Queueing Theory: Simulation in discrete time

EBB074A05

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2020:11:15

1 General info

This file contains the code and the results that go with these youtube movies:

- `https://youtu.be/DfYxayoQmjYc`
- `https://youtu.be/D8BIAoBICnw`
- `https://youtu.be/_BoagRyH5c0`

2 Simulation in Discrete time

2.1 one period, demand, service capacity, and queue

There is one server, jobs enter a queue in front of the server, and the server serves batches of customers, every hour say.

```python
L = 10
a = 5
d = 8
L = L + a -d
print(L)
```

```
7
```

```python
L = 3
a = 5
c = 7
d = min(c, L)
L += a -d
print(d, L)
```

```
3 5
```

2.2 two periods

```python
L = 3
a = 5
c = 7
```

```
d = min(c, L)
L += a - d

a = 6
d = min(c, L)
L += a - d
print(d, L)
```

```
5 6
```

2.3 simulate many periods, make vectors

```
import numpy as np

a = np.random.uniform(5, 8, size=5)
print(a)
```

```
[5.03506133 7.91548145 7.93660781 5.01231185 7.96954179]
```

2.4 Set seed

```
import numpy as np

np.random.seed(3)

a = np.random.uniform(5, 8, size=5)
print(a)
```

```
[6.65239371 7.12444347 5.87271422 6.53248282 7.67884086]
```

2.5 update with a for loop

```
num = 5

#a = np.random.uniform(5, 8, size=num)
#c = np.random.uniform(5, 8, size=num)
a = 9*np.ones(num)
c = 10*np.ones(num)
L = np.zeros_like(a)

L[0] = 20
for i in range(1, num):
    d = min(c[i], L[i-1])
    L[i] = L[i-1] + a[i] - d

print(L)
```

```
[20. 19. 18. 17. 16.]
```

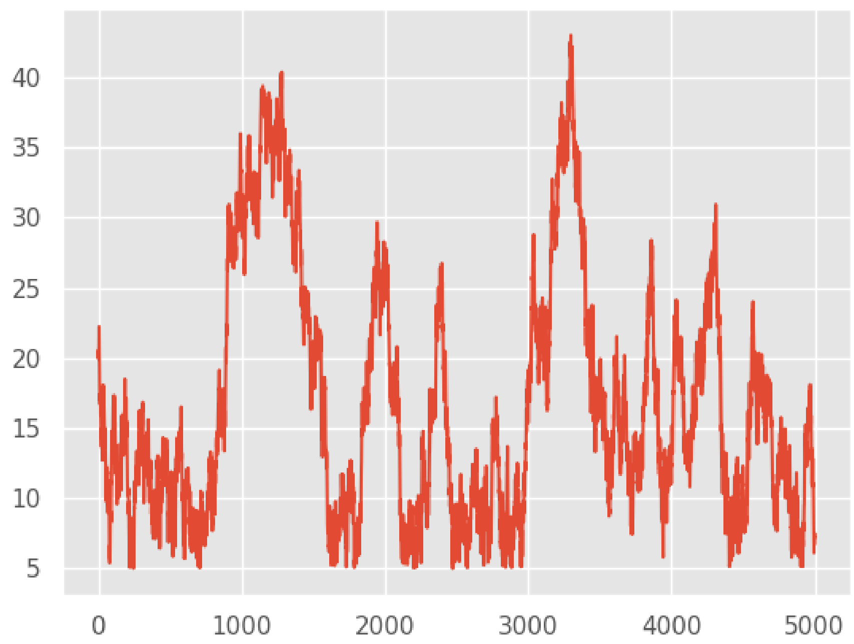
2.6 Compute mean and std of simulated queue length for $\rho \approx 1$

```
num = 5_000

np.random.seed(3)
a = np.random.uniform(5, 8, size=num)
c = (5+8)/1.99 * np.ones(num)
L = np.zeros_like(a)  # queue length at the end of a period

L[0] = 20
for i in range(1, num):
    d = min(c[i], L[i-1])
    L[i] = L[i-1] + a[i] - d

print(L.mean(), L.std())
```

```
16.78550665013682 8.695855000533511
```

2.7 plot the queue length process

```
import matplotlib.pyplot as plt

plt.clf()
plt.plot(L)
plt.savefig('queue-discrete_1.png')
'queue-discrete_1.png'
```



