Convergence of analysis results

Área de Mecánica Computacional

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1 Cantilever beam

Figure 1 shows a cantelever beam under a tip load P. The beam is of length l; moment of inertia I; height $2c$ and material properties corresponding to Poissons ratio and Young modulos ν and E respectively.

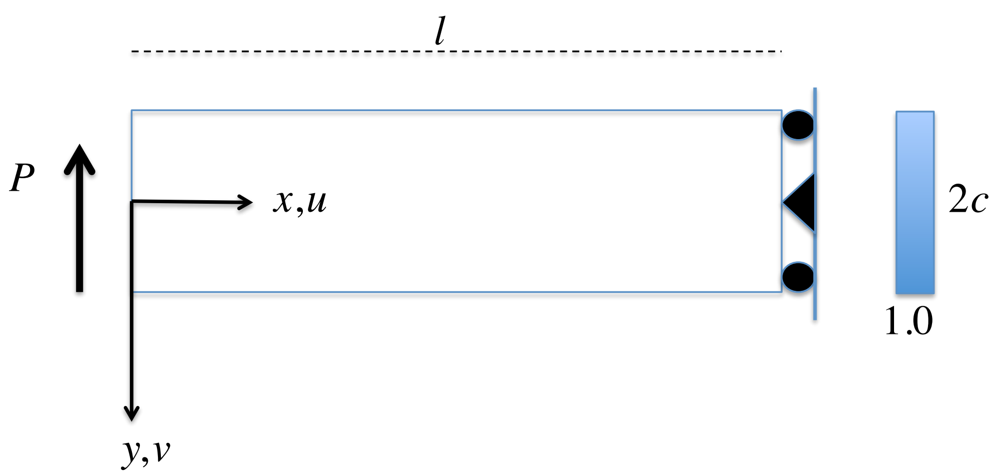


Figure 1: Cantelever beam.

The analytic solution (Timoshenko & Goodier, 1970) is given by:

$$u = -\frac{P}{2EI}x^2y - \frac{\nu P}{6EI}y^3 + \frac{P}{2IG}y^3 + \left(\frac{Pl^2}{2EI} - \frac{Pc^2}{2IG}\right)y$$

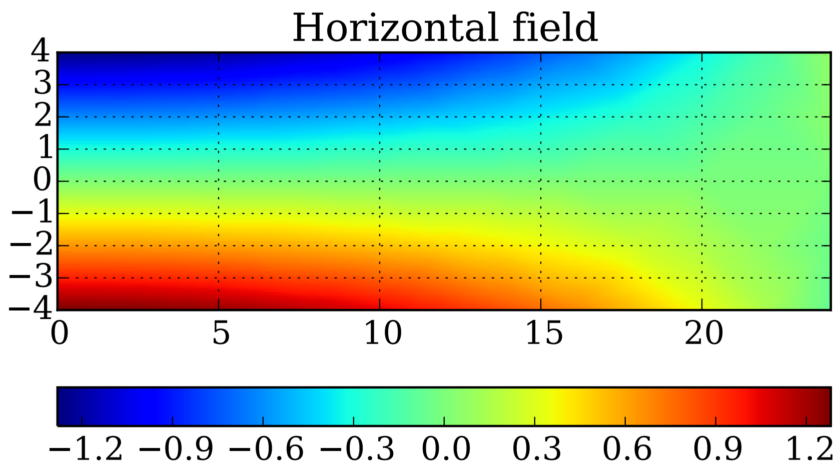
$$v = \frac{\nu P}{2EI}xy^2 + \frac{P}{6EI}x^3 - \frac{Pl^2}{2EI}x + \frac{Pl^3}{3EI}$$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \equiv -\frac{P}{EI}xy$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} \equiv \frac{\nu P}{EI}xy$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \equiv \frac{P}{2IG}\left(y^2 - c^2\right)$$

while the particular solution for parameters $E = 1000.0$, $\nu = 0.30$, $l = 24$ and $2c = 8$ is shwon below:



