Applications (사례 연구)

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Overview

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1. Robust statistics와 응용 사례: Basic applications

Basic applications (estimating μ and σ)

Consider observations from $X_i \sim N(\mu, \sigma^2)$. We need to estimate μ and σ^2.

- MLE (maximum likelihood estimator)

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} X_i \quad \text{and} \quad \hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \hat{\mu})^2$$

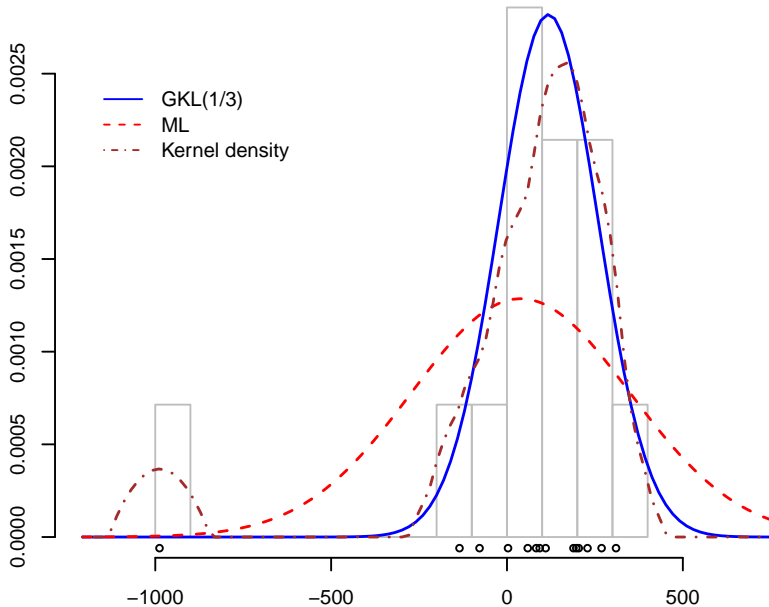- BUE (best unbiased estimator) or UMVUE

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \quad \text{and} \quad S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

- MDE (minimum distance estimator): KL, GKL, etc.
  MLE is a special case of GKL (Basu et al., 2011) which can have robustness. MDE is asymptotically fully efficient, but its calculation is quite complex. Thus, HL and Shamos are recommended (Talk-2).

1. Robust statistics와 응용 사례: Robust *t*-test

Darwin (1876) collected the data: the growth of pairs of corn (especially Zea May) seedings, one produced by self-fertilization and the other produced by cross-fertilization. For the data set, see Friendly et al. (2018).

Cross	23.500	12.000	21	22	19.125	21.500	22.125	20.375
	18.25	21.625	23.25	21	22.125	23.0	12	
Self	17.375	20.375	20	20	18.375	18.625	18.625	15.250
	16.50	18.000	16.25	18	12.750	15.5	**18**	
Difference	6.125	−8.375	1.000	2.000	0.750	2.875	3.500	5.125
	1.750	3.625	7.000	3.000	9.375	7.500	−6.000	

We can test $H_0 : \mu_x = \mu_y$ and $H_1 : \mu_x \neq \mu_y$, equivalently, $H_0 : \mu_d = 0$ and $H_1 : \mu_d \neq 0$, where $\mu_d = \mu_x - \mu_y$. A typical paired sample *t*-test with

$$T = \frac{\bar{D} - 0}{S_D / \sqrt{n}},$$

where $\bar{D}$ and S_D are the sample mean and standard deviation, which this becomes a one-sample *t*-test.

