What does $O(< expr >)$ mean?

1

What does $\Theta(< expr >)$ mean?

2

What does $\Omega(< expr >)$ mean?

3

Say that the input represents a positive integer, x, what is the size of n?

4

What does it mean by $O(1)$?

5

What would the pseudo code be for Euclid's algorithm?

6

What would the pseudo code be for Fast Modular Exponentiation?

7

What are some of the advantages of ElGamal encryption?

8

The complexity (i.e. space/running time) has the complexity proportional to $< expr >$.

2

The complexity (i.e. running time/space) is bounded by the $< expr >$.

1

$\lfloor \log_b x \rfloor + 1$ *Where b is the number representation, usually binary (so 2).*

4

The complexity (i.e. running time/space) is at least by the $< expr >$.

3

```
// Assume a >= b
hcf(a,b)
    if b = 0
        return a
    r = amodb
    return hcf(b,r)
```

Euclid's algorithm

6

It takes a constant time, no matter the amount of data, to perform the operation.

5

Sender Verification
Private key remains with owner
Public key is freely distributable
No secret channel needed at any point
No need for pre-shared keys

8

```
fme(a,b,k)
    d = a
    e = b
    s = 1
    While e > 0
        if e is odd
            s = (s.d)modk
        d = d^2modk
        e = ⌊e/2⌋
    return s
```

Fast Modular Exponentiation

7

What is the basic procedure for an encryption and decryption using publik key cryptography if Alice wants to send a message to Bob?

9

Describe public key generation in ElGamal encryption using p as the Prime Modulus and g as the Primitive root (as described in the COMP26120 lab)

10

Describe the encryption procedure used in the ElGamal cryptosystem given that person B wants to send message M to preson A

11

Describe the decryption process used in the ElGamal cryptosystem given that person A has received cyphertext (γ, δ) from person B, encrypted encrypted using the public key (p, g, g^a)

12

Consider the equation $a^x = y \bmod p$. If a is a primitive root of modulo p, then for every $y(1 \leq y < p)$, such an $x(1 \leq x < p)$ exists. What is x?

13

The [redacted] *is the inverse of exponentiation.*

14

Why can a private key in the ElGamal cryptosystem not, in practice, be recovered using the public key when p is large?

15

What is one way you can argue correctness of Euclid's algorithm?

16

Generate a large p and a g in $1 \leq g < p$
Generate a random integer a in $1 \leq a \leq p-2$
Compute $g^a \bmod p$. The public key is

$$(p, g, g^a)$$

The private key is a

Alice generates a private random integer a and Bob generates a private random integer b
Alice generates her public value $g^a \bmod p$
Bob generates his public value $g^b \bmod p$
Alice computes $g^{ab} = (g^a)^b \bmod p$
Bob computes $g^{ba} = (g^b)^a \bmod p$
Now they have a shared secret k since $k = g^{ab} = g^{ba}$

Use private key a to compute $(\gamma^{p-1-a}) \bmod p$
NOTE THAT: $(\gamma^{p-1-a}) = \gamma^{-a} = g^{-ak}$
Recover the message M by computing $(\gamma^{-a} \cdot \delta \bmod p)$
Note that this evaluates to $(g^{-ak} \cdot g^{ak} \cdot M \bmod p)$ or $1 \cdot M \bmod p$

Obtain A's public key (p, g, g^a)
Represent the message M as integers in the range $0, ..., p-1$
Select a random integer k from $1 \leq k \leq p-2$
Compute $\gamma = g^k \bmod p$ and $\delta = m \cdot (g^a)^k$
Send ciphertext $c = (\gamma, \delta)$ to A

The discrete logarithm is the inverse of exponentiation.

*X is the **discrete logarithm** of y with base a, modulo p.*

Let $r = amodb$. $hcf(a, b) = hcf(b, r)$ because all factors of a and b are also factors of b and r and vice versa. If they have the same factors, they have the same highest common factor.

To calculate a public key, y, a private key, x is needed. The equation for modular exponentiation can be used to generate the public key: $y = g^x \bmod p$ where g is a primitive root of the modulus p.
It is considered a one-way, or trapdoor function - easy to compute, hard to invert. For a large p, one of the few ways to figure out the private key x would be to calculate $g^x \bmod p$ for every x in $1 \leq x < p$ and find when one of these results matches y

What would half the correctness proof be for Euclid's algorithm?

