Supplementary Materials for Multi-tool Drilling Path Optimization by Multi-Agent Reinforcement Learning Approach

The details of all workpieces (Section I), as well as the hyperparameter settings of the participating algorithms (Section II), can be found in this document.

Section I: Workpiece details

- Table I – VIII are the serial numbers, coordinates, diameters, and process information of holes of 14, 29, 50, and 100 workpieces, respectively.
- The assignment of holes is summarized in Table IX.
- The colors of all the holes in this file are the same as the colors of the optimal paths of Fig. 7 in the main text.

TABLE I
THE DETAILS OF 14 HOLES PROBLEM / 2 PATHS

No.	Coordinate	Diameter/mm (Roughness)	Category	No.	Coordinate	Diameter/mm (Roughness)	Category
1	(10,10)	ϕ 3, (Ra 12.5)	Fixed hole / Path 2	8	(62.29,43.6)	ϕ 3, (Ra 12.5)	Decidable hole
2	(10,60)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	9	(62.29,26.4)	ϕ 5, (Ra 12.5)	Decidable hole
3	(18,53.5)	ϕ 3, (Ra 12.5)	Fixed hole / Path 2	10	(90,10)	ϕ 3, (Ra 12.5)	Fixed hole / Path 2
4	(18,42.5)	ϕ 12, (Ra 6.3)	Decidable hole	11	(82,16.5)	ϕ 12, (Ra 6.3)	Decidable hole
5	(32.32,12.66)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	12	(82,27.5)	ϕ 5, (Ra 12.5)	Decidable hole
6	(37.71,26.4)	ϕ 8, (Ra 6.3)	Decidable hole	13	(72.59,55.75)	ϕ 8, (Ra 6.3)	Fixed hole / Path 1
7	(37.71,43.6)	ϕ 8, (Ra 6.3)	Decidable hole	14	(90,60)	ϕ 8, (Ra 6.3)	Fixed hole / Path 2

TABLE II
THE DETAILS OF 29 HOLES PROBLEM / 2 PATHS

No.	Coordinate	Diameter/mm (Roughness)	Category	No.	Coordinate	Diameter/mm (Roughness)	Category
1	(115,176)	ϕ 8, (Ra 6.3)	Decidable hole	16	(128,120)	ϕ 5, (Ra 12.5)	Fixed hole / Path 2
2	(63,166)	ϕ 7, (Ra 6.3)	Decidable hole	17	(23,59)	ϕ 5, (Ra 12.5)	Fixed hole / Path 2
3	(4,209)	ϕ 4, (Ra 12.5)	Fixed hole / Path 1	18	(46,86)	ϕ 14, (Ra 6.3)	Decidable hole
4	(75,110)	ϕ 5, (Ra 12.5)	Fixed hole / Path 2	19	(104,95)	ϕ 5.8, (Ra 3.2)	Decidable hole
5	(75,203)	ϕ 5, (Ra 12.5)	Fixed hole / Path 2	20	(59,139)	ϕ 8, (Ra 6.3)	Decidable hole
6	(103,207)	ϕ 4, (Ra 12.5)	Fixed hole / Path 1	21	(83,177)	ϕ 4, (Ra 12.5)	Fixed hole / Path 1
7	(165,65)	ϕ 12, (Ra 6.3)	Decidable hole	22	(49,50)	ϕ 5, (Ra 12.5)	Fixed hole / Path 2
8	(149,163)	ϕ 5.8, (Ra 3.2)	Decidable hole	23	(184,124)	ϕ 13.5, (Ra 6.3)	Decidable hole
9	(79,226)	ϕ 5, (Ra 12.5)	Fixed hole / Path 2	24	(126,150)	ϕ 4, (Ra 12.5)	Fixed hole / Path 1
10	(71,131)	ϕ 5, (Ra 12.5)	Fixed hole / Path 2	25	(128,79)	ϕ 4, (Ra 12.5)	Fixed hole / Path 1
11	(84,55)	ϕ 13, (Ra 6.3)	Decidable hole	26	(49,213)	ϕ 4, (Ra 12.5)	Fixed hole / Path 1
12	(117,230)	ϕ 13, (Ra 6.3)	Decidable hole	27	(146,142)	ϕ 8, (Ra 6.3)	Decidable hole
13	(97,134)	ϕ 5, (Ra 12.5)	Fixed hole / Path 2	28	(126,191)	ϕ 8, (Ra 6.3)	Decidable hole
14	(51,70)	ϕ 4, (Ra 12.5)	Fixed hole / Path 1	29	(36,198)	ϕ 6.4, (Ra 6.3)	Decidable hole
15	(75,90)	ϕ 4, (Ra 12.5)	Fixed hole / Path 1				

