Problem 1

In [17]:
```
import numpy as np
from numpy import linalg as lg
from matplotlib import pyplot as plt
```

In [18]:
```
b = np.array([4.5, 6])
norm_b = lg.norm(b)
def f(x): return lg.norm(x - b)
def Df(x): return (x - b) / f(x)
x = np.array([0, 0])
```

In [19]:
```
# 2)
x = x - Df(x)
```

After 7 steps with constant step size 1, x reaches $\binom{4.2}{5.6}$. Since $\nabla f(x) = \frac{x-b}{||x-b||}$, $x^k - \nabla f(x^k)$ always takes steps of size 1 directly towards b. $||x^7 - b|| = 0.5 < 1$, so the sequence cannot converge.

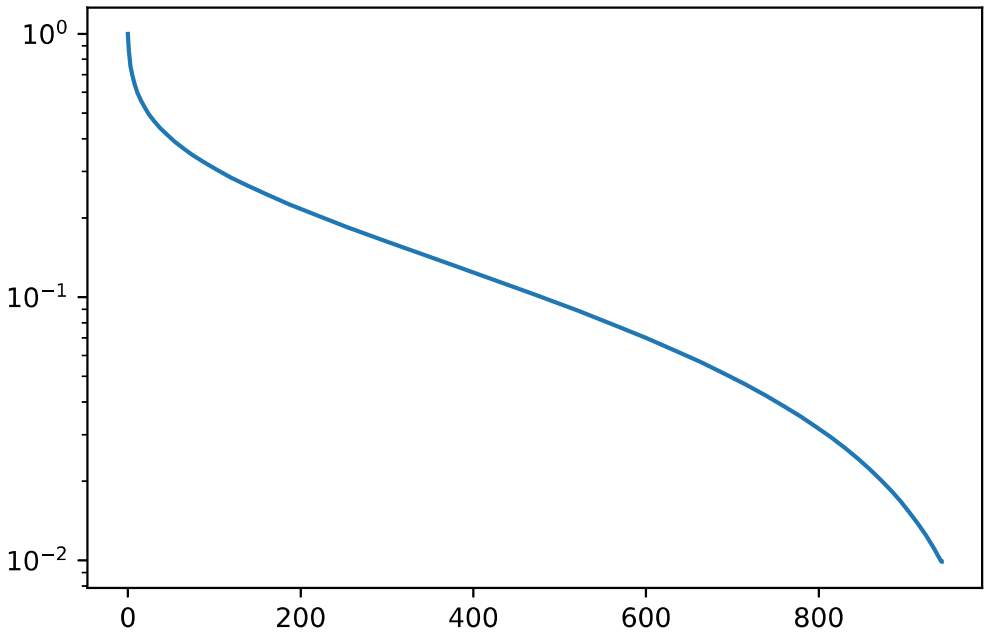
In [20]:
```
# 3)
k = 0
x = x - Df(x) * (5/6)**k
```

$||x^0 - b|| = 7.5 > 6 = \frac{1}{1-5/6} = \sum_{k=0}^{\infty}(\frac{5}{6})^k$. Therefore, it is impossible for the sequence to converge, the steps reduce in size too quickly.

In [21]:
```
# 4)
error = [1.0]
x, k = np.array([0, 0]), 0
while error[-1] > 0.01:
    x = x - Df(x) / (k+1)
    error.append(f(x) / norm_b)
    k = k + 1

plt.semilogy(np.arange(error.__len__()), error)
```

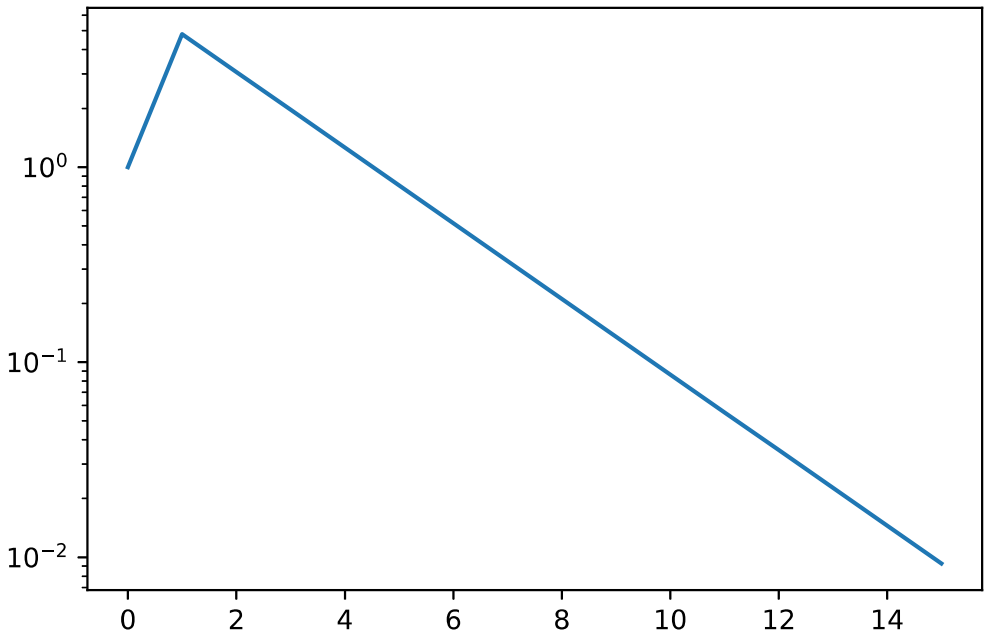
Out[21]:
```
[<matplotlib.lines.Line2D at 0x1feba96da90>]
```



In [22]:
```
def g(x): return ((x - b)**2).sum()
def Dg(x): return 2 * (x - b)
```

In [23]:
```
# 5)
error = [1.0]
x, k = np.array([0, 0]), 0
while error[-1] > 0.01:
    x = x - Dg(x) * 0.1
    error.append(g(x) / norm_b)
    k = k + 1
plt.semilogy(np.arange(error.__len__()), error)
```

Out[23]: [<matplotlib.lines.Line2D at 0x1fec21939a0>]



In [24]:
```
# 6)
x, k, z = np.array([0, 0]), 0, 1.0
while z > np.finfo(float).eps:
    z = z * (1/6)
    x = x - Dg(x) * z
    k = k + 1
```

After 20 steps, the step length has reached the machine epsilon. Unfortunately, $x^k - (\frac{1}{6})^k \nabla g(x)$ does not reach 1% of the optimal solution in 20 steps. Therefore, the sequence will not converge.

In [25]:
```
# 7)
error = [1.0]
x, k = np.array([0, 0]), 0
while error[-1] > 0.01:
    x = x - Dg(x) / (4*k + 4)
    error.append(g(x) / norm_b)
    k = k + 1
plt.semilogy(np.arange(error.__len__()), error)
```

Out[25]: [<matplotlib.lines.Line2D at 0x1fec1532cd0>]