17

$(a.b) mod k =$ [redacted]

18

Let p be a prime number. What is meant by a primitive root modulo p?

19

*What are the best, average and worst case complexities of **Bubble Sort**?*

20

*What are the best, average and worst case complexities of **Merge Sort**?*

21

Give pseudo code for merging 2 sorted lists, as part of merge sort.

22

Give pseudo code for MergeSort(L).

23

*What are the best, average and worst case complexities of **Quick Sort**?*

24

$$(a.b) mod k = (a mod k . b mod k) mod k$$

18

As $r = a \bmod b$, $\exists q$ *such that* $a = bq + r$,
$\therefore r = a - bq$.
Suppose x *is a factor of* a *and* b, *then* $\exists y and z$ *such that* $a = xy$, $b = xz$.
Hence: $r = xy - xzq$, $r = x(y - zq)$.
$\therefore x$ *is a factor of* r *(and also of* b *and* r*).*

17

Best: $O(n)$,
Average: $O(n^2)$,
Worst: $O(n^2)$

20

The numbers r_x *between 1 and* $p-1$ *that, when raised by the numbers between 1 and* $p-1$ *compute all the numbers between 1 and* $p-1$ *in some order with no repetitions.*

19

Merge(L_1, L_2*)*
    if $L_1 = []$ *return* L_2
    if $L_2 = []$ *return* L_1
    $x_1 = L_1[0]$
    $x_2 = L_2[0]$
    $L_1' = L_1[1 : |L_1| - 1]$
    $L_2' = L_2[1 : |L_2| - 1]$
    if $x_1 \leq x_2$
        return $[x_1]$+*Merge(*L_1' ,L_2*)*
    return $[x_2]$+*Merge(*L_1 ,L_2'*)*

Merge two sorted lists

22

Best: $O(n \log_2 n)$,
Average: $O(n \log_2 n)$,
Worst: $O(n \log_2 n)$

21

Best: $O(n \log_2 n)$,
Average: $O(n \log_2 n)$,
Worst: $O(n^2)$

24

MergeSort(L)
    if $|L| \leq 1$
        return L
    Split L into roughly equal halves, L_l *and* L_r
    return Merge(MergeSort(L_l*),MergeSort(*L_r*))*

MergeSort(L)

23

What would the pseudo code be for Quick Sort?

25

What is the minimum time for any sorting algorithm that uses only number comparisons?

26

What does saying that algorithm A runs in time g mean?

27

What is a permutation of a set?

28

What do we mean by a composition of two permutations?

29

What is the number of possible permutations on an n-element set?

30

In the context of a permutation, what do we mean by a transposition?

31

Convert this pair of simultaneous equations into matrix form

$$a_{1,1}x_1 + a_{1,2}x_2 = b_1$$
$$a_{2,1}x_2 + a_{2,2}x_2 = b_2$$

32

```
quicksort(L)
    if length of L ≤ 1
        return L
    remove the first element, x, from L
    L≤ := elements of L less than or equal to x
    L> := elements of L greater than x
    Ll := quicksort(L≤)
    Lr := quicksort(L>)
    return Ll + [x] + Lr
```

Quick Sort

25

$n \log_2 n$

26

Given an input of size n, the number of operations executed by A is bounded above by g(n).

27

A 1-to-1 map of the set onto itself. In basic terms, it is a set mapped to another order of itself. i.e $[0, 1, 2, 3, 4] \mapsto [2, 4, 1, 0, 3]$

28

The composition is the product of two permutations, α *and* β*, on a set n, given by* $\alpha \cdot \beta(n)$ *or* $\beta(\alpha(n))$

29

$n!$

30

A transposition is a special kind of permutation where only 2 elements in a set are affected (they are swapped). On a set X *a transposition* $\sigma = (i, j)$ *is given by*

$$\sigma(k) = \begin{cases} j & \text{if } k = i \\ i & \text{if } k = j \\ k & \text{ow.} \end{cases}$$

31

$$\begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

32

What is the determinant of the matrix:

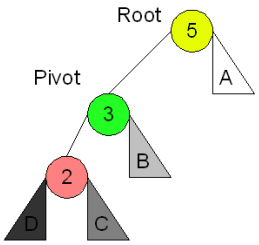
$$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$$

What is an upper triangular matrix and how do you calculate its determinant?

Which 4 operations have no effect on a matrix's determinant?