TABLE III
THE DETAILS OF 29 HOLES PROBLEM / 3 PATHS

No.	Coordinate	Diameter/mm (Roughness)	Category	No.	Coordinate	Diameter/mm (Roughness)	Category
1	(115,176)	ϕ 9, (Ra 6.3)	Decidable hole	16	(128,120)	ϕ 8, (Ra 12.5)	Fixed hole / Path 3
2	(63,166)	ϕ 10, (Ra 6.3)	Decidable hole	17	(23,59)	ϕ 8, (Ra 12.5)	Fixed hole / Path 3
3	(4,209)	ϕ 4, (Ra 12.5)	Fixed hole / Path 1	18	(46,86)	ϕ 14, (Ra 6.3)	Decidable hole
4	(75,110)	ϕ 8, (Ra 12.5)	Fixed hole / Path 3	19	(104,95)	ϕ 9.8, (Ra 6.3)	Decidable hole
5	(75,203)	ϕ 8, (Ra 12.5)	Fixed hole / Path 3	20	(59,139)	ϕ 8.5, (Ra 3.2)	Decidable hole
6	(103,207)	ϕ 4, (Ra 12.5)	Fixed hole / Path 1	21	(83,177)	ϕ 4, (Ra 12.5)	Fixed hole / Path 1
7	(165,65)	ϕ 12, (Ra 6.3)	Decidable hole	22	(49,50)	ϕ 5, (Ra 12.5)	Fixed hole / Path 2
8	(149,163)	ϕ 7.7, (Ra 3.2)	Decidable hole	23	(184,124)	ϕ 14, (Ra 6.3)	Decidable hole
9	(79,226)	ϕ 17, (Ra 12.5)	Decidable hole	24	(126,150)	ϕ 4, (Ra 12.5)	Fixed hole / Path 1
10	(71,131)	ϕ 8, (Ra 12.5)	Fixed hole / Path 3	25	(128,79)	ϕ 5, (Ra 12.5)	Fixed hole / Path 2
11	(84,55)	ϕ 14, (Ra 6.3)	Decidable hole	26	(49,213)	ϕ 4, (Ra 12.5)	Fixed hole / Path 1
12	(117,230)	ϕ 14, (Ra 6.3)	Decidable hole	27	(146,142)	ϕ 12, (Ra 6.3)	Decidable hole
13	(97,134)	ϕ 5, (Ra 12.5)	Fixed hole / Path 2	28	(126,191)	ϕ 12, (Ra 6.3)	Decidable hole
14	(51,70)	ϕ 5, (Ra 12.5)	Fixed hole / Path 2	29	(36,198)	ϕ 14.5, (Ra 3.2)	Decidable hole
15	(75,90)	ϕ 5, (Ra 12.5)	Fixed hole / Path 2				

TABLE IV
THE DETAILS OF 50 HOLES PROBLEM / 3 PATHS

No.	Coordinate	Diameter/mm (Roughness)	Category	No.	Coordinate	Diameter/mm (Roughness)	Category
1	(148,208)	ϕ 10, (Ra 6.3)	Decidable hole	26	(108,272)	ϕ 15, (Ra 12.5)	Decidable hole
2	(196,196)	ϕ 10, (Ra 6.3)	Decidable hole	27	(120,192)	ϕ 12.4, (Ra 6.3)	Decidable hole
3	(208,256)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1	28	(172,268)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2
4	(80,104)	ϕ 9.5, (Ra 6.3)	Decidable hole	29	(232,192)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3
5	(160,120)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3	30	(232,108)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2
6	(84,188)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1	31	(148,276)	ϕ 10, (Ra 6.3)	Decidable hole
7	(68,252)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2	32	(152,184)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1
8	(124,248)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3	33	(184,40)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2
9	(208,132)	ϕ 9.5, (Ra 6.3)	Decidable hole	34	(244,132)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1
10	(204,84)	ϕ 12.4, (Ra 6.3)	Decidable hole	35	(248,252)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1
11	(168,164)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1	36	(252,276)	ϕ 10, (Ra 3.2)	Decidable hole
12	(124,128)	ϕ 9.5, (Ra 3.2)	Decidable hole	37	(128,88)	ϕ 10, (Ra 3.2)	Decidable hole
13	(20,100)	ϕ 10, (Ra 6.3)	Decidable hole	38	(180,140)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2
14	(48,168)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3	39	(236,60)	ϕ 10, (Ra 6.3)	Decidable hole
15	(144,64)	ϕ 15, (Ra 12.5)	Decidable hole	40	(20,24)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1
16	(208,164)	ϕ 9.5, (Ra 6.3)	Fixed hole / Path 1	41	(40,68)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2
17	(108,92)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2	42	(84,40)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2
18	(68,132)	ϕ 13, (Ra 6.3)	Decidable hole	43	(20,256)	ϕ 12.4, (Ra 6.3)	Decidable hole
19	(52,52)	ϕ 10, (Ra 6.3)	Decidable hole	44	(120,60)	ϕ 10, (Ra 3.2)	Decidable hole
20	(228,232)	ϕ 13, (Ra 6.3)	Decidable hole	45	(156,40)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2
21	(248,168)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3	46	(128,156)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3
22	(168,228)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1	47	(100,128)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3
23	(64,228)	ϕ 14, (Ra 6.3)	Decidable hole	48	(100,220)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3
24	(32,208)	ϕ 13, (Ra 6.3)	Decidable hole	49	(192,112)	ϕ 9.2, (Ra 3.2)	Decidable hole
25	(28,152)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3	50	(224,148)	ϕ 9.2, (Ra 3.2)	Decidable hole

