Drill 9

We consider the binomial distribution for this drill. In the probability distribution summary (see `Distributions.pdf`), there are four formulas under the note of **Binomial** (n, p).

1. Prove the second one (the relation between the binomial and beta distributions). That is, prove the relation below:

$$\boxed{F_{\text{Bin}(n,p)}(x) = F_{\text{Beta}(n-\lfloor x\rfloor, \lfloor x\rfloor+1)}(1-p)}$$

You may check the R function below.

```
set.seed(0)   # Change this seed number.
n = 1 + round(100*runif(1))
p = runif(1)
x = min(50*runif(1), n)
c(x, n, p)
# Compare the following two:
c(pbinom(x, size=n, prob=p), pbeta(1-p, n-floor(x), floor(x)+1) )
```

Hints:

(a) How to generate a random sample from the binomial distribution using the uniform distribution between zero and one?

(b) $F_{\text{Bin}(n,p)}(x)$ is related to the above sample.

(c) $F_{\text{Bin}(n,p)}(x)$ is related to the order statistics of the above sample.

(d) The pmf $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$ is related to the pdf of the order statistics of the above sample.

2. Prove the third one (the relation between the binomial and negative binomial distributions):

$$\boxed{F_{\mathrm{Bin}(n,p)}(r-1) = 1 - F_{\mathrm{NB}(r,p)}(n-r),}$$

where $F_{\mathrm{NB}(r,p)}$ is the CDF of a negative binomial random variable with pmf $f(x) = \binom{r+x-1}{x} p^r (1-p)^x$ for $x = 0, 1, \ldots$.