EWS Blatt 10

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May 4, 2021

$\mathbb{E}\left[\left(\hat{h}(x) - \mathbb{E}\left[\hat{h}(x)\right]\right)^2\right]$ $\tilde{X} = [1 \quad X]$, $\tilde{y}^T = [1 \quad y^T]$ $\theta^* = \text{argmin}_\theta \ \|\tilde{y} - \tilde{X}\theta\| + \lambda\left(\|\tilde{I}\theta\|^2\right)$

$$\tilde{I} = \begin{bmatrix} 0 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix} = I - \begin{bmatrix} 1 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}$$

$$\begin{aligned} \hat{\theta} &= \text{argmin}_\theta \ R(\theta) \\ &= \text{argmin}_\theta \ \|\tilde{y} - \tilde{X}\theta\|^2 + \lambda\|\tilde{I}\theta\|^2 \\ &= \langle \tilde{y} - \tilde{X}\theta, \ \tilde{y} - \tilde{X}\theta\rangle + \lambda\langle \tilde{I}\theta, \ \tilde{I}\theta\rangle \\ &= \tilde{y}^T\tilde{y} + \theta^T\tilde{X}^T\tilde{X}\theta - 2\theta^T\tilde{X}^T\tilde{y} + \lambda\theta^T\tilde{I}^T\tilde{I}\theta \end{aligned}$$

$$\left(\tilde{X}^T\tilde{X} + \tilde{I}^T\tilde{I}\right)\hat{\theta} = \tilde{X}^T\tilde{y} \ \tilde{X}^T\tilde{X}$$