TABLE V
THE DETAILS OF 50 HOLES PROBLEM / 4 PATHS

No.	Coordinate	Diameter/mm (Roughness)	Category	No.	Coordinate	Diameter/mm (Roughness)	Category
1	(148,208)	ϕ 9.5, (Ra 6.3)	Decidable hole	26	(108,272)	ϕ 15, (Ra 12.5)	Decidable hole
2	(196,196)	ϕ 9.5, (Ra 6.3)	Decidable hole	27	(120,192)	ϕ 12.4, (Ra 6.3)	Decidable hole
3	(208,256)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1	28	(172,268)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3
4	(80,104)	ϕ 9.5, (Ra 6.3)	Decidable hole	29	(232,192)	ϕ 8.5, (Ra 12.5)	Fixed hole / Path 4
5	(160,120)	ϕ 8.5, (Ra 12.5)	Fixed hole / Path 4	30	(232,108)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2
6	(84,188)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1	31	(148,276)	ϕ 9.5, (Ra 6.3)	Decidable hole
7	(68,252)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2	32	(152,184)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1
8	(124,248)	ϕ 8.5, (Ra 12.5)	Fixed hole / Path 4	33	(184,40)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2
9	(208,132)	ϕ 9.5, (Ra 6.3)	Decidable hole	34	(244,132)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1
10	(204,84)	ϕ 12.4, (Ra 6.3)	Decidable hole	35	(248,252)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1
11	(168,164)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2	36	(252,276)	ϕ 10, (Ra 3.2)	Decidable hole
12	(124,128)	ϕ 9.5, (Ra 3.2)	Decidable hole	37	(128,88)	ϕ 8.5, (Ra 12.5)	Fixed hole / Path 4
13	(20,100)	ϕ 10, (Ra 6.3)	Decidable hole	38	(180,140)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2
14	(48,168)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3	39	(236,60)	ϕ 10, (Ra 6.3)	Decidable hole
15	(144,64)	ϕ 15, (Ra 12.5)	Decidable hole	40	(20,24)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1
16	(208,164)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2	41	(40,68)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3
17	(108,92)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3	42	(84,40)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3
18	(68,132)	ϕ 13, (Ra 6.3)	Decidable hole	43	(20,256)	ϕ 12.4, (Ra 6.3)	Decidable hole
19	(52,52)	ϕ 10, (Ra 6.3)	Decidable hole	44	(120,60)	ϕ 10, (Ra 3.2)	Decidable hole
20	(228,232)	ϕ 13, (Ra 6.3)	Decidable hole	45	(156,40)	ϕ 7, (Ra 12.5)	Fixed hole / Path 2
21	(248,168)	ϕ 8.5, (Ra 12.5)	Fixed hole / Path 4	46	(128,156)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3
22	(168,228)	ϕ 9, (Ra 12.5)	Fixed hole / Path 1	47	(100,128)	ϕ 6, (Ra 12.5)	Fixed hole / Path 3
23	(64,228)	ϕ 14, (Ra 6.3)	Decidable hole	48	(100,220)	ϕ 8.5, (Ra 12.5)	Fixed hole / Path 4
24	(32,208)	ϕ 13, (Ra 6.3)	Decidable hole	49	(192,112)	ϕ 12.4, (Ra 6.3)	Decidable hole
25	(28,152)	ϕ 8.5, (Ra 12.5)	Fixed hole / Path 4	50	(224,148)	ϕ 14, (Ra 6.3)	Decidable hole

