$E \to T(+E \mid \varepsilon)$

$T \to F(*T \mid \varepsilon)$

$F \to (E) \mid c \mid i$

$E \to TY$

$Y \to +E \mid \varepsilon$

$T \to FZ$

$Z \to *T \mid \varepsilon$

$F \to (E) \mid c \mid i$

BNF version.
(predictive)

c c i
3+4*5 endofinput.

$E \Rightarrow TY \Rightarrow FZY \Rightarrow cZY \Rightarrow cY \Rightarrow c+E \Rightarrow c+TY$

$\Rightarrow c+FZY \Rightarrow c+cZY \Rightarrow c+c*TY \Rightarrow c+c*FZY$

$\Rightarrow c+c*iZY \Rightarrow c+c*iY \Rightarrow c+c*i$

all terminals stop.

PREDICTIVE GRAMMAR

(1) No two productions for a single nonterminal that start with same token.
(left-factored)

(2) No Left-Recursion

$A \overset{*}{\Rightarrow} A\alpha$

$A \to B\alpha_1$

$B \to C\alpha_2$

$C \to A\alpha_3$

$A \Rightarrow B\alpha_1 \Rightarrow C\alpha_2\alpha_1 \Rightarrow A\alpha_3\alpha_2\alpha_1$

$\overset{*}{\Rightarrow} A\alpha_3\alpha_2\alpha_1\alpha_3\alpha_2\alpha_1$

$E \to T{*}{*}E$

$T \to E{+}E$

$E \Rightarrow T{*}{*}E \Rightarrow E{+}E{*}{*}E$

left-recursive.

DIRECT LEFT-RECURSION

$A \to A\alpha$

$A\beta$

$A\gamma$

δ

ϕ

$A \to A(\alpha|\beta|\gamma)\ \delta|\phi$

$A \to A\alpha$

$A \to \beta$

$A \to \beta\alpha^*$

$A \Rightarrow A\alpha \Rightarrow A\alpha^2 \Rightarrow A\alpha^3 \ldots\ldots$

$A\alpha^n \Rightarrow \beta\alpha^n$

$A \to A\alpha$
$A \to \beta$

$\Rightarrow$ BNF

$A \to \beta A'$
$A' \to \alpha A' \mid \varepsilon$

Algorithm: Eliminating Left Recursion

NONTERMINALS: $A_1\ A_2\ A_3 \ldots A_n$

```
for i = 1 to n
    for j = 1 to i-1
        if ∃ a production Ai → Aj γ
            replace that production by expanding
            Aj with all of the alternatives for Aj
    rewrite any direct left-recursion on Ai
```

$A \to Ax \mid ABy \mid Bz \mid x$

- $B \to Ax \mid Cw \mid Cx$
- $C \to Az \mid Bz \mid Cy \mid v$

$i=1$

$A \to A(x \mid By) \mid (Bz \mid x)$

(A = A, α = $(x \mid By)$, β = $(Bz \mid x)$)

- $A \to (Bz \mid x)A'$
- $A' \to (x \mid By)A' \mid \varepsilon$

$i=2$, $j=1$

$B \to (Bz \mid x)A'x \mid Cw \mid Cx$

$B \to Bz A'x \mid xA'x \mid Cw \mid Cx$

(α = $A'x$, β = $Cw \mid Cx$)

$B \to (x A'x \mid Cw \mid Cx)B'$

$B' \to$ [illegible] $z A' x B' \mid \varepsilon$

$i=3$

$j=1$, $j=2$

$C \to (Bz \mid x) A' z \mid Bz \mid Cy \mid v$

$C \to ((xA'x \mid zA'x \mid \underline{Cw} \mid \underline{Cx})z \mid x)A'z \mid$
$(xA'x \mid zA'x \mid \underline{Cw} \mid \underline{Cxz}) \mid \underline{Cy} \mid v$

factor out all terms starting with C

$C \to C\alpha \mid \beta$

$C \to \beta C'$

$C' \to \alpha C' \mid \varepsilon$

$E \to E + E$
$E - E$
$E * E$
$-E$
(E)
i
c

} low precedence

medium precedence

high " "

highest precedence PE

literals { i, c

construct from high to low precedence

LE: literal expression

$$LE \to i \mid c$$

PE: parenthesized expression

PE → (E) | LE  [LE as Type]

UE: unary expression

UE → -PE | PE

UE → -UE | PE

allows -x but not --x
not -----x

fix

ME: multiplicative expression

ME → UE*ME | UE | UE/ME | UE % ME

AE: additive expr.

AE → ME+AE | ME-AE | ME → ME((+|-)ME)*

E → AE