Lecture notes 9.5

Approximation methods in GR

If you remember from the first few weeks of class, exact solutions to GR are very tough to find! Approximation methods are, then, very useful.

There are a few commonly talked about in the gravitational wave literature:

1. **Post-Newtonian Expansion** (PN): improving our approximation of $v \ll c$, and computing the extra terms for $v \to c$. *This approximation is good when speeds are slow compared to c.* $v < 0.3c$.
2. **Post-Minkowski Expansion**: instead of linearized gravity ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h^2)$), we keep more terms. Up to h^3 or higher. *Similar to PN but valid for all speeds!*
3. **Self-Force** (GSF): we start with geodesics for a massless object, then slowly add corrections from its mass. *This approximation is good when the mass ratio is large.*
4. **Effective one body**: Kind of a combination of all of the other methods. In practice, very slow.

We will focus on PN and GSF.

The Post-Newtonian Expansion

Remember in our linearized gravity solution, we assumed that:

- The source is small compared to the size of the GWs.
- The speeds of the objects were much smaller than c.

To improve on this, we will try to expand in $\frac{v}{c}$, up to higher orders. We will still assume that the source is small (radius smaller than GW wavelength), and that the source is *weakly stressed* ($|T^{ij}|/T^{00} \sim (v/c)^2$).

Let's call $\epsilon = v/c$ for convenience. Then our goal is

- write the Einstein equations expanded out in ϵ
- Simplify!! Match terms on both sides with the same powers of ϵ
- Figure out how particles move following the corrected equations.
- Profit! We have better GWs

$$T^{00} = {}^0T^{00} + \epsilon({}^1T^{00}) + \epsilon^2({}^2T^{00}) + ...$$

We get corrected Newtonian gravity!

$$\ddot{x}^i = -\frac{GM}{r^2}\hat{x}^i[1 + \mathcal{O}(\epsilon^2) + \mathcal{O}(\epsilon^4)] + \hat{v}^i[\mathcal{O}(\epsilon^2) + \mathcal{O}(\epsilon^4)]$$

Technicalities: technically this approximation only works close to the source, we need to match with the post-Minkowskian approximation to get GWs. (People struggled with this for decades)

Later in class: Some effects only appear at certain orders in PN, this gives you a rough idea how detectable they are.

If you're curious, you can find results up to 4.5PN $(v/c)^9$ here: https://github.com/davidtrestini/PNpedia

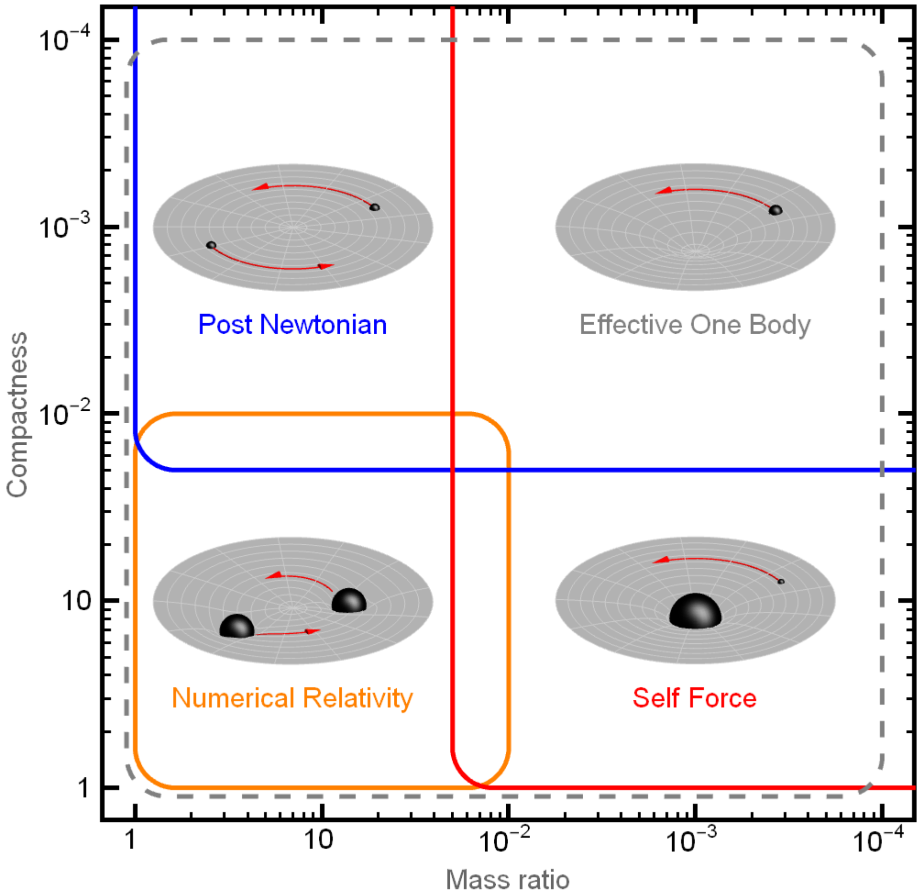

Figure 1: 9a552ad3770f8ce37a21d18c93a311fc.png