a presentation on

BOOLEAN ALGEBRA

team **LONELY HEARTS CLUB**

table of

CONTENTS

by
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CHAPTER 1

THE BASICS

The entire field of Boolean Algebra is based on 2 elements

0

1

and 3 operations

and	·
or	+
not	–

BOOLEAN EXPRESSIONS

Any finite combinations of Boolean elements and Boolean operators

e.g. $$xy + \bar{z}$$

Always evaluates to either 0 or 1

BOOLEAN FUNCTIONS

$B = \{0,1\}$

$B^n = \{(x_1, x_2, ..., x_n) : x_i \in B \text{ for } 1 \leq i \leq n\}$

$$F : B^n \to B$$

e.g. $F(x,y,z) = xy + \bar{z}$

by
Samia Zaman

CHAPTER 2

IDENTITIES

NAME	IDENTITY
Law of the Double Complement	$\overline{\overline{x}} = x$
Idempotent Law	$x + x = x$ $x.x = x$
Domination Law	$x + 1 = 1$ $x.0 = 0$
De Morgan's Law	$(\overline{x.y}) = \overline{x} + \overline{y}$ $(\overline{x + y}) = \overline{x}.\overline{y}$

IDENTITIES

The Absorption Law $x.(x+y) = x$

$x.(x+y)$	$= (x+0).(x+y)$	Identity Law
	$= x + 0.y$	Distributive Law
	$= x + y.0$	Commutative Law
	$= x + 0$	Domination Law
	$= x$	Identity Law

by
Mostafijur Rahman

CHAPTER 3

DUALS

$$x.(y+1) \rightarrow x+(y.0)$$

$$\bar{x}.1+(\bar{y}+z) \rightarrow (\bar{x}+0).(\bar{y}.z)$$

DUALITY PRINCIPLE

$$F = F^D$$

* independent of the function F

by
Tasmia Rahman

CHAPTER 4

MINTERM

$$y = x$$

$$y = \overline{x}$$

LITERAL

$$x_1 x_2 \cdots x_n$$

SUM OF PRODUCTS EXPANSION

TRUTH TABLE

x	0	0	0	0	1	1	1	1
y	0	0	1	1	0	0	1	1
z	0	1	0	1	0	1	0	1
F	0	1	0	0	1	0	1	0

$$F = \bar{x}\bar{y}z + x\bar{y}\bar{z} + xy\bar{z}$$

* aka **Conjunctive Normal Form**

by
Kazi Nafi

CHAPTER 5

FUNCTIONAL COMPLETENESS

1 $\{\cdot, +, \bar{\ }\}$

2 $\{\cdot, \bar{\ }\}$ since $x+y = \overline{\bar{x}\bar{y}}$ (De Morgan's Law)

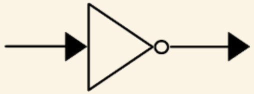
3 $\{\,|\,\}$ since $\bar{x} = x|x$, $xy = (x|y)|(x|y)$

by
Zinath Tasmia

CHAPTER 6

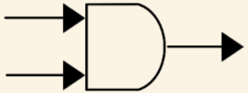
LOGIC GATES

The 3 basic gates



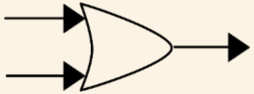
NOT

input	output
x	$\bar{x}$
0	1
1	0



AND

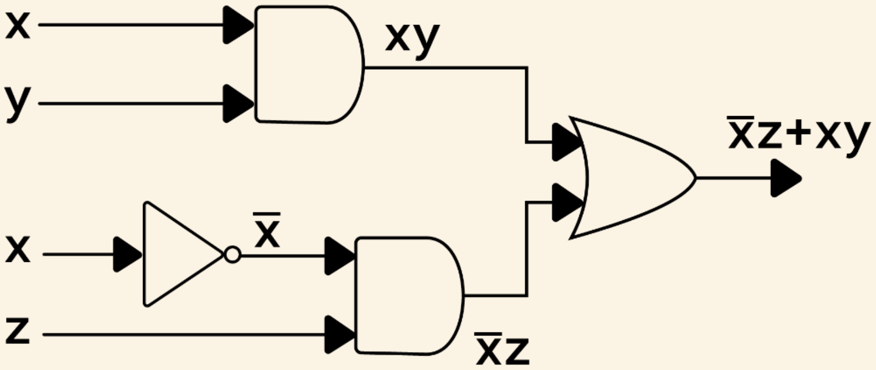
input		output
x	y	x+y
0	0	0
0	1	0
1	0	0
1	1	1



