Class -18
Tree based models

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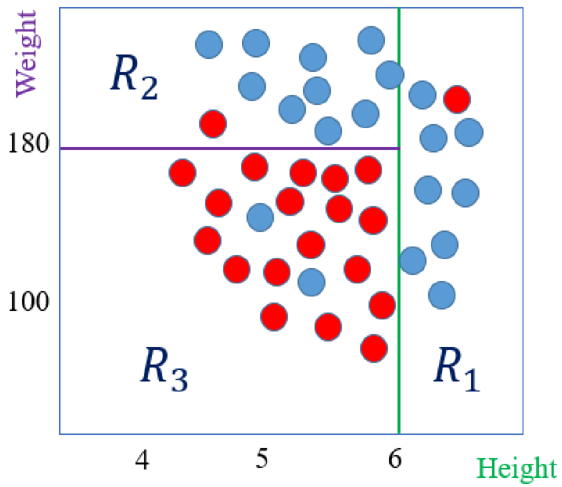
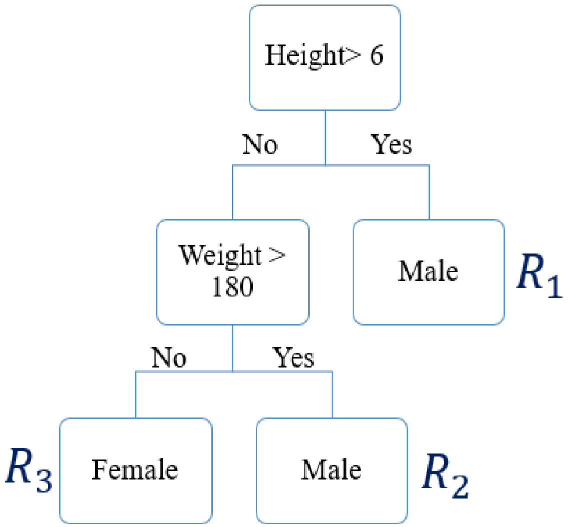
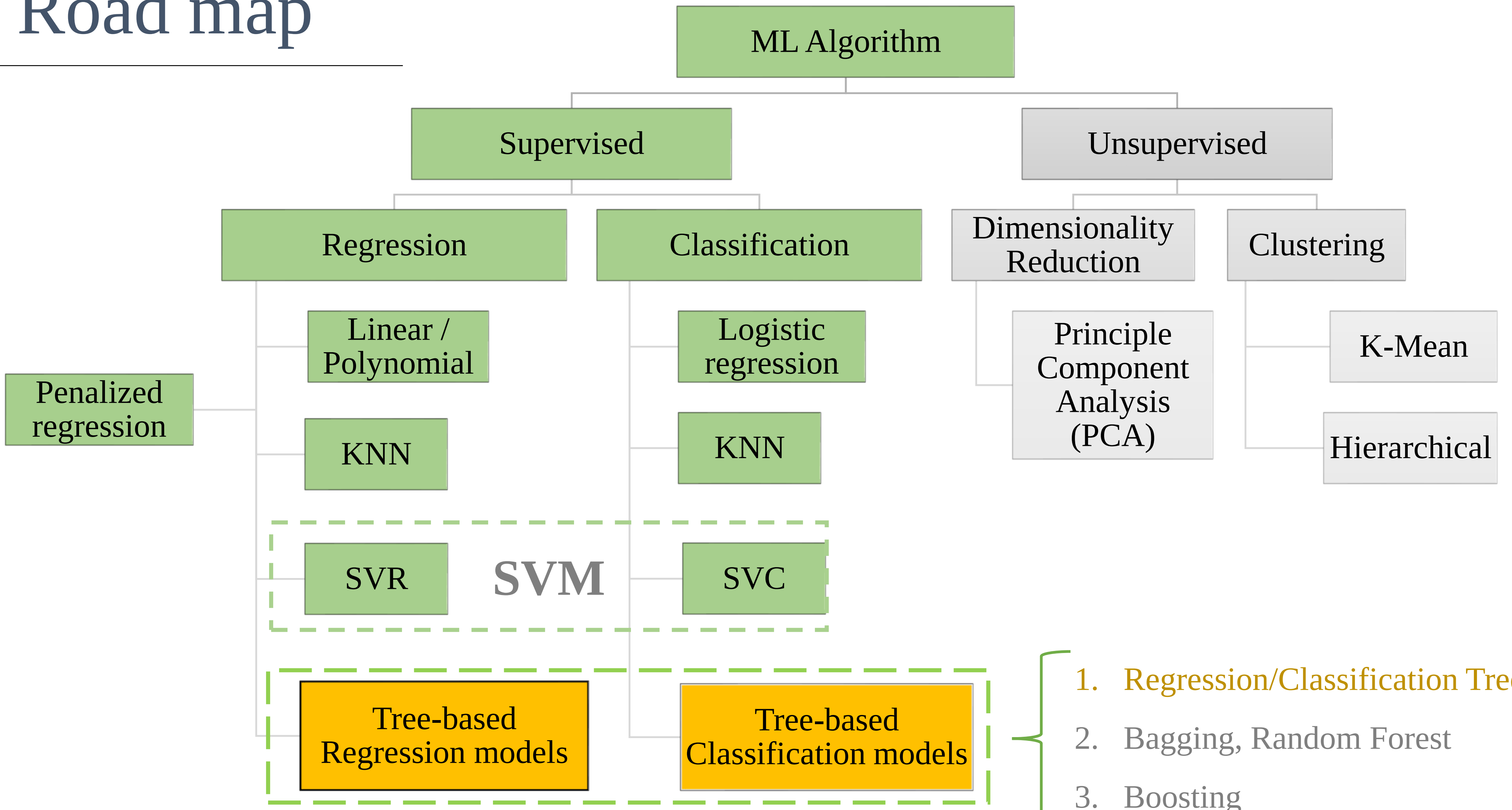




Road map



1. Regression/Classification Tree
2. Bagging, Random Forest
3. Boosting

Topics

Part I

1. Decision Tree Definitions
2. Decision Tree Metrics
   - MSE
   - Error Rate
   - Gini Index
   - Entropy

Part II

1. Regression Trees
2. Classification Trees

Part III

1. Pruning a tree
2. Tuning Hyperparameters

Part IV

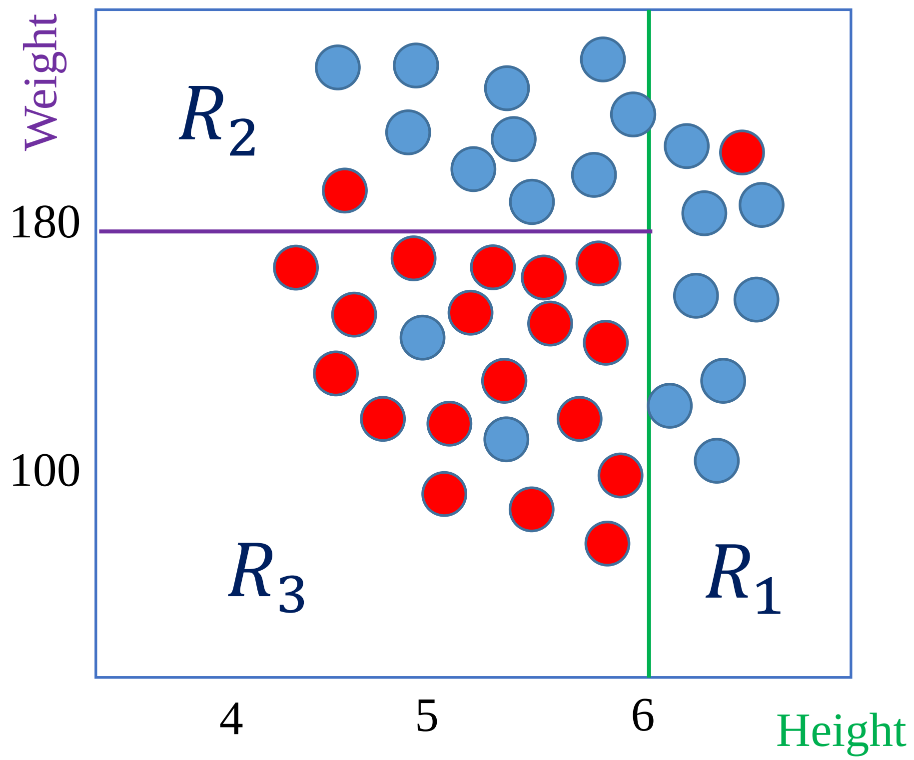
1. Pros and Cons
2. Applications in Finance