TABLE VI
THE DETAILS OF 100 HOLES PROBLEM / 3 PATHS

No.	Coordinate	Diameter/mm (Roughness)	Category	No.	Coordinate	Diameter/mm (Roughness)	Category
1	(138, 93.9)	ϕ 9.5, (Ra 6.3)	Decidable hole	51	(232.2, 118.3)	ϕ 9.5, (Ra 6.3)	Decidable hole
2	(284.8, 9.6)	ϕ 9.5, (Ra 6.3)	Decidable hole	52	(395.4, 92.3)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
3	(361, 175.1)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	53	(47.6, 82.5)	ϕ 9.5, (Ra 6.3)	Decidable hole
4	(55.7, 33.4)	ϕ 9.5, (Ra 6.3)	Decidable hole	54	(251.9, 3.5)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
5	(368.8, 66.6)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	55	(305.5, 175.2)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
6	(108.4, 96.5)	ϕ 9.5, (Ra 6.3)	Decidable hole	56	(95.3, 26.8)	ϕ 9.5, (Ra 6.3)	Decidable hole
7	(265.1, 145.2)	ϕ 9.5, (Ra 6.3)	Decidable hole	57	(222.8, 137.9)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
8	(128.6, 52.5)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	58	(209.7, 98.1)	ϕ 9.5, (Ra 6.3)	Decidable hole
9	(291.6, 113.2)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	59	(79.0, 194.6)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
10	(85.8, 126.5)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	60	(203.9, 190.6)	ϕ 13, (Ra 6.3)	Decidable hole
11	(125.1, 183.2)	ϕ 14.5, (Ra 12.5)	Decidable hole	61	(242.1, 100.7)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
12	(280.8, 169.8)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	62	(229.0, 181.0)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
13	(361.5, 32.9)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	63	(121.5, 115.2)	ϕ 14.5, (Ra 12.5)	Decidable hole
14	(368.3, 153.3)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	64	(258.8, 30.2)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
15	(144.7, 194.5)	ϕ 15, (Ra 12.5)	Decidable hole	65	(32.7, 26.5)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
16	(24.3, 76.2)	ϕ 9.7, (Ra 6.3)	Decidable hole	66	(14.1, 44.1)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
17	(103.4, 194.6)	ϕ 9.7, (Ra 6.3)	Decidable hole	67	(191.7, 68.7)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
18	(25.2, 124)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	68	(299.1, 79.2)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
19	(61.1, 67.3)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	69	(257.3, 59.9)	ϕ 14.5, (Ra 12.5)	Decidable hole
20	(250.6, 164.6)	ϕ 13, (Ra 6.3)	Decidable hole	70	(1.9, 67.4)	ϕ 14.5, (Ra 12.5)	Decidable hole
21	(92.8, 170)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	71	(385.1, 167.3)	ϕ 9.7, (Ra 6.3)	Decidable hole
22	(5.3, 90.7)	ϕ 9.5, (Ra 6.3)	Decidable hole	72	(80.2, 155.9)	ϕ 13, (Ra 6.3)	Decidable hole
23	(180.7, 171.1)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	73	(286.3, 55.8)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
24	(17.4, 142)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	74	(112.9, 166.6)	ϕ 13, (Ra 6.3)	Decidable hole
25	(267.4, 104.6)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	75	(83.9, 62.0)	ϕ 9.7, (Ra 6.3)	Decidable hole
26	(17.8, 3)	ϕ 14.5, (Ra 12.5)	Decidable hole	76	(389.3, 10.2)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
27	(285.8, 198.5)	ϕ 14.5, (Ra 12.5)	Decidable hole	77	(217.8, 161.9)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
28	(179.5, 96.2)	ϕ 13, (Ra 6.3)	Decidable hole	78	(392.2, 69.9)	ϕ 9.7, (Ra 6.3)	Decidable hole
29	(338.4, 149.8)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	79	(42.8, 104.8)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
30	(352, 117.9)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	80	(100.0, 5.0)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
31	(125.6, 6.1)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	81	(259.9, 80.1)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
32	(142.4, 172.8)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	82	(341.6, 14.3)	ϕ 14.5, (Ra 12.5)	Decidable hole
33	(391.3, 29.2)	ϕ 13, (Ra 6.3)	Decidable hole	83	(286.1, 150.5)	ϕ 13, (Ra 6.3)	Decidable hole
34	(308.5, 142.8)	ϕ 13, (Ra 6.3)	Decidable hole	84	(61.1, 138.4)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
35	(247.3, 196.9)	ϕ 9.5, (Ra 6.3)	Decidable hole	85	(321.3, 98.5)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
36	(56.3, 177)	ϕ 13, (Ra 6.3)	Decidable hole	86	(259.7, 183.0)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
37	(377.5, 49.8)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	87	(260.6, 124.6)	ϕ 13, (Ra 6.3)	Decidable hole
38	(29.8, 161.3)	ϕ 13, (Ra 6.3)	Decidable hole	88	(25.1, 95.6)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
39	(347.9, 82.1)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	89	(152.9, 13.4)	ϕ 13, (Ra 6.3)	Decidable hole
40	(234.2, 23.6)	ϕ 9.5, (Ra 6.3)	Decidable hole	90	(74.2, 102.5)	ϕ 13, (Ra 6.3)	Decidable hole
41	(395.5, 184.3)	ϕ 9.7, (Ra 6.3)	Decidable hole	91	(162.5, 165.1)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
42	(132.3, 28)	ϕ 9.7, (Ra 6.3)	Decidable hole	92	(118.7, 70.6)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
43	(344.7, 193)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	93	(178.7, 115.9)	ϕ 14.5, (Ra 12.5)	Decidable hole
44	(293.6, 33.7)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	94	(2.2, 120.7)	ϕ 13, (Ra 6.3)	Decidable hole
45	(172.1, 193)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	95	(364.0, 4.3)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
46	(180, 45)	ϕ 9.5, (Ra 6.3)	Decidable hole	96	(370.0, 108.2)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
47	(139.3, 136.8)	ϕ 9.7, (Ra 6.3)	Decidable hole	97	(77.6, 39.2)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
48	(387.4, 127.8)	ϕ 13, (Ra 6.3)	Decidable hole	98	(172.4, 144.2)	ϕ 13, (Ra 6.3)	Decidable hole
49	(83.8, 85.5)	ϕ 9.7, (Ra 6.3)	Decidable hole	99	(19.8, 181.0)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
50	(317.2, 47.4)	ϕ 9.5, (Ra 6.3)	Decidable hole	100	(395.0, 148.8)	ϕ 14.5, (Ra 12.5)	Decidable hole