1. Robust statistics와 응용 사례: Robust t-test

Theorem 1 (Park, 2018a)

Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2. Then we have

$$T_A = \sqrt{\frac{2n}{\pi}} \Phi^{-1}\left(\frac{3}{4}\right) \frac{\displaystyle\operatorname*{median}_{1 \le i \le n} X_i - \mu}{\displaystyle\operatorname*{median}_{1 \le i \le n} \left|X_i - \operatorname*{median}_{1 \le i \le n} X_i\right|} \xrightarrow{d} N(0, 1). \tag{1}$$

Theorem 2 (Jeong et al., 2018)

Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2. Then we have

$$T_B = \sqrt{\frac{3n}{2\pi}} \Phi^{-1}\left(\frac{3}{4}\right) \frac{\displaystyle\operatorname*{median}_{i \le j} \left(X_i + X_j\right) - 2\mu}{\displaystyle\operatorname*{median}_{i \le j} \left(|X_i - X_j|\right)} \xrightarrow{d} N(0, 1). \tag{2}$$

- The above Theorems work well with a large sample size because these are based on asymptotic standard normal distribution.
- Recently, Park and Wang (2018) developed the `rt.test` R package used the empirical distributions instead of the asymptotic standard normal distribution. Using the `rt.test`, we can carry out robustified *t*-test easily.
- Also, we can obtain the confidence intervals using the above robustified test statistics. By checking if zero is included inside each interval, we can test the hypothesis

$$H_0 : \mu_d = 0 \ \text{ and } \ H_1 : \mu_d \neq 0.$$
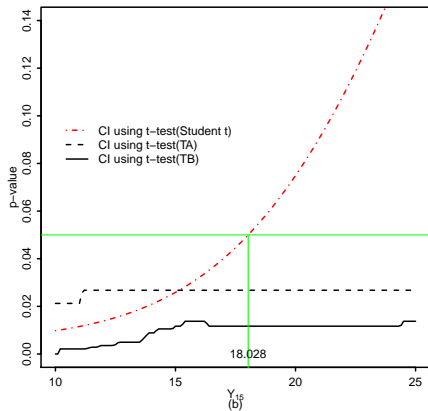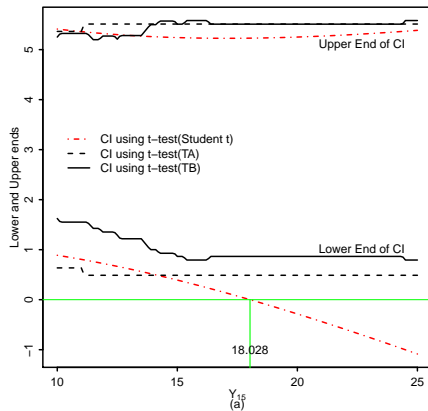
1. Robust statistics와 응용 사례: Robust *t*-test

This idea can be easily applied to control charting which is similar to a confidence interval.

- Phase I: use robustified control chart.
- Phase II: use conventional control chart.

This work is partially done (balanced case). See rcc function in rQCC.

```
> library("rQCC")
> help(rcc)
> tmp = c(
72, 84, 79, 49, 56, 87, 33, 42, 55, 73, 22, 60, 44, 80, 54, 74,
97, 26, 48, 58, 83, 89, 91, 62, 47, 66, 53, 58, 88, 50, 84, 69,
57, 47, 41, 46, 13, 10, 30, 32, 26, 39, 52, 48, 46, 27, 63, 34,
49, 62, 78, 87, 71, 63, 82, 55, 71, 58, 69, 70, 67, 69, 70, 94,
55, 63, 72, 49, 49, 51, 55, 76, 72, 80, 61, 59, 61, 74, 62, 57 )
> data2 = matrix(tmp, ncol=4, byrow=TRUE)
> rcc(data2, loc="HL2", scale="shamos")
     LCL      CL      UCL
36.99703 59.26250 81.52797
```

1. Robust statistics와 응용 사례: Robust Design/강건설계

Robust Design (Dual Response)

- The process mean response function.
$$\hat{M}(\mathbf{x}) = \hat{\beta}_0 + \sum_{i=1}^{k} \hat{\beta}_i x_i + \sum_{i=1}^{k} \hat{\beta}_{ii} x_i^2 + \sum_{i<j}^{k} \hat{\beta}_{ij} x_i x_j.$$

