Queueing Theory: Empirical distributions.

EBB074A05

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0.1 DONE Change the font size of emacs

0.2 DONE Change background to white

1 General info

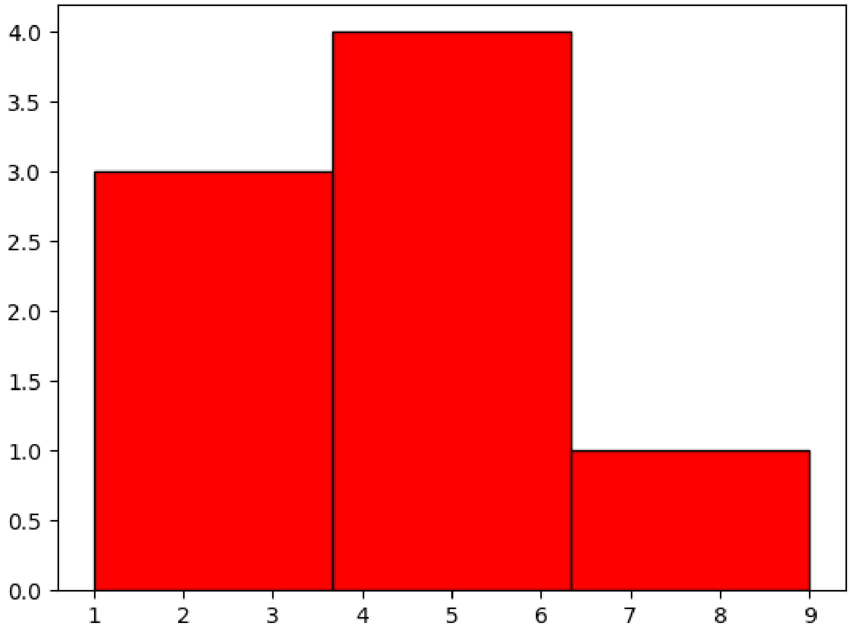
This file contains the code and the results that go with this youtube movie: `https://youtu.be/aKfv908uWqM`

2 Empirical distribution, how to make and plot

We want to know the fraction of periods the queue length is longer than some value q, say. For this we will make the empirical distribution of the queue lengths.

2.1 Plotting a PDF/histogram

```python
import matplotlib.pyplot as plt

x = [2, 5, 2, 1, 9, 5, 5, 5]

plt.clf()
plt.hist(x, bins=3, facecolor='red', edgecolor='black', linewidth=1)
plt.savefig('emp0.png')
'emp0.png'
```



2.1.1 DONE Explain: the `plt.clf()` is necessary to clear earlier plots.

2.1.2 DONE Change the number of bins from 3 to 7.

you can remove the `bins` argument altogether.

2.2 First naive idea

Given a set of measurements $x_1, \ldots, x_n$, the empirical CDF is defined as

$$F(x) = \sum_{i=1}^{n} I_{x_i \leq x}/n$$

This is a clean mathematical definition, but as if often the case with mathematical definitions, you should stay clear from using it to *compute* the CDF: the numerical performance is absolutely terrible.

```
x = [2, 5, 2, 1, 9, 5, 5, 5]

def F(y):
    tot = 0
    for xi in x:
        tot += xi <= y

    return  tot/len(x)

F(5.5)
```

```
0.875
```

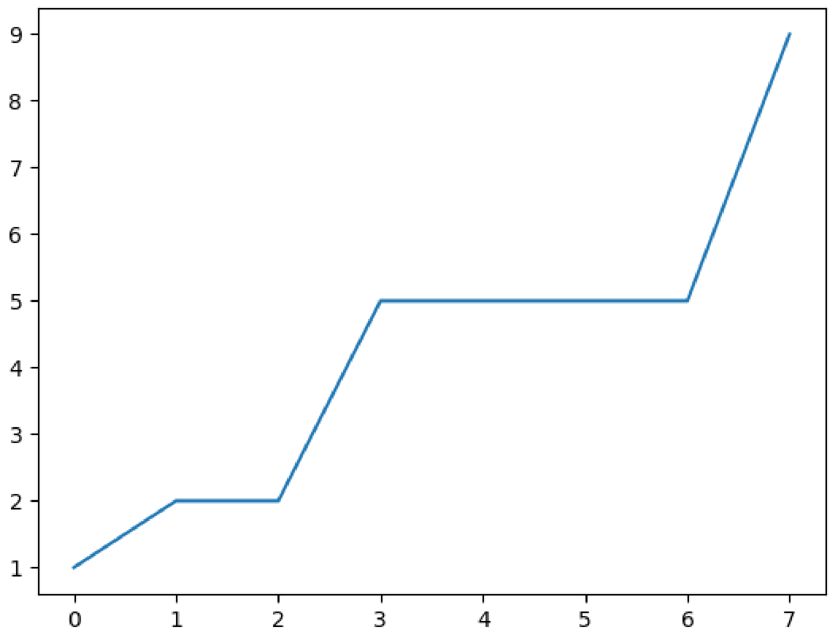
2.2.1 TODO Why is the numerical performance so bad?