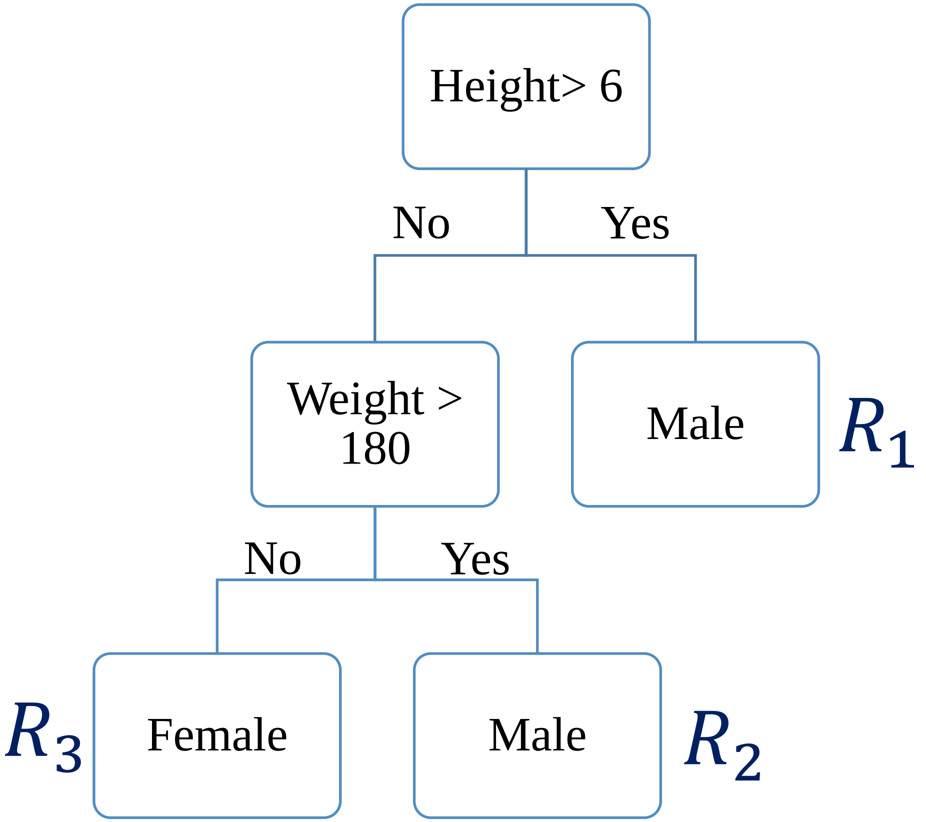
Part I

Decision Tree definitions and metrics

Decision Tree Definitions

- Decision Trees are sequence of simple questions seeking to partition datasets into homogeneous groups.
- DTs apply a top-down approach to data, trying to group and label observations that are similar.

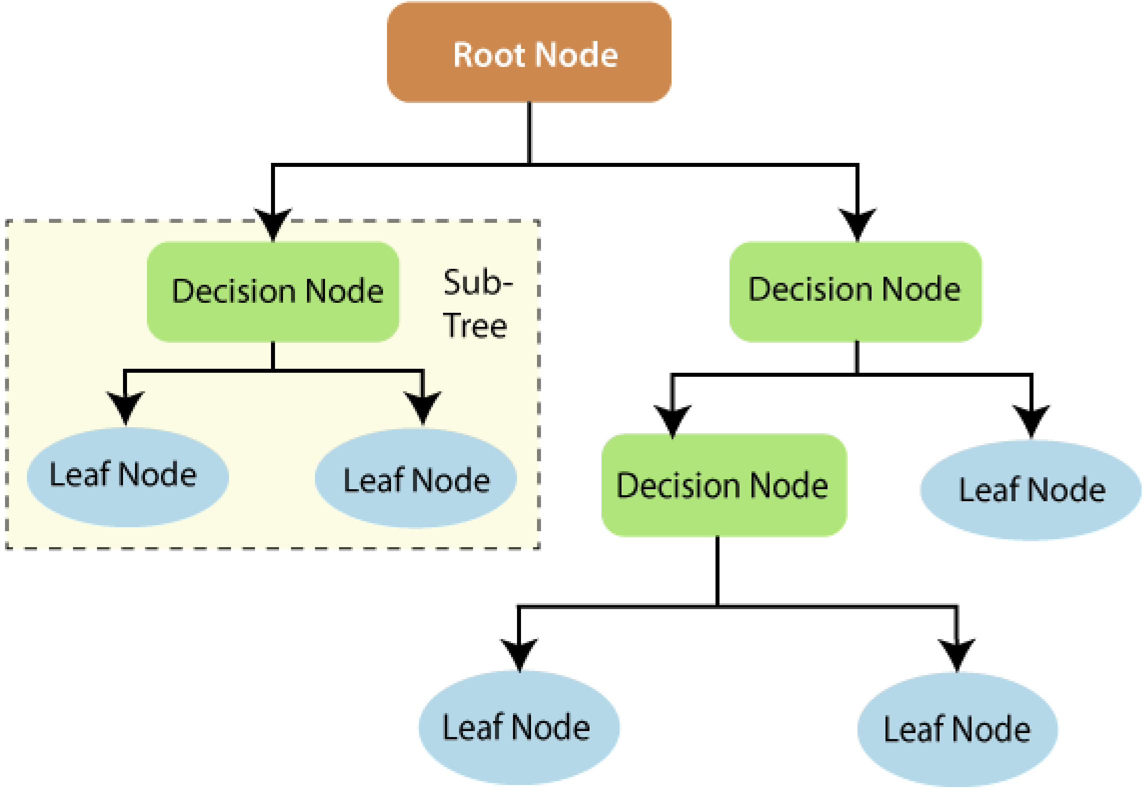




Decision Tree Definitions

- When the target variable consists of real numbers: regression trees
- When the target variable is categorical: classification trees
- Terminology:

  - ✓ Root node
  - ✓ Splitting
  - ✓ Branch
  - ✓ Decision node (internal node)
  - ✓ Leaf node (terminal node)
  - ✓ Sub-tree
  - ✓ Depth (level)

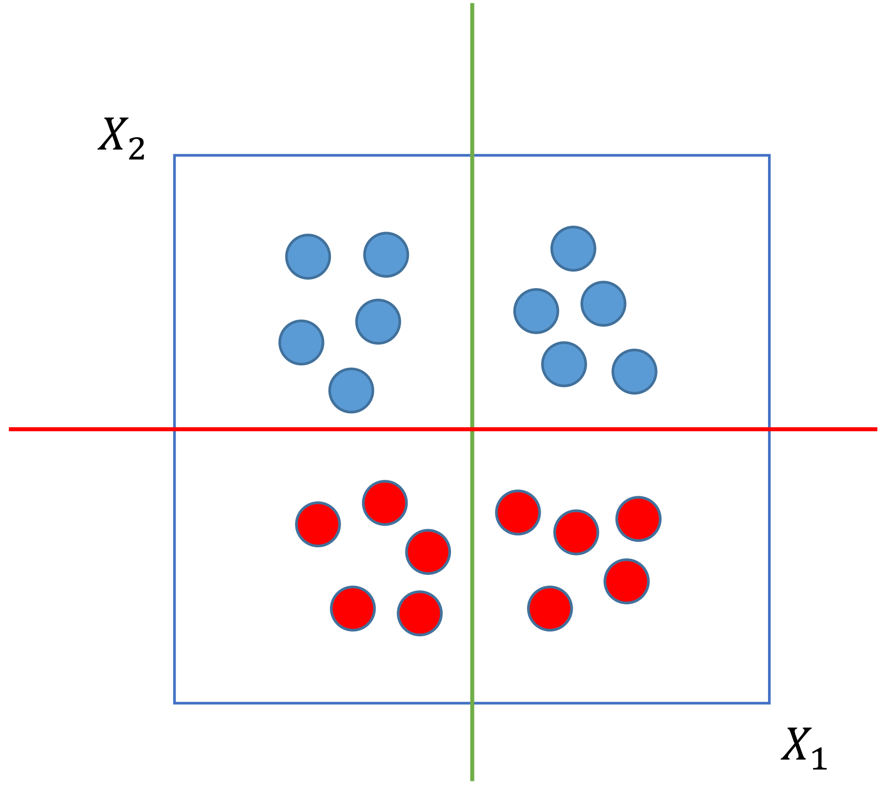


Decision Tree Metrics

- Regression trees: MSE
- Classification trees:
  1. Error rate
  2. Entropy
  3. Gini Index