TABLE VII
THE DETAILS OF 100 HOLES PROBLEM / 4 PATHS

No.	Coordinate	Diameter/mm (Roughness)	Category	No.	Coordinate	Diameter/mm (Roughness)	Category
1	(138, 93.9)	ϕ 9.5, (Ra 6.3)	Decidable hole	51	(232.2, 118.3)	ϕ 9.5, (Ra 6.3)	Decidable hole
2	(284.8, 9.6)	ϕ 9.5, (Ra 6.3)	Decidable hole	52	(395.4, 92.3)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
3	(361, 175.1)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	53	(47.6, 82.5)	ϕ 9.5, (Ra 6.3)	Decidable hole
4	(55.7, 33.4)	ϕ 9.5, (Ra 6.3)	Decidable hole	54	(251.9, 3.5)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
5	(368.8, 66.6)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4	55	(305.5, 175.2)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4
6	(108.4, 96.5)	ϕ 9.5, (Ra 6.3)	Decidable hole	56	(95.3, 26.8)	ϕ 9.5, (Ra 6.3)	Decidable hole
7	(265.1, 145.2)	ϕ 9.5, (Ra 6.3)	Decidable hole	57	(222.8, 137.9)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
8	(128.6, 52.5)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	58	(209.7, 98.1)	ϕ 9.5, (Ra 6.3)	Decidable hole
9	(291.6, 113.2)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	59	(79.0, 194.6)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
10	(85.8, 126.5)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	60	(203.9, 190.6)	ϕ 13, (Ra 6.3)	Decidable hole
11	(125.1, 183.2)	ϕ 14.5, (Ra 12.5)	Decidable hole	61	(242.1, 100.7)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
12	(280.8, 169.8)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	62	(229.0, 181.0)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
13	(361.5, 32.9)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	63	(121.5, 115.2)	ϕ 14.5, (Ra 12.5)	Decidable hole
14	(368.3, 153.3)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	64	(258.8, 30.2)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
15	(144.7, 194.5)	ϕ 15, (Ra 12.5)	Decidable hole	65	(32.7, 26.5)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
16	(24.3, 76.2)	ϕ 9.7, (Ra 6.3)	Decidable hole	66	(14.1, 44.1)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
17	(103.4, 194.6)	ϕ 9.7, (Ra 6.3)	Decidable hole	67	(191.7, 68.7)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
18	(25.2, 124)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	68	(299.1, 79.2)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4
19	(61.1, 67.3)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	69	(257.3, 59.9)	ϕ 14.5, (Ra 12.5)	Decidable hole
20	(250.6, 164.6)	ϕ 13, (Ra 6.3)	Decidable hole	70	(1.9, 67.4)	ϕ 14.5, (Ra 12.5)	Decidable hole
21	(92.8, 170)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	71	(385.1, 167.3)	ϕ 9.7, (Ra 6.3)	Decidable hole
22	(5.3, 90.7)	ϕ 9.5, (Ra 6.3)	Decidable hole	72	(80.2, 155.9)	ϕ 13, (Ra 6.3)	Decidable hole
23	(180.7, 171.1)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	73	(286.3, 55.8)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4
24	(17.4, 142)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	74	(112.9, 166.6)	ϕ 13, (Ra 6.3)	Decidable hole
25	(267.4, 104.6)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	75	(83.9, 62.0)	ϕ 9.7, (Ra 6.3)	Decidable hole
26	(17.8, 3)	ϕ 14.5, (Ra 12.5)	Decidable hole	76	(389.3, 10.2)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4
27	(285.8, 198.5)	ϕ 14.5, (Ra 12.5)	Decidable hole	77	(217.8, 161.9)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
28	(179.5, 96.2)	ϕ 13, (Ra 6.3)	Decidable hole	78	(392.2, 69.9)	ϕ 9.7, (Ra 6.3)	Decidable hole
29	(338.4, 149.8)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	79	(42.8, 104.8)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
30	(352, 117.9)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	80	(100.0, 5.0)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
31	(125.6, 6.1)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	81	(259.9, 80.1)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4
32	(142.4, 172.8)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	82	(341.6, 14.3)	ϕ 14.5, (Ra 12.5)	Decidable hole
33	(391.3, 29.2)	ϕ 13, (Ra 6.3)	Decidable hole	83	(286.1, 150.5)	ϕ 13, (Ra 6.3)	Decidable hole
34	(308.5, 142.8)	ϕ 13, (Ra 6.3)	Decidable hole	84	(61.1, 138.4)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
35	(247.3, 196.9)	ϕ 9.5, (Ra 6.3)	Decidable hole	85	(321.3, 98.5)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
36	(56.3, 177)	ϕ 13, (Ra 6.3)	Decidable hole	86	(259.7, 183.0)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
37	(377.5, 49.8)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	87	(260.6, 124.6)	ϕ 13, (Ra 6.3)	Decidable hole
38	(29.8, 161.3)	ϕ 13, (Ra 6.3)	Decidable hole	88	(25.1, 95.6)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4
39	(347.9, 82.1)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	89	(152.9, 13.4)	ϕ 13, (Ra 6.3)	Decidable hole
40	(234.2, 23.6)	ϕ 9.5, (Ra 6.3)	Decidable hole	90	(74.2, 102.5)	ϕ 13, (Ra 6.3)	Decidable hole
41	(395.5, 184.3)	ϕ 9.7, (Ra 6.3)	Decidable hole	91	(162.5, 165.1)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4
42	(132.3, 28)	ϕ 9.7, (Ra 6.3)	Decidable hole	92	(118.7, 70.6)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
43	(344.7, 193)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	93	(178.7, 115.9)	ϕ 14.5, (Ra 12.5)	Decidable hole
44	(293.6, 33.7)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4	94	(2.2, 120.7)	ϕ 13, (Ra 6.3)	Decidable hole
45	(172.1, 193)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4	95	(364.0, 4.3)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
46	(180, 45)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4	96	(370.0, 108.2)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4
47	(139.3, 136.8)	ϕ 9.7, (Ra 6.3)	Decidable hole	97	(77.6, 39.2)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
48	(387.4, 127.8)	ϕ 13, (Ra 6.3)	Decidable hole	98	(172.4, 144.2)	ϕ 13, (Ra 6.3)	Decidable hole
49	(83.8, 85.5)	ϕ 9.7, (Ra 6.3)	Decidable hole	99	(19.8, 181.0)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4
50	(317.2, 47.4)	ϕ 9.5, (Ra 6.3)	Decidable hole	100	(395.0, 148.8)	ϕ 14.5, (Ra 12.5)	Decidable hole

