MLE of $\mathcal{N}(\mu, \sigma^2)$

① Show that the MLEs of $\mathcal{N}(\mu, \sigma^2)$ are

$$\hat{\mu} = \bar{X}$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

② Show that $\hat{\sigma}^2$ is biased. ($\hat{\mu}$ is unbiased)

That is, $E[\hat{\mu}] = \mu$

But $E[\hat{\sigma}^2] \neq \sigma^2$

$$s^2 = \frac{1}{n-1} \sum (X_i - \bar{X})^2 \text{ is unbiased}$$

that is $E[s^2] = \sigma^2$.

③ s^2 is unbiased for σ^2.

But s is biased for σ.

Thus make s unbiased ✓

Prove the above theoretically.

Show the above using Monte Carlo simulations.

MLE of Double exponential (Laplace)

Show that MLE of μ is given by the median