Chapter 2

Motion in One Dimension

# 2.1 The Important Stuff

## 2.1.1 Position, Time and Displacement

We begin our study of motion by considering objects which are very small in comparison to the size of their movement through space. When we can deal with an object in this way we refer to it as a particle. In this chapter we deal with the case where a particle moves along a straight line.

The particle’s location is specified by its coordinate, which will be denoted by x or y. As the particle moves, its coordinate changes with the time, t. The change in position from x1 to x2 of the particle is the displacement ∆x, with ∆x = x2 − x1.

## 2.1.2 Average Velocity and Average Speed

When a particle has a displacement ∆x in a change of time ∆t, its average velocity for that time interval is

~~v~~ = ∆x = x2 − x1 (2.1)



∆t t2 − t1

The average speed of the particle is absolute value of the average velocity and is given by

Distance travelled

~~s~~ =  (2.2)

∆t

In general, the value of the average velocity for a moving particle depends on the initial and final times for which we have found the displacements.

## 2.1.3 Instantaneous Velocity and Speed

We can answer the question “how fast is a particle moving at a particular time t?” by finding the instantaneous velocity. This is the limiting case of the average velocity when the time

27

interval ∆t include the time t and is as small as we can imagine:

∆x dx

v = lim = (2.3)



∆t→0 ∆t dt

The instantaneous speed is the absolute value (magnitude) of the instantaneous velocity.

If we make a plot of x vs. t for a moving particle the instantaneous velocity is the slope of the tangent to the curve at any point.

## 2.1.4 Acceleration

When a particle’s velocitychanges, then we way that the particle undergoes an acceleration.

If a particle’s velocity changes from v1 to v2 during the time interval t1 to t2 then we define the average acceleration as

~~v~~ = ∆x = x2 − x1 (2.4)



∆t t2 − t1

As with velocity it is usually more important to think about the instantaneous acceleration, given by

∆v dv

a = lim = (2.5)



∆t→0 ∆t dt

If the acceleration a is positive it means that the velocity is instantaneously increasing; if a is negative, then v is instantaneously decreasing. Oftentimes we will encounter the word deceleration in a problem. This word is used when the sense of the acceleration is opposite that of the instantaneous velocity (the motion). Then the magnitude of acceleration is given, with its direction being understood.

## 2.1.5 Constant Acceleration

A very useful special case of accelerated motion is the one where the acceleration a is constant. For this case, one can show that the following are true:

v = v0 + at (2.6) x = x0 + v0t + at2 (2.7)

|  |  |  |  |
| --- | --- | --- | --- |
| v2 x | =  = | v02 + 2a(x − x0) x0 + 12(v0 + v)t | (2.8)  (2.9) |

In these equations, we mean that the particle has position x0 and velocity v0 at time t = 0; it has position x and velocity v at time t.

These equations are valid only for the case of constant acceleration.

## 2.1.6 Free Fall

An object tossed up or down near the surface of the earth has a constant downward acceleration of magnitude 9.80 sm2. This number is always denoted by g. Be very careful about the sign; in a coordinate system where the y axis points straight up, the acceleration of a freely–falling object is

ay = −9.80 sm2 = −g (2.10)

Here we are assuming that the air has no effect on the motion of the falling object. For an object which falls for a long distance this can be a bad assumption.

Remember that an object in free–fall has an acceleration equal to −9.80 sm2 while it is moving up, while it is moving down, while it is at maximum height... always!



# 2.2 Worked Examples

## 2.2.1 Average Velocity and Average Speed



1. Boston Red Sox pitcher Roger Clemens could routinely throw a fastball at ahorizontal speed of 160 kmhr . How long did the ball take to reach home plate 18.4m away? [HRW5 2-4]

We assume that the ball moves in a horizontal straight line with an average speed of 160 km/hr. Of course, in reality this is not quite true for a thrown baseball.

We are given the average velocity of the ball’s motion and also a particular displacement, namely ∆x = 18.4m. Equation 2.1 gives us:

∆x ∆x ~~v~~ =  =⇒ ∆t = 

∆t ~~v~~

But before using it, it might be convenient to change the units of ~~v~~. We have:

~~v~~ = 160 hr · 1km · 3600s! = 44.4 ms km 1000m 1hr



Then we find:

∆x 18.4m

∆t = ~~v~~ = 44.4 ms = 0.414s



The ball takes 0.414 seconds to reach home plate.



1. Taking the Earth’s orbit to be a circle of radius 1.5 × 108 km, determine the speed of the Earth’s orbital motion in (a) meters per second and (b) miles per

second. [Wolf 2-18]

1. This is not straight line motion of course, but we can sill find an average speed by dividing the distance traveled (around a circular path) by the time interval. Here, the distance traveled by the Earth as it goes once around the Sun is the circumference of the orbit,

C = 2πR = 2π(1.5 × 108 km) = 9.42 × 108 km = 9.42 × 1011 m

and the time interval over which that takes place is one year,

24hr 3600s

1yr = 365.25day ! = 3.16 × 107 s



1day 1hr

so the average speed is

s = Ct = 93.42.16××1010117 ms × [[1]](#footnote-1) m = 2.99 10 s



1. To convert this to mis , use 1mi = 1.609km. Then

 × 4 m 1×mi = 18.6 mis s = 2.99 10 s 1.609 103 m

## 2.2.2 Acceleration



3. An electron moving along the x axis has a position given by x = (16te−t)m, where t is in seconds. How far is the electronfrom the origin when it momentarily stops? [HRW6 2-20]

To find the velocity of the electron as a function of time, take the first derivative of x(t):

dx −t −t −t m

v = dt = 16e − 16te = 16e (1 − t) s

again where t is in seconds, so that the units for v are ms .

