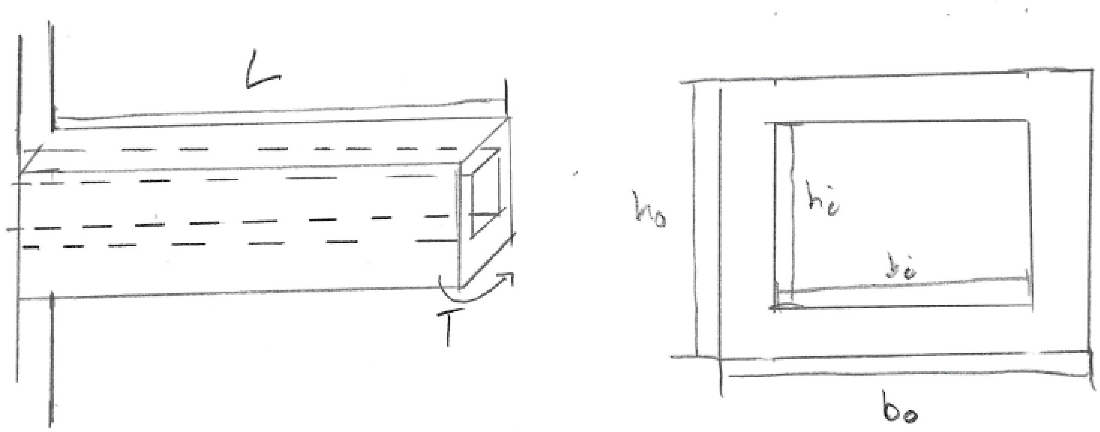
ENGN 0310

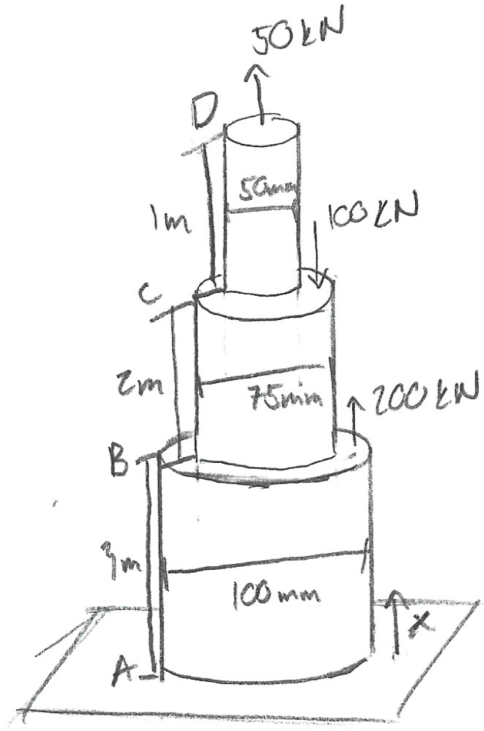
Midterm Exam

Instructions

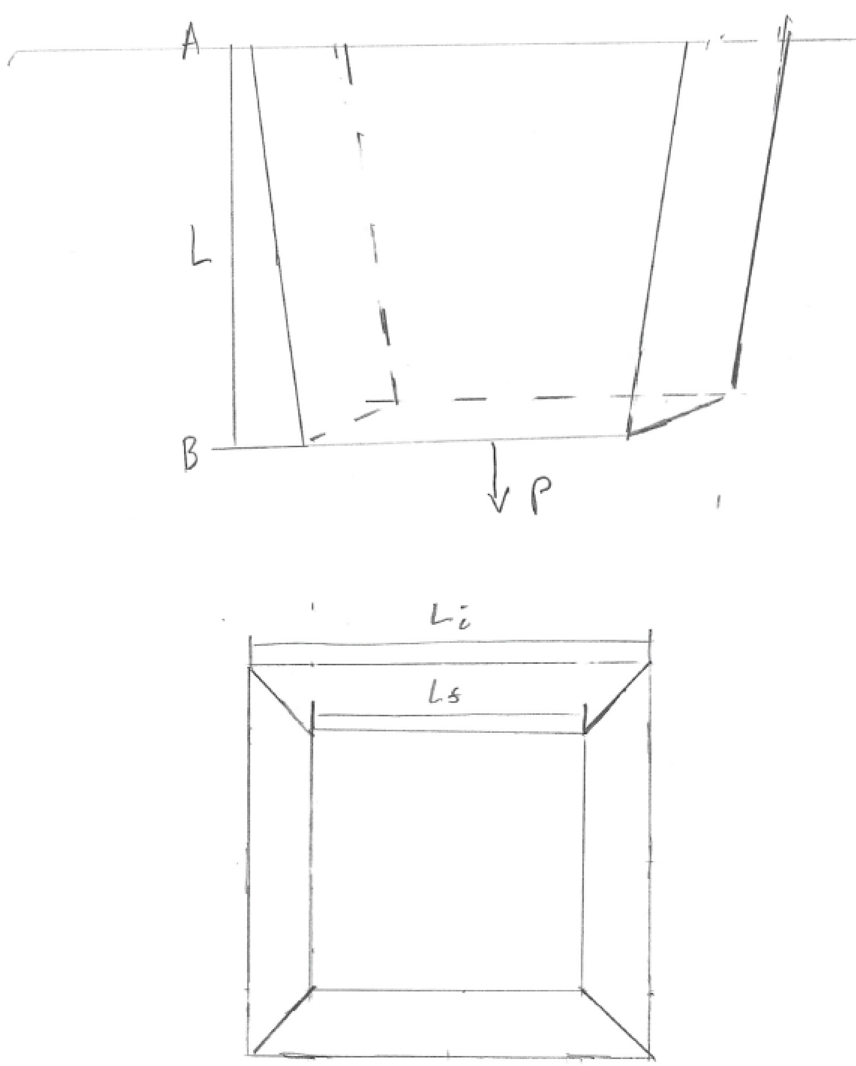
- Your solutions must be submitted in the form of a single file to Canvas (pdf preferred), with the addition of a Mathematica file if you so choose.
- This exam is open book and open notes, but consulting other sources is not allowed. Your exam should be entirely your own work and consulting other students is not allowed.
- Late submission will be penalized, but if problems arise during submission to Canvas you can email your solutions to Andrew_Bagnoli@brown.edu within the time limit of your exam for full credit.
- Once you begin your exam you will have three hours to submit your work for full credit. Note that all exams are due by 11:59pm on Sunday, October 24^{th} and cannot be submitted after that date, so if you start your exam after 8:59pm you will not have the full time to complete your exam.
- In the interests of fairness to those taking the exam at times when the Professor and/or TAs are not available questions cannot be answered during the exam, whether in person or by email.



