What does $O(< expr >)$ mean?

1

What does $\Theta(< expr >)$ mean?

2

What does $\Omega(< expr >)$ mean?

3

Say that the input represents a positive integer, x, what is the size of n?

4

What does it mean by $O(1)$?

5

MergeSort and QuickSort of examples of what algorithmic technique?

6

*What are the three steps of **Divide-and-conquer**?*

7

*What is the **Greedy** algorithm? Give an example in term of the knapsack problem.*

8

The complexity (i.e. running time/space) is bounded by the $< expr >$.

1

The complexity (i.e. space/running time) has the complexity proportional to $< expr >$.

2

The complexity (i.e. running time/space) is at least by the $< expr >$.

3

$\lfloor \log_b x \rfloor + 1$ *Where b is the number representation, usually binary (so 2).*

4

It takes a constant time, no matter the amount of data, to perform the operation.

5

Divide-and-conquer

6

- We divide the input into parts.
- Solve the parts (often recusrively).
- Combine the solutions to give the final result.

Three steps of Divide-and-conquer

7

A greedy algorithm is an algorithmic paradigm that follows the problem solving heuristic of making the locally optimal choice at each stage with the hope of finding a global optimum.
With the knapscak problem, organise all possible items by their value to weight ratio. Then iterate through this list, adding them to the knapsack where possible.

8

Dynamic Programming is a ________ method - we solve all ________ first and then ________ to solve the given problem.

9

Give some examples of optimisation problems with ***dynamic programming****.*

10

Define ***tractable*** *and* ***intractable problems****.*

11

What would the pseudo code be for Euclid's algorithm?

12

What would the pseudo code be for Fast Modular Exponentiation?

13

What are some of the advantages of ElGamal encryption?

14

What is the basic procedure for an encryption and decryption using publik key cryptography if Alice wants to send a message to Bob?

15

Describe public key generation in ElGamal encryption using p as the Prime Modulus and g as the Primitive root (as described in the COMP26120 lab)

16

Dynamic Programming is a bottom-up method - we solve all smaller problems first and then combine them to solve the given problem.

** Some path-finding algorithms use dynamic programming, e.g. Floyd's algorithm.*
** Some text similarity tests, e.g. longest common subsequence.*
** Knapsack problems: The O/1 knapsack can be solved using dynamic programming.*
** Constructiong optimal search trees.*
** Some travelling sales person problems.*
** Genome matching and protein-chain matching.*

Optimisation problems with dynamic problems.

Tractable: An algorithm that can be solved in ***polynomial*** *time.*
Intractable: A problem that can be solved in theory (e.g. given large but finite resources, especially time), but for which in practice any solution takes too many resources to be useful.


```
// Assume a >= b
hcf(a,b)
    if b = 0
        return a
    r = amodb
    return hcf(b,r)
```

Euclid's algorithm


```
fme(a,b,k)
    d = a
    e = b
    s = 1
    While e > 0
        if e is odd
            s = (s.d)modk
        d = d^2 modk
        e = ⌊e/2⌋
    return s
```

Fast Modular Exponentiation

Sender Verification
Private key remains with owner
Public key is freely distributable
No secret channel needed at any point
No need for pre-shared keys

Alice generates a private random integer a and Bob generates a private random integer b
Alice generates her public value $g^a \bmod p$
Bob generates his public value $g^b \bmod p$
Alice computes $g^{ab} = (g^a)^b \bmod p$
Bob computes $g^{ba} = (g^b)^a \bmod p$
Now they have a shared secret k since $k = g^{ab} = g^{ba}$

Generate a large p and a g in $1 \leq g < p$
Generate a random integer a in $1 \leq a \leq p - 2$
Compute $g^a \bmod p$. The public key is

$$(p, g, g^a)$$

The private key is a

Describe the encryption procedure used in the ElGamal cryptosystem given that person B wants to send message M to preson A

17

Describe the decryption process used in the ElGamal cryptosystem given that person A has received cyphertext (γ, δ) from person B, encrypted encrypted using the public key (p, g, g^a)

18

Consider the equation $a^x = y \bmod p$. If a is a primitive root of modulo p, then for every $y(1 \leq y < p)$, such an $x(1 \leq x < p)$ exists. What is x?

19

The is the inverse of exponentiation.

20

Why can a private key in the ElGamal cryptosystem not, in practice, be recovered using the public key when p is large?

21

What is one way you can argue correctness of Euclid's algorithm?

22

What would half the correctness proof be for Euclid's algorithm?