Now the electron “momentarily stops” when the velocity v is zero. From our expression for v we see that this occurs at t = 1s. At this particular time we can find the value of x:

x(1s) = 16(1)e−1 m = 5.89m

The electron was 5.89m from the origin when the velocity was zero.

(a) From Eq. 2.3 we find v(t) from x(t):

dx d − 3 − t2 v(t) = = (20t 5t ) = 20 15 dt dt



where, if t is in seconds then v will be in ms . The velocity v will be zero when

20 − 15t2 = 0

which we can solve for t:

 2 ⇒ t2 = 20 = 1.33s2

15t = 20 =

15

(The units s2 were inserted since we know t2 must have these units.) This gives:

t = ±1.15s

(We should be careful... t may be meaningful for negative values!) (b) From Eq. 2.5 we find a(t) from v(t):

dv d − 2) = −30t a(t) = = (20 15t dt dt



where we mean that if t is given in seconds, a is given in sm2. From this, we see that a can be zero only at t = 0.

1. From the result is part (b) we can also see that a is negative whenever t is positive. a is positive whenever t is negative (again, assuming that t < 0 has meaning for the motion of this particle).
2. Plots of x(t), v(t), and a(t) are given in Fig. 2.1.



5. In an arcade video game a spot is programmed to move across the screen according to x = 9.00t−0.750t3, where x is distance in centimeters measured from the left edge of the screen and t is time in seconds. When the spot reaches a screen edge, at either x = 0 or x = 15.0cm, t is reset to 0 and the spot starts moving again according to x(t). (a) At what time after starting is the spot instantaneously at rest? (b) Where does this occur? (c) What is its acceleration when this occurs? (d) In what direction is it moving just prior to coming to rest? (e) Just after? (f) When does it first reach an edge of the screen after t = 0? [HRW5 2-31]

1. This is a question about the instantaneous velocity of the spot. To find v(t) we calculate:

dx d − 3 .00 − 2.25t2 v(t) = = (9.00t 0.750t ) = 9 dt dt



where this expression will give the value of v in cms when t is given in seconds.

2.5

2.0

1.5

1.0

0.5

0.0

-0.5

-1.0

3.0

**t**

-60

-40

-20

0

20

**x, (m)**

-1.0

-0.5

0.0

0.5

1.0

1.5

2.0

2.5

3.0

-100

-50

0

**v, (m/s)**

**t**

2.5

3.0

-1.0

-0.5

0.0

0.5

1.0

1.5

2.0

**t**

-100

-50

0

**a, (m/s**

**2**

**)**

Figure 2.1: Plot of x(t), v(t), and a(t) for Example 4.

We want to know the value of t for which v is zero, i.e. the spot is instantaneously at rest. We solve:

 − 2 = 0 =⇒ t2 = 9.00 = 4.00s2

9.00 2.25t

2.25

(Here we have filled in the proper units for t2 since by laziness they were omitted from the first equations!) The solutions to this equation are

t = ±2.00s

but since we are only interested in times after the clock starts at t = 0, we choose t = 2.00s.

1. In this part we are to find the value of x at which the instantaneous velocity is zero. In part (a) we found that this occurred at t = 3.00s so we calculate the value of x at t = 2.00s:

x(2.00s) = 9.00 · (2.00) − 0.750 · (2.00)3 = 12.0cm

(where we have filled in the units for x since centimeters are implied by the equation). The dot is located at x = 12.0cm at this time. (And recall that the width of the screen is

15.0cm.)

1. To find the (instantaneous) acceleration at all times, we calculate:

dv d − 2 − .50t a(t) = = (9.00 2.25t ) = 4 dt dt



where we mean that if t is given in seconds, a will be given in sm2. At the time in question (t = 2.00s) the acceleration is

a(t = 2.00s) = −4.50 · (2.00) = −9.00

that is, the acceleration at this time is −9.00 sm2.

1. From part (c) we note that at the time that the velocity was instantaneously zero the acceleration was negative. This means that the velocity was decreasing at the time. If the velocity was decreasing yet instantaneously equal to zero then it had to be going from positive to negative values at t = 2.00s. So just before this time its velocity was positive.
2. Likewise, from our answer to part (d) just after t = 2.00s the velocity of particle had to be negative.
3. We have seen that the dot never gets to the right edge of the screen at x = 15.0cm. It will not reverse its velocity again since t = 2.00s is the only positive time at which v = 0. So it will keep moving to back to the left, and the coordinate x will equal zero when we have:

x = 0 = 9.00t − 0.750t3

Factor out t to solve:

t(9.00 − 0.750t2) = 0 =⇒ t(9= 0.00 − 0.750t2) = 0 orotherwise.