Control how a Decision Tree decides to split the data

They all measure **impurity**



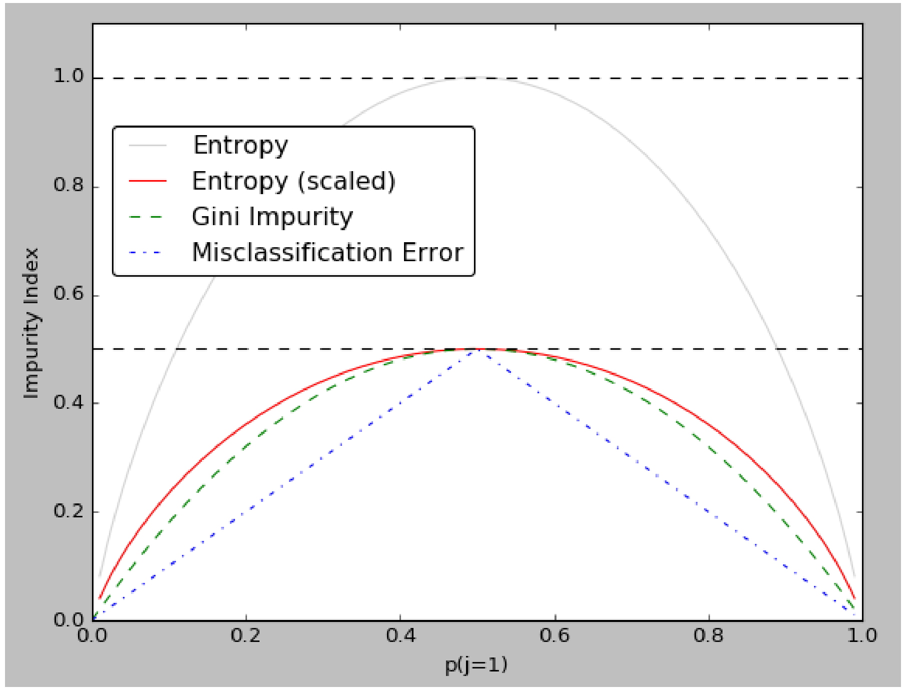
Decision Tree Metrics

- Entropy: Measures the impurity or randomness (uncertainty) in the data points
- Gini Index: Measure how often a randomly chosen element would be incorrectly labeled

- For both Entropy and Gini, 0 expresses all the elements belong to a specified class (pure)
- Different decision tree algorithms utilize different impurity metrics

$$entropy = -\sum_j p_j \log_2(p_j)$$

$$Gini = 1 - \sum_j p_j^2$$



Part II

Regression / Classification Trees!

How does a decision tree work?

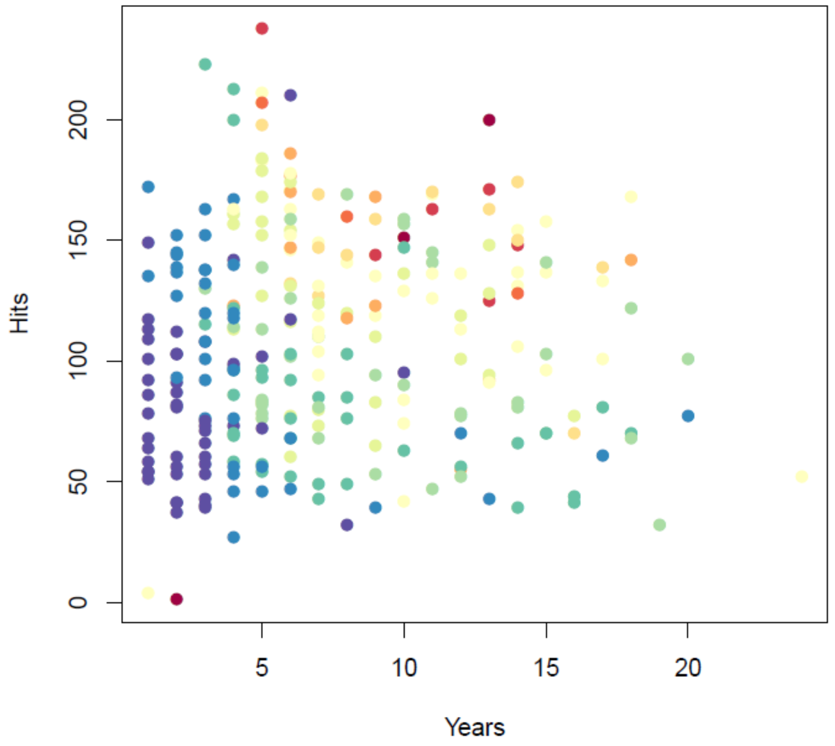
Regression Trees

- Baseball Salary is color-coded from low (blue, green) to high (yellow, red)

- DTs apply a top-down approach to data, trying to group and label observations that are similar.

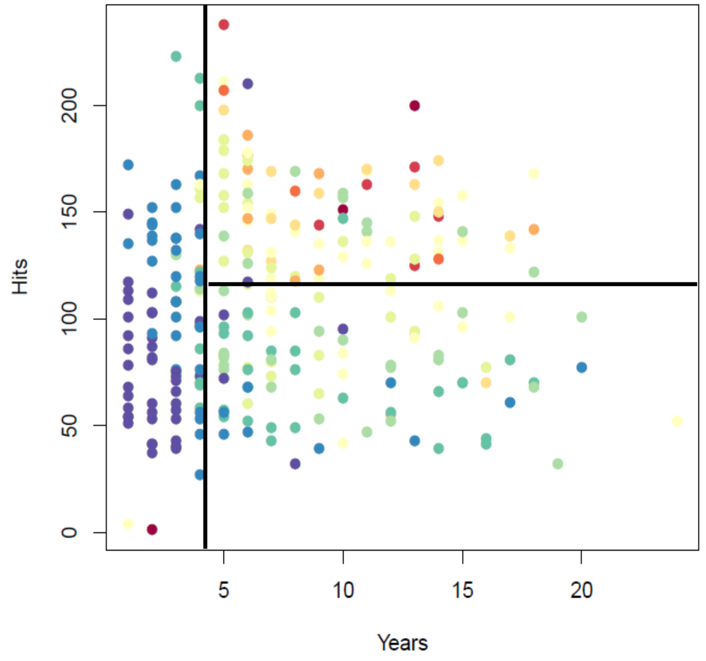
- The main questions in every decision-making process:

1. Which feature to start with?
2. Where to put the split (cut off)?



Interpreting the results

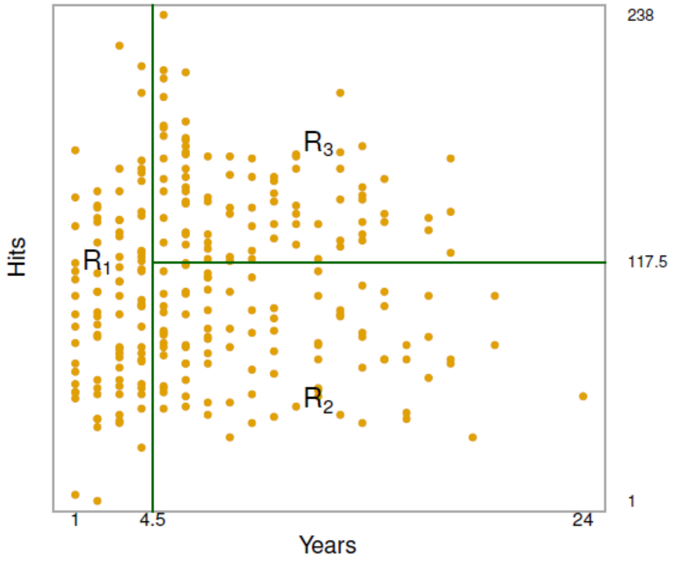
- Based on color-coded salary, it seems that years is the most important factor in determining salary.
- For less experienced players, the number of hits seems irrelevant.
- Among more experienced players thought, players with more hits tend to have higher salaries.
- As one can see, the model is very easy to display, interpret and explain.



Tree building process

- Divide the feature space into J distinct and non-overlapping regions,
- For every observation that falls into the region R_j, we make the same prediction, which is simply the mean of the target values for the training observations in R_j.
- The goal is to find rectangles $R_1, R_2, \ldots, R_J$ that minimize the RSS:

$$\sum_{j=1}^{J} \sum_{i \in R_j} (y_i - \hat{y}_{R_j})^2$$



- Where $\hat{y}_{R_j}$ is the mean target for the training observations within the j^{th} rectangle.

