

Drill 9

We consider the binomial distribution for this drill. In the probability distribution summary (see [Distributions.pdf](#)), there are four formulas under the note of **Binomial**(n, p). Among them, it would be most difficult to prove the second one (the relation between the binomial and beta distributions). Others may not be difficult. Prove the relation:

$$F_{\text{Bin}(n,p)}(x) = F_{\text{Beta}(n-x, x+1)}(1-p).$$

You may check the R function below.

```
1 set.seed(1) # Change this seed number.
2 n = 1 + round(50*runif(1))
3 p = runif(1)
4 x = sample(0:n, size=1)
5 # Compare the following two:
6 pbinom(x, size=n, prob=p)
7 pbeta(1-p, n-x, x+1)
```

Hints:

1. How to generate a random sample from the binomial distribution using the uniform distribution between zero and one?
2. $F_{\text{Bin}(n,p)}(x)$ is related to the above sample.
3. $F_{\text{Bin}(n,p)}(x)$ is related to the order statistics of the above sample.
4. The pmf $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$ is related to the pdf of the order statistics of the above sample.