- The process variance response function.
$$\hat{V}(\mathbf{x}) = \hat{\eta}_0 + \sum_{i=1}^{k} \hat{\eta}_i x_i + \sum_{i=1}^{k} \hat{\eta}_{ii} x_i^2 + \sum_{i<j}^{k} \hat{\eta}_{ij} x_i x_j.$$

In the above, we need to estimate $M(\mathbf{x})$, $V(\mathbf{x})$, β and η

- The $M(\mathbf{x})$ and $V(\mathbf{x})$ can be estimated using robust estimators such as HL and Shamos estimators. For more details (HL and other estimators), see Talk-2 at ▸ 2018/Seminar
- The β and η can be estimated using the regression method.

1. Robust statistics와 응용 사례: Robust Design/강건설계

Refer to Park and Leeds (2016) and Talk-2 at ▸ 2018/Seminar

Method A: $\hat{M}(\mathbf{x})$ using the sample **mean** and
$\hat{V}(\mathbf{x})$ using the sample **variance**.
(BASELINE – without contamination!)

Method B: $\hat{M}(\mathbf{x})$: **median** and $\hat{V}(\mathbf{x})$: the squared **MAD**

Method C: $\hat{M}(\mathbf{x})$: **median** and $\hat{V}(\mathbf{x})$: the squared **IQR**

Method D: $\hat{M}(\mathbf{x})$: **HL (Hodges-Lehmann)** and
$\hat{V}(\mathbf{x})$: the squared **Shamos**

Method E: $\hat{M}(\mathbf{x})$: **median** and $\hat{V}(\mathbf{x})$: the squared **Shamos**

Method F: $\hat{M}(\mathbf{x})$: **HL** and $\hat{V}(\mathbf{x})$: the squared **MAD**.

Method G: $\hat{M}(\mathbf{x})$: **HL** and $\hat{V}(\mathbf{x})$: the squared **IQR**.

(a)

(b)
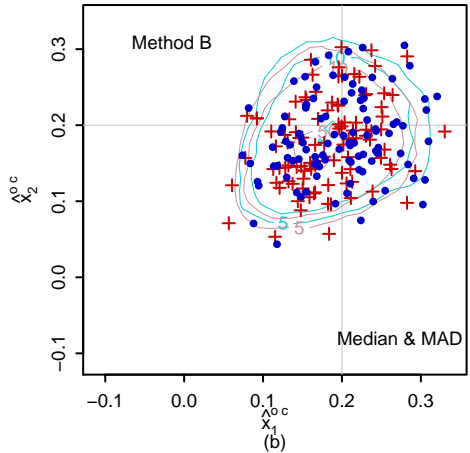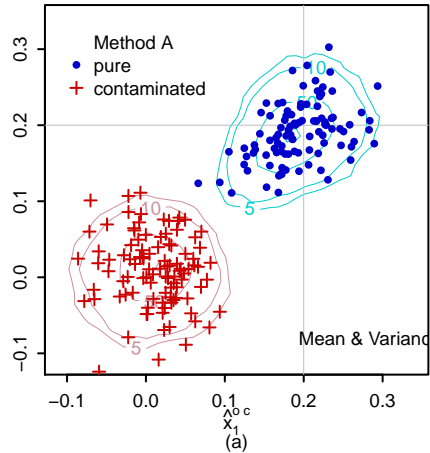
2. Missing · Incomplete: RD with unbalanced samples

Recall: Robust Design (Dual Response)

$$\text{Mean response} : \hat{M}(\mathbf{x}) = \hat{\beta}_0 + \sum_{i=1}^{k} \hat{\beta}_i x_i + \sum_{i=1}^{k} \hat{\beta}_{ii} x_i^2 + \sum_{i<j}^{k} \hat{\beta}_{ij} x_i x_j$$

$$\text{Variance response} : \hat{V}(\mathbf{x}) = \hat{\eta}_0 + \sum_{i=1}^{k} \hat{\eta}_i x_i + \sum_{i=1}^{k} \hat{\eta}_{ii} x_i^2 + \sum_{i<j}^{k} \hat{\eta}_{ij} x_i x_j$$

