Supply Chain Design for Facility Location based on Minimum Distance Approach

Chanseok PARK

Applied Statistics Laboratory
Department of Industrial Engineering
Pusan National University

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Overview

1. Motivation from traditional method based on center of gravity
2. Euclidean distance
3. L_1 distance
4. Proposed new distance — location-driven approach
5. Future Work

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Center of Gravity

Center of Gravity

Assume that there are n customers and each of them is located at the ith location given by the coordinate (x_i, y_i).

$$u = \frac{x_1 + x_2 + \cdots + x_n}{n} \quad \text{and} \quad v = \frac{y_1 + y_2 + \cdots + y_n}{n} \tag{1}$$

More general version is considering weights for each customer.
For example, give more weights for big buyers. Thus, we can consider $w_i(x_i, y_i)$ and then we have

$$u = \frac{w_1 x_1 + w_2 x_2 + \cdots + w_n x_n}{W} \text{ and } v = \frac{w_1 y_1 + w_2 y_2 + \cdots + w_n y_n}{W},$$

where $W = \sum_{i=1}^{n} w_i$. We can check that this includes (1) as a special case with $w_i = 1/n$.

Center of Gravity

We can rewrite

$$u = \frac{w_1}{W}x_1 + \frac{w_2}{W}x_2 + \cdots + \frac{w_n}{W}x_n \text{ and } v = \frac{w_1}{W}y_1 + \frac{w_2}{W}y_2 + \cdots + \frac{w_n}{W}y_n$$

and $\sum_{i=1}^{n} w_i/W = 1$.
Thus, without loss of generality, we use

$$u = w_1x_1 + w_2x_2 + \cdots + w_nx_n \text{ and } v = w_1y_1 + w_2y_2 + \cdots + w_ny_n$$

where $\sum_{i=1}^{n} w_i = 1$.
The above center of gravity method is widely used for determining the facility location.

- Ballou, R. H. (1973). Potential error in the **center of gravity** approach to facility location, Transportation Journal.
- Esnaf S. et al. (2008). Fuzzy C-Means and **Center of Gravity** Combined Model for A Capacitated Planar Multiple Facility Location Problem
- Many other papers ....

Center of Gravity

- We can think that the center of gravity is the optimal location which minimizes the total or overall distance from all the customers.
  But, it is actually **NOT**.
- Recently, it is proved that this center of gravity **CANNOT** optimize any of proper distances.
- Some researchers think that (u, v) minimizes the following

$$D_{\mathrm{CG}}(u, v) = \sum_{i=1}^{n} w_i \Big\{ (x_i - u)^2 + (y_i - v)^2 \Big\}$$

  The (u, v) optimizes $D_{\mathrm{CG}}(u, v)$.
  But, $D_{\mathrm{CG}}(u, v)$ is **NOT** a proper distance, which is a kind of a common mistake.

Euclidean distance

- The most widely used distance is Euclidean distance. With weights w_i, it is given by

$$D_{\mathrm{E}}(u,v) = \sum_{i=1}^{n} w_i \sqrt{(x_i - u)^2 + (y_i - v)^2}.$$

  One can easily check that $(u,v) = (\sum_{i=1}^{n} w_i x_i, \sum_{i=1}^{n} w_i y_i)$ does **NOT** optimize $D_{\mathrm{E}}(u,v)$.
- The optimal location (u,v) based on $D_{\mathrm{E}}(u,v)$ is NOT in closed form.
- Why the center-of-gravity method is widely used in SCM?
  In my humble opinion, many are fooled by its name CENTER.
- NOTE:
  - In statistics, the sample mean is the same as the center of gravity.
  - In physics, the balancing position is the center of gravity.

  Thus, many think that the optimal location based on $D_{\mathrm{CG}}(u,v)$ optimized the Euclidean distance $D_{\mathrm{E}}(u,v)$. But, it is NOT true.

Euclidean distance

Is Euclidean distance appropriate in supply chain design?
Answer: Yes and No.

- **Yes** in an open space because this guarantees the SHORTEST distance between any two points in an open space.
- **No** with constraints (for example, taxicab geometry like urban area). For an urban area, the L_1 distance is more appropriate, which has many nick names such as rectilinear, snake, taxicab, Manhattan, city-block distances.
- This L_1 distance has a long history in mathematics and statistics. In mathematics, it is known as Minkowski distance. In statistics, the optimal location based on L_1 is the same as the median.