**-1**

**-10.0**

**-5.0**

**0.0**

**5.0**

**10.0**

**15.0**

**x, cm**

**0**

**1**

**2**

**4**

**3**

**t, s**

Figure 2.2: Plot of x vs t for moving spot. Ignore the parts where x is negative!

The first solution is the time that the dot started moving, so that is not the one we want. The second case gives:

 − 2) = 0 =⇒ t2 = 9.00 = 12.0s2

(9.00 0.750t

0.750

which gives t = 3.46s

since we only want the positive solution. So the dot returns to x = 0 (the left side of the screen) at t = 3.46s.

If we plot the original function x(t) we get the curve given in Fig. 2.2 which shows that the spot does not get to x = 15.0cm before it turns around. (However as explained in the problem, the curve does not extend to negative values as the graph indicates.)

## 2.2.3 Constant Acceleration



1. The head of a rattlesnake can accelerate 50 sm2 in striking a victim. If a car could do as well, how long would it take to reach a speed of 100 kmhr from rest?

[HRW5 2-33]

First, convert the car’s final speed to SI units to make it easier to work with:

100 hr = 100 hr ! · 1km · 3600s! = 27.8 ms km km 1000m 1hr



The acceleration of the car is 50 sm2 and it starts from rest which means that v0 = 0. As we’ve found, the final velocity v of the car is 27.8 ms . (The problem actually that this is final speed but if our coordinate system points in the same direction as the car’s motion, these are the same thing.) Equation 2.6 lets us solve for the time t:

 v = v0 + at =⇒ t = v − v0

a Substituting, we find t = 27.508 mssm2− 0 = 0.55s

If a car had such a large acceleration, it would take 0.55s to attain the given speed.



1. A body moving with uniform acceleration has a velocity of 12.0 cms when its x coordinate is 3.00cm. If its x coordinate 2.00s later is −5.00cm, what is the

magnitude of its acceleration? [Ser4 2-25]

In this problem we are given the initial coordinate (x = 3.00cm), the initial velocity (v0 = 12.0 cms ), the final x coordinate (x = −5.00cm) and the elapsed time (2.00s). Using Eq. 2.7 (since we are told that the acceleration is constant) we can solve for a. We find:

x = x0 + v0t + at2 =⇒ at2 = x − x0 − v0t

Substitute things:

at2 = −5.00cm − 3.00cm − 12.0 cms (2.00s) = −32.0cm

Solve for a:

a = − t2 = −(2.00s)2 = −16.0cms2 2( 32.0cm) 2( 32.0cm)



The x acceleration of the object is −16. cms2 . (The magnitude of the acceleration is 16.0 cms2 .)



1. A jet plane lands with a velocity of 100 ms and can accelerate at a maximum rate of −5.0 ms2 as it comes to rest. (a) From the instant it touches the runway, what is the minimum time needed before it stops? (b) Can this plane land at a small airport where the runway is 0.80km long? [Ser4 2-31]
2. The data given in the problem is illustrated in Fig. 2.3. The minus sign in the acceleration indicates that the sense of the acceleration is opposite that of the motion, that is, the plane is decelerating.

The plane will stop as quickly as possible if the acceleration does have the value −5.0 sm2, so we use this value in finding the time t in which the velocity changes from v0 = 100 ms to v = 0. Eq. 2.6 tells us:

t = v − v0

a Substituting, we find:

t = (0 − 100 ms ) = 20s



*a* = -5.0 m/s2



m/s

100

*v*

=

0

x

Figure 2.3: Plane touches down on runway at 100 ms and comes to a halt.

The plane needs 20s to come to a halt.

1. The plane also travels the shortest distance in stopping if its acceleration is −5.0 sm2. With x0 = 0, we can find the plane’s final x coordinate using Eq. 2.9, using t = 20s which we got from part (a):

x = x0 + 21(v0 + v)t = 0 + (100 ms + 0)(20s) = 1000m = 1.0km

The plane must have at least 1.0km of runway in order to come to a halt safely. 0.80km is not sufficient.



9. A drag racer starts her car from rest and accelerates at 10.0 sm2 for the entire distance of 400m ( mile). (a) How long did it take the car to travel this distance? (b) What is the speed at the end of the run? [Ser4 2-33]

1. The racer moves in one dimension (along the x axis, say) with constant acceleration a = 10.0 sm2. We can take her initial coordinate to be x0 = 0; she starts from rest, so that v0 = 0. Then the location of the car (x) is given by:

x = x0 + v0t + at2

= = 0 + 0 + 21at2 = (10.0 sm2)t2

We want to know the time at which x = 400m. Substitute and solve for t:

m2)t2 =t2 = 2(400m) = 80.0s2 400m = (10.0 s 

⇒

s

which gives

t = 8.94s .

The car takes 8.94s to travel this distance.

1. We would like to find the velocity at the end of the run, namely at t = 8.94s (the time we found in part (a)). The velocity is:

v = v0 + at

= 0 + (10.0 sm2)t = (10.0 sm2)t

cm

1.0

Path of

electron

Voltage source

Accelerating

region

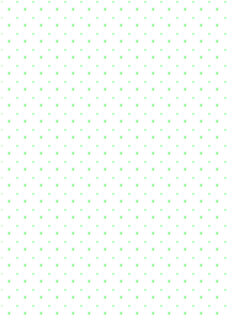


Figure 2.4: Electron is accelerated in a region between two plates, in Example 10.