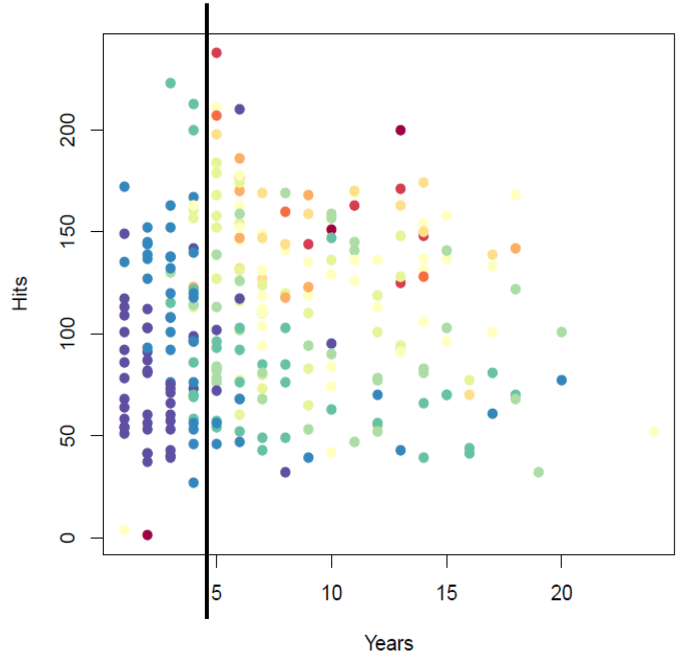
Tree building process: Recursive Binary Splitting

- How does the algorithm select X_j and the split s ?
- X_j and s are selected such that splitting the feature space into the regions $\{X|X_j < s\}$ and $\{X|X_j \geq s\}$ leads to the largest possible reduction in RSS.

$$R_1(j,s) = \{X|X_j < s\} \text{ and } R_2(j,s) = \{X|X_j \geq s\}$$

- Seeking for the value of j and s that minimized the following equation:

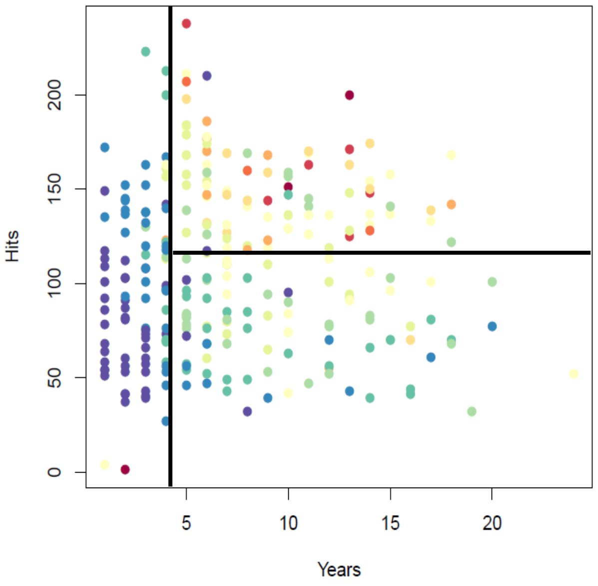
$$\sum_{i:\ x_i \in R_1(j,s)} (y_i - \hat{y}_{R_1})^2 + \sum_{i:\ x_i \in R_2(j,s)} (y_i - \hat{y}_{R_2})^2$$

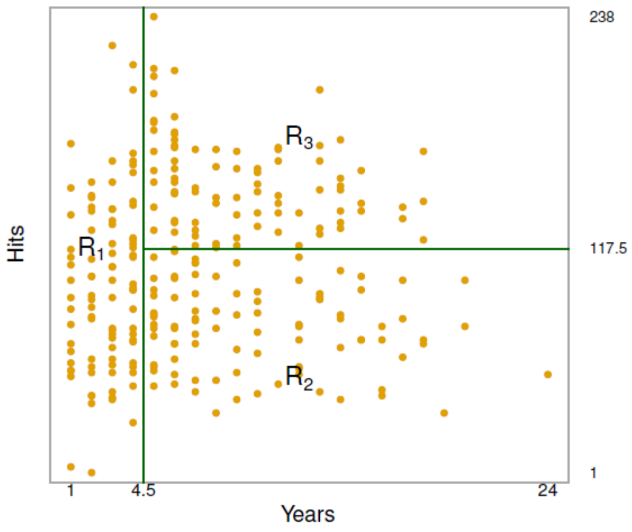


- The best split is made at that particular step, rather than looking ahead and picking a split that will lead to a better tree in some future step.

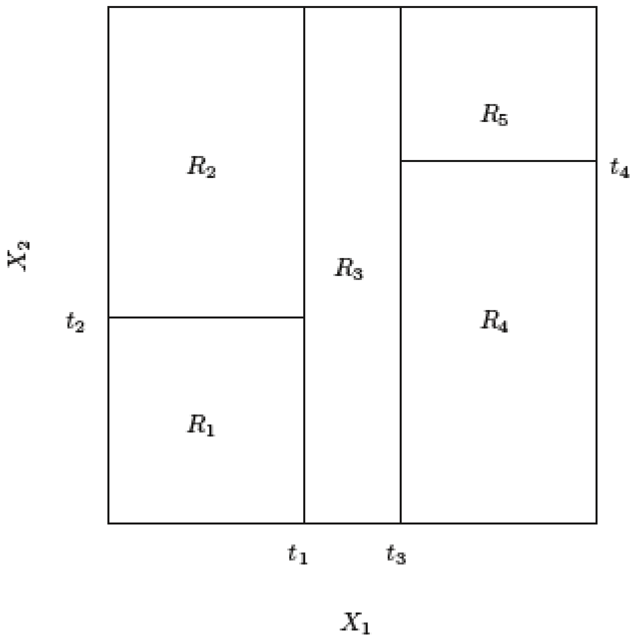
Tree building process: Recursive Binary Splitting

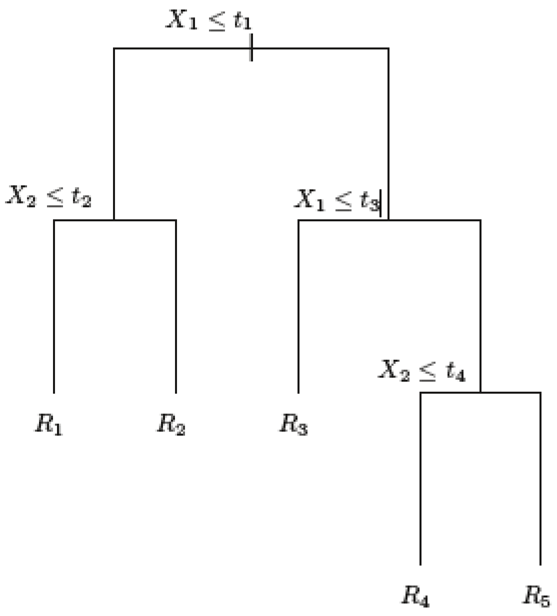
- Next, the algorithm repeats the process, looking for the best feature and best split in order to split the data further to minimize the RSS within each of the resulting regions.
- The process continues until a stopping criterion is reached; for instance, continue until no region contains more than a fixed number of observations.