L_1 distance

- A conventional L_1 is given by

$$D_{L_1}(u,v) = \sum_{i=1}^{n} \Big\{ |x_i - u| + |y_i - v| \Big\}$$

- The problem with the above is that weights are not considered and the optimal location with L_1 is not unique with the even number of customers.

New L_1 distance with weights w_i

- One can think that w_i are added as below

$$D_{L_1}(u,v) = \sum_{i=1}^{n} w_i \Big\{ |x_i - u| + |y_i - v| \Big\}.$$

- We proposed a new method based on the statistical quantile function for uniqueness.

Proposed new distance — location-driven approach

A national question is:

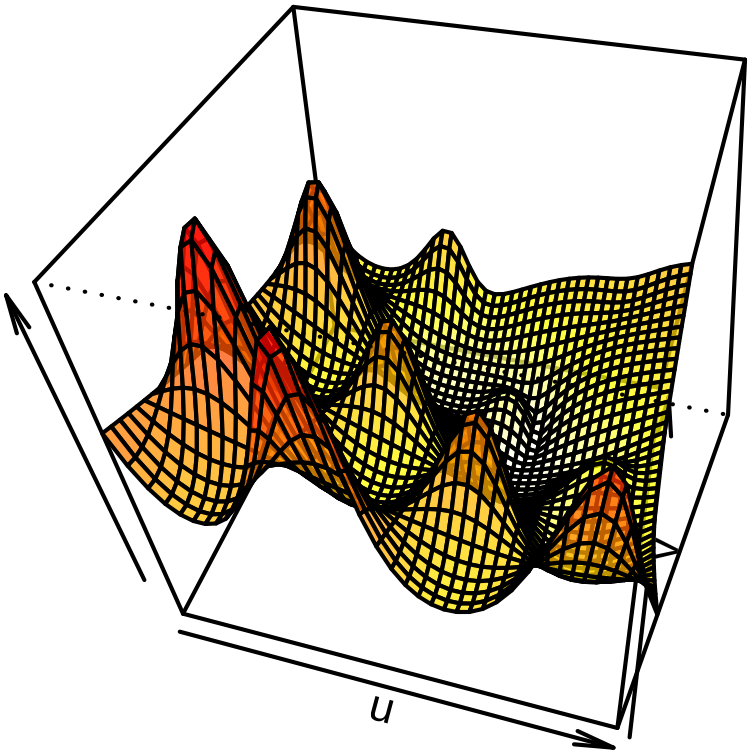
- Are the weights w_i are constant?
  No, in practice, they depends on the location.
- We proposed a new method.
  We coined a new name: **location-driven approach**.
  The weights are function of (u, v). Thus, it is given by $w_i(u, v)$.
- Another extension is to use L_p distance with this location-driven weights, $w_i(u, v)$. Then the object function is given by