At t = 8.94s, the velocity is

v = (10.0 sm2)(8.94s) = 89.4 ms

The speed at the end of the run is 89.4 ms .



10. An electron with initial velocity v0 = 1.50 × 105 ms enters a region 1.0cm long where it is electrically accelerated, as shown in Fig. 2.4. It emerges with velocity v = 5.70 × 106 ms . What was its acceleration, assumed constant? (Such a process occurs in the electron gun in a cathode–ray tube, used in television receivers

and oscilloscopes.) [HRW5 2-39]

We are told that the acceleration of the electron is constant, so that Eqs. 2.6–2.9 can be used.

Here we know the initial and final velocities of the electron (v0 and v). If we let its initial coordinate be x0 = 0 then the final coordinate is x = 1.0cm = 1.0×10−2 m. We don’t know the time t for its travel through the accelerating region and of course we don’t know the (constant) acceleration, which is what we’re being asked in this problem. We see that we can solve for a if we use Eq. 2.8:

 v2 = v02 + 2a(x − x0) =⇒ a = v2 −−v02

2(x x0)

Substitute and get:

(5.70 × 106 ms )2 − (1.50 × 105 ms )2

a = 2(1.0 × 10−2 m)

= 1.62 × 1015 sm2

The acceleration of the electron is 1.62 × 1015 sm2 (while it is in the accelerating region).



1. A world’s land speed record was set by Colonel John P. Stapp when onMarch 19, 1954 he rode a rocket–propelled sled that moved down a track at 1020 kmh . He and the sled were brought to a stop in 1.4s. What acceleration did he experience? Express your answer in g units. [HRW5 2-41]

For the period of deceleration of the rocket sled (which lasts for 1.4s) were are given the initial velocity and the final velocity, which is zero since the sled comes to rest at the end. First, convert his initial velocity to SI units:

km = (1020 kmh ) 101km3 m! 36001h s! = 283.3 ms



v0 = 1020 h

The Eq. 2.6 gives us the acceleration a:

|  |  |
| --- | --- |
|  | v = v0 + at =⇒ a = − 0 t |
| Substitute: | 0 283.3 m |

v v

 a = −1.4s s = −202.4 sm2

The acceleration is a negative number since it is opposite to the sense of the motion; it is a deceleration. The magnitude of the sled’s acceleration is 202.4 sm2. To express this as a multiple of g, we note that

|ga| = 2029.80.4smsm22 = 20.7



so the magnitude of the acceleration was |a| = 20.7g. That’s a lotta g’s!



1. A subway train is traveling at 80 kmh when it approaches a slower train 50m ahead traveling in the same direction at 25 kmh . If the faster train begins decelerating at 2.1 sm2 while the slower train continues at constant speed, how soon and at what relative speed will they collide? [wolf 2-73]

First, convert the initial speeds of the trains to units of ms . We find:

80 kmh = 22.2 ms 25 kmh = 6.94 ms .

The situation of the trains at t = 0 (when the rear train begins to decelerate) is shown in Fig. 2.5. We choose the origin of the x axis to be at the initial position of the rear train; then the initial position of the front train is x = 50m. If we call the x–coordinate of the rear train x1, then since it has initial velocity 22.2 ms and acceleration −2.1 sm2 (note the minus sign!) the equation for x1(t) is

x1(t) = (22.2 ms )t + (−2.1 sm2)t2 = (22.2 ms )t + (−1.05 sm2)t2

Meanwhile, the front car has an initial velocity of 6.94 ms and no acceleration, so its coordinate (x2) is given by

x2(t) = 50m + (6.94 ms )t



m/s

22.2

*a*

= -2.1 m/s

2

m/s

6.94

1



2

50

m

*x*

*2*

*x*

*1*

Figure 2.5:

Two subway trains in Example 12.

The trains will collide if there is ever a time at which their coordinates are equal. So we want to see if there is a t which gives the condition:

(22.2 ms )t + (−1.05 sm2)t2 = 50m + (6.94 ms )t

This is a quadratic equation, for which we can use the quadratic formula. Neglecting the units for simplicity, we can rearrange the terms and rewrite it as

1.05t2 − 15.28t + 50 = 0

and the quadratic formula gives the answers as

58s

= = (

−

b

±

√



b

2

−

4

ac



15

.

28

±

q



(15

.

28)

2

−

4(1

.

05)(50)



9

.

2a 2(1.05) 4.97s

This is a little confusing because there are two possible answers! (Both values of t are positive.) But the answer we want is the first one, 4.97s — after the collision, the second time is not relevant1. So the trains will collide t = 4.97s after the rear car begins to decelerate.

At the time we have found, the velocity of the rear train is

v = v0 + at = 22.2 ms + (−2.1 sm2)(4.97s) = 11.8 ms

and the velocity of the front train remains 6.94 ms . So at the time of the collision, the rear train is going faster by a difference of

∆v = 11.8 ms − 6.94 ms = 4.8 ms

That is the relative speed at which the collision takes place.

