac{M^{1/2}}{R^{3/2}} \quad (\text{Kepler})$$

$$\frac{d}{dt}\left(\pi f_{GW} = \frac{M^{1/2}}{R^{3/2}}\right)$$

$$\dot{R} = -\frac{64}{5}\frac{\mu M^2}{R^3}$$

$$\pi f_{GW} \dot{f}_{GW} = -\frac{3}{2}\frac{M^{1/2}}{R^{5/2}}\dot{R}$$

$$\dot{f}_{GW} = \frac{96}{5} M^{2/3} \mu \pi^{8/3} f_{GW}^{11/3}$$

$$\boxed{\text{Chirp Mass} : \mathcal{M} = M_c \equiv \mu^{3/5} M^{2/5}}$$

\mathcal{M}

$$\boxed{\dot{f}_{GW} = \frac{96}{5}\pi^{8/3} M_c^{5/3} f^{11/3}}$$

GW Amplitude:

$$A(t) \sim \frac{\mu M}{r R(t)} f^2(t)$$

increasing

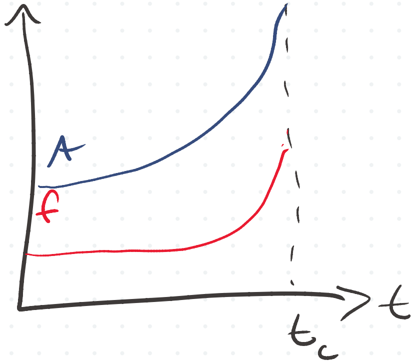
↳ shrinking

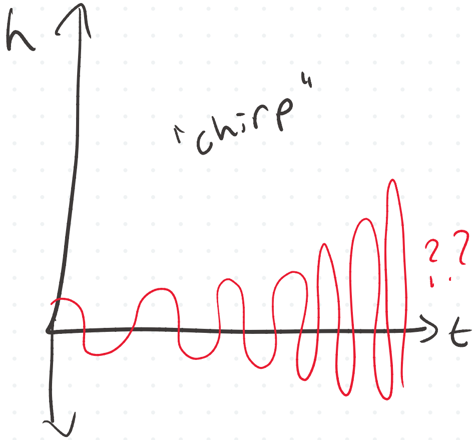
$$= 2\frac{M_c^{5/3}}{r}(\pi f_{GW})^{2/3}$$

$$R \sim (t_c - t)^{1/4}$$

$$f_{GW} \sim R^{-3/2} \sim (t_c - t)^{-3/8}$$

$$A \sim f^{2/3} \sim (t_c - t)^{-1/4}$$





So far we have assumed:

$v \ll c$

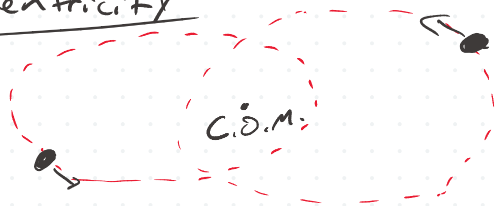
$R \gg M$

sources are points

Let's relax these:

- eccentricity
- $v \to c$, $a \to M$
- spin
- matter

Eccentricity



To get GWs:
- write mass distribution into T^{00}
- Calculate $\ddot{I}_{jk}$

$$R(\theta) = \frac{a(1-e^2)}{1 + e\cos\theta}$$

a : semi major axis

e : [-1, 1] eccentricity

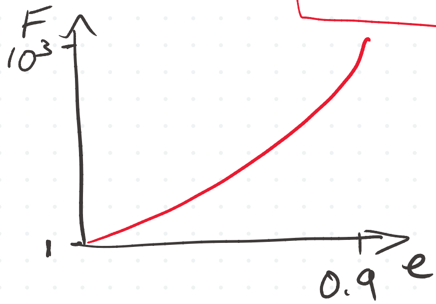
$e < 0 \Rightarrow$ hyperbolic / parabolic path
<u>unbound</u>! Not an orbit

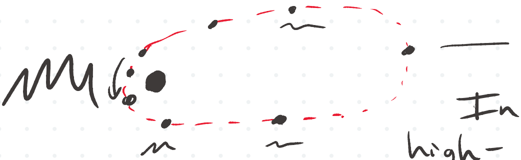
$e = 0 \Rightarrow$ circular 

$e = 1 \Rightarrow$ head-on collision 

We can derive:

$$\langle \dot{E} \rangle = \langle \dot{E} \rangle_{e=0} \underbrace{(1-e^2)^{-7/2}\left[1 + \frac{73}{24}e^2 + \frac{37}{96}e^4 + \dots\right]}_{F(e)}$$





In practice, these high-eccentricity binaries only emit strongly at closest approach (peri apsis).

Next time:
$\dot{e}$ of orbit?