

Fourier Coefficients

Input file: **standard input**
Output file: **standard output**
Time limit: 8 seconds
Memory limit: 1024 megabytes

This is an **interactive problem**. Your program will interact with the judge via standard input and output. The judge's execution may take up to 1.3 seconds.

You are given an integer N . The judge secretly chooses a function

$$f(x) := \sum_{k=0}^{N-1} A_k \cos(kx),$$

where each A_0, A_1, \dots, A_{N-1} is an integer with $0 \leq A_k < 998244353$.

You must determine the values of A_0, A_1, \dots, A_{N-1} by interacting as described below:

You will output N pairs of integers $(X_1, Y_1), \dots, (X_N, Y_N)$. Each pair must satisfy

$$0 \leq X_i \leq Y_i < 998244353, \quad Y_i \neq 0.$$

The judge will then respond with N integers Z_1, \dots, Z_N , where

$$Z_i = f(\arccos(X_i/Y_i)) \bmod 998244353.$$

Detailed definition of Z_i . Under the constraints on X_i, Y_i , the value $f(\arccos(X_i/Y_i))$ is a rational number. Write it in lowest terms as P_i/Q_i ; one can show $Q_i \not\equiv 0 \pmod{998244353}$. Then Z_i is defined to be the unique integer $0 \leq Z_i < 998244353$ satisfying

$$Z_i Q_i \equiv P_i \pmod{998244353}.$$

Such a Z_i always exists and is unique.

Input

- All inputs are integers.
- $1 \leq N \leq 5 \times 10^5$.

Interaction Protocol

This is an interactive problem. Your program will interact with the judge via standard input and output.

First, read the integer N from standard input:

N

Then output N query pairs (X_i, Y_i) in the following format, satisfying the constraints above:

$X_1 Y_1$
 $X_2 Y_2$
 \vdots
 $X_N Y_N$

If your output is valid, the judge will reply with N lines:

$$\begin{matrix} Z_1 \\ Z_2 \\ \vdots \\ Z_N \end{matrix}$$

If your output is invalid, you will receive:

-1

If you receive “-1”, your program must terminate immediately.

Finally, after receiving the Z_i , output the hidden coefficients A_0, A_1, \dots, A_{N-1} in order:

$$\begin{matrix} A_0 \\ A_1 \\ \vdots \\ A_{N-1} \end{matrix}$$

Note

- **After every output operation, print a newline and flush standard output.** If you fail to flush, you may receive a TLE verdict.
- If you produce an invalid output at any point or your program terminates unexpectedly, the verdict is undefined.
- Immediately terminate your program after printing the answer (or after reading “-1”). Otherwise, the verdict is undefined.
- Extra newlines or any deviation from the specified format will be judged as invalid.
- The judge is non-adaptive: the values A_0, \dots, A_{N-1} are fixed at the start and do not change during the interaction.

Example

Suppose $N = 2$ and $(A_0, A_1) = (3, 2)$. A possible interaction is shown below.

Input	Your Output	Explanation
2		You read N .
	0 1 1 1	You query two valid pairs (X_i, Y_i) .
3 5		The judge returns $Z_1 = 3, Z_2 = 5$.
	3 2	You output the recovered (A_0, A_1) .