## 2.2.4 Free Fall



y

v

0

*y*

=

0 m

*y*

=50

m

Figure 2.6: Object thrown upward reaches height of 50m.

13. (a) With what speed must a ball be thrown vertically from ground level to rise to a maximum height of 50m? (b) How long will it be in the air? [HRW5 2-61]

(a) First, we decide on a coordinate system. I will use the one shown in Fig. 2.6, where the y axis points upward and the origin is at ground level. The ball starts its flight from ground level so its initial position is y0 = 0. When the ball is at maximum height its coordinate is y = 50m, but we also know its velocity at this point. At maximum height the instantaneous velocity of the ball is zero. So if our “final” point is the time of maximum height, then v = 0.

So for the trip from ground levelto maximum height, we know y0, y, v and the acceleration a = −9.8 sm2 = −g, but we don’t know v0 or the time t to get to maximum height.

From our list of constant–acceleration equations, we see that Equation 2.8 will give us the initial velocity v0:

v2 = v02 + 2a(y − y0) =⇒ v02 = v2 − 2a(y − y0)

Substitute, and get: v02 = (0)2 − 2(−9.8 sm2)(50m − 0) = 980 ms22 The next step is to “take the square root”. Since we know that v0 must be a positive number, we know that we should take the positive square root of 980 ms22. We get:

v0 = +31 ms

The initial speed of the ball is 31 ms

(b) We want to find the total time that the ball is in flight. What do we know about the ball when it returns to earth and hits the ground? We know that its y coordinate is equal to zero. (So far, we don’t know anything about the ball’s velocity at the the time it returns to ground level.) If we consider the time between throwing and impact, then we do know y0, y, v0 and of course a. If we substitute into Eq. 2.7 we find:



1However it would be relevant if the trains were on parallel tracks; then the collision would not take place and we could find the times at which they were side-by-side and their relative velocities at those times.

8.00

m/s

y

30.0

m

Figure 2.7: Ball is thrown straight down with speed of 8.00 ms , in Example 14.

0 = 0 + (31 ms )t + (−9.8 sm2)t2

It is not hard to solve this equation for t. We can factor it to give:

t[(31 ms ) + (−9.8 sm2)t] = 0

which has two solutions. One of them is simply t = 0. This solution is an answer to the question we are asking, namely “When does y = 0?” because the ball was at ground level at t = 0. But it is not the solution we want. For the other solution, we must have:

(31 ms ) + (−9.8 sm2)t = 0

which gives

t = 2(319.8 smms2 ) = 6.4s

The ball spends a total of 6.4 seconds in flight.



1. A ball is thrown directly downward with an initial speed of 8.00 ms from a height of 30.0m. When does the ball strike the ground? [Ser4 2-46]

We diagram the problem as in Fig. 2.7. We have to choose a coordinate system, and here I will put the let the origin of the y axis be at the place where the ball starts its motion (at the top of the 30m height). With this choice, the ball starts its motion at y = 0 and strikes the ground when y = −30m.

We can now see that the problem is asking us: At what time does y = −30.0m? We have v0 = −8.00 ms (minus because the ball is thrown downward!) and the acceleration of the the ball is a = −g = −9.8 sm2, so at any time t the y coordinate is given by

y = y0 + v0t + 21at2 = (−8.00 ms )t − gt2

v

0

y

4.00

m

Figure 2.8: Student throws her keys into the air, in Example 15.

But at the time of impact we have

y = −30.0m = (−8.00 ms )t − 21gt2 = (−8.00 ms )t − (4.90 sm2)t2 ,

an equation for which we can solve for t. We rewrite it as:

(4.90 sm2)t2 + (8.00 ms )t − 30.0m = 0

which is just a quadratic equation in t. From our algebra courses we know how to solve this; the solutions are:

30.0m)

−

(8

.

00

m



s

)

±

q



(8

.

00

m



s

)

2

−

4(4

.

90

m



s

2

)(

−



2(4

.

90

m



)

t =

s2

and a little calculator work finally gives us: t = −

3.42s 1.78s

Our answer is one of these ...which one? Obviously the ball had to strike the ground at some positive value of t, so the answer is t = 1.78s.

The ball strikes the ground 1.78s after being thrown.



1. A student throws a set of keys vertically upward to her sorority sister in awindow 4.00m above. The keys are caught 1.50s later by the sister’s outstretched hand. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught? [Ser4 2-47]
2. We draw a simple pictureof the problem; such a simple pictureis given in Fig. 2.8. Having a picture is important, but we should be careful not to put too much into the picture; the problem did not say that the keys were caught while they were going up or going down. For all we know at the moment, it could be either one!

We will put the origin of the y axis at the point where the keys were thrown. This simplifies things in that the initial y coordinate of the keys is y0 = 0. Of course, since this is a problem about free–fall, we know the acceleration: a = −g = −9.80 sm2.

What mathematical information does the problem give us? We are told that when t = 1.50s, the y coordinate of the keys is y = 4.00m. Is this enough information to solve the problem? We write the equation for y(t):

y = y0 + v0t + at2 = v0t − gt2

where v0 is presently unknown. At t = 1.50s, y = 4.00m, so:

4.00m = v0(1.50s) − (9.80 sm2)(1.50s)2 .