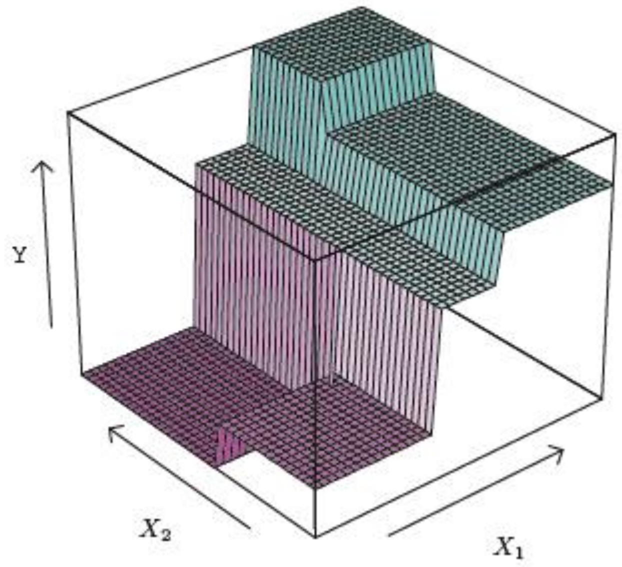




A Five-Region Example of Recursive Binary Splitting







The output of recursive binary splitting on a two-dimensional example

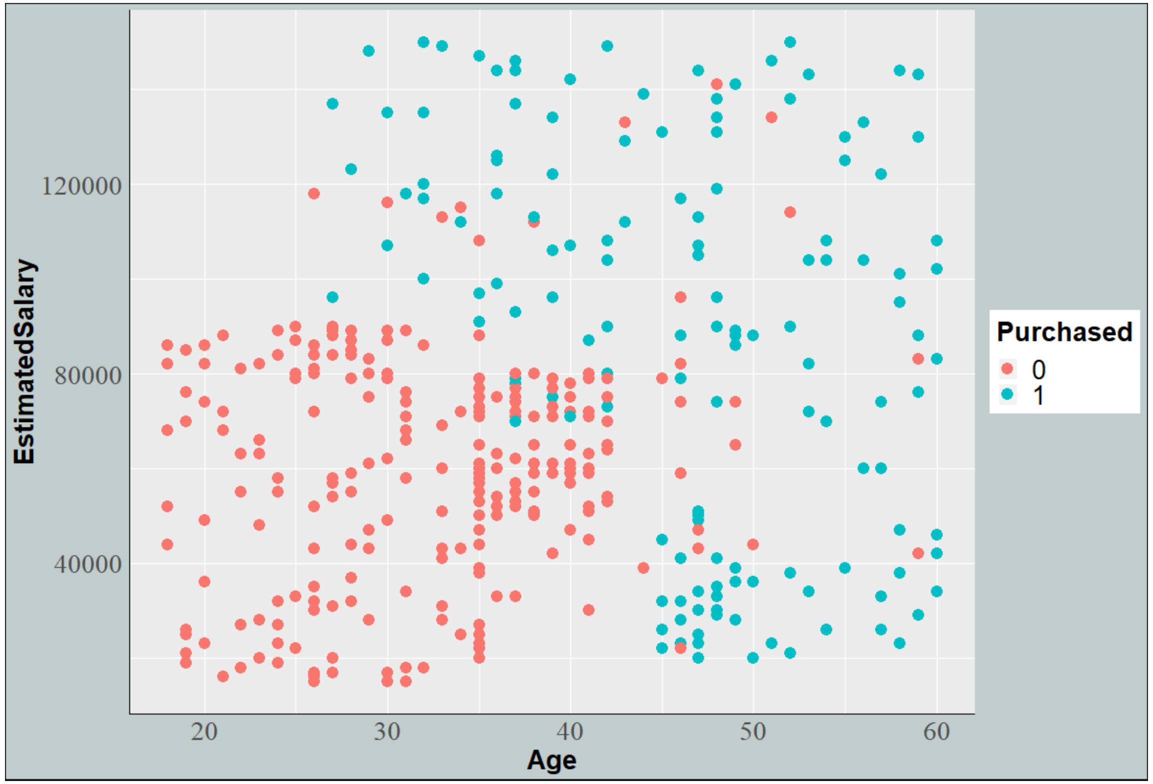
A tree corresponding to the partition in the left panel.

A perspective plot of the prediction surface corresponding to that tree.

Classification Trees

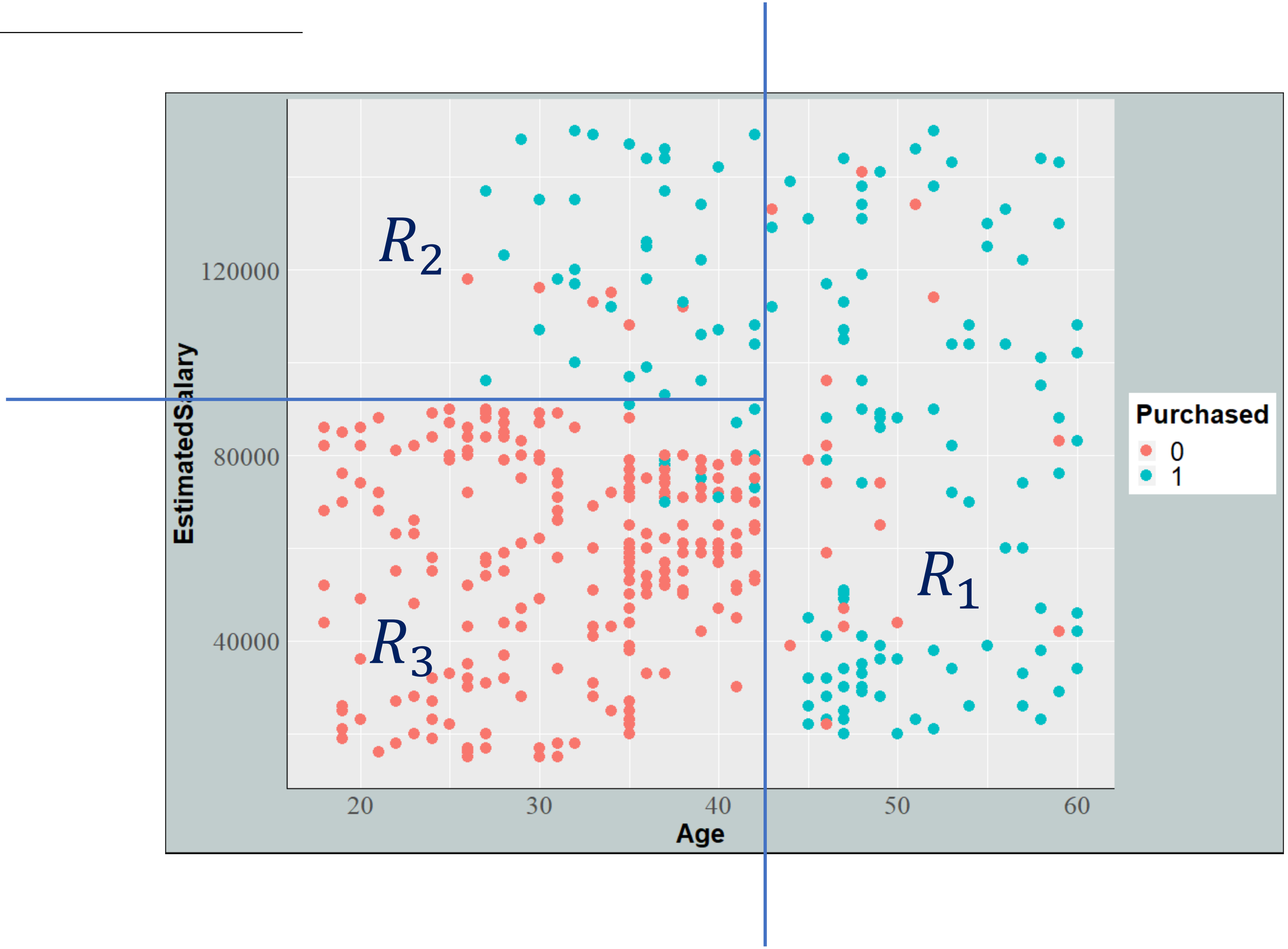
- Classification trees are very similar to regression trees, except that it is used to predict a qualitative response rather thana quantitative one.
- The prediction of the algorithm at each terminal node will be the category with the majority of data points i.e., the most commonly occurring class.

Classification Trees (example)



Classification Trees

Classification Trees



Classification Trees (details)

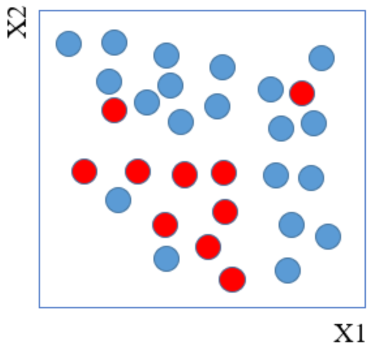
- Just as in the regression setting, the recursive binary splitting is used to grow a classification tree. However, instead of RSS we will be using one of the following impurity metrics:

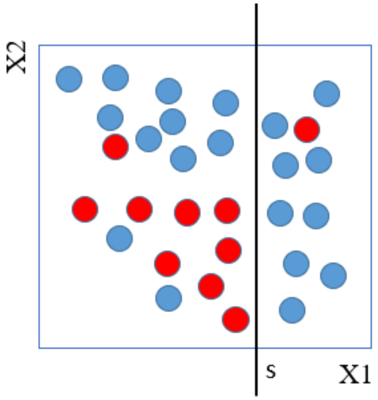
  1. Classification error rate: $\longrightarrow$ $$E = 1 - \max_k(\hat{p}_{mk})$$
  2. Gini index: $\longrightarrow$ $$G = \sum_{k=1}^{K} \hat{p}_{mk}(1 - \hat{p}_{mk})$$
  3. Cross entropy: $\longrightarrow$ $$D = -\sum_{k=1} \hat{p}_{mk} \log \hat{p}_{mk}$$

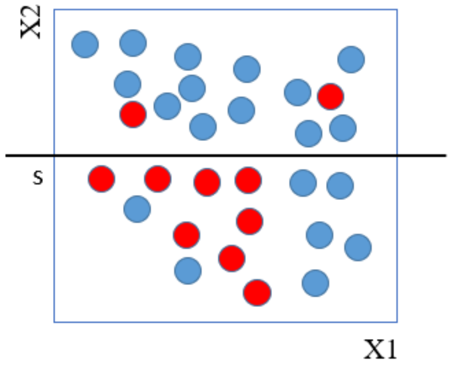
- $\hat{p}_{mk}$ represents the proportion of training observations in the m^{th} region from the k^{th} class.
- Classification error rate is not sufficiently sensitive to node purity and in practice either Gini or Cross entropy is preferred.