TABLE VIII
THE DETAILS OF 100 HOLES PROBLEM / 5 PATHS

No.	Coordinate	Diameter/mm (Roughness)	Category	No.	Coordinate	Diameter/mm (Roughness)	Category
1	(138, 93.9)	ϕ 9.5, (Ra 6.3)	Decidable hole	51	(232.2, 118.3)	ϕ 9.5, (Ra 6.3)	Decidable hole
2	(284.8, 9.6)	ϕ 9.5, (Ra 6.3)	Decidable hole	52	(395.4, 92.3)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
3	(361, 175.1)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	53	(47.6, 82.5)	ϕ 9.5, (Ra 6.3)	Decidable hole
4	(55.7, 33.4)	ϕ 9.5, (Ra 6.3)	Decidable hole	54	(251.9, 3.5)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
5	(368.8, 66.6)	ϕ 5.3, (Ra 12.5)	Fixed hole / Path 5	55	(305.5, 175.2)	ϕ 5.3, (Ra 12.5)	Fixed hole / Path 5
6	(108.4, 96.5)	ϕ 9.5, (Ra 6.3)	Decidable hole	56	(95.3, 26.8)	ϕ 9.5, (Ra 6.3)	Decidable hole
7	(265.1, 145.2)	ϕ 9.5, (Ra 6.3)	Decidable hole	57	(222.8, 137.9)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
8	(128.6, 52.5)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4	58	(209.7, 98.1)	ϕ 9.5, (Ra 6.3)	Decidable hole
9	(291.6, 113.2)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4	59	(79.0, 194.6)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4
10	(85.8, 126.5)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	60	(203.9, 190.6)	ϕ 13, (Ra 6.3)	Decidable hole
11	(125.1, 183.2)	ϕ 14.5, (Ra 12.5)	Decidable hole	61	(242.1, 100.7)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
12	(280.8, 169.8)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	62	(229.0, 181.0)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
13	(361.5, 32.9)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	63	(121.5, 115.2)	ϕ 14.5, (Ra 12.5)	Decidable hole
14	(368.3, 153.3)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4	64	(258.8, 30.2)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
15	(144.7, 194.5)	ϕ 15, (Ra 12.5)	Decidable hole	65	(32.7, 26.5)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
16	(24.3, 76.2)	ϕ 9.7, (Ra 6.3)	Decidable hole	66	(14.1, 44.1)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
17	(103.4, 194.6)	ϕ 9.7, (Ra 6.3)	Decidable hole	67	(191.7, 68.7)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
18	(25.2, 124)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	68	(299.1, 79.2)	ϕ 5.3, (Ra 12.5)	Fixed hole / Path 5
19	(61.1, 67.3)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4	69	(257.3, 59.9)	ϕ 14.5, (Ra 12.5)	Decidable hole
20	(250.6, 164.6)	ϕ 13, (Ra 6.3)	Decidable hole	70	(1.9, 67.4)	ϕ 14.5, (Ra 12.5)	Decidable hole
21	(92.8, 170)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4	71	(385.1, 167.3)	ϕ 9.7, (Ra 6.3)	Decidable hole
22	(5.3, 90.7)	ϕ 9.5, (Ra 6.3)	Decidable hole	72	(80.2, 155.9)	ϕ 13, (Ra 6.3)	Decidable hole
23	(180.7, 171.1)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	73	(286.3, 55.8)	ϕ 5.3, (Ra 12.5)	Fixed hole / Path 5
24	(17.4, 142)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	74	(112.9, 166.6)	ϕ 13, (Ra 6.3)	Decidable hole
25	(267.4, 104.6)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4	75	(83.9, 62.0)	ϕ 9.7, (Ra 6.3)	Decidable hole
26	(17.8, 3)	ϕ 14.5, (Ra 12.5)	Decidable hole	76	(389.3, 10.2)	ϕ 5.3, (Ra 12.5)	Fixed hole / Path 5
27	(285.8, 198.5)	ϕ 14.5, (Ra 12.5)	Decidable hole	77	(217.8, 161.9)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
28	(179.5, 96.2)	ϕ 13, (Ra 6.3)	Decidable hole	78	(392.2, 69.9)	ϕ 9.7, (Ra 6.3)	Decidable hole
29	(338.4, 149.8)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	79	(42.8, 104.8)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
30	(352, 117.9)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4	80	(100.0, 5.0)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
31	(125.6, 6.1)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	81	(259.9, 80.1)	ϕ 5.3, (Ra 12.5)	Fixed hole / Path 5
32	(142.4, 172.8)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3	82	(341.6, 14.3)	ϕ 14.5, (Ra 12.5)	Decidable hole
33	(391.3, 29.2)	ϕ 13, (Ra 6.3)	Decidable hole	83	(286.1, 150.5)	ϕ 13, (Ra 6.3)	Decidable hole
34	(308.5, 142.8)	ϕ 13, (Ra 6.3)	Decidable hole	84	(61.1, 138.4)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
35	(247.3, 196.9)	ϕ 9.5, (Ra 6.3)	Decidable hole	85	(321.3, 98.5)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
36	(56.3, 177)	ϕ 13, (Ra 6.3)	Decidable hole	86	(259.7, 183.0)	ϕ 7.5, (Ra 12.5)	Fixed hole / Path 3
37	(377.5, 49.8)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	87	(260.6, 124.6)	ϕ 13, (Ra 6.3)	Decidable hole
38	(29.8, 161.3)	ϕ 13, (Ra 6.3)	Decidable hole	88	(25.1, 95.6)	ϕ 5.3, (Ra 12.5)	Fixed hole / Path 5
39	(347.9, 82.1)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1	89	(152.9, 13.4)	ϕ 13, (Ra 6.3)	Decidable hole
40	(234.2, 23.6)	ϕ 9.5, (Ra 6.3)	Decidable hole	90	(74.2, 102.5)	ϕ 13, (Ra 6.3)	Decidable hole
41	(395.5, 184.3)	ϕ 9.7, (Ra 6.3)	Decidable hole	91	(162.5, 165.1)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4
42	(132.3, 28)	ϕ 9.7, (Ra 6.3)	Decidable hole	92	(118.7, 70.6)	ϕ 5, (Ra 12.5)	Fixed hole / Path 1
43	(344.7, 193)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2	93	(178.7, 115.9)	ϕ 14.5, (Ra 12.5)	Decidable hole
44	(293.6, 33.7)	ϕ 5.3, (Ra 12.5)	Fixed hole / Path 5	94	(2.2, 120.7)	ϕ 13, (Ra 6.3)	Decidable hole
45	(172.1, 193)	ϕ 9.5, (Ra 6.3)	Decidable hole	95	(364.0, 4.3)	ϕ 4.5, (Ra 12.5)	Fixed hole / Path 4
46	(180, 45)	ϕ 9.5, (Ra 6.3)	Decidable hole	96	(370.0, 108.2)	ϕ 5.3, (Ra 12.5)	Fixed hole / Path 5
47	(139.3, 136.8)	ϕ 9.7, (Ra 6.3)	Decidable hole	97	(77.6, 39.2)	ϕ 6, (Ra 12.5)	Fixed hole / Path 2
48	(387.4, 127.8)	ϕ 13, (Ra 6.3)	Decidable hole	98	(172.4, 144.2)	ϕ 13, (Ra 6.3)	Decidable hole
49	(83.8, 85.5)	ϕ 9.7, (Ra 6.3)	Decidable hole	99	(19.8, 181.0)	ϕ 5.3, (Ra 12.5)	Fixed hole / Path 5
50	(317.2, 47.4)	ϕ 9.5, (Ra 6.3)	Decidable hole	100	(395.0, 148.8)	ϕ 14.5, (Ra 12.5)	Decidable hole

