

Class 7- Polynomial Regression



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➔ Polynomial regression model

The polynomial regression model is a special case of multiple linear regression models!

- Create new variables $X_1 = X$, $X_2 = X^2$, ... etc and then treat as **multiple linear regression**.
- Not really interested in the coefficients; more interested in the fitted function!

$$\hat{f}(X) = f_{\mathbf{w},b}(X) = b + w_1x + w_2x^2 + \dots w_dx^d$$

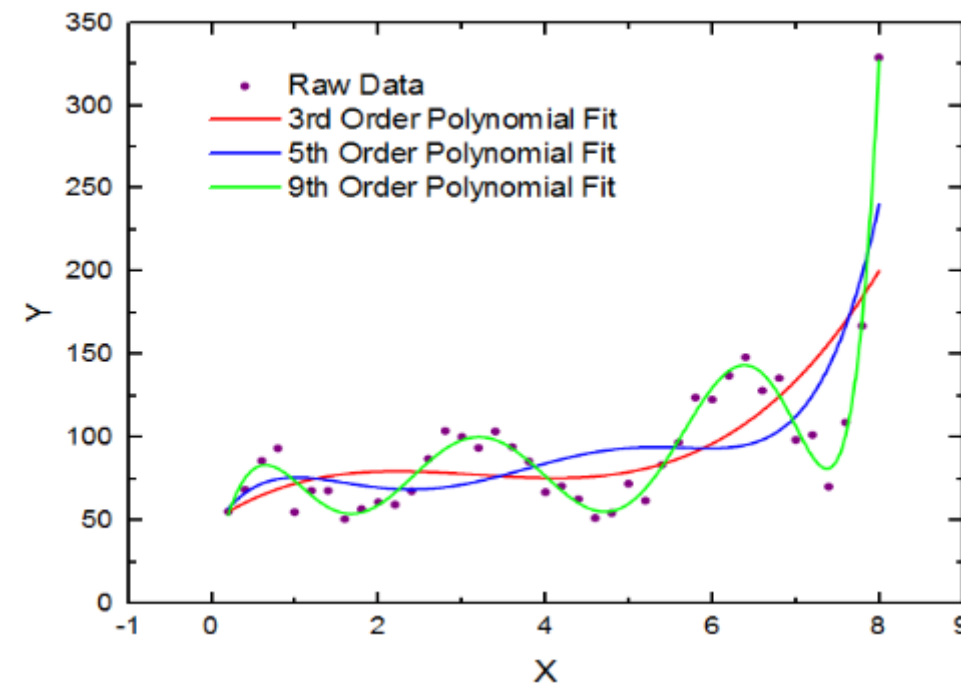
- \mathbf{W} is a d -dimensional vector of parameters
- b is a real number
- d is the polynomial degree of the model (we either fix the d at some reasonably low value, else use **cross-validation** to choose d)

➔ The optimization problem

The optimization problem is defined as:

$$\text{Min}_{w,b} \text{MSE} = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2 = \frac{1}{N} \sum_{i=1}^N (y_i - f_{w,b}(X_i))^2$$

- We use the same **loss function** as in linear regression!
- The solution to this optimization problem is w^* and b^*
- Now we can make predictions!