Unbalanced Data Set

i	x_{i1}	x_{i2}				Y_{ir_i}				$\overline{Y}_i$	S_i^2
1	-1	-1	84.3	57.0	56.5					65.93	253.06
2	0	-1	75.7	87.1	71.8	43.8	51.6			66.00	318.28
3	1	-1	65.9	47.9	63.3					59.03	94.65
4	-1	0	51.0	60.1	69.7	84.8	74.7			68.06	170.35
5	0	0	53.1	36.2	61.8	68.6	63.4	48.6	42.5	53.46	139.89
6	1	0	46.5	65.9	51.8	48.4	64.4			55.40	83.11
7	-1	1	65.7	79.8	79.1					74.87	63.14
8	0	1	54.4	63.8	56.2	48.0	64.5			57.38	47.54
9	1	1	50.7	68.3	62.9					60.63	81.29

Theorem 3

Let $Y_1, \ldots, Y_r$ be a random sample of size r from the probability density function $f(y)$ with a finite fourth moment and let $\mu = E(Y)$ and $\theta_k = E(Y - \mu)^k$, $k = 2, 3, 4$. Then we have

$$\mathrm{Var}(\overline{Y}) = \frac{1}{r}\mu \ \text{ and } \ \mathrm{Var}(S^2) = \frac{1}{r}(\theta_4 - \frac{r-3}{r-1}\theta_2^2).$$

Especially, if Y_i's have <u>independent and identical normal distribution</u>, then $\mathrm{Var}(S^2) = 2\sigma^4/(r-1)$.

Proof.

See Casella and Berger (2002). □

- $\mathrm{Var}(\overline{Y}) \propto 1/r$.
- $\mathrm{Var}(\overline{S^2}) \propto$?. Under the normality, $\mathrm{Var}(\overline{S^2}) \propto 1/(r-1)$.

2. Missing · Incomplete: RD with unbalanced samples

Under the normality assumption, we can solve this problem by using the weighted least squares (WLS) regression instead of the ordinary least squares (OLS) regression (Cho and Park, 2005).

OLS versus WLS

- OLS: $\mathbf{Y} \sim N(\mathbf{X}\beta, \sigma^2\mathbf{I})$ and $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$.
- WLS: $\mathbf{Y} \sim N(\mathbf{X}\beta, \sigma^2\mathbf{W}^{-1})$ and $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{W}\mathbf{X})^{-1}\mathbf{X}'\mathbf{W}\mathbf{Y}$.

Mean and Variance responses

- Mean response: $\mathbf{W} = \operatorname{diag}(r_1, r_2, \ldots, r_n)$
- Variance response: $\mathbf{W} = \operatorname{diag}(r_1 - 1, r_2 - 1, \ldots, r_n - 1)$

where n is the number of the design points.

What if normality is **not** satisfied

$\operatorname{Var}(\overline{S^2})$ is **not** proportional to $1/(r - 1) \Rightarrow$ **multiple imputation**.

Sorting machine

small   normal   large
y < a   a < y < b   y > b
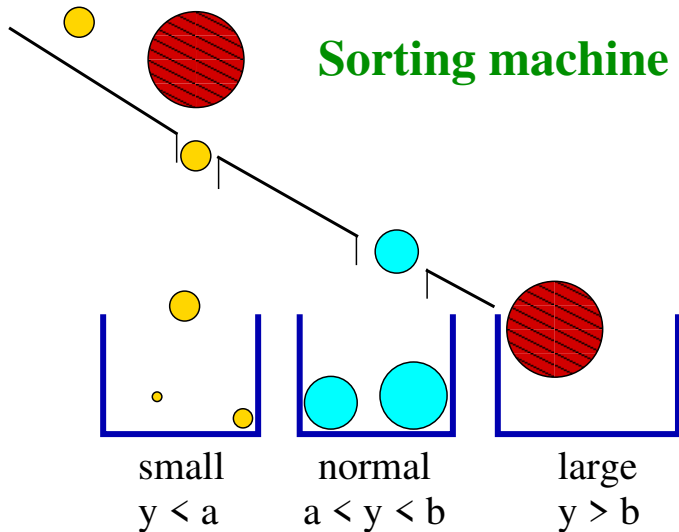
2. Missing · Incomplete: RD with incomplete/grouped data