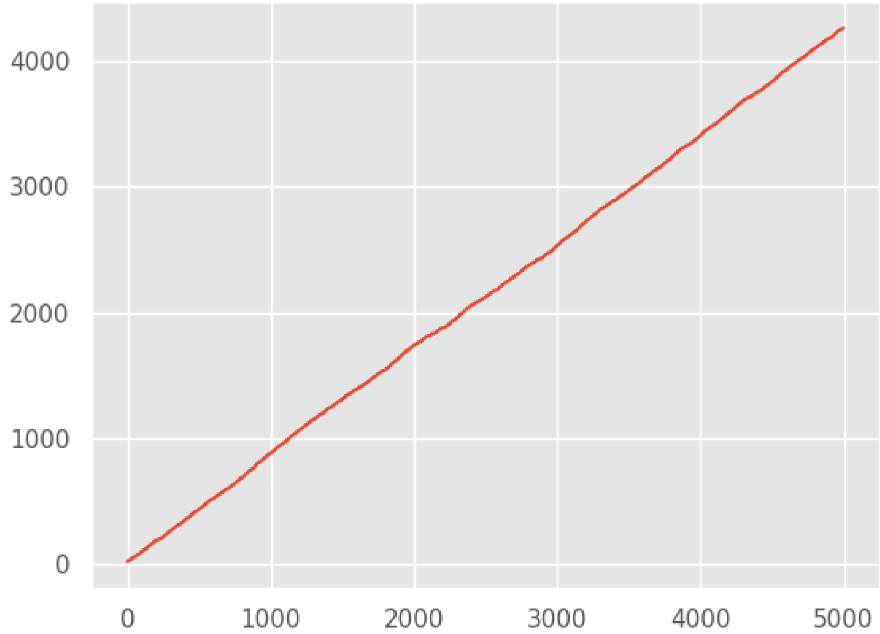
2.8 Compute mean and std of simulated queue length for $\rho = 1/2$

```
num = 5_000

np.random.seed(3)
a = np.random.uniform(5, 8, size=num)
c = (5+8) * np.ones(num)
L = np.zeros_like(a)  # queue length at the end of a period

L[0] = 20
for i in range(1, num):
    d = min(c[i], L[i-1])
    L[i] = L[i-1] + a[i] - d

print(L.mean(), L.std())
```

```
6.5051538887388 0.890452952271703
```

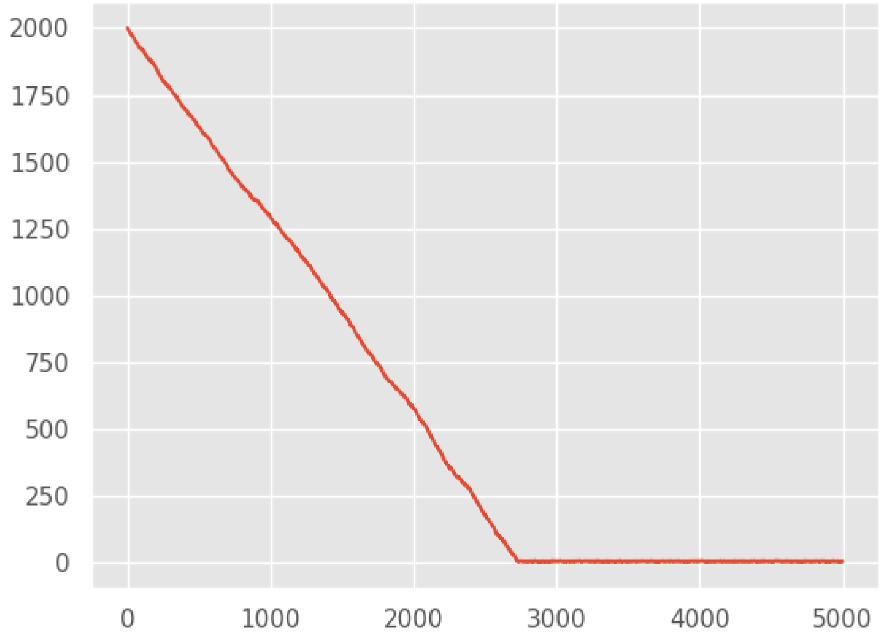
2.9 show the drift when $\rho > 1$

```
num = 5_000

np.random.seed(3)
a = np.random.uniform(5, 8, size=num)
c = (5+8)/2.3 * np.ones(num)
L = np.zeros_like(a)  # queue length at the end of a period

L[0] = 20
for i in range(1, num):
    d = min(c[i], L[i-1])
    L[i] = L[i-1] + a[i] - d


plt.clf()
plt.plot(L)
plt.savefig('queue-discrete_2.png')
'queue-discrete_2.png'
```