TABLE IX
THE SUMMARY OF HOLE ASSIGNMENT

Workpieces	Paths	The summary of holes	Size
14 holes	2	Fixed holes / Path 1: [2,5,13]	3
		Fixed holes / Path 2: [1,3,10,14]	4
		Decidable holes: [4,6,7,8,9,11,12]	7
29 holes	2	Fixed holes / Path 1: [21,24,3,6,26,15,25,14] Fixed holes / Path 2: [13,22,4,10,16,5,17,9]	8×2
		Decidable holes: [7,27,12,23,2,28,19,29,11,18,8,1,20]	13
	3	Fixed holes / Path 1: [21,24,3,6,26] Fixed holes / Path 2: [15,25,14,13,22] Fixed holes / Path 3: [4,10,16,5,17]	5×3
		Decidable holes: [9,7,27,12,23,2,28,19,29,11,18,8,1,20]	14
50 holes	3	Fixed holes / Path 1: [32,40,22,34,35,6,3,16,11] Fixed holes / Path 2: [30,45,33,7,38,42,28,17,41] Fixed holes / Path 3: [47,14,46,8,5,48,29,21,25]	9×3
		Decidable holes: [37,31,49,27,50,26,43,19,44,15,1,36,23,2,4,18,24,39,13,9,20,10,12]	23
	4	Fixed holes / Path 1: [32,40,22,34,35,6,3] Fixed holes / Path 2: [16,11,30,45,33,7,38] Fixed holes / Path 3: [42,28,17,41,47,14,46] Fixed holes / Path 4: [8,5,48,29,21,25,37]	7×4
		Decidable holes: [31,49,27,50,26,43,19,44,15,1,36,23,2,4,18,24,39,13,9,20,10,12]	22
100 holes	3	Fixed holes / Path 1: [18,92,52,29,67,62,39,84,85,24,61,43,65,64,66,13,37] Fixed holes / Path 2: [57,97,3,80,31,79,10,54,86,23,12,77,32,21,95,59,30] Fixed holes / Path 3: [9,8,14,19,25,91,88,44,68,5,81,73,55,99,96,76,45]	17×3
		Decidable holes: [46,50,1,100,53,34,93,4,22,58,83,82,7,69,48,56,17,16,74,72,42,41,87,15,40,94,35,2,78,71,49,26,89,70,36,47,6,28,60,20,38,90,98,11,51,75,63,27,33]	49
	4	Fixed holes / Path 1: [18,92,52,29,67,62,39,84,85,24,61,43,65] Fixed holes / Path 2: [64,66,13,37,57,97,3,80,31,79,10,54,86] Fixed holes / Path 3: [23,12,77,32,21,95,59,30,9,8,14,19,25] Fixed holes / Path 4: [91,88,44,68,5,81,73,55,99,96,76,45,46]	14×4
		Decidable holes: [50,1,100,53,34,93,4,22,58,83,82,7,69,48,56,17,16,74,72,42,41,87,15,40,94,35,2,78,71,49,26,89,70,36,47,6,28,60,20,38,90,98,11,51,75,63,27,33]	48
	5	Fixed holes / Path 1: [18,92,52,29,67,62,39,84,85,24] Fixed holes / Path 2: [61,43,65,64,66,13,37,57,97,3] Fixed holes / Path 3: [80,31,79,10,54,86,23,12,77,32] Fixed holes / Path 4: [21,95,59,30,9,8,14,19,25,91] Fixed holes / Path 5: [88,44,68,5,81,73,55,99,96,76]	10×5
		Decidable holes: [45,46,50,1,100,53,34,93,4,22,58,83,82,7,69,48,56,17,16,74,72,42,41,87,15,40,94,35,2,78,71,49,26,89,70,36,47,6,28,60,20,38,90,98,11,51,75,63,27,33]	50

Section II: Hyperparameter Settings

This section provides detailed configurations of all participating methods: DQN, MFVFD, LGA_mTSP, and MAAC, including agent system settings and hyperparameter configurations.

DQN [1]: In the deep Q-network method, we adopt a single-agent approach, treating the set of all decidable holes as a single agent. For a MTdDPO problem with J paths and M decidable holes, DQN modules are described as follows.