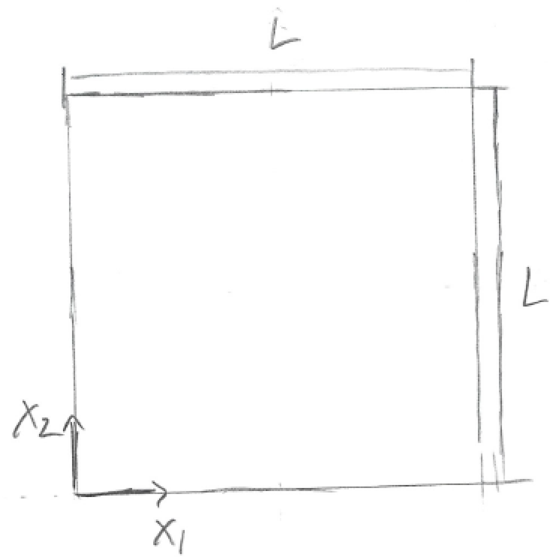
1. Consider bar in the figure above, with a rectangular cross section with a rectangular hole, where the outer height is h_o, the outer base is b_o, the height of the hole is h_i, and the base of the hole is b_i. The shear modulus of the bar is G. If a torque T is applied at the end of the bar then what is the twist at the end of the bar? Note: The Polar Moment of Inertia of a rectangle with base b and height h is given by $J = \frac{bh}{12}\left(b^2 + h^2\right)$, this does not have to be derived.



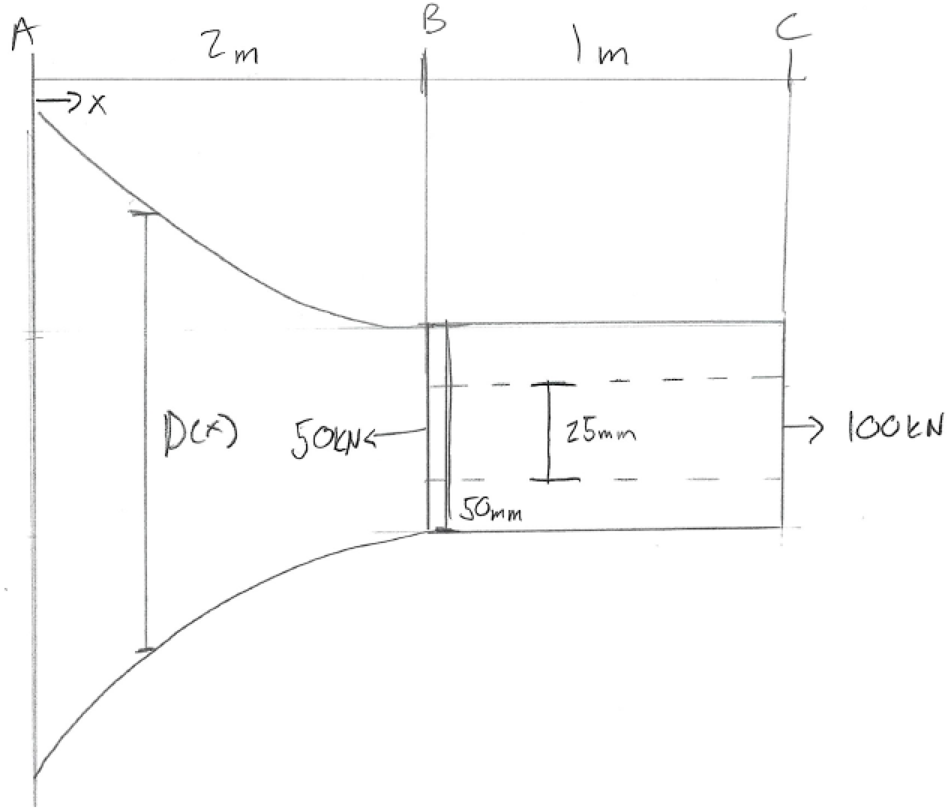
2. Consider bar in the figure above, where section AB has a length of $3m$, a diameter of $100mm$, and is made of a material with a Young's modulus of $72GPa$. Section BC has a length of $2m$, a diameter of $75mm$, and is made of a material with a Young's modulus of $117GPa$. Section CD has a length of $1m$, a diameter of $50mm$, and is made of a material with a Young's modulus of $200GPa$. Plot the displacement as a function of x, as defined in the figure above as the distance from point A.