2.10 Start with a large queue, take ρ< 1, show the drift

```
num = 5_000

np.random.seed(3)
a = np.random.uniform(5, 8, size=num)
c = (5+8)/1.8 * np.ones(num)
L = np.zeros_like(a)   # queue length at the end of a period

L[0] = 2_000
for i in range(1, num):
    d = min(c[i], L[i-1])
    L[i] = L[i-1] + a[i] - d


plt.clf()
plt.plot(L)
plt.savefig('queue-discrete_3.png')
'queue-discrete_3.png'
```



Things to memorize:

- if the capacity is equal or less than the arrival rate, the queue lenght will explode.
- If the capacity is larger than the arrival rate, the queue length will stay around 0 (between quotes).
- If we start with a huge queue, but the service capacity is larger than the arrival rate, then the queue will drain rather fast, in fact, about linear.

2.11 Queues with blocking.

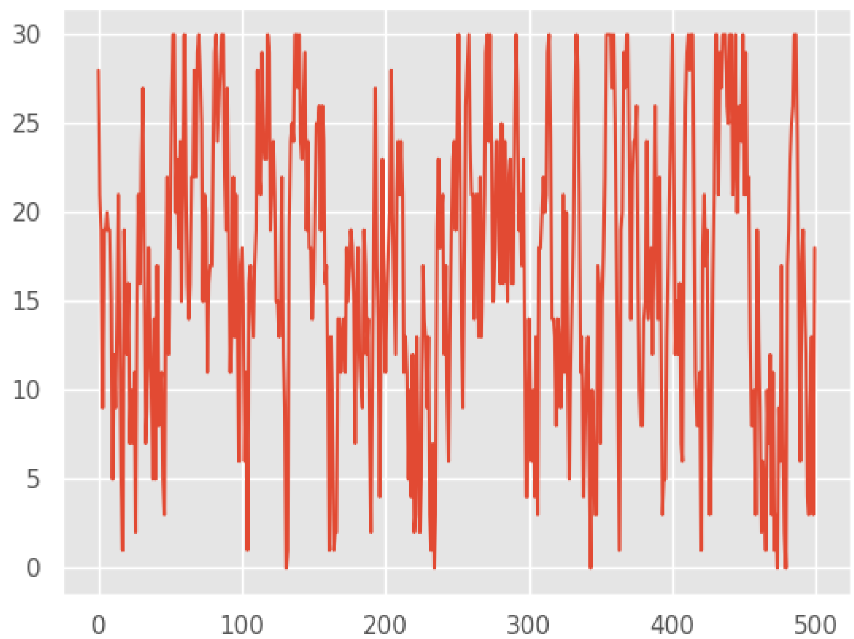
We have a queue subject to blocking. When the queue exceeds K, say, then whatever of the batch of items coming in exceeds K is rejected. This is the so-called complete reject rule. Two more assumptions: service occurs before arrival, and jobs arriving in a period cannot be served.

```
num = 500

np.random.seed(3)
a = np.random.randint(0, 20, size=num)
c = 10*np.ones(num)
L = np.zeros_like(a)  # queue length at the end of a period
loss = np.zeros_like(a)  # queue length at the end of a period

K = 30 # max people in queue, otherwise they leave

L[0] = 28
for i in range(1, num):
    d = min(c[i], L[i-1])
```

```
    loss[i] = max(L[i-1] + a[i] - d - K, 0) # service before arrivals.
    L[i] = L[i-1] + a[i] - d - loss[i]

lost_fraction = sum(loss)/sum(a)
print(lost_fraction)
```

```
0.026064291920069503
```

```
plt.clf()
plt.plot(L)
plt.savefig('queue-discrete_loss.png')
'queue-discrete_loss.png'
```



If we would assume that departures occur at the end of a period (hence, after the arrivals), then the code has to be as follows:

```
for i in range(1, num):
    d = min(c[i], L[i-1])
    loss[i] = max(L[i-1] + a[i] - K, 0) # service at end of period
    L[i] = L[i-1] + a[i] - d - loss[i]

lost_fraction = sum(loss)/sum(a)
print(lost_fraction)
```