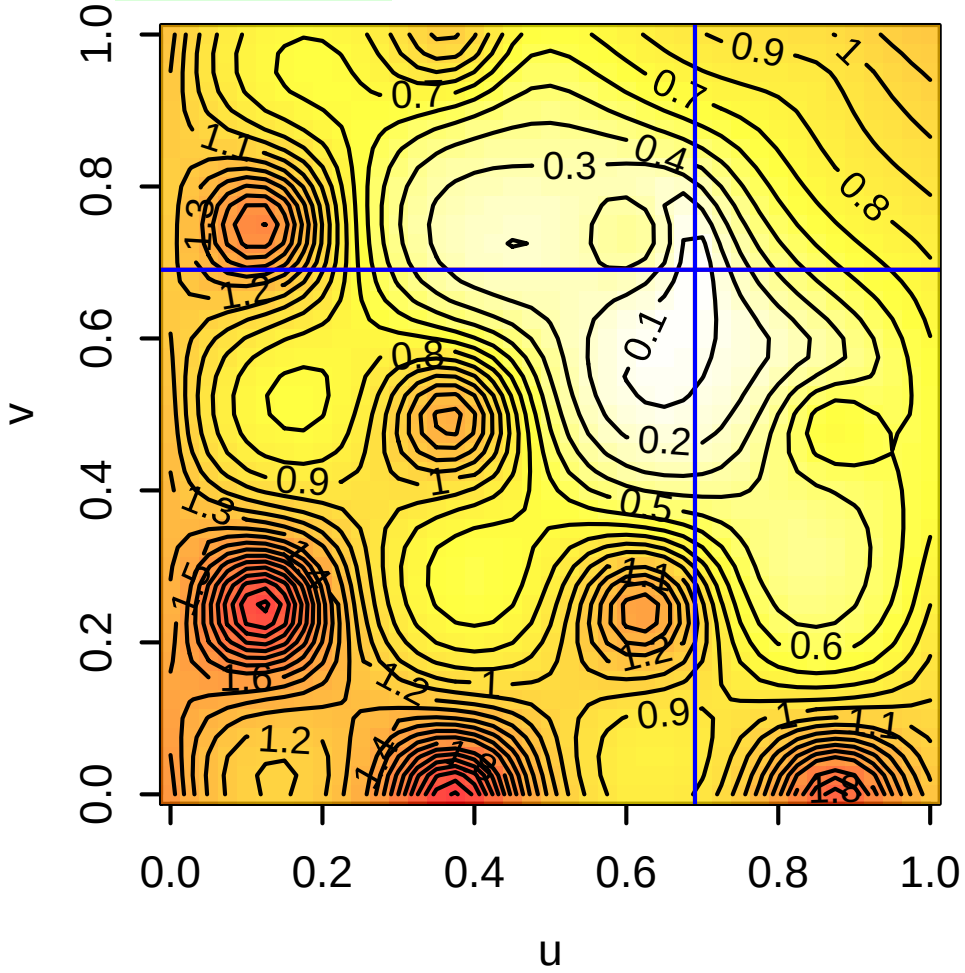
$$D_{L_p}(u, v) = \sum_{i=1}^{n} w_i(u, v) \Big\{ |x_i - u|^p + |y_i - v|^p \Big\}^{1/p}.$$

- Then, is the optimization of $D_{L_p}(u, v)$ stable?
  For L_2, the fixed point method works well.