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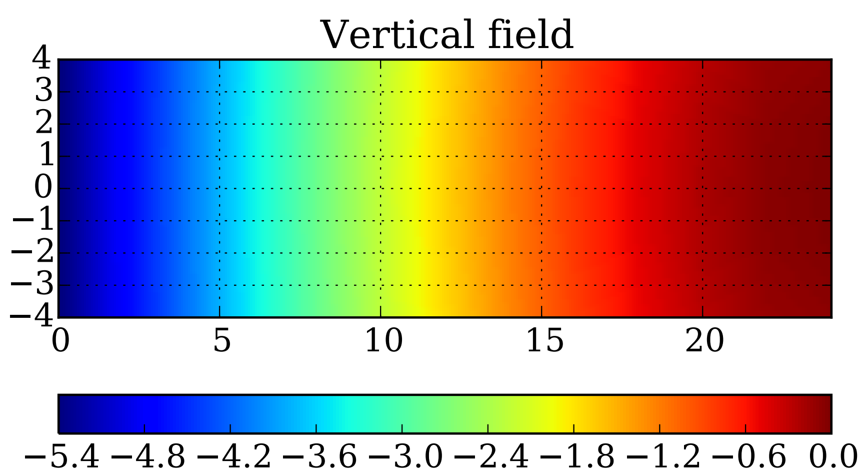

Figure 2: Horizontal field.

The beam was also analyzed with the FEM using solid lineal square elements under plane stress conditions. We conducted 5 different analysis with the following set of characteristic element size $h = [6.0, 3.0, 1.5, 1.0, 0.5]$. The meshes are shown in fig. 3

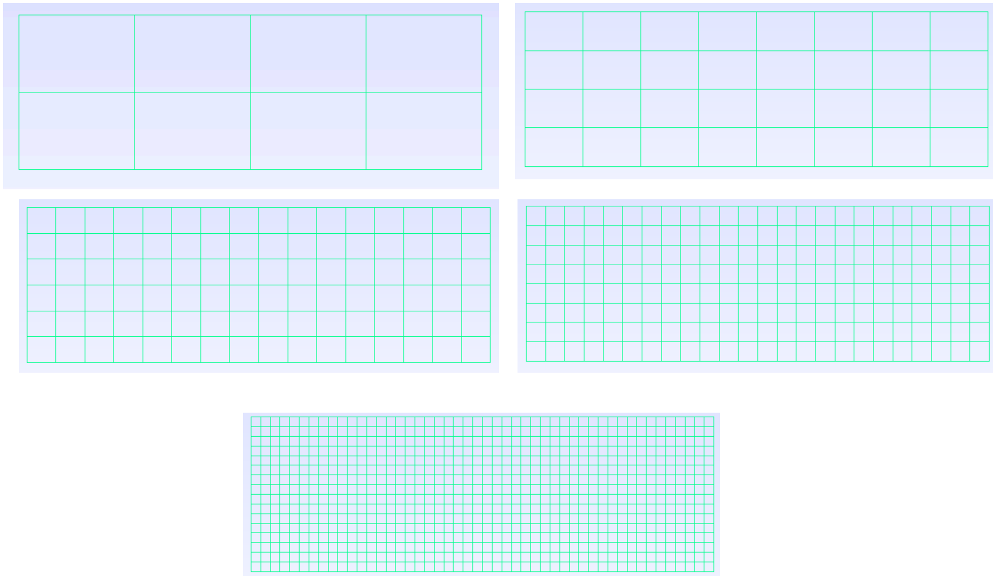

Figure 3: Finite element meshes.

The horizontal and vertical displacement contours for the FE-solution and the analytic solution are compared in fig. 4 for the case $h = 1.0$. Notice that in the finite elment models the load is applied as nodal point loads of the same magnitud distributed along all the nodes at $x = 0$. This implies a uniform load distribution instead of the parabolic load consitent with the shear stress in the analytic solution.