Incomplete Data with grouping

			Full observations					Interval observations		
i	x_{i1}	x_{i2}	y_{i1}	y_{i2}	y_{i3}	y_{i4}	y_{i5}	$(-\infty, 45)$	$[45, 55]$	$(55, \infty)$
1	-1	-1	55.1	61.4	53.5	72.4	62.6	2	17	81
2	0	-1	65.5	59.2	60.4	57.3	65.0	2	58	40
3	1	-1	58.7	63.3	56.9	49.4	67.7	4	18	78
4	-1	0	61.3	52.1	54.3	47.3	57.9	6	46	48
5	0	0	54.5	47.8	49.8	44.4	51.8	8	84	8
6	1	0	45.0	54.1	62.0	59.0	55.8	6	47	47
7	-1	1	50.5	56.0	54.3	60.2	47.8	5	33	62
8	0	1	52.3	60.5	53.8	62.1	57.9	7	43	50
9	1	1	75.5	44.0	83.7	58.0	56.5	4	23	73

- **All** observations: Clearly the best
- Full observations only: (measurement cost is expensive).
- Interval observations only: (measurement cost is cheap or free).

Which of <u>full</u> or <u>interval</u> is better? (It depends on the sample size).

2. Missing · Incomplete: RD with incomplete/grouped data

Recall: Robust Design (Dual Response)

$$\text{Mean response} : \hat{M}(\mathbf{x}) = \hat{\beta}_0 + \sum_{i=1}^{k} \hat{\beta}_i x_i + \sum_{i=1}^{k} \hat{\beta}_{ii} x_i^2 + \sum_{i<j}^{k} \hat{\beta}_{ij} x_i x_j$$

$$\text{Variance response} : \hat{V}(\mathbf{x}) = \hat{\eta}_0 + \sum_{i=1}^{k} \hat{\eta}_i x_i + \sum_{i=1}^{k} \hat{\eta}_{ii} x_i^2 + \sum_{i<j}^{k} \hat{\eta}_{ij} x_i x_j$$

For the application of incomplete/grouped data to the robust design, see Lee and Park (2006). (Below, $n = 5$ full obs. and 100 interval obs.)

Empirical bias and MSE of the mean and variance under considered method

Estimate	Full obs. only		All observations		Interval obs. only	
	$\hat{\mu}_1$	$\hat{\sigma}_1^2$	$\hat{\mu}_2$	$\hat{\sigma}_2^2$	$\hat{\mu}_3$	$\hat{\sigma}_3^2$
Bias	0.0956	-0.9653	0.0059	2.8018	0.0009	4.5426
Variance	20.2007	4777.45	1.2125	762.0400	1.3103	989.9277
MSE	20.2098	4778.38	1.2125	769.8901	1.3103	1010.5630

2. Missing · Incomplete: RD with incomplete/grouped data

- Full observations are costly.
  Interval observations are cheap or free.
- Curvature of profile likelihood can be used for precision of estimators,
  which can be used for equivalent sample sizes with the same precision
  (Lee and Park, 2006).
- The parameter estimation with incomplete/grouped data is tricky.
  The EM, MCEM, QEM can be used (Park, 2018b).
  The MI can also be used but this is also tricky.
  In general, the model is known, the MLE (EM) is better than the MI.
- This idea can be applied to various applications with parameter
  estimates.

