Assignment #1 Part 1
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Logical Definitions for OWL2 Direct Semantics

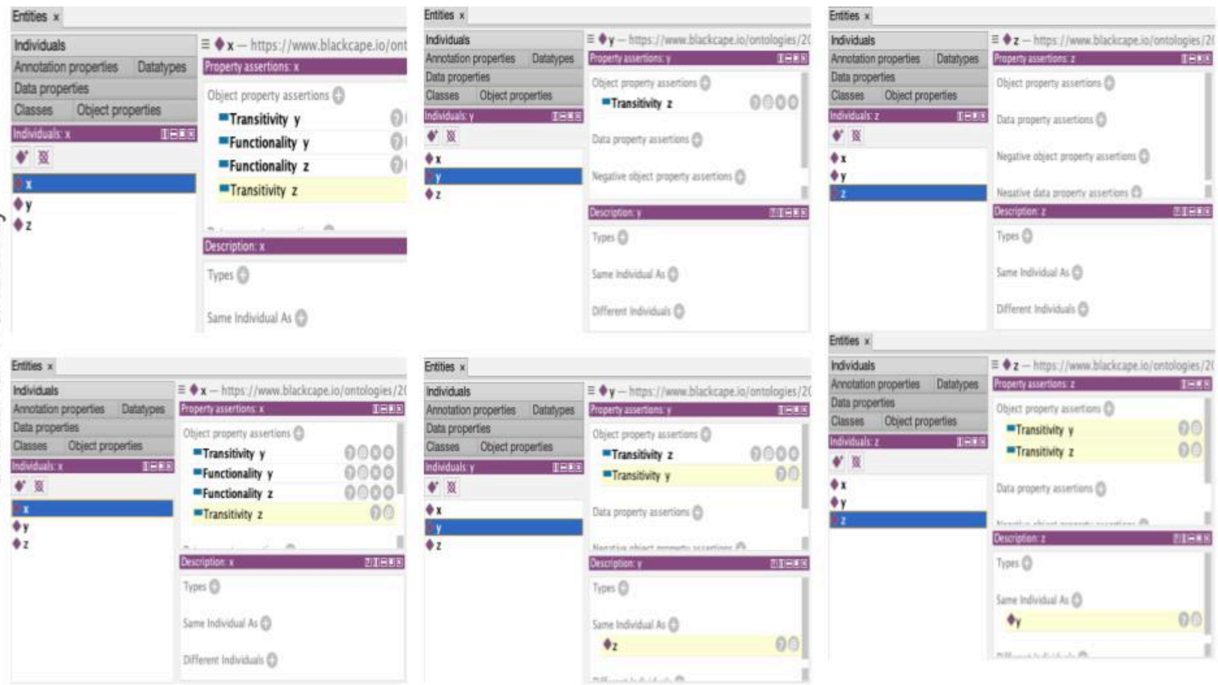
Functional: If xRy and xRz, then y=z. If x has birthdate y and x has birthdate z, then y=z.
Inverse Functional: If xRy and zRy, then x=z. If x has social security number y and z has social security number y, then x=z.
Transitive: If xRy and yRz, then xRz. If x is contained in y and y is contained in z, then x is contained in z.
Symmetric: If xRy, then yRx. If x is a friend of y, then y is a friend of x.
Asymmetric: If xRy, then it is not the case that yRx If x is the parent of y, then it is not the case that y is the parent of x.
Reflexive: xRx, x is as tall as itself.
Irreflexive: It is not the case that xRx. No x is taller than itself

Functional Transitivity (X^{NS})

Suppose property R is both functional and transitive.
By functionality, for any x, y, z: if xRy $\wedge$ xRz, then it follows that y=z, and by transitivity if xRy $\wedge$ yRz then it also must follow that xRz.
Consider the case when y $\neq$ z, then by transitivity (xRy $\wedge$ yRz $\Rightarrow$ xRz) and by functionality, [xRy $\wedge$ xRz) $\Leftrightarrow$ y=z] , but since y$\neq$z and y=z cannot both be true, we have demonstrated a set of assignments where P as both a functional and transitive property is inconsistent.

Creating two distinct properties 'Transitivity' and 'Functionality' and providing three variables and assertions based on the function signatures shows a configuration where functional transitive property is consistent. However, for an ontology containing an object property which is defined as both functional and transitive, the reasoners will abend.