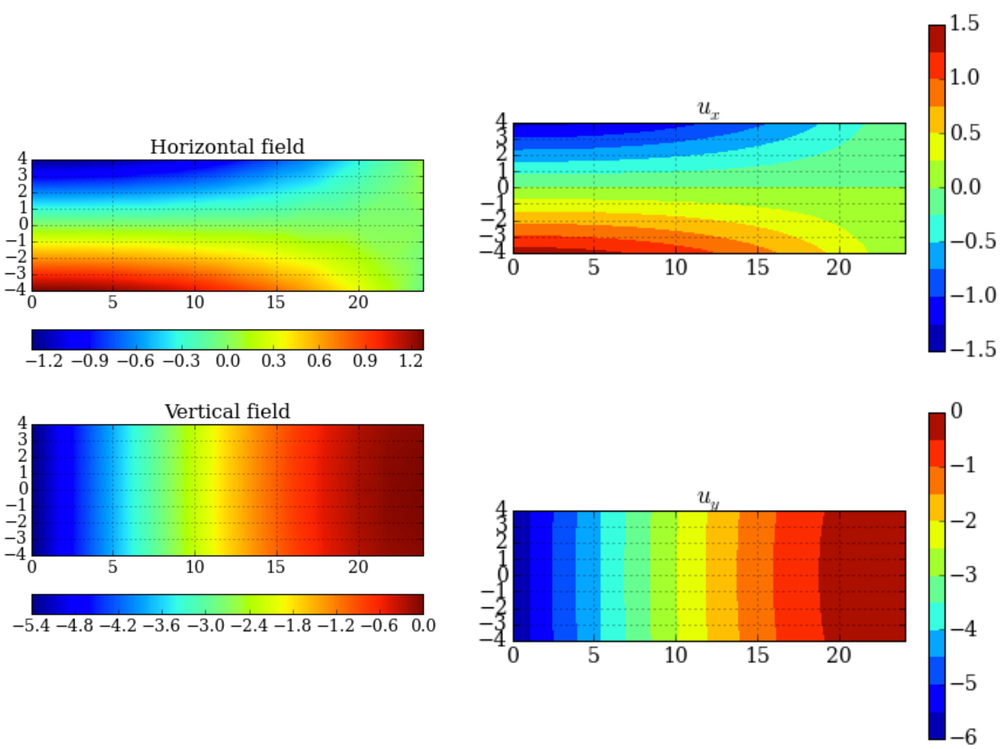

Figure 4: Analytic (left) vs Numerical (right) solution.

$$\prod_{FE} = -\frac{1}{2}U^T K U$$

$$\prod_{Exa} = \frac{\prod_{n-1}^2 - \prod_n \prod_{n-2}}{2\prod_{n-1} - \prod_n - \prod_{n-2}}$$

Using $n = 5$ we have $\prod_{Exa} = -154.09$.

$$\|\vec{u}_{Exa} - \vec{u}_{FE}\| \equiv (\prod_{Exa} - \prod_{FE})^{1/2}$$

h	$\prod_{FE}$	$\|\vec{u}_{Exa} - \vec{u}_{FE}\|$	$\frac{\|\vec{u}_{Exa} - \vec{u}_{FE}\|}{\|\vec{u}_{Exa}\|}$
6.0	-118.414	5.973	0.481
3.0	-139.273	3.849	0.310
1.5	-145.866	2.868	0.231
1.0	-147.501	2.567	0.207
0.5	-148.811	2.298	0.185

Table 1: Convergence of anlysis results

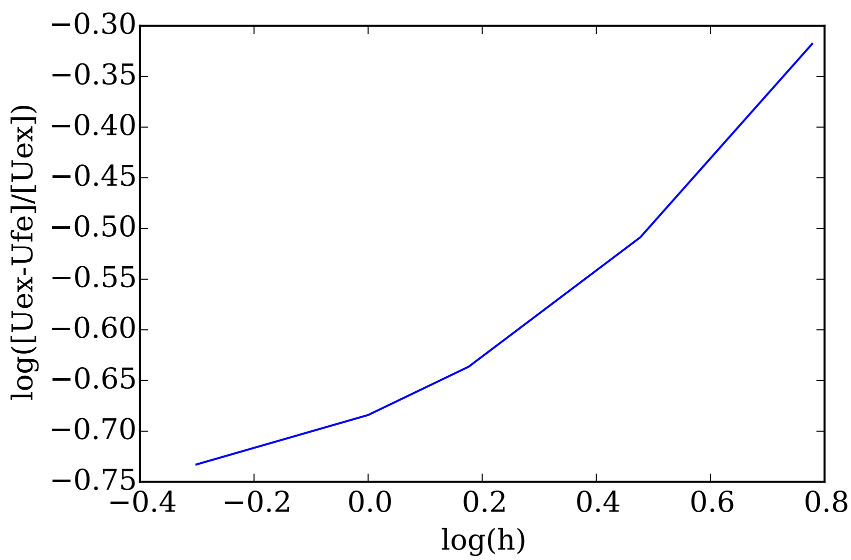

Figure 5: Energy norm of the error

$$\log\left(\|\vec{u}_{Exa} - \vec{u}_{FE}\|\right) = \log c + k \log h$$

References

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