Now we can solve for v0. Rearrange this equation to get:

v0(1.50s) = 4.00m + (9.80 sm2)(1.50s)2 = 15.0m .

So:

15.0m m

v0 = 1.50s = 10.0 s

1. We want to find the velocity of the keys at the time they were caught, that is, at t = 1.50s. We know v0; the velocity of the keys at all times follows from Eq. 2.6,

v = v0 + at = 10.0 ms − 9.80 sm2t

So at t = 1.50s, v = 10.0 ms − 9.80 sm2(1.50s) = −4.68 ms .

So the velocity of the keys when they were caught was −4.68 ms . Note that the keys had a negative velocity; this tells us that the keys were moving downward at the time they were caught!





16. A ball is thrown vertically upward from the ground with an initial speed of 15.0 ms . (a) How long does it take the ball to reach its maximum altitude? (b) What is its maximum altitude? (c) Determine the velocity and acceleration of the ball at t = 2.00s. [Ser4 2-49]

1. An illustration of the data given in this problem is given in Fig. 2.9. We measure the coordinate y upward from the place where the ball is thrown so that y0 = 0. The ball’s acceleration while in flight is a = −g = −9.80 sm2. We are given that v0 = +15.0 ms .

The ball is at maximum altitude when its (instantaneous) velocity v is zero (it is neither going up nor going down) and we can use the expression for v to solve for t:

 v = v0 + at =⇒ t = v − v0

a

v

o

= +15.0

m/s

v = 0

m/s

y

a = -9.80

m/s

2

Figure 2.9: Ball is thrown straight up with initial speed 15.0 ms .

Plug in the values for the top of the ball’s flight and get:

t = (0) − (15.0 ms ) = 1.53s .



The ball takes 1.53s to reach maximum height.



1. Now that we have the value of t when the ball is at maximum height we can plug it into Eq. 2.7 and find the value of y at this time and that will be the value of the maximum height. But we can also use Eq. 2.8 since we know all the values except for y. Solving for y we find:

 v2 = v02 + 2ay =⇒ y = v2 − v02

2a Plugging in the numbers, we get

y = (0)2 − (15.0 ms )2 = 11.5m



The ball reaches a maximum height of 11.5m .



1. At t = 2.00s (that is, 2.0 seconds after the ball was thrown) we use Eq. 2.6 to find:

v = v0 + at = (15.0 ms ) + (−9.80 ms2)(2.00s) = −4.60 ms .

so at t = 2.00s the ball is on its way back down with a speed of 4.60 ms .

As for the next part, the acceleration of the ball is always equal to −9.80 sm2 while it is in flight.





17. A baseball is hit such that it travels straight upward after being struck by the bat. A fan observes that it requires 3.00s for the ball to reach its maximum height. Find (a) its initial velocity and (b) its maximum height. Ignore the

effects of air resistance. [Ser4 2-51]

v

0

v=0

t = 3.00 s

Figure 2.10: Ball is hit straight up; reaches maximum height 3.00s later.

1. An illustration of the data given in the problem is given in Fig. 2.10.

For the period from when the ball is hit to the time it reaches maximum height, we know the time interval, the acceleration (a = −g) and also the final velocity, since at maximum height the velocity of the ball is zero. Then Eq. 2.6 gives us v0:

v = v0 + at =⇒ v0 = v − at

and we get: v0 = 0 − (−9.80 sm2)(3.00s) = 29.4 ms

The initial velocity of the ball was +29.4 ms .



1. To find the value of the maximum height, we need to find the value of the y coordinate at time t = 3.00s. We can use either Eq. 2.7 or Eq. 2.8. the latter gives:

 2 v02 + 2a(y − y0) =⇒ (y − y0) = v2 − v02

v =

2a

Plugging in the numbers we find that the change in y coordinate for the trip up was:

y − y0 = 02 − (29.4 ms )2 = 44.1m .



The ball reached a maximum height of 44.1m .





18. A parachutist bails out and freely falls 50m. Then the parachute opens, and thereafter she decelerates at 2.0 sm2. She reaches the ground with a speed of 3.0 ms . (a) How long was the parachutist in the air? (b) At what height did the fall

begin? [HRW5 2-84]

(a) This problem gives several odd bits of information about the motion of the parachutist! We organize the information by drawing a diagram, like the one given in Fig. 2.11. It is

**(**

**a**

**)**

**(**

**b**

**)**

**v=0**

**50**

**m**

Free Fall

Deceleration

**v=3.0 m/s (c)**

Figure 2.11: Diagram showing motion of parachutist in Example 18.

very important to organize our work in this way!

At the height indicated by (a) in the figure, the skydiver has zero initial speed. As she falls from (a) to (b) her acceleration is that of gravity, namely 9.80 sm2 downward. We know that (b) is 50m lower than (a) but we don’t yet know the skydiver’s speed at (b). Finally, at point (c) her speed is 3.0 ms and between (b) and (c) her “deceleration” was 2.0 sm2, but we don’t know the difference in height between (b) and (c).

How can we start to fill in the gaps in our knowledge?

We note that on the trip from (a) to (b) we do know the starting velocity, the distance travelled and the acceleration. From Eq. 2.8 we can see that this is enough to find the final velocity, that is, the velocity at (b).

