#### The 3rd Universal Cup Stage 11: Sumiyosi, October 5-6, 2024

# Problem D. Two Box

Time limit: 6 seconds

Memory limit: 1024 megabytes

You are given a sequence of non-negative integers  $A = (A_1, A_2, \dots, A_N)$  of length N and Q queries. The i-th query is described as follows:

• Change  $A_{x_i}$  to  $y_i$ , and then compute the answer to the following problem based on the updated sequence A.

There are two boxes, one white and one black, and M balls numbered from 1 to M. Initially, all balls are in the white box.

You perform the following operation N times:

• Choose an integer x that satisfies  $1 \le x \le M$ . Move ball x from its current box to the other box.

After the *i*-th operation, all the numbers on the balls in the black box must be less than or equal to  $A_i$ . Compute the number of possible sequences of operations that satisfy this condition, modulo 998244353.

Process the queries in order.

#### Constraints

- $1 \le N, Q \le 3 \times 10^4$
- $1 \le M \le 15$
- $1 \le x_i \le N$
- $1 \leq A_i, y_i \leq M$

#### Input

The input is given in the following format from standard input:

### Output

Output Q lines. On the i-th line, output the answer to the i-th query.

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## **Examples**

standard input	standard output
3 3 2	5
1 3 1	14
3 2	
1 3	
6 8 1	2100
3 8 7 7 1 6	
1 4	
12 10 8	2741280
1 3 2 6 3 6 7 7 5 5 4 7	3007680
12 4	1503840
7 10	1916160
4 2	1972800
9 8	728640
9 9	1821600
8 3	621440
4 9	
10 2	

### Note

For the first sample case:

For the first query, A = (1, 3, 2). In this case, possible sequences of operations include, for example:

- Choose x = 1. Move ball 1 from the white box to the black box. The black box now contains ball 1.
- Choose x = 3. Move ball 3 from the white box to the black box. The black box now contains balls 1 and 3.
- Choose x = 3. Move ball 1 from the black box back to the white box. The black box now contains ball 1.

Other possible sequences of x are (1, 1, 1), (1, 1, 2), (1, 2, 1), and (1, 2, 2), totaling 4 additional possibilities. Therefore, there are 5 possible sequences of operations.