2.3 A better idea

```python
sorted(x)
```

1	2	2	5	5	5	5	9

```python
plt.clf()
plt.plot(sorted(x))
plt.savefig("emp00.png")
"emp00.png"
```



2.4 Yet better idea

```python
def cdf_better(x):
    x = sorted(x)
    n = len(x)
    y = range(1, n + 1)
    y = [z / n for z in y]   # normalize
    return x, y


x = [2, 5, 2, 1, 8, 5, 5]
x, F = cdf_better(x)
F
```

0.14285714285714285 0.2857142857142857 0.42857142857142855 0.5714285714285714 0.7142857142857143

2.4.1 DONE Explain

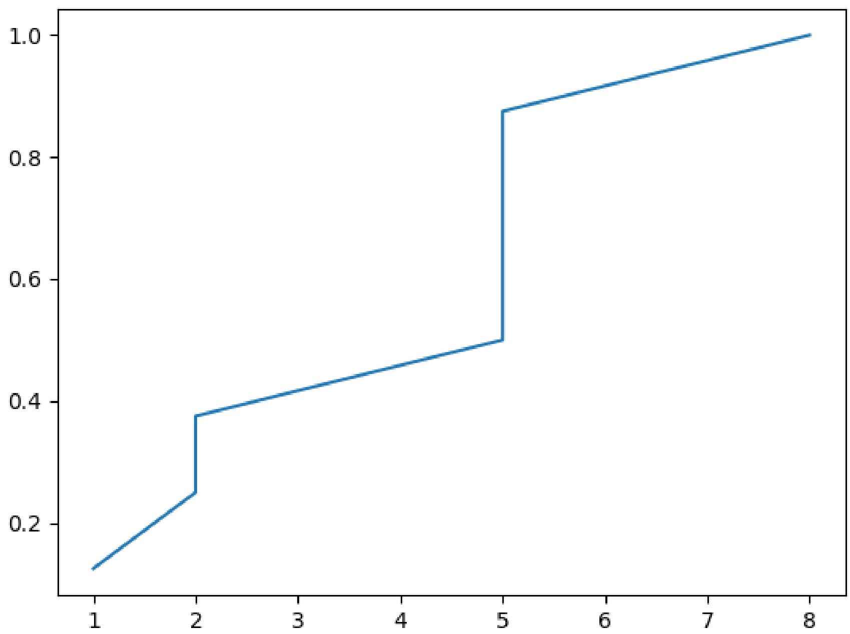
Why the ~n = len(x)