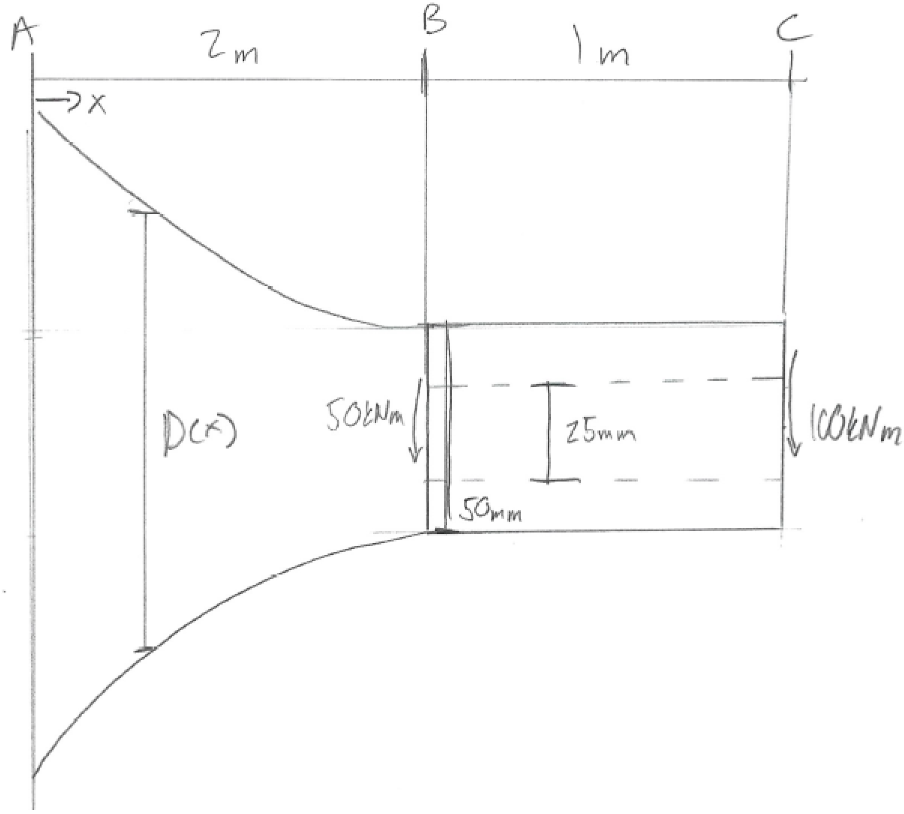
3. Consider the bar in the figure above, with a linearly tapered square cross section with initial side length L_i and final side length L_f, that is made of a material with Young's Modulus E and density ρ. If a force P is applied at point B, then what is the net displacement at B? Assume that the force of gravity cannot be ignored.



4. Consider the square in the figure above, with side length L and the bottom left corner defined as the origin. Draw the square after it undergoes the displacement field $\mathbf{u}(\mathbf{X}) = (\alpha X_1, \beta X_2)$, with locations of the vertices and side lengths clearly labeled.



5. Consider the bar in the figure above, where $D(x) = (50mm)e^{2-x}$ is the diameter of section AB. Section of the bar AB, which has a length of $2m$, is made of a material with $E_{AB} = 120GPa$ and $\alpha_{AB} = 8.4 \times 10^{-6} \frac{1}{°C}$. Section of the bar BC, which has a length of $1m$, is made of a material with $E_{BC} = 200GPa$ and $\alpha_{BC} = 10 \times 10^{-6} \frac{1}{°C}$. The force on point B is $50kN$ and the force on C is $100kN$. The outer diameter of the bar in the section BC is $50mm$ and the hole has a diameter of $25mm$. What is the net displacement at point C when the temperature is $80°C$? Assume that the bar is initially at $20°C$.



6. Consider the same bar from problem five, but now instead of forces, torques are applied at B and C, with magnitudes of $50kNm$ and $100kNm$ respectively. The shear modulus of section AB is $44GPa$ and the shear modulus of BC is $77GPa$. What is the net twist at point C? There is no change in temperature in this problem.