Illustrative Example: Tensile Testing Equipment
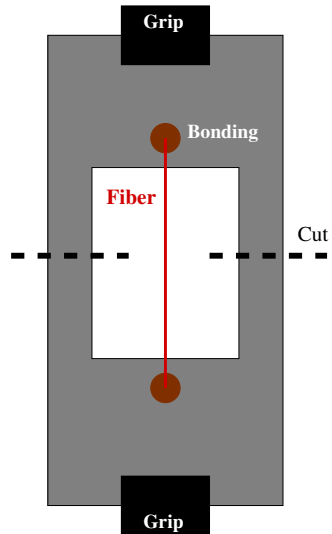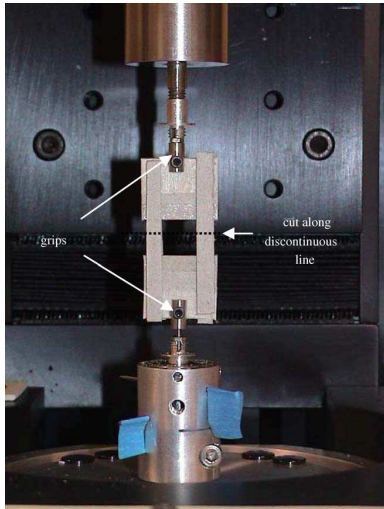
2. Missing · Incomplete: Competing risks

Most multi-modal strength analyses of materials have been studied based on the so-called weakest link theory which requires two assumptions (Beetz, 1982; Goda and Fukunaga, 1986):

Assumptions

A1 The material contains inherently many strength-limiting defects, and its strength depends on the weakest defect of all of them.

A2 There are no interactions among the defects.

What if the above assumptions are **not** satisfied

multiple imputation can be considered.

Strength data with three fracture causes (modes)

Strength	mode	Strength	mode	Str.	mode	Str.	mode
54	{3}	7	{1, 2, 3}	86	{2}	104	{1}
143	{2}	81	{3}	141	{1}	89	{3}
97	{3}	52	{3}	79	{3}	9	{3}
104	{3}	40	{3}	23	{3}	111	{1, 2, 3}
71	{1, 2}	82	{2}	8	{3}	150	0
98	{1}	3	{3}	17	{3}	79	{2}
24	{2}	130	{2}	41	{2}	94	{2}
138	{3}	5	{3}	43	{2, 3}	150	0
38	{3}	32	{2}	9	{3}	77	{2}
78	{3}	16	{3}	92	{2}	76	{3}
150	0	33	{3}	80	{2}	100	{2}
46	{3}	137	{1, 2}	92	{3}	108	{2}
109	{1}	71	{1}	60	{2}	88	{1}
7	{3}	11	{3}	150	0	150	0
42	{2}	6	{3}	43	{3}	124	{1, 2}

2. Missing · Incomplete: Competing risks

Specimens in tensile strength experiments are broken down due to

- several causes (**competing risks**)
- with the cause of fracture not properly identified (**missing**)
- along with censoring due to time and cost considerations on experiments.

For example, the fracture causes are due to:

- a surface defect (**mode 1**),
- an inner defect (**mode 2**), and
- an end effect at the clamp to hold the specimen (**mode 3**).
- The censored observations are denoted by **0**.
  (The observations were censored at **150** in the previous data set).

NOTE: In competing risks literature, **missing cause** is called **masking**.

2. Missing · Incomplete: Competing risks

The competing risks can be modeled as a system in series.
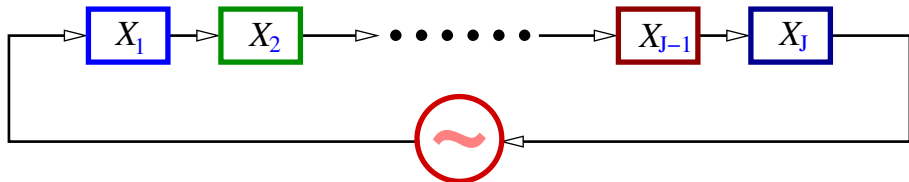
Let $X = \min(X_1, X_2, \ldots, X_J)$. Then the cdf of X is easily obtained as

$$
\begin{aligned}
F(x|\boldsymbol{\Theta}) &= 1 - P[X > x] = 1 - P[\min(X_1, X_2, \ldots, X_J) > x] \\
&= 1 - P[X_1 > x, X_2 > x, \ldots, X_J > x] \\
&= 1 - P[X_1 > x] \cdot P[X_2 > x] \cdots\cdots P[X_J > x] \\
&= 1 - \prod_{j=1}^{J} \left\{ 1 - F_j(x|\boldsymbol{\theta}_j) \right\},
\end{aligned}
$$

where $\boldsymbol{\Theta} = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_J)$.

