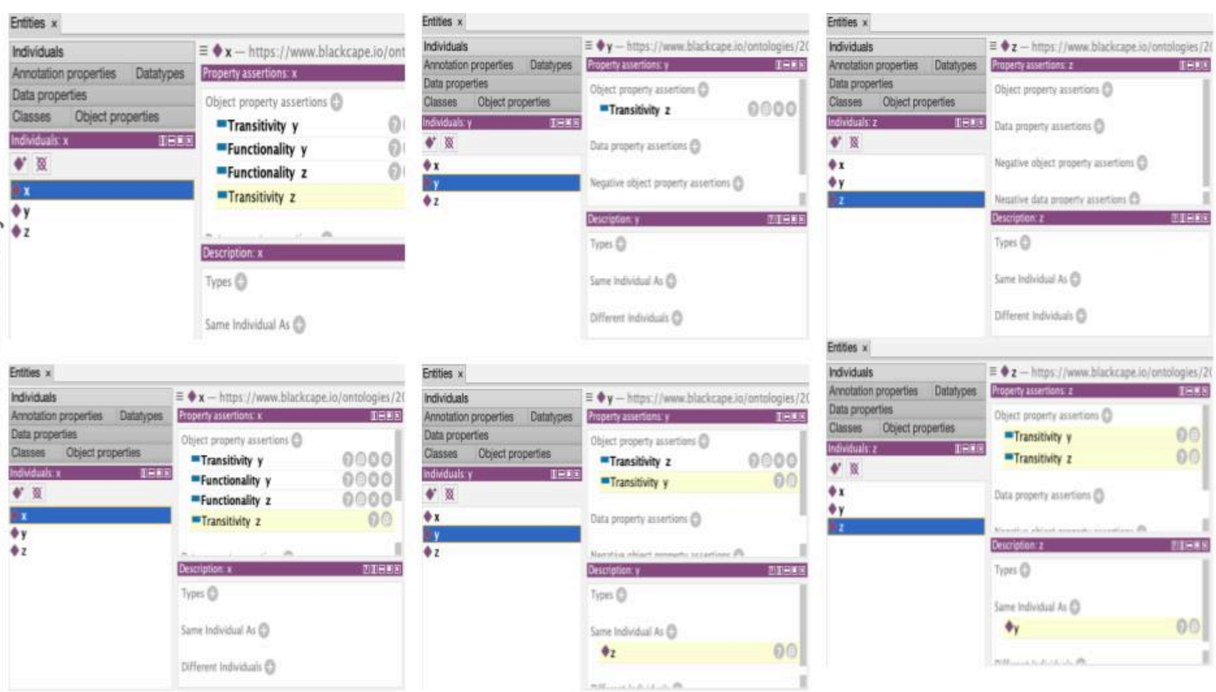
Functional: If xRy and xRz, then y=z. If x has birthdate y and x has birthdate z, then y=z.
Inverse Functional: If xRy and zRy, then x=z. If x has social security number y and z has social security number y, then x=z.
Transitive: If xRy and yRz, then xRz. If x is contained in y and y is contained in z, then x is contained in z.
Symmetric: If xRy, then yRx. If x is a friend of y, then y is a friend of x.
Asymmetric: If xRy, then it is not the case that yRx If x is the parent of y, then it is not the case that y is the parent of x.
Reflexive: xRx, x is as tall as itself.
Irreflexive: It is not the case that xRx. No x is taller than itself

Functional Transitivity (X^{NS})

Suppose property R is both functional and transitive.
By functionality, for any x, y, z: if xRy ∧ xRz, then it follows that y=z, and by transitivity if xRy ∧ yRz then it also must follow that xRz.
Consider the case when y ≠ z, then by transitivity (xRy ∧ yRz ⇒ xRz) and by functionality, [xRy ∧ xRz) ⇔ y=z] , but since y≠z and y=z cannot both be true, we have a contradiction. ⊥

Functional Transitivity



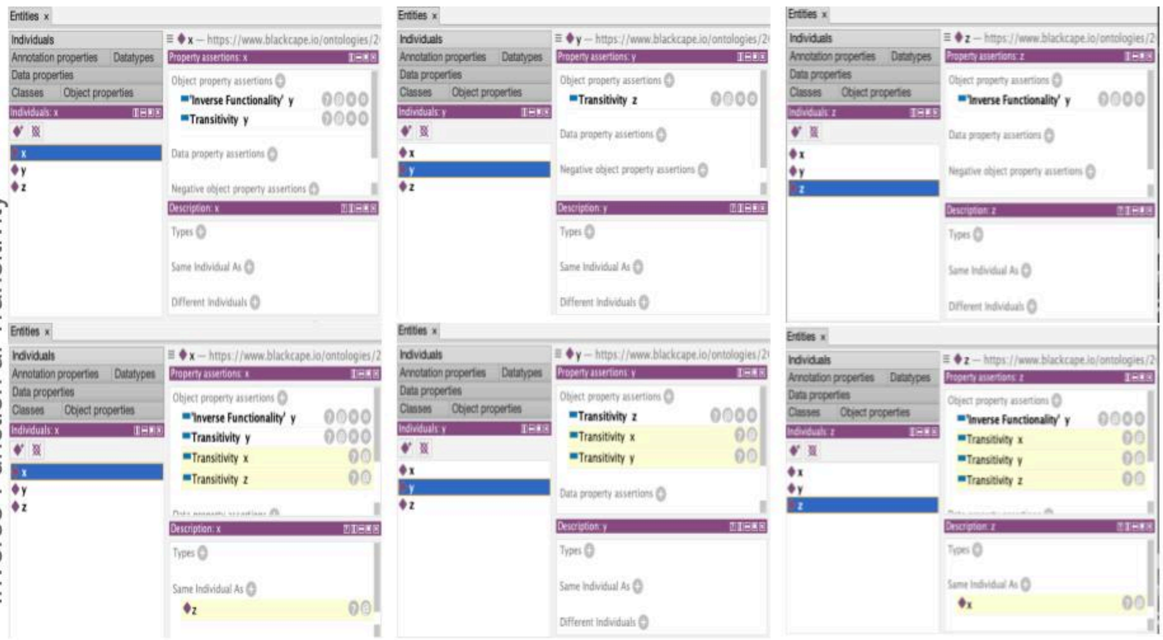
Inverse-Functional Transitivity (X^{NS})

Suppose property R is both inverse functional and transitive.
If R is Inverse Functional, then (xRy ∧ zRy) ⇔ x=z
If R is Transitive, then (xRy ∧ yRz) ⇒ xRz

* Note that ff R is Reflexive, then (xRy ∧ yRx) and we know from the initial state of the table that Transitive Reflexivity is satisfiable.

By inverse functionality, If xRy ∧ zRy, then x=z and by transitivity, if xRy ∧ yRz then it follows that xRz.
Consider the case when x ≠ z, then then (xRy ∧ yRz ⇒ xRz) holds true but Inverse functionality fails due to contradiction since it is obviously not the case that (x ≠ z) and (x = z). ⊥

Inverse-Function al Transitivity



Asymmetric Transitivity **(X^{NS})**

Irreflexive Transitivity **(X^{NS})**

Asymmetric Symmetry **(X^{UNSAT})**

Asymmetric Reflexivity **(X^{UNSAT})**

Irreflexive Reflexivity **(X^{UNSAT})**