- Some pilot studies are in the next pages.

Pilot studies with proposed new distance

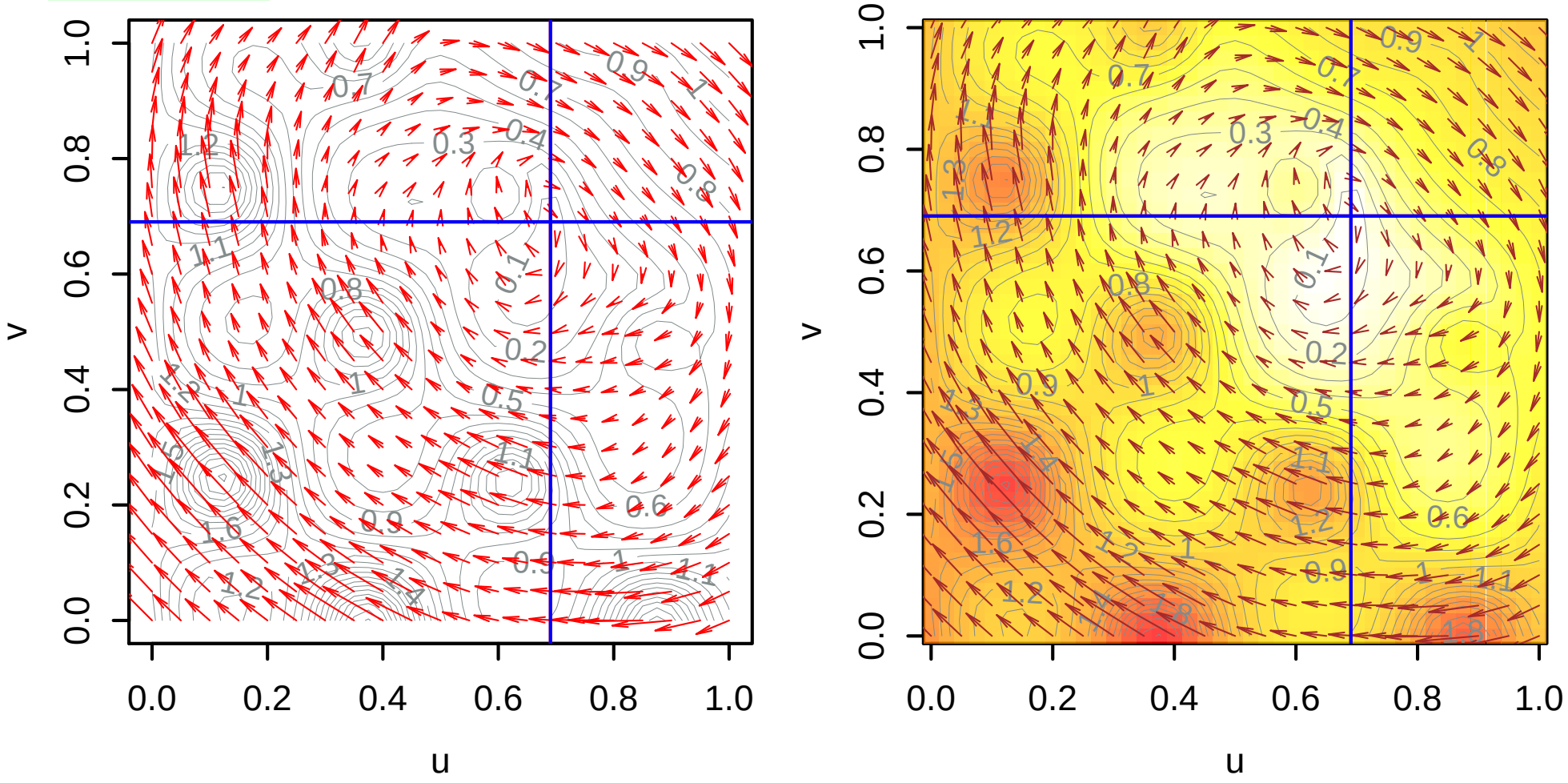
We considered a very complex function.
Actually the optimal location is around $(0.7, 0.7)$.