2. Missing · Incomplete: Competing risks

- Let p be the dimension of $\boldsymbol{\theta}_j$. Then $\boldsymbol{\Theta} = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_J)$ has $J \times p$ parameters (curse of high dimensionality).
- Masking (missing cause) make the likelihood function more complex.
- EM method can help the likelihood function simple by **partitioning** the likelihood (low dimension).

Brief Literature Review

- Cox (1959):   Exponential with only two causes but no masking.
- Herman and Patell (1971):   Exponential with multiple causes but no masking.
- Miyakawa (1984):   Exponential with only two causes, only complete masking, but no censoring.
- Usher and Hodgson (1988), Usher and Guess (1989), Guess et al. (1991), and Reiser et al. (1995):   general masking problem, but mainly on Exponential.

2. Missing · Incomplete: Competing risks

Recent work

- Park and Kulasekera (2004): Exponential and Weibull with multiple causes, censoring, and masking (only complete masking). They provide the closed-form MLEs for Exponential. For Weibull, the closed-form MLE is available only when the common shape parameter is estimated by the likelihood function. They also show the bound for the Weibull shape estimate.
- Albert and Baxter (1995): EM for Exponential with partial masking.
- Park (2005) and Park and Padgett (2006): EM for various distributions including Exponential, Weibull, (log)normal and Wald with multiple causes, censoring, and general masking.
- Extension to Load-sharing problem: Park (2010, 2013).

2. Missing · Incomplete: Competing risks

## Revisit: Strength data with three fracture causes (modes)

Mode	Weibull		Lognormal		Wald	
	$\lambda^{(j)}$	$\alpha^{(j)}$	$\mu^{(j)}$	$\sigma^{(j)}$	$\mu^{(j)}$	$\lambda^{(j)}$
Surface	2.615×10^{-10}	4.308	5.0390	0.360	165.4	1172.5
Inner defect	8.916×10^{-6}	2.329	4.853	0.6732	169.4	261.6
End effect	1.131×10^{-2}	0.880	4.693	1.7034	7478.7	30.2

MSEs under the Competing Models

Model	Weibull	Lognormal	Wald
MSE	0.1322911	**0.1314757**	0.1396442

Note: For R program, refer to `Talk-R.r` at ▸ 2020/Talk-R

Weibull plots with the previous results
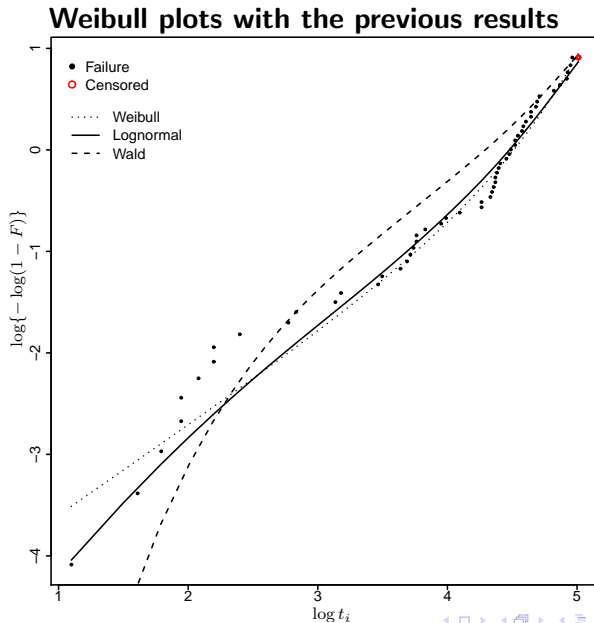
2. Missing · Incomplete: Load-Sharing

Load-sharing models

- Consider a multi-component system connected in parallel, in which components fail one by one.
- Total load or traffic applied to the system is redistributed among the remaining surviving components as component fails.
- Kim and Kvam (2004) provided an important first step in drawing parametric inference on load-sharing properties. (high-dimensional, but no closed-form solution).