(1) State & Input Layer: The state of the agent is the distribution of decidable holes among the paths, with the state space size being J^M. Therefore, in the input layer, a $1 \times M$ vector is input, where each element corresponds to the path to which each decidable hole belongs.

(2) Action & Output layer: The actions of the agent are analogous to the state, both represented by a $1 \times M$ vector, where each element corresponds to the path to which each decidable hole will transfer to. However, to limit the agent's actions and avoid excessive randomness in the DQN algorithm, each element of the action vector is constrained to the values -1, 0, or 1. These values indicate three behaviors of decidable holes, i.e., the decidable hole transfers to the previous path, remains in the current path, or transfers to the next path (based on the path numbering), respectively. Hence, the size of the action space is 3^M. Thus, in the output layer, the softmax activation function is employed to generate a $3 \times M$ matrix. Based on the values in this matrix, the optimal behavior for each decidable hole is selected, resulting in a $1 \times M$ vector as the action output.

(3) Reward: In each iteration step, the reward calculation follows (6) in the manuscript.

(4) Hyperparameter settings of DQN: The following hyperparameter are all set the same for all workpieces: (i) The hidden layer utilizes a 2×64 network with the ReLU activation function, (ii) The maximum episodes = 2000, (iii) Network learning rate α_{net} = 0.001. Other hyperparameters are set as Table X.

TABLE X
HYPERPARAMETER SETTINGS OF DQN

Workpiece	Size of Inputted vector	Size of Experience Replay Buffer	Maximum Iteration Steps (Termination condition)
14-2	7	1000	1000
29-2	13	10000	3000
29-3	14	50000	3000
50-3	23	50000	10000
50-4	22	50000	10000
100-3	49	100000	50000
100-4	48	100000	50000
100-5	50	100000	50000

MFVFD [2]: Mean-Field theory and Value Function Decomposition (MFVFD) is a MARL approach where the Q-value update function is factorized into local Q-functions and a mean-field Q-function for agents. In MFVFD, the agent system solves the optimal policy using the same episode training as DQN. Therefore, for each agent in MFVFD, the state, action, and reward are set the same as those of MM, and the hyperparameters of structure of hidden layers (2×64 network with the ReLU activation function), episode (2000), maximum iteration steps (as shown in Table X) are set the same as those in DQN.

LGA_mTSP [3]: Given the similarity in the encoding of feasible solutions and the availability of process methods between the MTdDPO and the existing multi-depot multi-traveling salesman problem with constraints, and in order to evaluate the performance of the heuristic algorithm on MTdDPO problems, we choose the Learning Genetic Algorithm for multi-traveling salesman problem (LGA_mTSP), which is a heuristic algorithm based on learning curves, for comparative analysis. The following hyperparameters are all set the same for all workpieces according to [3]: (i) Iteration number = 200, (ii) Crossover rate = 0.8, (iii) Mutation rate = 0.3, (iv) Elitism rate = 0.3, (v) Selection method is tournament selection. Other hyperparameters are set as Table XI.

TABLE XI
HYPERPARAMETER SETTINGS OF LGA_MTSP

Workpiece	Size of Population	Learning Interval	Learning State (F: Fast, M: Medium, S: Slow)
14-2	30	[10, 20]	F: [10, 13], M: [14, 16], S: [17, 20]
29-2	50	[12, 25]	F: [12, 15], M: [16, 20], S: [21, 25]
29-3	50	[10, 20]	F: [10, 13], M: [14, 16], S: [17, 20]
50-3	100	[12, 25]	F: [12, 15], M: [16, 20], S: [21, 25]
50-4	100	[10, 20]	F: [10, 13], M: [14, 16], S: [17, 20]
100-3	200	[12, 25]	F: [12, 15], M: [16, 20], S: [21, 25]
100-4	200	[12, 25]	F: [12, 15], M: [16, 20], S: [21, 25]
100-5	200	[10, 20]	F: [10, 13], M: [14, 16], S: [17, 20]

MAAC [4]: In Multi-Agent Reinforcement Learning with Actor-Attention-Critic (MAAC), each decidable hole is treated as an independent agent, with the states, actions, and reward settings identical to those in MM. The configurations for the experience replay buffer size and maximum iteration steps are aligned with those in Table X. Furthermore, the following hyperparameter settings are consistent with the original literature: the Adam optimizer's learning rate is set to 0.001, the discount factor is 0.99, and a temperature setting of 0.01 is used for the soft actor-critic. Notably, for smaller-scale agent problems involving 12, 29, and 50 holes, the hidden layer size is set to 128, which mirrors the original paper. However, for the larger 100-hole problem, we reduce the hidden layer size to 64 due to GPU memory constraints (the experiments were conducted using an NVIDIA RTX 3090). Additionally, we standardize the batch size across all cases to 128.

References

[1] V. Mnih *et al.*, "Human-level control through deep reinforcement learning," *nature*, vol. 518, no. 7540, pp. 529-533, 2015.
[2] T. Zhang, Q. Ye, J. Bian, G. Xie, and T.-Y. Liu, "MFVFD: A Multi-Agent Q-Learning Approach to Cooperative and Non-Cooperative Tasks," in *IJCAI*, 2021, pp. 500-506.
[3] M. Belhor, A. El-Amraoui, A. Jemai, and F. Delmotte, "Learning-based metaheuristic approach for home healthcare optimization problem," *Computer Systems Science and Engineering*, vol. 45, pp. 1-19, 2023.
[4] S. Iqbal, and F. Sha, "Actor-attention-critic for multi-agent reinforcement learning," In *ICML*, pp. 2961–2970, 2019.