Decision Tree Metrics (Simple Example)

Node	*Gini* $1-\sum_j p_j^2$	*Cross entropy* $-\sum_j p_j \log(p_i)$	*Error rate* $1-\max(p_i)$
Entire data before split	$\left\{1-\left(\left(\frac{10}{30}\right)^2+\left(\frac{20}{30}\right)^2\right)\right\} = 0.44$	$-\left\{\left(\frac{10}{30} log \frac{10}{30}+\frac{20}{30} log \frac{20}{30}\right)\right\} = 0.64$	0.333
Root node: X1 > s	$\frac{20}{30} * \left\{1-\left(\left(\frac{9}{20}\right)^2+\left(\frac{11}{20}\right)^2\right)\right\} +$ $\frac{10}{30} * \left\{1-\left(\left(\frac{1}{10}\right)^2+\left(\frac{9}{10}\right)^2\right)\right\} =$ $\frac{20}{30} * 0.495 + \frac{10}{30} * 0.18 = \mathbf{0.39}$	$-\frac{20}{30}\left\{\left(\frac{9}{20}\log_2\frac{9}{20}+\frac{11}{20}\log_2\frac{11}{20}\right)\right\} +$ $-\frac{10}{30}\left\{\left(\frac{1}{10}\log_2\frac{1}{10}+\frac{9}{10}\log_2\frac{9}{10}\right)\right\} =$ $\frac{20}{30} * 0.69 + \frac{10}{30} * 0.325 = \mathbf{0.57}$	$\frac{20}{30} * \frac{9}{20} +$ $\frac{10}{30} * \frac{1}{10} =$ $\mathbf{0.333}$
Root node: X2 > s	$\frac{15}{30} * \left\{1-\left(\left(\frac{2}{15}\right)^2+\left(\frac{13}{15}\right)^2\right)\right\} +$ $\frac{15}{30} * \left\{1-\left(\left(\frac{8}{15}\right)^2+\left(\frac{7}{15}\right)^2\right)\right\} =$ $\frac{15}{30} * 0.231 + \frac{15}{30} * 0.497 = \mathbf{0.37}$	$-\frac{15}{30}\left\{\left(\frac{2}{15}\log_2\frac{2}{15}+\frac{13}{15}\log_2\frac{13}{15}\right)\right\} +$ $-\frac{15}{30}\left\{\left(\frac{8}{15}\log_2\frac{8}{15}+\frac{7}{15}\log_2\frac{7}{15}\right)\right\} =$ $\frac{15}{30} * 0.39 + \frac{15}{30} * 0.69 = \mathbf{0.54}$	$\frac{15}{30} * \frac{2}{15} +$ $\frac{15}{30} * \frac{7}{15} =$ $\mathbf{0.3}$

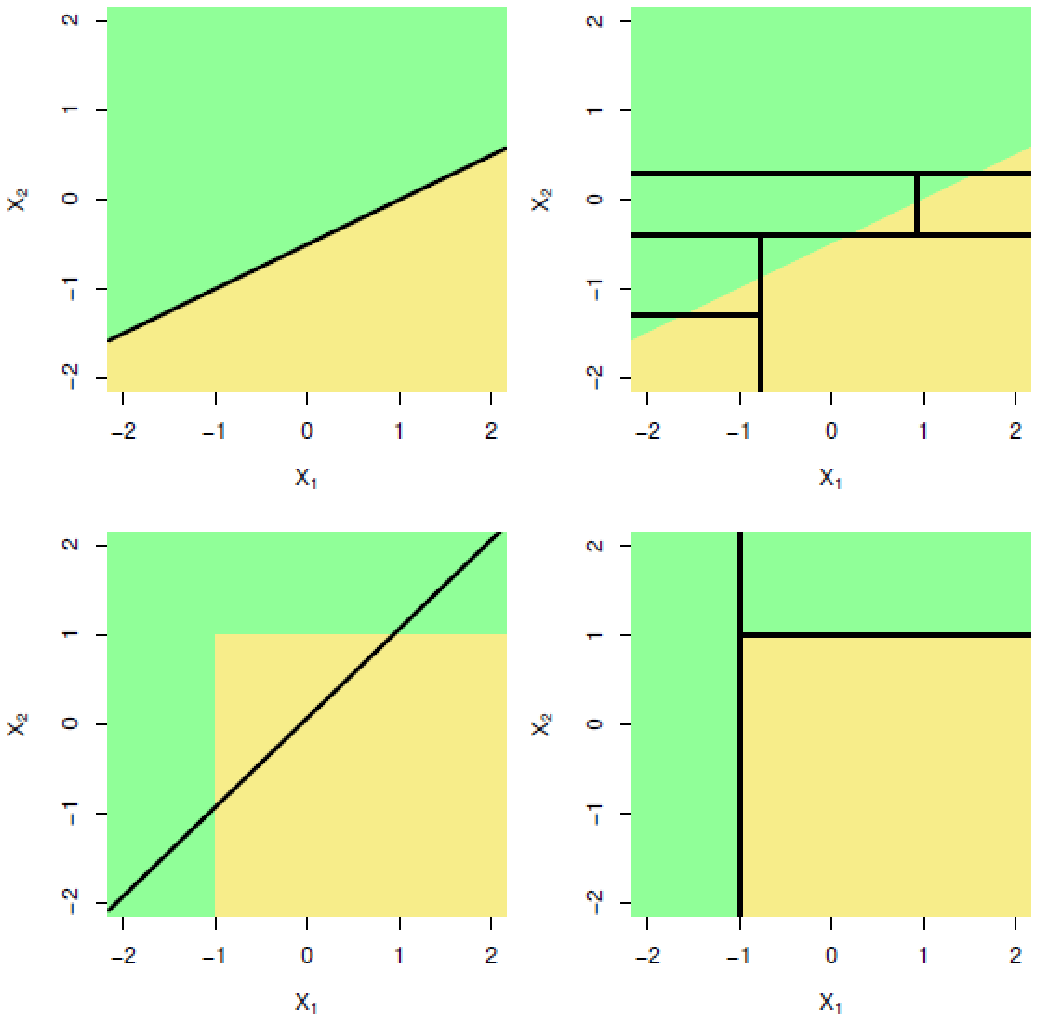






Trees Versus Linear Models

Left column: linear model; Right column: tree-based model



Top Row: True linear boundary
Bottom row: true non-linear boundary.

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Part III

Pruning a tree

Tunning hyper parameters

Pruning a tree

- Decision tree may produce good predictions on the training set, but is likely to overfit the data, leading to poor test set performance. (why?)
- A smaller tree with fewer splits may lead to lower variance and better interpretation at the cost of a little bias.
- This strategy may result in smaller trees, but is too short-sighted:

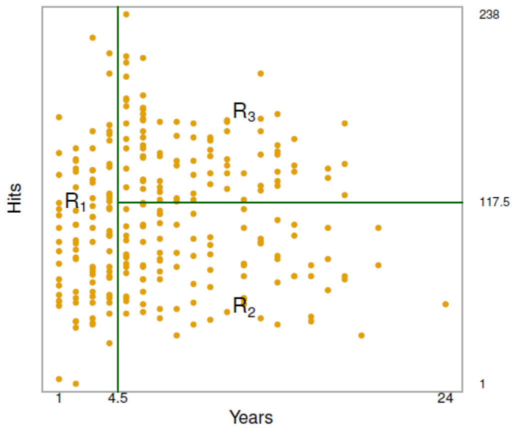
" a seemingly worthless split early on in the tree might be followed by a very good split, a split that leads to a large reduction in RSS/Gini/Cross entropy later on"

- A better strategy is to grow a very large tree T_0, and then prune it back in order to obtain a subtree.
- Cost complexity pruning is used to do this.

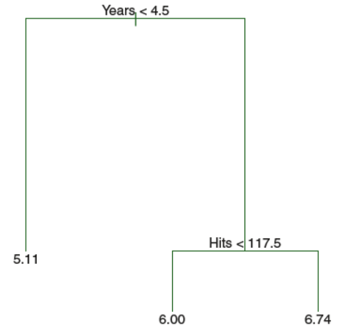
Cost complexity pruning (weakest link pruning)

- Consider a sequence of trees indexed by a nonnegative tuning parameter α.
- For each value of α there corresponds a subtree $T \subset T_0$ such that the following objective function is minimized.

$$\sum_{m=1}^{|T|} \sum_{i:\, x_i \in R_m} (y_i - \hat{y}_{R_m})^2 + \alpha |T|$$



  - $|T|$ indicates the number of terminal nodes of the tree T
  - R_m is the rectangle corresponding to m^{th} terminal node and
  - $\hat{y}_{Rm}$ is the mean of the training observations in R_m



- α controls the bias variance trade off and is determined by cross validation.
- Lastly, we return to full data set and obtain the subtree corresponding to α

Building a Regression Tree algorithm

Algorithm 8.1 *Building a Regression Tree*

1. Use recursive binary splitting to grow a large tree on the training data, stopping only when each terminal node has fewer than some minimum number of observations.

2. Apply cost complexity pruning to the large tree in order to obtain a sequence of best subtrees, as a function of α.

3. Use K-fold cross-validation to choose α. That is, divide the training observations into K folds. For each $k = 1, \ldots, K$:

   (a) Repeat Steps 1 and 2 on all but the kth fold of the training data.