23

$(a.b) modk =$

24

Obtain A's public key (p, g, g^a)
Represent the message M as integers in the range $0, ..., p-1$
Select a random integer k *from* $1 \leq k \leq p-2$
Compute $\gamma = g^k \bmod p$ *and* $\delta = m \cdot (g^a)^k$
Send ciphertext $c = (\gamma, \delta)$ *to A*

17

Use private key a *to compute* $(\gamma^{p-1-a}) \bmod p$
NOTE THAT: $(\gamma^{p-1-a}) = \gamma^{-a} = g^{-ak}$
Recover the message M by computing $(\gamma^{-a} \cdot \delta \bmod p)$
Note that this evaluates to $(g^{-ak} \cdot g^{ak} \cdot M \bmod p)$ *or* $1 \cdot M \bmod p$

18

X is the ***discrete logarithm*** *of* y *with base* a*, modulo* p*.*

19

The discrete logarithm is the inverse of exponentiation.

20

To calculate a public key, y*, a private key,* x *is needed. The equation for modular exponentiation can be used to generate the public key:* $y = g^x \bmod p$ *where* g *is a primitive root of the modulus* p*.*
It is considered a one-way, or trapdoor function - easy to compute, hard to invert. For a large p*, one of the few ways to figure out the private key* x *would be to calculate* $g^x \bmod p$ *for every* x *in* $1 \leq x < p$ *and find when one of these results matches* y

21

Let $r = amodb$*.* $hcf(a, b) = hcf(b, r)$ *because all factors of a and b are also factors of b and r and vice versa. If they have the same factors, they have the same highest common factor.*

22

As $r = a \bmod b$*,* $\exists q$ *such that* $a = bq + r$*,*
$\therefore r = a - bq$*.*
Suppose x *is a factor of* a *and* b*, then* $\exists y and z$ *such that* $a = xy$*,* $b = xz$*.*
Hence: $r = xy - xzq$*,* $r = x(y - zq)$*.*
$\therefore x$ *is a factor of* r *(and also of* b *and* r*).*

23

$(a.b)modk = (amodk.bmodk)modk$

24

Let p be a prime number. What is meant by a primitive root modulo p?

25

What are the best, average and worst case complexities of ***Bubble Sort****?*

26

What are the best, average and worst case complexities of ***Merge Sort****?*

27

Give pseudo code for merging 2 sorted lists, as part of merge sort.

28

Give pseudo code for MergeSort(L).

29

What are the best, average and worst case complexities of ***Quick Sort****?*

30

What would the pseudo code be for Quick Sort?

31

What is the minimum time for any sorting algorithm that uses only number comparisons?

32

Best: $O(n)$,
Average: $O(n^2)$,
Worst: $O(n^2)$

26

The numbers r_x *between 1 and* $p-1$ *that, when raised by the numbers between 1 and* $p-1$ *compute all the numbers between 1 and* $p-1$ *in some order with no repetitions.*

25

Merge(L_1, L_2*)*
    if $L_1 = []$ *return* L_2
    if $L_2 = []$ *return* L_1
    $x_1 = L_1[0]$
    $x_2 = L_2[0]$
    $L_1' = L_1[1 : |L_1| - 1]$
    $L_2' = L_2[1 : |L_2| - 1]$
    if $x_1 \leq x_2$
        return $[x_1]$+*Merge(*L_1' ,L_2*)*
    return $[x_2]$+*Merge(*L_1 ,L_2'*)*

Merge two sorted lists

28

Best: $O(n \log_2 n)$,
Average: $O(n \log_2 n)$,
Worst: $O(n \log_2 n)$

27

Best: $O(n \log_2 n)$,
Average: $O(n \log_2 n)$,
Worst: $O(n^2)$

30

MergeSort(L)
    if $|L| \leq 1$
        return L
    Split L into roughly equal halves, L_l *and* L_r
    return Merge(MergeSort(L_l*),MergeSort(*L_r*))*

MergeSort(L)

29

$n \log_2 n$

32

quicksort(L)
    if length of $L \leq 1$
        return L
    remove the first element, x, from L
    $L_{\leq}$:= *elements of L less than or equal to x*
    $L_{>}$:= *elements of L greater than x*
    L_l := *quicksort(*$L_{\leq}$*)*
    L_r := *quicksort(*$L_{>}$*)*
    return L_l + *[x]* + L_r

Quick Sort

31

What does saying that algorithm A runs in time g mean?

33

What is a permutation of a set?

34

What do we mean by a composition of two permutations?

35

What is the number of possible permutations on an n-element set?

36

In the context of a permutation, what do we mean by a transposition?

