# Add to 15 and Tic Tac Toe

The Add to 15 game is equivalent to tic tac toe. I do mean "equivalent", not just similar in that it involves 9 moves and generally ends in a tie.

There are 8 winning combinations. You can generate this list systematically, by asking first, what combinations start with 9 as the highest number? It cannot include an 8, or a 7 or a 6, but it can include a 5. Then the next number is a 1. So 9-5-1 is on the list. Can there by another group including a 9? Yes, 9-4-2. Can there be another one? Answer: no. In this way, you can generate the following list

9 5 1

9 4 2

8 6 1

8 5 2

8 4 3

7 6 2

7 5 3

6 5 4

Now, think about tic tac toe. Consider the board. First of all, as I said, there are 9 places for moves. There are 3 rows, 3 columns and 2 diagonals. This is 8 ways of winning. However, we are not done with our proof of the equivalence. The next step is to assign a number to each of the 9 places so that each row and each column and each diagonal are one of the 9 winning add to 15 combinations. It is possible to make guesses, but this also can be done systematically. Let's make a count of how many groups each number appears in. The number 9 is in 2 groups. The number 8 is in 3 groups. Count them up and you will get

9 is in 2

8 is in 3

7 is in 2

6 is in 3

5 is in 4

4 is in 3

3 is in 2

2 is in 3

1 is in 2

This set of facts gives us a way to assign numbers to each of the places on the tic tac toe board. Follow along.

I will start with the middle box. It is in 4 winning sets of boxes: the middle row, the middle column and both diagonals. This means that 5 belongs there. Now let's move to the upper left corner. At this point, it is important to realize that there are symmetries involved and there is more than one assignment of numbers to boxes that will work. I will put an 8 in the upper left corner. This is reasonable because this box is in 3 winning sets: the top row, leftmost column and the diagonal going from top left to bottom right. When I do this, it means that the lower right box must be a 2 to finish the adding up to 15: 8 plus 5 plus 2 is 15. So I/we have assigned 3 numbers. Let's put a 6, another number in 3 winning groups, in the upper right box. I can now finish up the assignments.

8 + ? + 6 for the top row means the middle top must be a 1.

6 + 5 + ? for the diagonal going from upper right to lower left means the lower left must be a 4.

8 + ? + 4 for the left column means the middle number must be a 3

3 + 5 + ? for the middle row means the middle right column must be a 7

1 + 5 + ? for the middle column means the lower row, middle number is a 9.

This completes the proof of equivalence. Now, your challenge can be to implement a tic tac toe game.