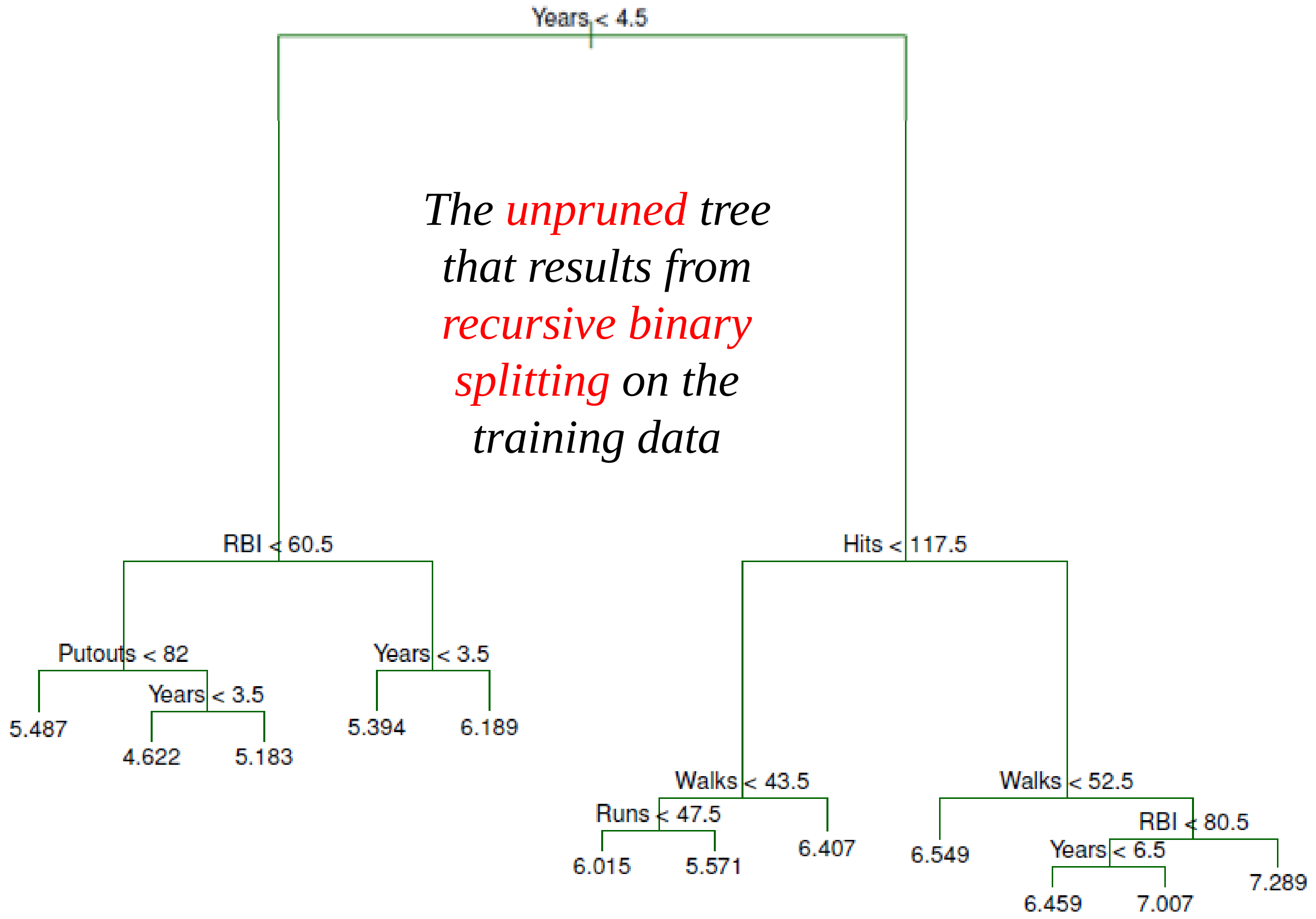
   (b) Evaluate the mean squared prediction error on the data in the left-out kth fold, as a function of α.

   Average the results for each value of α, and pick α to minimize the average error.

4. Return the subtree from Step 2 that corresponds to the chosen value of α.

Source: An introduction to Statistical Learning

Salary example continued



The unpruned tree that results from recursive binary splitting on the training data

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Finding the optimal α or T

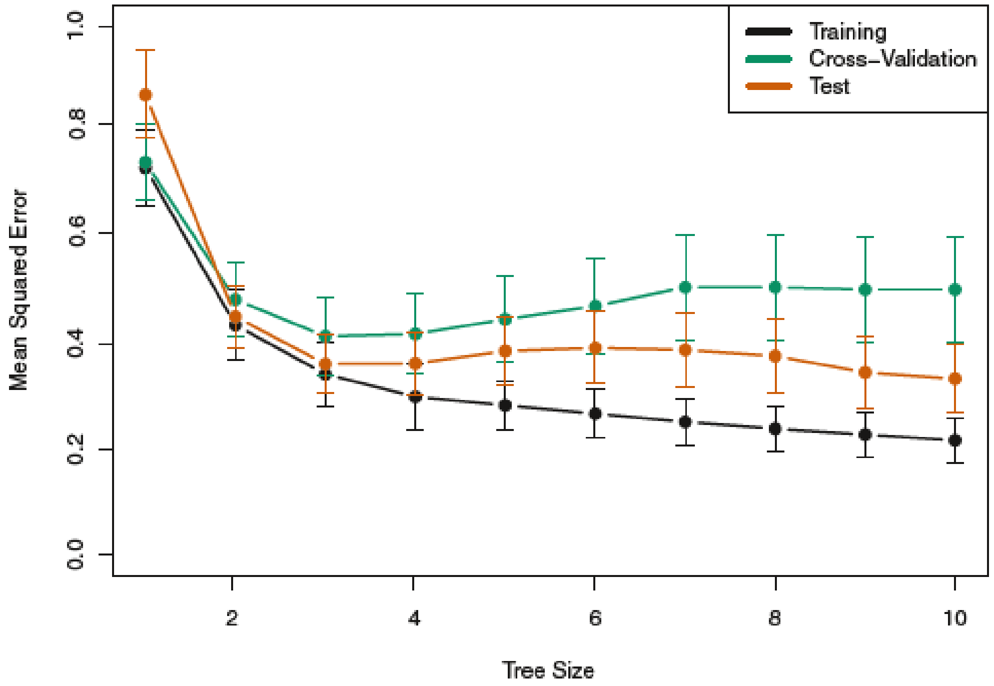
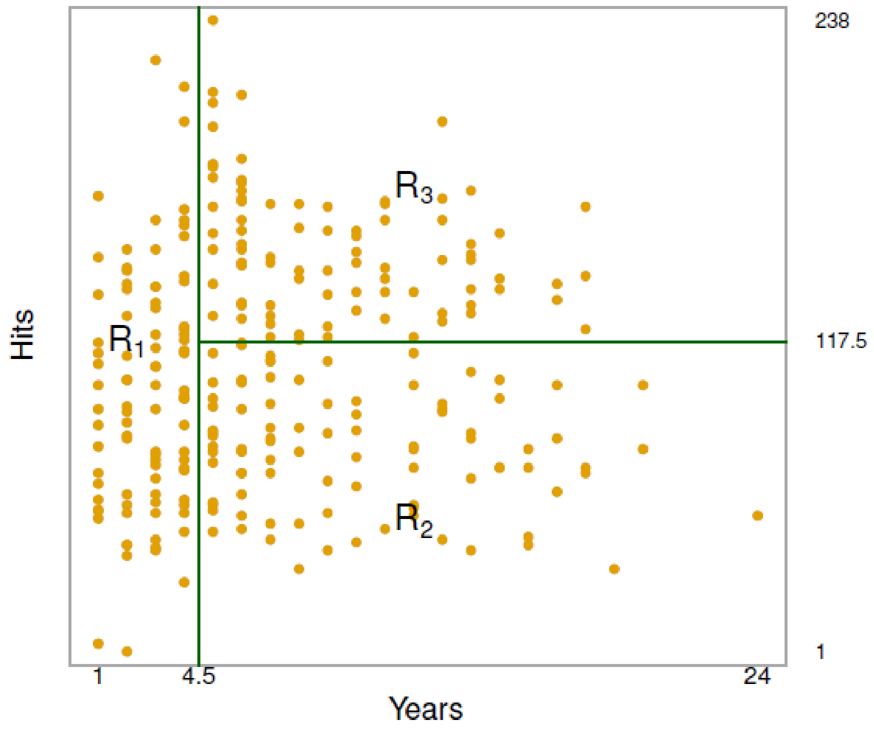


FIGURE 8.5. *Regression tree analysis for the* **Hitters** *data. The training, cross-validation, and test MSE are shown as a function of the number of terminal nodes in the pruned tree. Standard error bands are displayed. The minimum cross-validation error occurs at a tree size of three.*