37

Convert this pair of simultaneous equations into matrix form

$$a_{1,1}x_1 + a_{1,2}x_2 = b_1$$
$$a_{2,1}x_2 + a_{2,2}x_2 = b_2$$

38

What is the determinant of the matrix:

$$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$$

39

What is an upper triangular matrix and how do you calculate its determinant?

40

A 1-to-1 map of the set onto itself. In basic terms, it is a set mapped to another order of itself. i.e $[0, 1, 2, 3, 4] \mapsto [2, 4, 1, 0, 3]$

34

Given an input of size n, the number of operations executed by A is bounded above by g(n).

33

$n!$

36

The composition is the product of two permutations, α *and* β*, on a set n, given by* $\alpha \cdot \beta(n)$ *or* $\beta(\alpha(n))$

35

$$\begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

38

A transposition is a special kind of permutation where only 2 elements in a set are affected (they are swapped). On a set X *a transposition* $\sigma = (i, j)$ *is given by*

$$\sigma(k) = \begin{cases} j & \text{if } k = i \\ i & \text{if } k = j \\ k & \text{ow.} \end{cases}$$

37

It is a matrix where all of its entries below the diagonal are zero.

$$\begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ 0 & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{n,n} \end{pmatrix}$$

Its determinant is calculated by taking the product of the entries on the diagonal. i.e $a_{1,1} \cdot a_{2,2} \cdot ... \cdot a_{n,n}$

40

$a_1a_4 - a_2a_3$

Often denoted as:

$$\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix}$$

The original system of equations to which the matrix corresponds only has a unique solution if the determinant is non-zero.

39

Which 4 operations have no effect on a matrix's determinant?

41

In a tree, if node u is the ***parent (ancestor)*** *node of v, then v is a ( ) of u. Two children of the same parent are .*

42

What is a tree?

43

In a tree, an external node is known as . It has no .

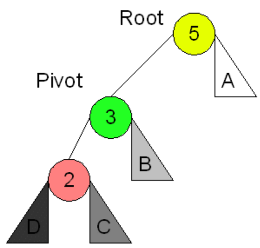
44

In a tree, an internal node has one or more .

45

What do we do here?

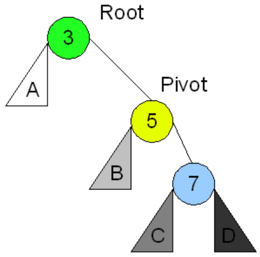
Left Left Case



46

What do we do here?

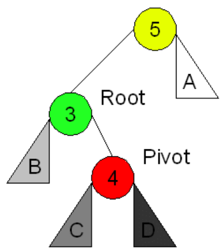
Right Right Case



47

What do we do here?

Left Right Case



48

In a tree, if node u is the ***parent (ancestor)*** *node of v, then v is a child (descendent) of u. Two children of the same parent are siblings.*

42

Transposing two rows
Transposing two columns
Adding a multiple of one row to another
Adding a multiple of one column to another
Also note that if all entries in any row or column are 0 then the determinant is 0

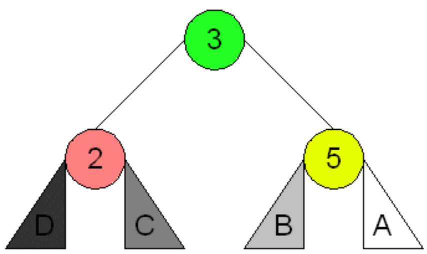
41

In a tree, an external node is known as a leaf node. It has no children.

44

A tree T is a non-empty set of nodes storing useful information in a parent-child relationship with the following properties:
T has a special node r referred to as the root.
Each node v of T different from r has a parent node u.

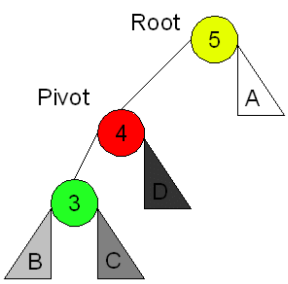
43



46

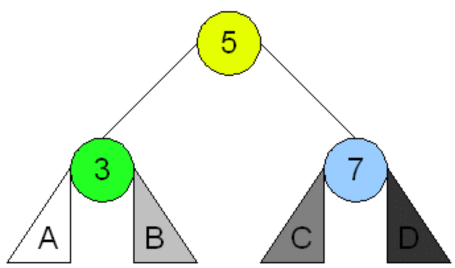
In a tree, an internal node has one or more children.

45



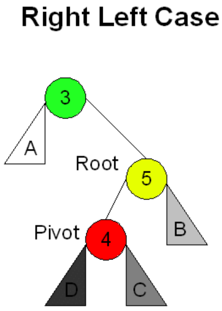
Then a left-left!

48



47

What do we do here?



