

Assignment #1 (Linear Algebra)

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Name: , ID:

Problem 1: Basic Vector Operations

(points)

Let two vectors $\mathbf{a} = (1 \ 2 \ 3)^T$ and $\mathbf{b} = (-8 \ 1 \ 2)^T$, answer the following equations:

- (1) Calculate the ℓ_2 norm of \mathbf{a} and \mathbf{b} .
- (2) Calculate the Euclidean distance between \mathbf{a} and \mathbf{b} (i.e. ℓ_2 norm of $\mathbf{a} - \mathbf{b}$).
- (3) Are \mathbf{a} and \mathbf{b} orthogonal? State your reason.

Problem 2: Basic Matrix Operations

(points)

Suppose $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$, answer the following questions:

- (1) Calculate A^{-1} and $\det(A)$.
- (2) The Rank of A is?
- (3) The trace of A is?
- (4) Calculate $A + A^T$.
- (5) Is A an orthogonal matrix? State your reason.
- (6) Calculate all the eigenvalue λ and corresponding eigenvectors of A .
- (7) Diagonalize the matrix A .
- (8) Calculate the $\ell_{2,1}$ norm $\|A\|_{2,1}$ and the Frobenius norm (i.e. ℓ_2 norm) $\|A\|_F$.
- (9) Calculate the nuclear norm $\|A\|_*$ and the spectral norm $\|A\|_2$.

Problem 2: Linear Equations

(points)

Please give some proper steps to show how you get the answer.

Let $x = (x_1, x_2, x_3)^T$ and

$$\begin{cases} 2x_1 + 2x_2 + 3x_3 = 1 \\ x_1 - x_2 = -1 \\ -x_1 + 2x_2 + x_3 = 2 \end{cases}$$

Answer the following questions:

- (1) Solve the linear equations

- (2) Write it into matrix form(i.e. $Ax = b$) and we will use the same A and b in the following questions.
- (3) The Rank of A is?
- (4) Calculate A^{-1} and $\det(A)$
- (5) Use (4) to solve the linear equations
- (6) Calculate the inner product and outer product of x and b .(i.e. $\langle x, b \rangle$ and $x \otimes b$)
- (7) Calculate the ℓ_1, ℓ_2 and ℓ_∞ norm of b
- (8) Suppose $y = (y_1, y_2, y_3)$, calculate $y^T Ay$, $\nabla_y y^T Ay$.
- (9) We add one linear equation $-x_1 + 2x_2 + x_3 = 2$ into linear equations above. Write it into matrix form(i.e. $A_1x = b$)
- (10) The rank of A_1 is?
- (11) Could these linear equations $A_1x = b$ be solved? State reasons.