Use a coordinate system (y) which has its origin at level (b), and the y axis pointing upward. Then the initial y coordinate is y0 = 50m and the the initial velocity is v0 = 0. The final y coordinate is y = 0 and the acceleration is constant at a = −9.80 sm2. Then using Eq. 2.8 we have:

v2 = v02 + 2a(y − y + 0) = 0 + 2(−9.80 sm2)(0 − 50m) = 980 ms22

which has the solutions v = ±31.3 ms

but here the skydiver is obviously moving downward at (b), so we must pick

v = −31.3 ms

for the velocity at (b).

While we’re at it, we can find the time it took to get from (a) to (b) using Eq. 2.6, since we know the velocities and the acceleration for the motion. We find:

 v = v0 + at =⇒ t = v − v0

a

Substitute:

t = (−31−9.3.80ms sm−2 0) = 3.19s

The skydiver takes 3.19s to fall from (a) to (b).

Now we consider the motion from (b) to (c). For this part of the motion we know the initial and final velocities. We also know the acceleration, but we must be careful about how it is expressed. During this part of the trip, the skydiver’s motion is always downward (velocity is always negative) but her speed decreases from 31.9 ms to 3.0 ms . The velocity changes from −31.3 ms to −3.0 ms so that the velocity has increased. The acceleration is positive; it is in the opposite sense as the motion and thus it was rightly called a “deceleration” in the problem. So for the motion from (b) to (c), we have

a = +2.0 sm2

We have the starting and final velocities for the trip from (b) to (c) so Eq. 2.6 lets us solve for the time t:

 v = v0 + at =⇒ t = v − v0

a Substitute: t = −3.0 ms = 14.2s

−

(

−

31

.

3

m



s

)



+2

.

0

m



s2

Now we are prepared to answer part (a) of the problem. The time of the travel from (a) was 3.19s; the time of travel from (b) to (c) was 14.2s. The total time in the air was

tTotal = 3.19s + 14.2s = 17.4s

(b) Let’s keep thinking about the trip from (b) to (c); we’ll keep the origin at the same place as before (at (b)). Then for the trip from (b) to (c) the initial coordinate is y0 = 0. The initial velocity is v0 = −31.9 ms and the final velocity is v = −3.0 ms . We have the acceleration, so Eq. 2.8 gives us the final coordinate y:

 v2 = v02 + 2a(y − y0) =⇒ y − y0 = v2 − v02

2a

Substitute:

y − y0 = (−3.0 ms )2 − ( 31.3 ms )2 = −243m

−



m



2(+2.0 s2)

Since we chose y0 = 0, the final coordinate of the skydiver is y = −243m.

We have used the same coordinate system in both parts, so overall the skydiver has gone from y = +50m to y = −243m. The change in height was

∆y = −243m − 50m = −293m

So the parachutist’s fall began at a height of 293m above the ground.





19. A stone falls from rest from the top of a high cliff. a second stone is thrown downward from the same height 2.00s later with an initial speed of 30.0 ms . If both stones hit the ground simultaneously, how high is the cliff? [Ser4 2-54]

This is a “puzzle”–type problem which goes beyond the normal substitute–and–solve type; it involves more organization of our work and a clear understanding of our equations. Here’s the way I would attack it.

We have two different falling objects here with their own coordinates; we’ll put our origin at the top of the cliff and call the y coordinate of the first stone y1 and that of the second stone y2. Each has a different dependence on the time t.

For the first rock, we have v0 = 0 since it falls from rest and of course a = −g so that its position is given by

y1 = y0 + v0t + 21at2 = −gt2

This is simple enough but we need to remind ourselves that here t is the time since the first stone started its motion. It is not the same as the time since the second stone starts its motion. To be clear, let’s call this time t1. So we have:

y1 = −21gt21 = −4.90 sm2t21

Now, for the motion of the second stone, if we write t2 for the time since it started its motion, the facts stated in the problem tell us that its y coordinate is given by:

y2 = y0 + v0t2 + 21at22 = (−30.0 ms )t2 − 12gt22

So far, so good. The problem tells us that the first stone has been falling for 2.0s longer than the second one. This means that t1 is 2.0s larger than t2. So:

t1 = t2 + 2.0s =⇒ t2 = t1 − 2.0s

(We will use t1 as our one time variable.) Putting this into our last equation and doing some algebra gives

y2 = (−30.0 ms )(t1 − 2.0s) − (9.80 sm2)(t1 − 2.0s)2

|  |  |
| --- | --- |
| =  =  = | (−30.0 ms )(t1 − 2.0s) − (4.90 sm2)(t12− 4.0st1 + 4.0s2)2  (−4.90 sm2)t21 + (−30.0 ms + 19.6 ms )t1 + (60.0m − 19.6m)  (−4.90 sm2)t21 + (−10.4 ms )t1 + (40.4m) |

We need to remember that this expression for y2 will be meaningless for values of t1 which are less than 2.0s. With this expression we can find values of y1 and y2 using the same time coordinate, t1.

