# Convergence of analysis results

Área de Mecánica Computacional

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## 1 Cantilever beam

Figure 1 shows a cantelever beam under a tip load $P$. The beam is of length $l$; moment of inertia $I$; height $2c$ and material properties corresponding to Poissons ratio and Young modulos $\nu$ and $E$ respectively.

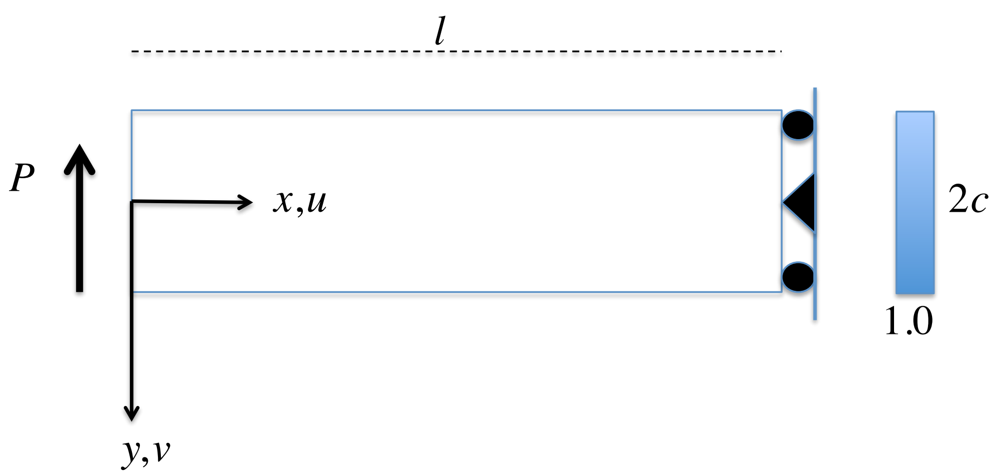



Figure 1: Cantelever beam.

The analytic solution (Timoshenko & Goodier, 1970) is given by:

$$u = -\frac{P}{2EI}x^2y - \frac{\nu P}{6EI}y^3 + \frac{P}{2IG}y^3 + \left(\frac{Pl^2}{2EI} - \frac{Pc^2}{2IG}\right)y$$

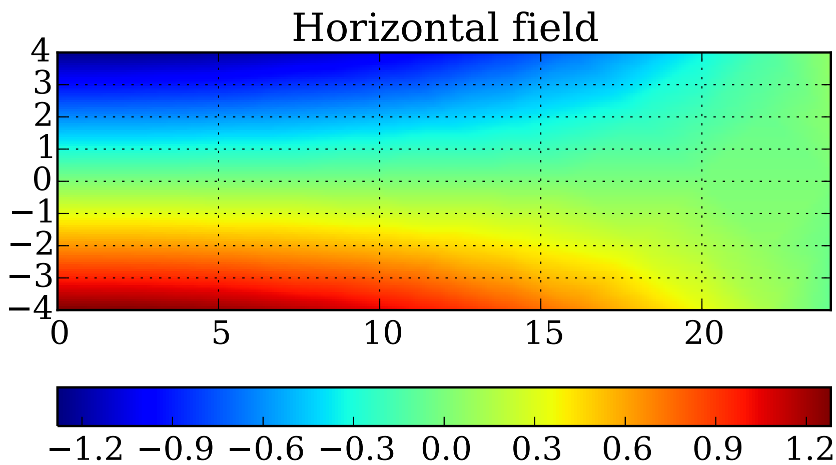
$$v = \frac{\nu P}{2EI}xy^2 + \frac{P}{6EI}x^3 - \frac{Pl^2}{2EI}x + \frac{Pl^3}{3EI}$$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \equiv -\frac{P}{EI}xy$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} \equiv \frac{\nu P}{EI}xy$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \equiv \frac{P}{2IG}\left(y^2 - c^2\right)$$

while the particular solution for parameters $E = 1000.0$, $\nu = 0.30$, $l = 24$ and $2c = 8$ is shwon below:

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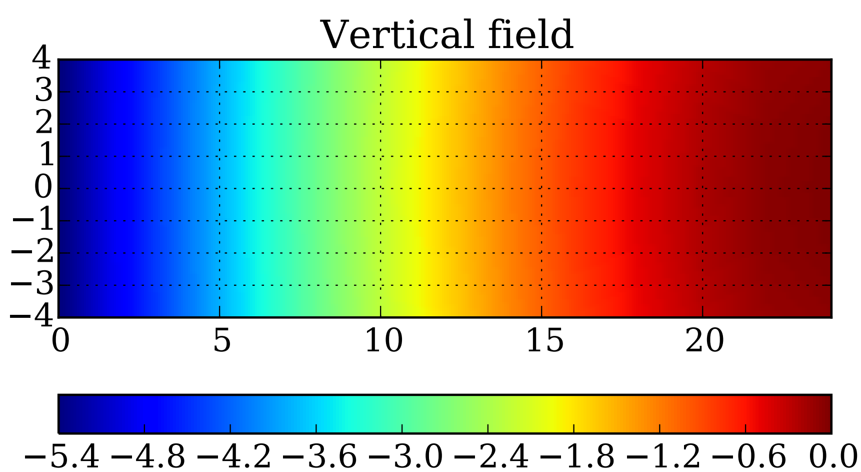


Figure 2: Horizontal field.

The beam was also analyzed with the FEM using solid lineal square elements under plane stress conditions. We conducted 5 different analysis with the following set of characteristic element size $h = [6.0, 3.0, 1.5, 1.0, 0.5]$. The meshes are shown in fig. 3

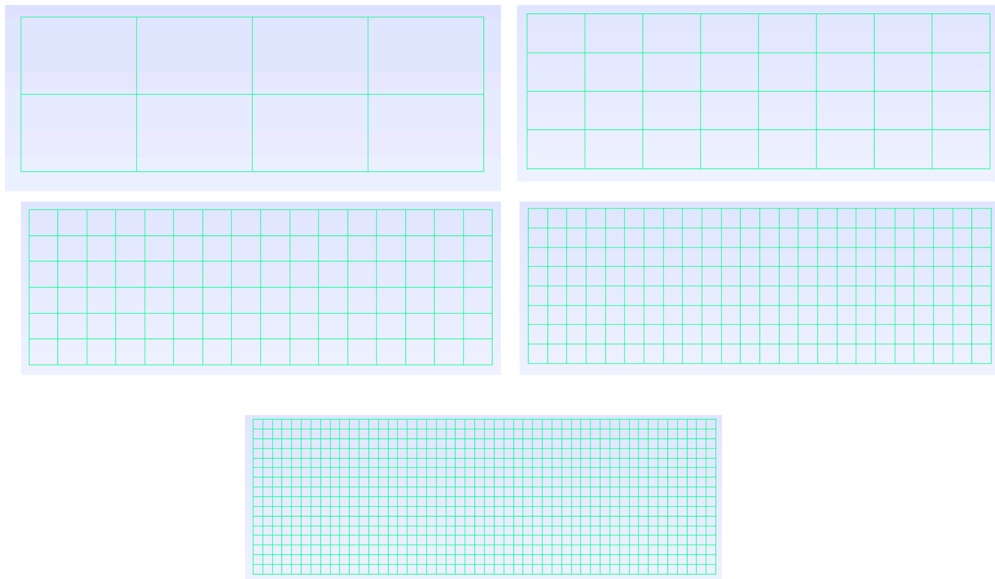

Figure 3: Finite element meshes.

The horizontal and vertical displacement contours for the FE-solution and the analytic solution are compared in fig. 4 for the case $h = 1.0$. Notice that in the finite elment models the load is applied as nodal point loads of the same magnitud distributed along all the nodes at $x = 0$. This implies a uniform load distribution instead of the parabolic load consitent with the shear stress in the analytic solution.