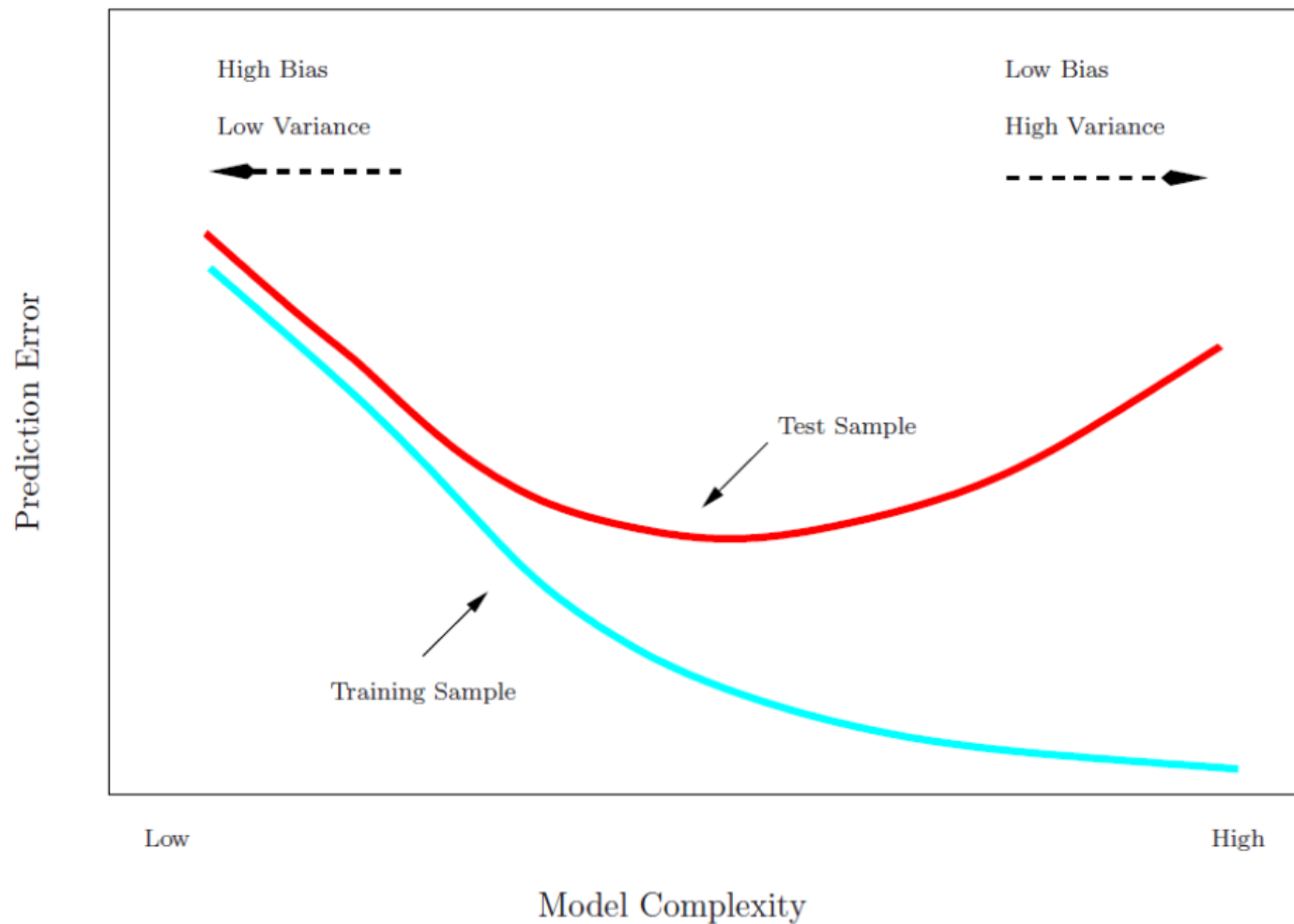




Polynomial Regression Evaluation Metrics

$$MSE = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y})^2$$

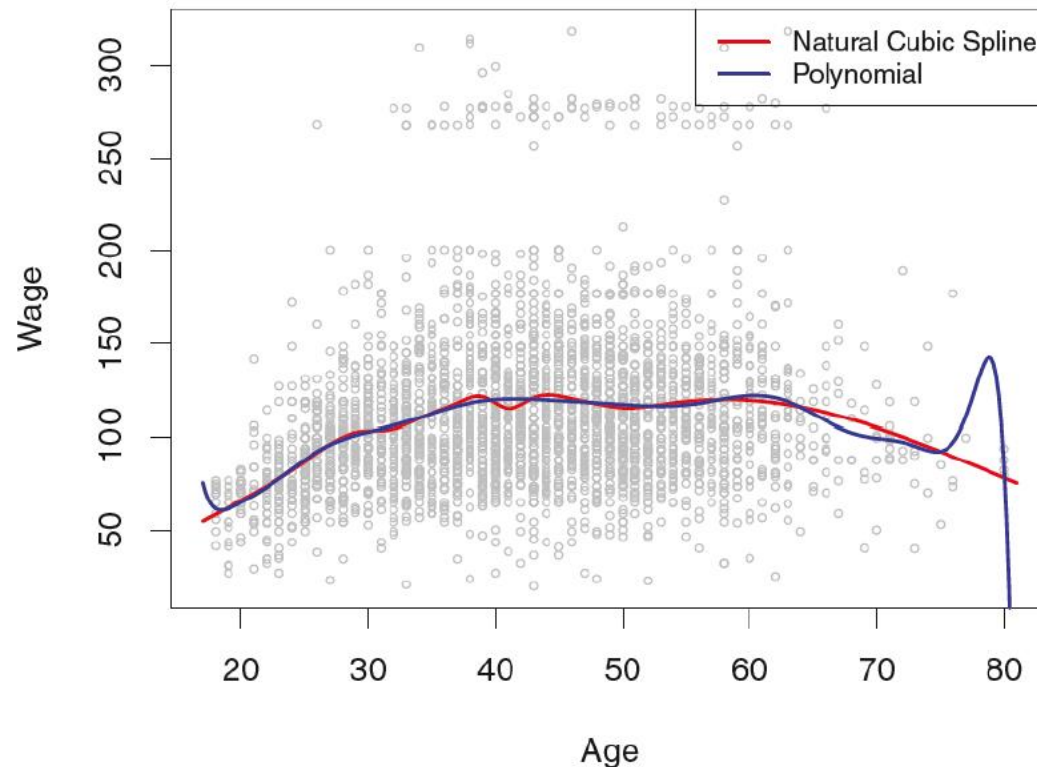
$$RMSE = \sqrt{MSE}$$





Caveats!

- In general quadratic loss functions are **sensitive to outliers**
- Polynomials have **notorious tail behavior**
- Polynomials are **global fit**!
- **Solution**: piecewise polynomial, splines and local regressions.





Piecewise polynomials and splines!