Recent work

- Park (2010): Closed-form solution of Kim and Kvam (2004). But, only for exponential model.
- Park (2013): Extended to Weibull, etc. It may not be possible to obtain a closed-form solution. EM algorithm is used for partitioning (low-dimensional)

2. Missing · Incomplete: Grouped Data

Observed frequencies of intermittent inspection data (Nelson, 1982)

Inspection time	Observed failures
$0 \sim 6.12$	5
$6.12 \sim 19.92$	16
$19.92 \sim 29.64$	12
$29.64 \sim 35.40$	18
$35.40 \sim 39.72$	18
$39.72 \sim 45.24$	2
$45.24 \sim 52.32$	6
$52.32 \sim 63.48$	17
$63.48 \sim$	73

- These censored and grouped data can also be regarded as interval-censored data.

- Grouped data are popular in engineering experiments (Seo and Yum, 1993; Shapiro and Gulati, 1998; Xiong and Ji, 2004; Meeker, 1986; Nelson, 1990; Sun, 2006; Lee and Park, 2006; Park et al., 2017).

2. Missing · Incomplete: Grouped Data

Handling interval data is picky

- MLE is not in closed form.
- EM can be applied for normal, but not for Weibull.
- Thus, it is very picky to handle interval data.

Recall EM algorithm

- **E-step:** $Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)}) = \int \ell^c(\boldsymbol{\theta}|\mathbf{y}, \mathbf{z}) p(\mathbf{z}|\mathbf{y}, \boldsymbol{\theta}^{(t)}) d\mathbf{z}$

- **M-step:** $\boldsymbol{\theta}^{(t+1)} = \arg \max_{\boldsymbol{\theta}} Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)})$

MC-EM (Wei and Tanner, 1990a,b) and **Q-EM** (Park, 2018b) can ease this problem.

- **MC-EM**: uses stochastic Monte Carlo integration (more flexible).
- **Q-EM**: uses deterministic quantile integration (less flexible, but more accurate).

Note: For the analysis of the grouped data in (Nelson, 1982), refer to

Robust control chart with **unbalanced** samples

- For balanced sample, see rQCC package (Park and Wang, 2020b).
  Due to missing, original sample can be unbalanced.
- For control chart with **unbalanced** samples, refer to Park and Wang (2020a).
  Note: this method is not robust.
- Thus, the future work would be:
  How to combine the methods of (Park and Wang, 2020b) (robust) and Park and Wang (2020a) (unbalanced) so that a new method is robust with unbalanced samples.
  $\implies$ Robust control chart with unbalanced samples.

3. Future work: Competing risks

Extension of Competing risks models

- Competing risks problems are solved with moderate tailed distribution. Thus, heavier tailed distributions can be considered, which may handle contaminated observations.

- Masking and censoring with competing risks are considered. Interval data with competing risks can be challenging.

- Load-sharing with Weibull distributions are solved. Various other distributions should be considered.

- EM works very well with competing risks model. Then, does MI with competing risks work well? We need to compare these two methods. Since MI is less model-dependent (robust to miss-specifications), MI can be preferred if MI performs well.

3. Future work: Multiple Imputation

Robust Imputation with Robustness property

- Robust Imputation:
  Imputation is closely related to bootstrap (Efron, 1994). Robust bootstrap can be considered to obtain robustness property.

- Robust pooled estimate:
  At the final step of the MI, we need to pool m estimates. The sample mean is widely used, which can be improved with robust estimate.

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