Now, the problem tells us that at some time (t1) the coordinates of the two stones are equal. We don’t yet yet know what that time or coordinate is but that is the information contained in the statement “both stones hit the ground simultaneously”. We can find this time by setting y1 equal to y2 and solving:

(−4.90 sm2)t21 = (−4.90 sm2)t12+ (−10.4 ms )t1 + (40.4m)

*vA=0* A

B

C

*v*

*B*

*v*

*C*

30.0

m

1.50

s

Figure 2.12: Diagram for the falling object in Example 20.

Fortunately the t2 term cancels in this equation making it a lot easier. We get:

(−10.4 ms )t1 + (40.4m) = 0

which has the solution

40.4m t1 = 10.4 ms = 3.88s

So the rocks will have the same location at t1 = 3.88s, that is, 3.88s after the first rock has been dropped.

What is that location? We can find this by using our value of t1 to get either y1 or y2 (the answer will be the same). Putting it into the expression for y1 we get:

y1 = −4.90 sm2t21 = (−4.90 sm2)(3.88s)2 = −74m

So both stones were 74m below the initial point at the time of impact. The cliff is 74m high.





1. A falling object requires 1.50s to travel the last 30.0m before hitting the ground. From what height above the ground did it fall? [Ser4 2-68]

This is an intriguing sort of problem... very easy to state, but not so clear as to where to begin in setting it up!

The first thing to do is draw a diagram. We draw the important points of the object’s motion, as in Fig. 2.12. The object has zero velocity at A; at B it is at a height of 30.0m above the ground with an unknown velocity. At C it is at ground level, the time is 1.50s later than at B and we also don’t know the velocity here. Of course, we know the acceleration: a = −9.80 sm2!!

We are given all the information about the trip from B to C, so why not try to fill in our knowledge about this part? We know the final and initial coordinates, the acceleration and the time so we can find the initial velocity (that is, the velocity at B). Let’s put the origin at ground level; then, y0 = 30.0m, y = 0 and t = 1.50s, and using

y = y0 + v0t + at2

we find: v0t = (y − y0) − 12at2 = (0 − (30.0m)) − (−9.80 sm2)(1.50)2 = −19.0m

so that

v0 = − t = −(1.50s) = −12.5 ms . ( 19.0m) ( 19.0m)



This is the velocity at point B; we can also find the velocity at C easily, since that is the final velocity, v:

v = v0 + at = (−12.5 ms ) + (−9.80 sm2)(1.50s) = −27.3 ms

Now we can consider the trip from the starting point, A to the point of impact, C. We don’t know the initial y coordinate, but we do know the final and initial velocities: The initial velocity is v0 = 0 and the final velocity is v = −27.3 ms , as we just found. With the origin set at ground level, the final y coordinate is y = 0. We don’t know the time for the trip, but if we use:

v2 = v02 + 2a(y − y0)

we find:

(y − y0) = (v2 − v02) (−27.3 ms )2 − (0)2 = −38.2m



=

2a 

and we can rearrange this to get:

y0 = y + 38.2m = 0 + 38.2m = 38.2m

and the so the object started falling from a height of 38.2m .



There are probably cleverer ways to do this problem, but here I wanted to give you the slow, patient approach!



1. A student is staring idly out her dormitory window when she sees a waterballoon fall past. If the balloon takes 0.22s to cross the 130cm–high window, from what height above the top of the window was it dropped? [Wolf 2-78]

I will set up the vertical coordinate y as shown in Fig. 2.13. The origin is at the place where the balloon was dropped, and we don’t know how far above the window that is. Note, the y axis points downward here, so that y as a function of time is given by y = 21gt2.

Using this system, yet the y coordinate of the top of the window bet y1 and the bottom of the window be y2. Suppose the balloon crosses the top of the window at t1 and the bottom of the window at t2. The problem tells us that

y2 − y1 = 1.30m and t2 − t1 = 0.22s

Using the equation of motion for the balloon, we have

y1 = 12gt21 and y2 = 12gt22

0

y

*y1, t1*

*y2, t2*

Figure 2.13: Diagram for the falling object in Example 21.

In fact at this point the problem is really solved because we have four equations for the four unknowns y1, y2, t1 and t2. We just need to do some math! One way to solve the equations is to substitute for the y’s as:

y2 − y1 = 12gt22 − 12gt21 = g(t22 − t21) = 1.30m

But here we can factor the term t22 − t21 to give:

g(t22 − t21) = 21g(t2 + t1)(t2 − t1) = 1.30m

This gives us t2 + t1:

2(1.30m) 2(1.30m) t2 + t1 = (9.80 sm2)(t2 − t1) = s = 1.206s



Adding this to the equation t2 − t1 = 0.22s gives

2t2 = 1.43 =⇒ t2 = 0.71s =⇒ t1 = 0.49

And then the equation for y1 gives us

y1 = 12gt21 = (9.80 sm2)(0.492s)2 = 1.19m

so that the balloon began its fall 1.19m above the top of the window.



52 CHAPTER 2. MOTION IN ONE DIMENSION

1. . (a) If the position of a particle is given by x = 20t − 5t3, where x is in meters and t is in seconds, when if ever is the particle’s velocity zero? (b) When is its acceleration a zero? (c) When is a negative? Positive? (d) Graph x(t), v(t), and a(t). [HRW5 2-28] [↑](#footnote-ref-1)