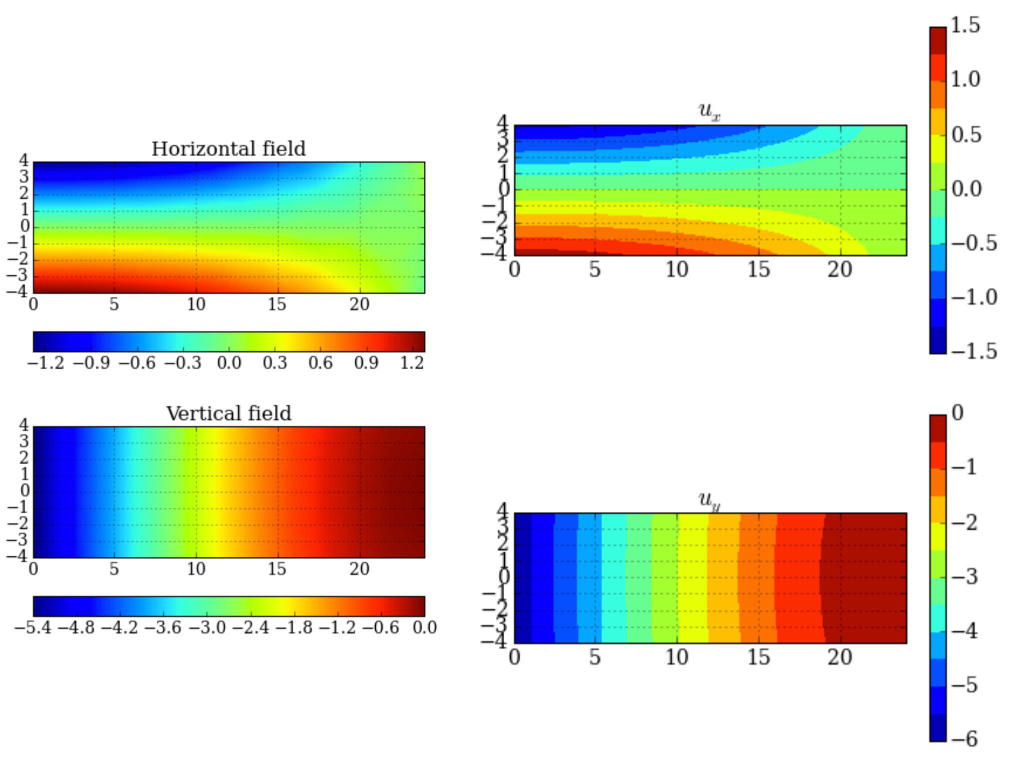


Figure 4: Analytic (left) vs Numerical (right) solution.

$$\prod_{FE} = -\frac{1}{2} U^T K U$$

$$\prod_{Exa} = \frac{\prod_{n-1}^2 - \prod_n \prod_{n-2}}{2\prod_{n-1} - \prod_n - \prod_{n-2}}$$

Using $n = 5$ we have $\prod_{Exa} = -154.09$.

$$\|\vec{u}_{Exa} - \vec{u}_{FE}\| \equiv (\prod_{Exa} - \prod_{FE})^{1/2}$$

| $h$ | $\prod_{FE}$ | $\|\vec{u}_{Exa} - \vec{u}_{FE}\|$ | $\frac{\|\vec{u}_{Exa} - \vec{u}_{FE}\|}{\|\vec{u}_{Exa}\|}$ |
|---|---|---|---|
| 6.0 | $-118.414$ | 5.973 | 0.481 |
| 3.0 | $-139.273$ | 3.849 | 0.310 |
| 1.5 | $-145.866$ | 2.868 | 0.231 |
| 1.0 | $-147.501$ | 2.567 | 0.207 |
| 0.5 | $-148.811$ | 2.298 | 0.185 |

Table 1: Convergence of anlysis results

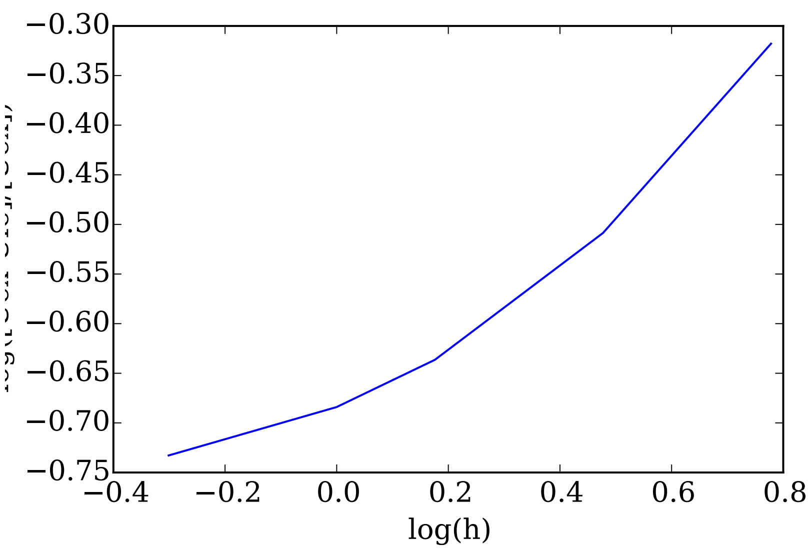


Figure 5: Energy norm of the error

# References

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