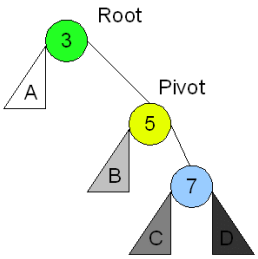
What do we do here?

Left Left Case



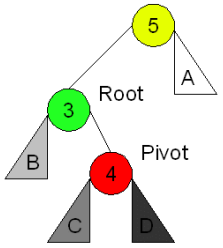
What do we do here?

Right Right Case



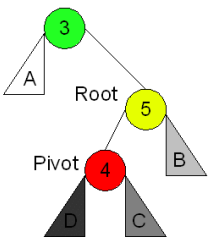
What do we do here?

Left Right Case



What do we do here?

Right Left Case



What does a Depth First Search use?

It is a matrix where all of its entries below the diagonal are zero.

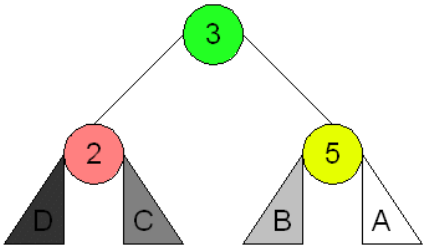
$$\begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ 0 & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{n,n} \end{pmatrix}$$

Its determinant is calculated by taking the product of the entries on the diagonal. i.e $a_{1,1} \cdot a_{2,2} \cdot \ldots \cdot a_{n,n}$

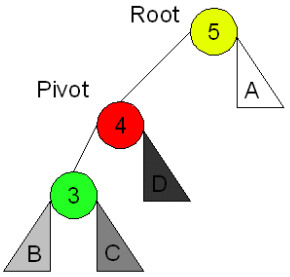
$a_1 a_4 - a_2 a_3$
Often denoted as:

$$\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix}$$

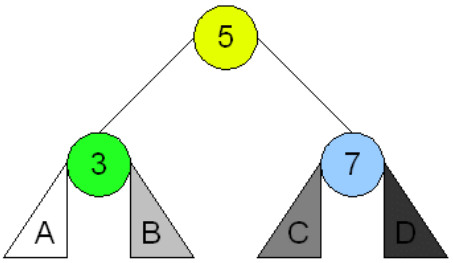
The original system of equations to which the matrix corresponds only has a unique solution if the determinant is non-zero.



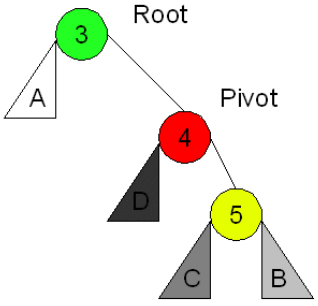
Transposing two rows
Transposing two columns
Adding a multiple of one row to another
Adding a multiple of one column to another
Also note that if all entries in any row or column are 0 then the determinant is 0



Then a left-left!



A stack!



Then a right-right!

What does a Breadth First Search use?

41

What is the running time of Dijkstra's algorithm?

42

What's the running time of a depth first search when the graph is an adjacency matrix?

43

What's the running time of a depth first search when the graph is an adjacency list?

44

How do we insert an element into a heap?

45

How do we remove the smallest element from a (min) heap?

46

Dijkstras algorithm is a [redacted] *algorithm for graph structures.*

47

Give the pseudocode for Dijkstra's algorithm w/ priority queue

48

$O(E + V \log(v))$

42

A queue!

41

$O(V + E)$

44

$O(V^2)$ *since finding neighbours takes* $O(V)$ *time.*

43

We move the last element from the heap to the first element (we can override the first element since we've removed it). Now we 'down heap' by swapping the moved node with its smallest child until it is smaller than both its children or it has no children.

46

First, you insert it at the next space in the heap (last element of current row, or a new row), then you keep swapping it with its parent if the parent is larger than it.

45

```
function Dijkstra(Graph, Source):
    dist[source] = 0, create Q
    for each vertex v in Graph:
        if v != source, dist[v] = INFINITY
        prev[v] = UNDEFINED
        Q.addWithPriority(v, dist[v])
    while Q != empty
        u = Q.extractMin()
        for each neighbour v of u:
            alt = dist[u] + length(u, v)
            if alt ¡ dist[v]
                dist[v] = alt, prev[v] = u
                Q.decreasePriority(v, alt)
    return dist, prev
```

Code for dijkstras

48

Dijkstras algorithm is a shortest path algorithm for graph structures.

47