49

What does a Depth First Search use?

50

What does a Breadth First Search use?

51

What is the running time of Dijkstra's algorithm?

52

What's the running time of a depth first search when the graph is an adjacency matrix?

53

What's the running time of a depth first search when the graph is an adjacency list?

54

How do we insert an element into a heap?

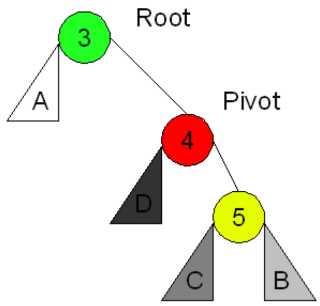
55

How do we remove the smallest element from a (min) heap?

56

A stack!

50



Then a right-right!

49

$O(E + V \log(v))$

52

A queue!

51

$O(V + E)$

54

$O(V^2)$ *since finding neighbours takes* $O(V)$ *time.*

53

We move the last element from the heap to the first element (we can override the first element since we've removed it). Now we 'down heap' by swapping the moved node with its smallest child until it is smaller than both its children or it has no children.

56

First, you insert it at the next space in the heap (last element of current row, or a new row), then you keep swapping it with its parent if the parent is larger than it.

55

What is a priority queue?

57

What is a heap?

58

A heap-based priority queue consists of:
: A complete binary tree with keys that satisfy the heap-order property.
: A reference to the last node in T.
: A comparator that defines the total order relation among keys.

59

*What is an **AVL Tree**?*

60

*What is **graph colouring**?*

61

The minimum number of colours require to colour a graph is its .

62

Graph colouring is an example of an NP-complete problem. The only algorithms known for NP-complete problems are .

63

*Describe the difference between a **directed** and **undirected graph**. Give an example of each.*

64

*A **heap** is a binary that stores a collection of keys at its internal nodes that satifies two addtional properties:*
A relational property that affects how the keys are stored and a structural property.
It allows insertions and removals to be performed in logarithmic time.

58

*A **priority queue** P is a container of elements with keys associated to them at the time of insertion.*

- $insertItem(k, e)$: Inserts an element e with key k into P. - $removeMin()$: Returns and removes from P an element with the smallest key.

Two fundamental methods in a priority queue.

57

An AVL tree is another balanced binary search tree. Named after their inventors, Adelson-Velskii and Landis, they were the first dynamically balanced trees to be proposed. Like red-black trees, they are not perfectly balanced, but pairs of sub-trees differ in height by at most 1, maintaining an $O(logn)$ search time.

60

A heap-based priority queue consists of:
Heap: A complete binary tree with keys that satisfy the heap-order property.
Last: A reference to the last node in T.
Comp: A comparator that defines the total order relation among keys.

59

The minimum number of colours require to colour a graph is its chromatic number.

62

In graph theory, graph coloring is a special case of graph labeling; a coloring of graph with k colours is allocation of the colours to the nodes of the graph, such that each node has just one colour and nodes linked by an edge have different colours.

61

An undirected graph is one where more than one edge is possible between a pair of nodes.
Directed example: Trees, Utility networks.
Undirected example: Graphic models, Social networks, Transport links.

64

Graph colouring is an example of an NP-complete problem. The only algorithms known for NP-complete problems are exponential.

63

An edge is ______ on a node if it has the node as source or target (directed) or if it links the node to another (undirected).

65

Define the terms ***sparse*** *and* ***dense*** *in the context of graphs.*

66

What is a ***connected component*** *of a graph G?*

67

Dijkstras algorithm is a ______ algorithm for graph structures.

68

Give the pseudocode for Dijkstra's algorithm w/ priority queue

69

A graph is sparse if it has few edges, more specifically, $Edges = O(Nodes)$.
It is dense if most pairs of nodes are joined by edges. More specifically, $Edges = O(Nodes^2)$

66

An edge is incident on a node if it has the node as source or target (directed) or if it links the node to another (undirected).

65

Dijkstras algorithm is a shortest path algorithm for graph structures.

68

The largest connected sub-graph (i.e. the one that cannot be expanded with additional nodes without becoming disconnected.)

67

```
function Dijkstra(Graph, Source):
    dist[source] = 0, create Q
    for each vertex v in Graph:
        if v != source, dist[v] = INFINITY
        prev[v] = UNDEFINED
        Q.addWithPriority(v, dist[v])
    while Q != empty
        u = Q.extractMin()
        for each neighbour v of u:
            alt = dist[u] + length(u, v)
            if alt ¡ dist[v]
                dist[v] = alt, prev[v] = u
                Q.decreasePriority(v, alt)
    return dist, prev
```

Code for dijkstras

69