Functional Transitivity

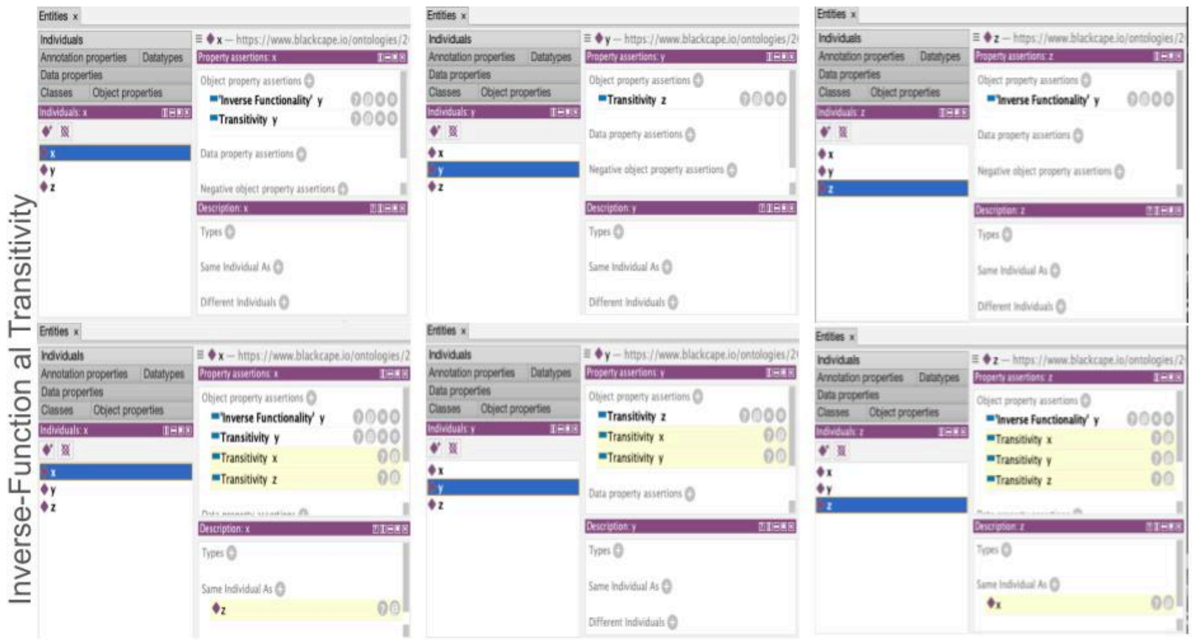


Inverse-Functional Transitivity (X^{NS})

Suppose property R is both inverse functional and transitive.
If R is Inverse Functional, then $(xRy \wedge zRy) \Leftrightarrow x=z$
If R is Transitive, then $(xRy \wedge yRz) \Rightarrow xRz$
* Note that ff R is Reflexive, then $(xRy \wedge yRx)$ and we know from the initial state of the table that Transitive Reflexivity is satisfiable.

By inverse functionality, If $xRy \wedge zRy$, then $x=z$ and by transitivity, if $xRy \wedge yRz$ then it follows that xRz.
Consider the case when $x \neq z$, then then $(xRy \wedge yRz \Rightarrow xRz)$ holds true but Inverse functionality fails due to contradiction since it is obviously not the case that $(x \neq z)$ and $(x = z)$. So we are able to observe at least one set of variable assignments where an inverse-functional transitive property is inconsistent.

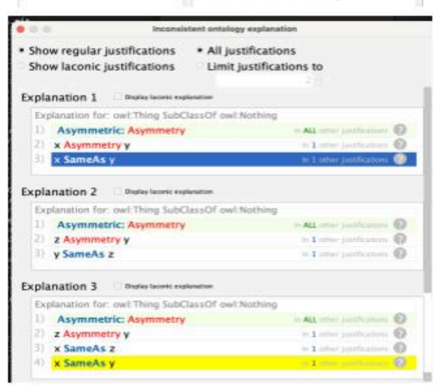
Creating two distinct properties ‘Transitivity’ and ‘InverseFunctionality’, as demonstrated below, and providing three variables and assertions based on the function signatures shows a configuration where functional transitive property is consistent (namely where $(x=z,y=z,x=y)$. However, for an ontology containing an object property which is defined as both inverse-functional and transitive, the reasoners will abend.



Asymmetric Transitivity **(X^{NS})**

If property R is both asymmetric and transitive, then [xRy ∧ ¬(yRx)] ∧ [(xRy ∧ yRz) ⇒ xRz]. This breaks down where the assignments of variables such that x=y, y=z, and x=z, while it allows transitivity to hold at the same time causes asymmetry to be inconsistent.

This is demonstrated in Protege. When the property Asymmetry is actually set as an asymmetric property, the reasoner identifies the inconsistency.



Irreflexive Transitivity **(X^{NS})**

Suppose property R is both reflexive and transitive, then $\neg(xRx) \wedge [(xRy \wedge yRz) \Rightarrow xRz]$. We can readily show an inconsistency when y=x, in which case transitivity holds but irreflexivity fails to hold.

Unsatisfiable Semantics

Asymmetric Reflexivity **(X^{UNSAT})**

$[xRy \wedge \neg(yRx)] \wedge (xRx)$. If we consider the case when $y = x$, then xRx and $\neg(yRx)$ will evaluate as inconsistent.

Asymmetric Symmetry **(X^{UNSAT})**

Property R being both asymmetric and symmetric is intuitively a non-starter. $xRy \Leftrightarrow yRx$ and $xRy \wedge \neg(yRx)$ cannot be consistent since by symmetry $xRy \wedge yRx$ and asymmetry requires that $\neg(yRx)$ which results in this property P being unsatisfiable in all cases

Irreflexive Reflexivity **(X^{UNSAT})**

Property R being both Reflexive and Irreflexive is intuitively a non-starter. Reflexivity specifies that xRx and irreflexivity specifies the negation, $\neg(xRx)$. Property P defined as such, cannot be consistent in any case since $(xRx) \wedge \neg(xRx)$ can never be true.