Pilot studies with proposed new distance

Quiver plot with the same function. Again, the optimal location is around $(0.7, 0.7)$.

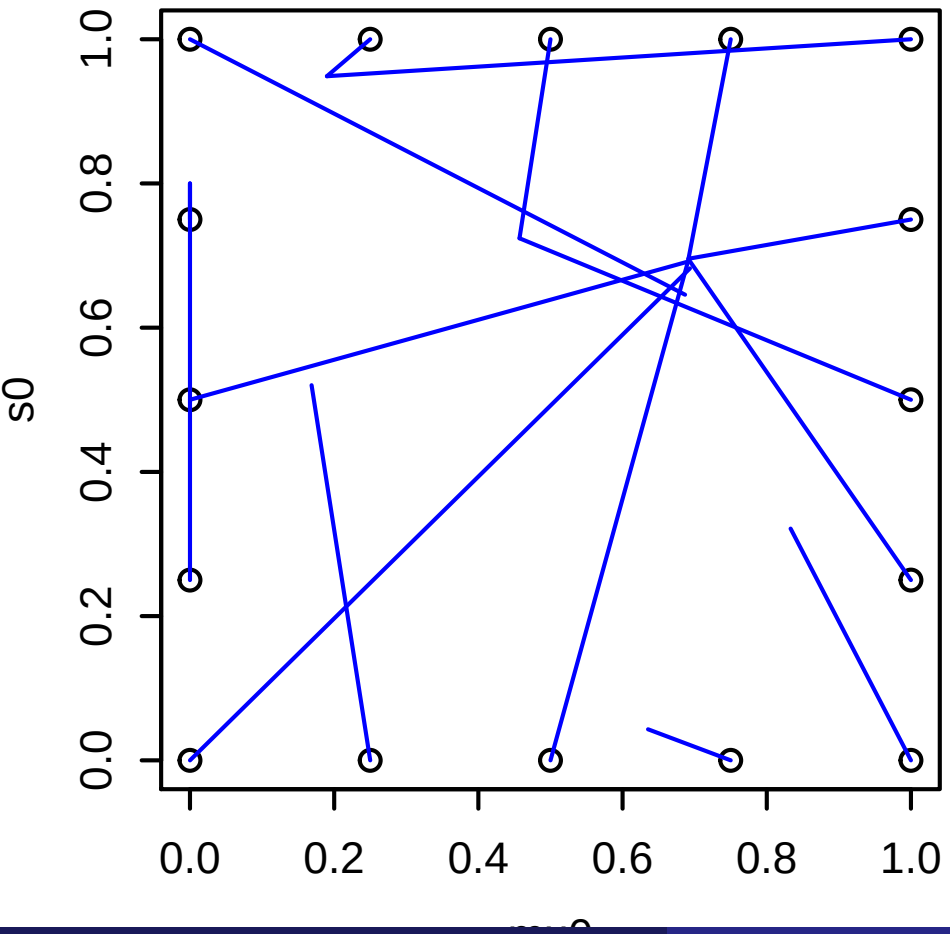


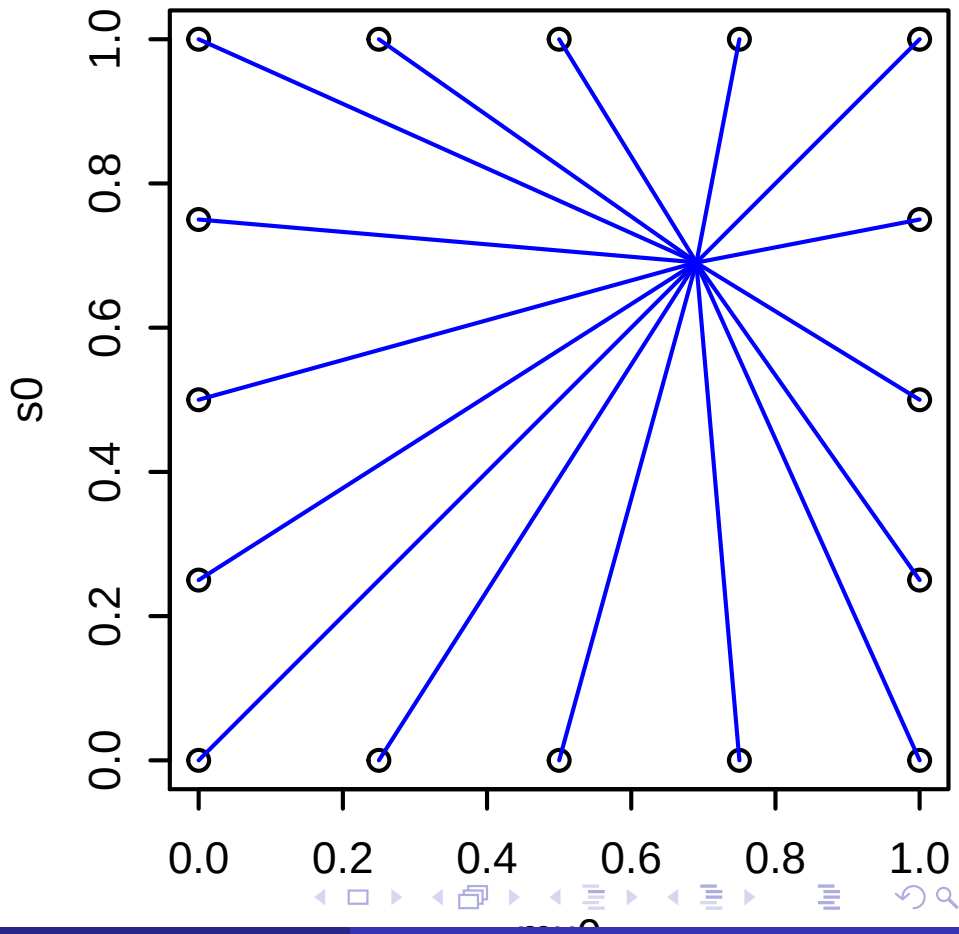
Pilot studies with proposed new distance

Solution paths with different initial conditions.

The left one is obtained using the Newton-Raphson method.

The right one is obtained using the proposed method based on the fixed point method, which is **very successful**.





Future Work

- Entropy is widely used in statistics. This provides optimal estimate which is widely known as the maximum likelihood estimate. Thus, we are planning to extend this supply chain design with the use of entropy.
- Location-driven approach works very with the Euclidean distance. But, from pilot study, the location-driven approach with L_1 distance does not provide a stable optimization process. Genetic algorithm works but, not so satisfactory.
- For stable optimization process, we are planning to use the particle swam optimization (PSO) and differential evolution (EV) and others.

A conventional L_1 distance provides a robust estimate with 50% breakdown point.

- Is the L_1 distance with weight w_i still robust? $\Rightarrow$ Yes.
- Then, does it has a 50% breakdown point. $\Rightarrow$ Yes and No because it depends on the weights.
- Then, can we have a formula of the breakdown point as a function of the weights? Maybe possible, but it seems related to discrete convex problem.