Drill 7

We consider the simple linear regression model

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i,$$

where we assume that ϵ_i are independent and distributed with $N(0, \sigma^2)$.

1. We want to test $H_0 : \beta_1 = 0$ versus $H_1 : \beta_1 \neq 0$ using the F test statistic which is given by
$$F = \frac{\text{SSR}/(p-1)}{\text{SSE}/(n-p)} = \frac{\text{MSR}}{\text{MSE}} \sim F(p-1, n-p)$$
under $H_0 : \beta_1 = 0$. In this Drill, we have $p = 2$.

   (a) Obtain the simulated power function of the F test with the significance level $\alpha = 0.05$. Here, we assume $\sigma = 5$ and the values of X_i are given by $X_i = i$ where $i = 1, 2, \ldots, 10$.

   (b) Obtain the theoretical power function of the F test. (Hint: the non-central F distribution in `Note08.pdf` can be used).

   (c) Compare the theoretical and simulated power functions. (The results should be similar to the following plot).
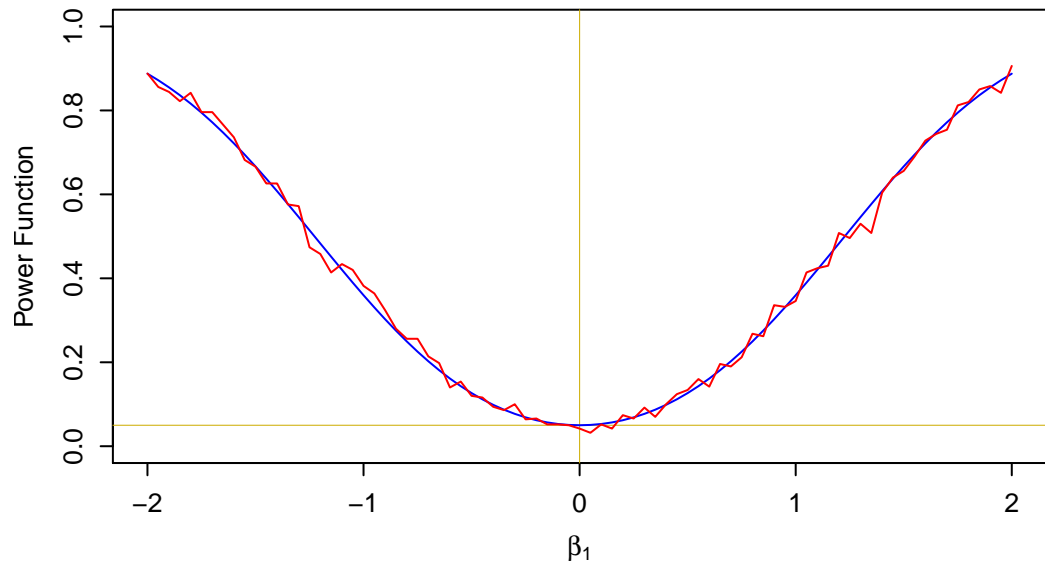
2. We want to test $H_0 : \beta_1 = 0$ versus $H_1 : \beta_1 \neq 0$ using the t test statistic which is given by

$$\frac{\hat{\beta}_1 - \beta_1}{\sqrt{\text{MSE}/S_{xx}}} \sim t(n-p)$$

under $H_0 : \beta_1 = 0$. Here, we have $p = 2$.

(a) Obtain the simulated power function of the t test with the significance level $\alpha = 0.05$. Here, we assume $\sigma = 5$ and the values of X_i are given by $X_i = i$ where $i = 1, 2, \ldots, 10$.

(b) Obtain the theoretical power function of the t test. (Hint: the non-central t distribution in Drill-5 can be helpful).

(c) Compare the theoretical and simulated power functions.

3. Plot ROC curves (theoretical and simulated curves).