# Observable Error in Mass, Momentum, and Energy Conservative Dynamical Low-rank Integration Schemes for the Vlasov-Poisson Equation

April Herwig

February 12, 2025

**Abstract**

Vlasov-Poisson has (infinitely) many conserved quantities. Low-rank tensor approximation [3]. Does this conserve more observables than we expect?

## Contents

**1 Introduction** **1**

**2 The Vlasov-Poisson Equation and Conservation Properties** **2**

**3 A Low-rank Tensor Approximation Scheme** **2**

**4 Numerical Example: Landau Damping** **2**

**5 Conclusion** **2**

## 1 Introduction

Efficient numerical solutions to kinetic equations is the basis for many fields in physics and has applications such as fusion energy generation in magnetically confined chambers, or dosage calculation in radiation therapy. Kinetic models are dependent on both physical space coordinates $x \in \Omega \subset \mathbb{R}^3$ and velocity coordinates $v \in \mathbb{R}^3$. A grid-based discretization with $n$ grid points in each dimension therefore incurs a computational cost of order $O(n^6)$. Hence, direct solutions for the (kinetic) density function are prohibitively expensive.

Recently, dynamical low-rank solvers for kinetic equations such as the Vlasov-Poisson equation have been proposed [2]. These methods are based on [4], in which matrix differential equations are approximated by low-rank matrices using a Galerkin principle. These solvers have been shown to be successful [1], but without additional care the projection to low-rank tensor products destroys important physical structure in the system. This can be alleviated by altering the approximation space such that some moments of the velocity coordinate always lie in the space spanned by the low-rank matrices [3]. This is sufficient to guarantee that continuity equations for mass, momentum, and energy are conserved up to an error that is linear in the time step size.

In the following report we examine whether the conservative dynamical low-rank integrator also preserves further observables and if so, to what order. The report is structured as follows: in section 2 the Vlasov-Poisson equation is introduced and conservation properties are discussed; in section 3 the conservative dynamical low-rank integrator is shown; in section 4 the integrator is demonstrated on an analytically understood example system.

if some observ-able works, mention here

# 2 The Vlasov-Poisson Equation and Conservation Properties

# 3 A Low-rank Tensor Approximation Scheme

# 4 Numerical Example: Landau Damping

# 5 Conclusion

# References


[1] Fabio Cassini and Lukas Einkemmer. "Efficient 6D Vlasov simulation using the dynamical low-rank framework Ensign". In: *Computer Physics Communications* 280 (2022), p. 108489. ISSN: 0010-4655. DOI: `10.1016/j.cpc.2022.108489`. URL: `http://dx.doi.org/10.1016/j.cpc.2022.108489`.

[2] Lukas Einkemmer and Christian Lubich. *A low-rank projector-splitting integrator for the Vlasov–Poisson equation*. 2018. eprint: `1801.01103`. URL: `https://arxiv.org/abs/1801.01103`.

[3] Lukas Einkemmer, Alexander Ostermann, and Carmela Scalone. "A robust and conservative dynamical low-rank algorithm". In: *Journal of Computational Physics* 484 (2023), p. 112060. ISSN: 0021-9991. DOI: `https://doi.org/10.1016/j.jcp.2023.112060`.

[4] Othmar Koch and Christian Lubich. "Dynamical Low-Rank Approximation". In: *SIAM Journal on Matrix Analysis and Applications* 29.2 (2007), pp. 434–454. DOI: `10.1137/050639703`. URL: `https://doi.org/10.1137/050639703`.