

Drill 9

We consider the binomial distribution for this drill. In the probability distribution summary (see `Distributions.pdf`), there are four formulas under the note of **Binomial**(n, p). Among the four, it would be most difficult to prove the second one (the relation between the binomial and beta distributions). Others may not be difficult. Prove the relation below:

$$F_{\text{Bin}(n,p)}(x) = F_{\text{Beta}(n-\lfloor x \rfloor, \lfloor x \rfloor + 1)}(1 - p).$$

You may check the R function below.

```
1 set.seed(0) # Change this seed number.
2 n = 1 + round(100*runif(1))
3 p = runif(1)
4 x = min(50*runif(1), n)
5 c(x, n, p)
6 # Compare the following two:
7 c(pbinom(x, size=n, prob=p), pbeta(1-p, n-floor(x), floor(x)+1) )
```

Hints:

1. How to generate a random sample from the binomial distribution using the uniform distribution between zero and one?
2. $F_{\text{Bin}(n,p)}(x)$ is related to the above sample.
3. $F_{\text{Bin}(n,p)}(x)$ is related to the order statistics of the above sample.
4. The pmf $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$ is related to the pdf of the order statistics of the above sample.