Information for ROB 501 Exam-II (Final)

Date is Monday, December 17, 2018, 1:30 PM–3:20 PM

The Rooms are assigned as follows:
Rooms (First letter of last name)
DOW 1013 (A-P)   DOW 1014 (R-Z)

Remark: When you receive your exam, please PRINT your name on it and COPY the honor pledge. **DO NOT OPEN THE EXAM UNTIL TOLD TO DO SO. DO NOT COUNT PAGES.**

RULES:

1. CLOSED TEXTBOOK
2. CLOSED CLASS NOTES
3. CLOSED HOMEWORK
4. CLOSED HANDOUTS
5. 3 SHEETS OF NOTE PAPER (Front and Back), US Letter Size. You can write anything you want on your "cheat sheets"
6. NO CALCULATORS, CELL PHONES, HEADPHONES, SMART WATCHES, etc.

Material Covered

- Lecture 1 through the material on Real Analysis. The exam will emphasize material since Exam 1.
- To be clear, for Real Analysis, you are responsible for open sets, closed sets, interior of a set, closure of a set, limit points, Cauchy sequences, completeness, contraction mapping theorem, subsequences, compact sets and existence of min and max of a continuous function on a compact set, convex sets, convex functions.
- There are no questions from the end of term lecture on QPs, and LPs.
- You may have to invert by hand a 2×2 matrix.
- HW #1 through HW #10, except remove Lagrange multipliers and 'SVD for image processing'; emphasis is definitely on material since Exam 1.
- There are no "Word Problems" like the HW problem where you designed a KF using LIDAR measurements.
- Least squares problems of all types (inner product spaces, weighted least squares[1], over determined, under determined, RLS, BLUE, MVE). `RLS with forgetting factor is NOT on the exam.`
- Matrix Inversion Lemma, Symmetric matrices, Orthogonal matrices, QR factorizations, positive definite matrices, Schur complement.
- SVD, matrix 2-norm, approximate a matrix by one of lower rank, relation of $A = U\Sigma V^\top$ to e-values and e-vectors of $A^\top A$ and $AA^\top$. (see SVD handout and proof),
- Probability at the level of BLUE, MVE, and the handout on Gaussian Random Vectors is on the exam.
- General probability is NOT on the exam (what is a density, what is a cumulative distribution function, P(A|B), etc.)
- The Kalman filter is covered. There is no question on the Extended Kalman Filter (EKF).
- Modified Gram Schmidt is NOT on the exam.
- Hermetian matrices are NOT on the exam (present in SVD handout).

Type of Questions

- The format is very similar to Exam 1. There is an A+ problem.
- The level of difficulty will be as close to that of our Exam 1 as possible. **There are no written proofs on the final exam. There may be short answer questions as on last year's final exam.** The problem will have two or three parts, each worth five points. I give an example below and you can find other examples on the 2016 and 2017 Final Exams.

[1] Means the inner product uses a positive definite matrix other than the identity.

Sample of the Short-Answer Problem The following are short answer questions. You are not supposed to give a proof; only give a few short reasons why something is TRUE or FALSE.

(a) **(5 Points)** Suppose that $(\mathcal{X}, ||\cdot||)$ is a finite-dimensional normed space and $S \subset \mathcal{X}$ is a subset. Suppose that $P : S \to S$ satisfies $\forall\ x, y \in S$, $||P(x) - P(y)|| \le 0.8||x - y||$. Then, for any $x_0 \in S$, the sequence $x_{k+1} = P(x_k)$ converges and has a limit in S. **T or F**.

Answer: $\boxed{\text{False}}$. By the proof of the Contraction Mapping Theorem, the sequence (x_k) is Cauchy. Because $\mathcal{X}$ is finite dimensional, it is complete, and thus Cauchy sequences have limits. But because S was not stated to be closed, the limit may not be an element of S.

Remark: Suppose you had answered $\boxed{\text{False}}$, because S is not closed. `This would earn 3 or 4 points, probably 4`. You understood the essence of the question. What you left out was a reason that the sequence (x_k) should converge to something at all.

Remark: Suppose you had incorrectly answered $\boxed{\text{True}}$. By the proof of the Contraction Mapping Theorem, the sequence (x_k) is Cauchy. Because $\mathcal{X}$ is finite dimensional, it is complete, and thus Cauchy sequences have limits. `This would earn at least 2 points, and maybe 3`. You have analyzed why the sequence should converge. You missed the fact that S is not necessarily closed and hence many not contain the limit point of the sequence.

(b) **(5 Points)** [A variation on the above problem:] Suppose that $(\mathcal{X}, ||\cdot||)$ is a finite-dimensional normed space and $S \subset \mathcal{X}$ is a closed subset. Suppose that $P : S \to S$ satisfies $\forall\ x, y \in S$, $||P(x) - P(y)|| \le 0.8||x - y||$. Then, for any $x_0 \in S$, the sequence $x_{k+1} = P(x_k)$ converges and has a limit in S. **T or F**.

Answer: $\boxed{\text{True}}$. Because $\mathcal{X}$ is finite dimensional, it is complete. S is then complete because it is a closed subset of a complete normed space. Hence, all the hypotheses of the Contraction Mapping Theorem are met, and thus the result is true. *The point here is to check that the hypotheses of the theorem are met and then apply it.*

Remark: Suppose you had answered $\boxed{\text{True}}$, by the Contraction Mapping Theorem. `This would earn only 2 or 3 points`. The problem is you did not check one of the key hypotheses of the Contraction Mapping Theorem, namely that S is complete. That is really what the question is about, because P being a contraction is rather obvious.

Grading: 1 point for the **T or F** part and 4 points for the reasoning. As illustrated above, even if you get the T or F wrong, the reasons will be graded to see if you understood something about the problem.