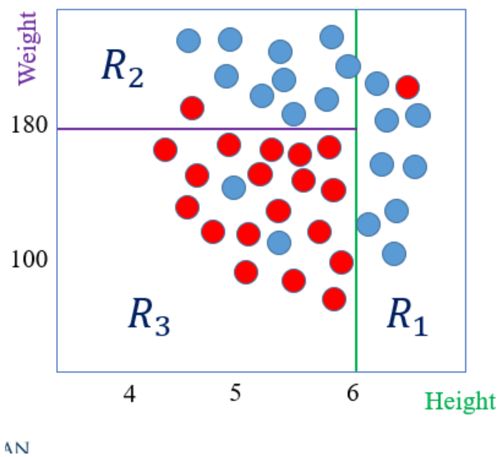
The optimal (pruned) tree

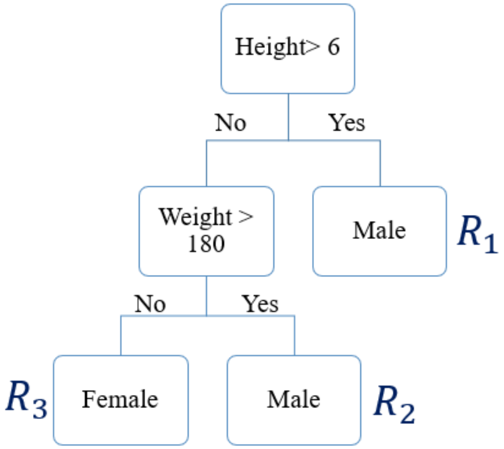


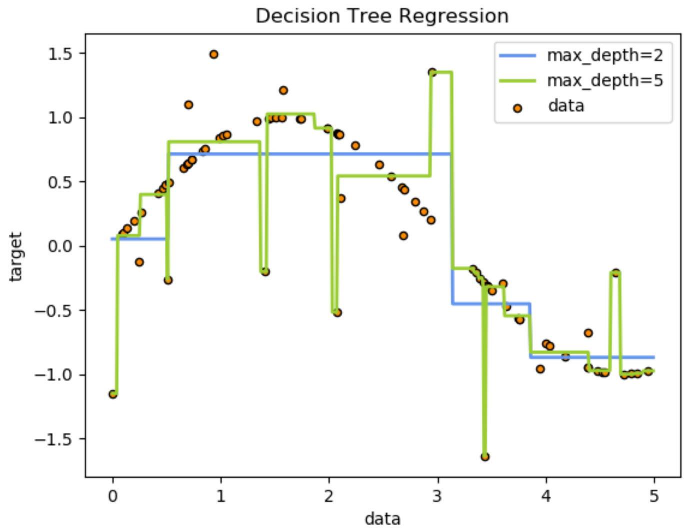
Other hyperparameters

➢ To avoid overfitting, regularization parameters can be added to the model such as:

- Maximum depth of the tree
- Minimum population at a node
- Maximum number of decision nodes







Part IV

Pros and Cons

Applications in finance

CART's Pros and Cons

Pros:

- Very simple and interpretable models used for both regression and classification.
- Popular supervised machine learning models because of providing a visual explanation for the prediction
- Mirror human decision-making process. Easy to explain to people. Even easier than linear regression!
- Can easily handle categorical data without the need to create dummy variables
- handles data in its raw form (no preprocessing needed) and can use the same variables more than once in different parts of the same DT, which may uncover complex interdependencies between sets of variables.

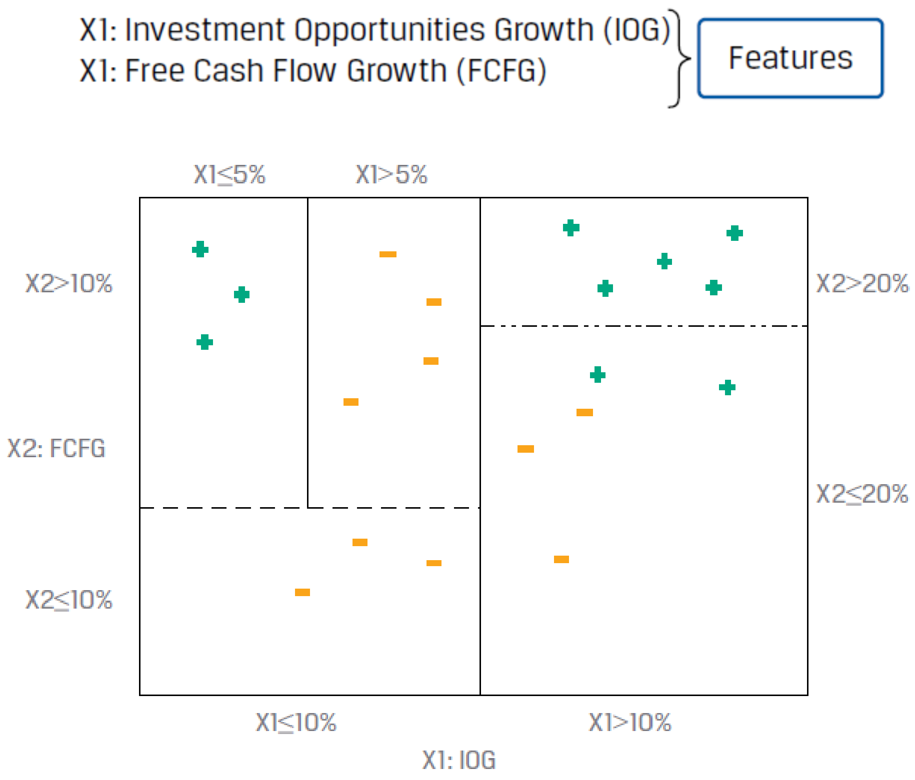
Cons:

- Poor level of predictive accuracy and can easily overfit the data if used without stopping criteria.

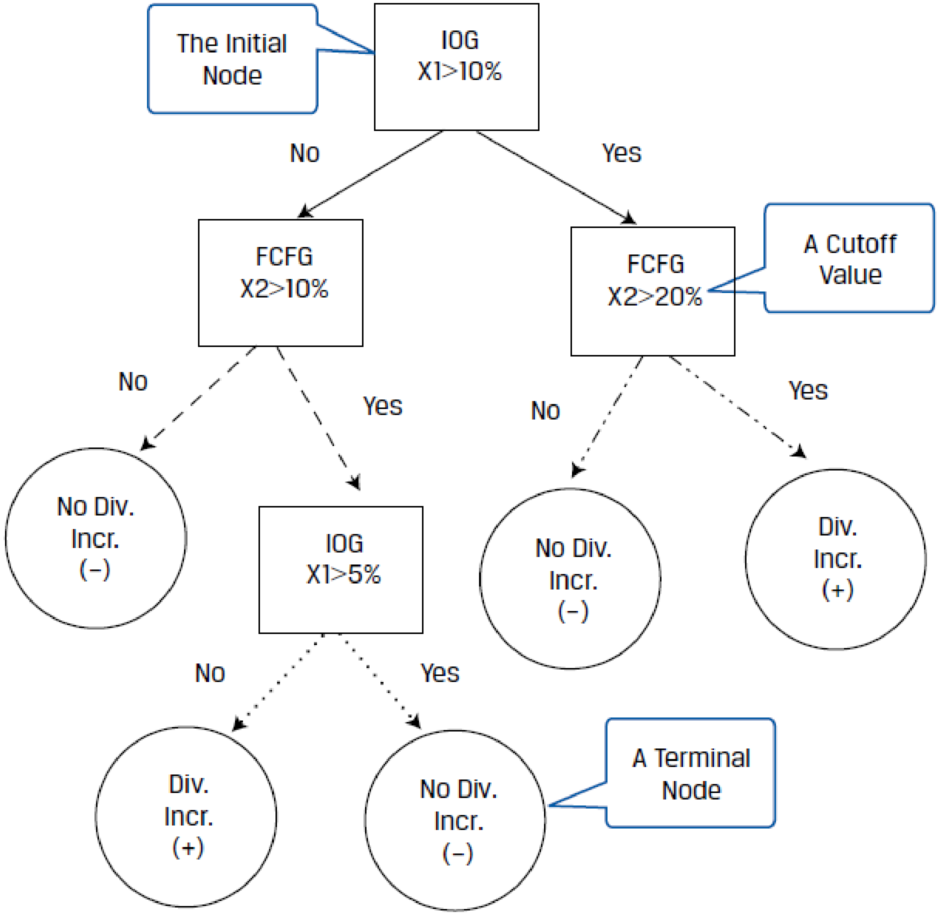
CART's Applications in finance

- Enhancing detection of fraud in financial statements,
- Generating consistent decision processes in equity and fixed-income selection
- Simplifying communication of investment strategies to clients.
- Portfolio allocation problems.

CART example in finance



Source: CFA PROGRAM. Level II . Reading 7



Students' questions

1. Item 1