2.4.2 TODO Explain

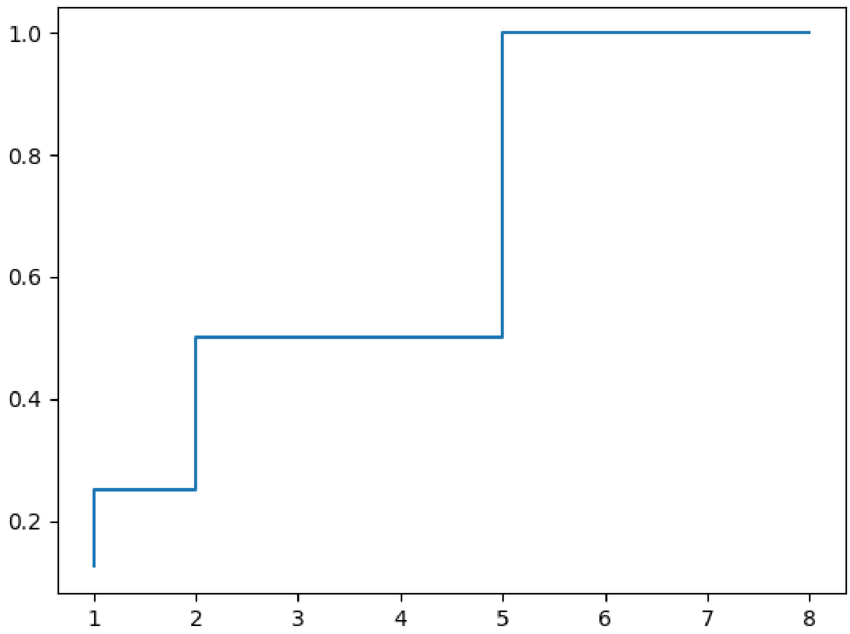
You should know that `for` loops in R and python are quite slow. We use this in the list comprehension in the line in which we `#normalize`. For larger amounts of data it is better to use `numpy`. This we do below.

2.5 Plot the cdf

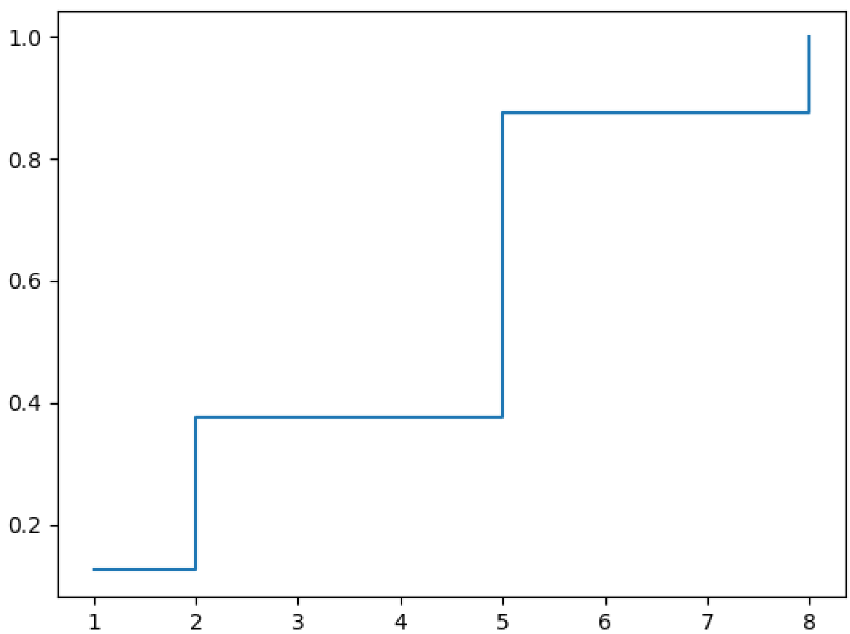
```
x = [2, 5, 2, 1, 8, 5, 5, 5]
x, F = cdf_better(x)

plt.clf()
plt.plot(x, F)
plt.savefig(fname)
fname
```




```
plt.clf()
plt.step(x, F)
plt.savefig(fname)
fname
```




```
plt.clf()
plt.plot(x, F, drawstyle="steps-post")
plt.savefig(fname)
fname
```



2.6 Faster with numpy

```
import numpy as np

def cdf(x):
    y = np.arange(1, len(x) + 1) / len(x)
    x = np.sort(x)
    return x, y

x = [2, 5, 2, 1, 8, 5, 5]
x, F = cdf(x)
F
```

0.14285714	0.28571429	0.42857143	0.57142857	0.71428571	0.85714286	1

2.7 Remove duplicate values

Finally, we can make the computation of the cdf significantly faster with using the following numpy functions.

```
unique, count = np.unique(np.sort(x), return_counts=True)
unique, count
```

array	((1 2 5 8))	array	((1 2 3 1))

```
count.cumsum()/7
```

0.14285714	0.42857143	0.85714286	1

```
def cdf_fastest(x):
    # remove multiple occurences of the same value
    unique, count = np.unique(np.sort(x), return_counts=True)
    x = unique
    y = count.cumsum() / count.sum()
    return x, y

x = [2, 5, 2, 1, 8, 5, 5]
x, F = cdf_fastest(x)
F
```

0.14285714	0.42857143	0.85714286	1