

Drill 9

We consider the binomial distribution for this drill. In the probability distribution summary (see `Distributions.pdf`), there are four formulas under the note of **Binomial** (n, p) .

1. Prove the second one (the relation between the binomial and beta distributions).

That is, prove the relation below:

$$F_{\text{Bin}(n,p)}(x) = F_{\text{Beta}(n-\lfloor x \rfloor, \lfloor x \rfloor + 1)}(1 - p)$$

Hints:

- (a) How to generate a random sample from the binomial distribution using the uniform distribution between zero and one?
 - (b) $F_{\text{Bin}(n,p)}(x)$ is related to the above sample.
 - (c) $F_{\text{Bin}(n,p)}(x)$ is related to the order statistics of the above sample.
 - (d) The pmf $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$ is related to the pdf of the order statistics of the above sample.
2. Prove the third one (the relation between the binomial and negative binomial distributions):

$$F_{\text{Bin}(n,p)}(r-1) = 1 - F_{\text{NB}(r,p)}(n-r),$$

where $F_{\text{NB}(r,p)}$ is the CDF of a negative binomial random variable with pmf $f(x) = \binom{r+x-1}{x} p^r (1-p)^x$ for $x = 0, 1, \dots$