OR

input		output
x	y	x+y
0	0	0
0	1	1
1	0	1
1	1	1

COMBINATION OF GATES



CIRCUIT EXAMPLE

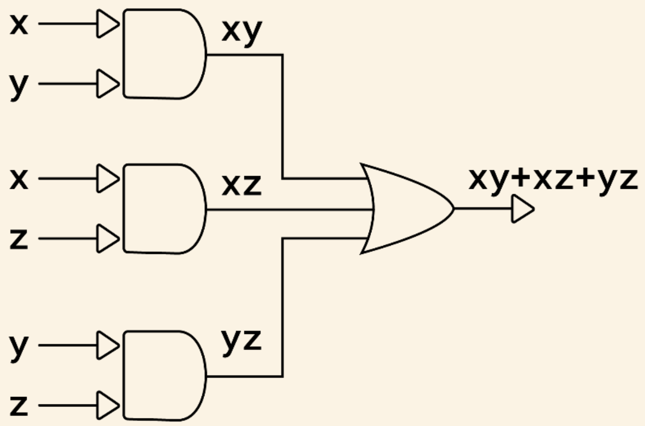
AN APPLICATION

Majority Voting Circuit

x	0	0	0	0	1	1	1	1
y	0	0	1	1	0	0	1	1
z	0	1	0	1	0	1	0	1
M	0	0	0	1	0	1	1	1

TRUTH TABLE

$$M = xy+yz+zx$$

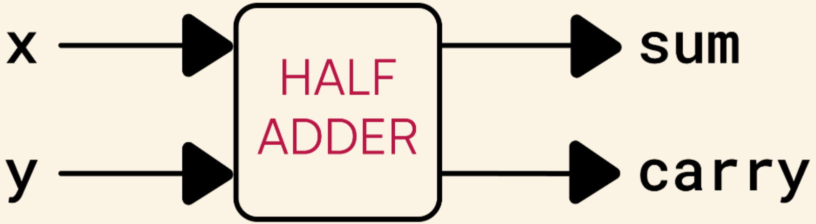


CIRCUIT

by
Rifat Sultana

CHAPTER 7

HALF ADDER

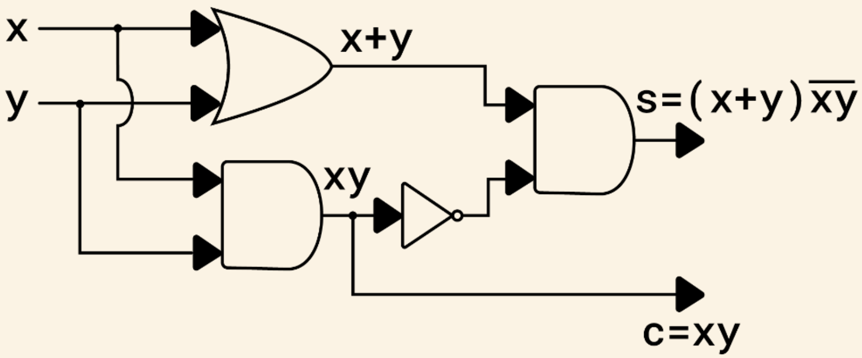


BLOCK DIAGRAM

HALF ADDER

input		output	
x	y	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

TRUTH TABLE



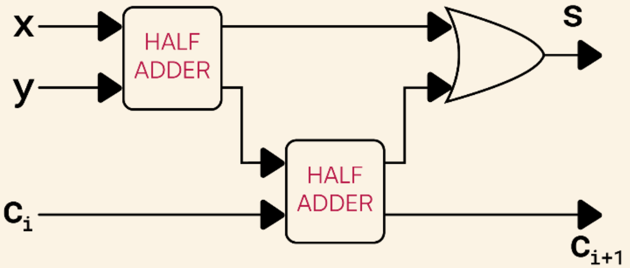
CIRCUIT

FULL ADDER

input			output	
x	y	c_i	s	c_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$s = xyc_i+xyc_i+xyc_i+xyc_i$$

$$c_{i+1} = xyc_i+xyc_i+xyc_i+xyc_i$$



TRUTH TABLE

SOP EXPANSION

CIRCUIT

CHAPTER 8

by
Tanzeem Ahmed

K MAP

Working Principle:

1. select nxn K Map (n = number of variables)
2. put 1s and 0s
3. identify blocks
4. cover the blocks
5. express function

K MAP

for 2 variables

	y	$\bar{y}$
x	xy	$x\bar{y}$
$\bar{x}$	$\bar{x}y$	$\bar{x}\bar{y}$

for 3 variables

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
x	xyz	$xy\bar{z}$	$x\bar{y}\bar{z}$	$x\bar{y}z$
$\bar{x}$	$\bar{x}yz$	$\bar{x}y\bar{z}$	$\bar{x}\bar{y}\bar{z}$	$\bar{x}\bar{y}z$

for 4 variables

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
wx	$xyzw$	$xy\bar{z}w$	$x\bar{y}\bar{z}w$	$x\bar{y}zw$
$\bar{w}x$	$xyz\bar{w}$	$xy\bar{z}\bar{w}$	$x\bar{y}\bar{z}\bar{w}$	$x\bar{y}z\bar{w}$
$\bar{w}\bar{x}$	$\bar{x}yz\bar{w}$	$\bar{x}y\bar{z}\bar{w}$	$\bar{x}\bar{y}\bar{z}\bar{w}$	$\bar{x}\bar{y}z\bar{w}$
$w\bar{x}$	$\bar{x}yzw$	$\bar{x}y\bar{z}w$	$\bar{x}\bar{y}\bar{z}w$	$\bar{x}\bar{y}zw$

K MAP

e.g. $xy + \bar{x}y = y$

	y	$\bar{y}$
x	1	0
$\bar{x}$	1	0

Step 1

→

	y	$\bar{y}$
x	1	0
$\bar{x}$	1	0

Step 2

→

	y	$\bar{y}$
x	1	0
$\bar{x}$	1	0

Step 3

by
Irteza Asif

CHAPTER 9

QUINE-MCCLUSKEY METHOD

Working Principle:

1. arrange minterms
2. identify single-bit changes
3. repeat until we get all prime implicants
4. make prime implicant table
5. compare the prime implicants
6. select the essentials

QUINE-MCCLUSKEY METHOD

for 2 variables

Group of 1s	String	Minterm
0	00	$\bar{x}\bar{y}$
1	01	$\bar{x}y$
	10	$x\bar{y}$
2	11	xy

for 3 variables

Group of 1s	String	Minterm
0	000	$\bar{x}\bar{y}\bar{z}$
1	001	$\bar{x}\bar{y}z$
	010	$\bar{x}y\bar{z}$
	100	$x\bar{y}\bar{z}$
2	011	$\bar{x}yz$
	101	$x\bar{y}z$
	110	$xy\bar{z}$
3	111	xyz

QUINE-MCCLUSKEY METHOD

e.g. $xyz+x\bar{y}z+\bar{x}yz+\bar{x}\bar{y}\bar{z}$

Group of 1s	String	Minterm
0	000	$\bar{x}\bar{y}\bar{z}$
2	~~011~~	$\cancel{\bar{x}yz}$
	~~101~~	$\cancel{x\bar{y}z}$
3	~~111~~	$\cancel{xyz}$

Step 1

String	Minterm
~~-11~~	$\cancel{yz}$
~~1-1~~	$\cancel{xz}$

Step 2

String	Minterm
--1	z

Step 3

QUINE-MCCLUSKEY METHOD

e.g. $$xyz+x\bar{y}z+\bar{x}yz+\bar{x}\bar{y}\bar{z} = z+\bar{x}\bar{y}\bar{z}$$

		Minterms			
		xyz	$x\bar{y}z$	$\bar{x}yz$	$\bar{x}\bar{y}\bar{z}$
ESP	z				
	$\bar{x}\bar{y}\bar{z}$				

Step 1

		Minterms			
		xyz	$x\bar{y}z$	$\bar{x}yz$	$\bar{x}\bar{y}\bar{z}$
ESP	z	✓	✓	✓	
	$\bar{x}\bar{y}\bar{z}$				✓

Step 2

FIN