Problem 1

In [1]:
```
using DelimitedFiles, LinearAlgebra, Statistics, Plots
cd("C:\\Users\\april\\Documents\\schoolwork\\Numerik")
flatten(x) = [x[i] for i in eachindex(x)]

X, headers = readdlm("hitters.x.csv", ',', header=true)
y = readdlm("hitters.y.csv", ',', skipstart=1)
n, d = size(X)
```

Out[1]:
```
(263, 19)
```

1. Scaling the features to have variance 1 drastically decreases the norm of each feature, which in turn decreases the variance $\mathbb{E}\left[\left(\hat{h}(x) - \mathbb{E}\left[\hat{h}(x)\right]\right)^2\right]$. It also allows comparison between elements of θ.

In [2]:
```
# 1.
scale = 1 ./ sqrt.(var(X, dims=2)) |> flatten |> Diagonal
X, y = scale*X, scale*y;
```

2. We can write $\tilde{X} = [1\ X]$, $\tilde{y}^T = [1\ y^T]$ and for theta we write $\hat{\theta} = \text{argmin}_\theta\ ||\tilde{y} - \tilde{X}\theta||^2 + \lambda||\tilde{I}\,\theta||^2$, with

$$\tilde{I} = \begin{bmatrix} 0 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix} = I - \begin{bmatrix} 1 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}.$$

This is solved almost identically to the least squares regression: we have

$$\begin{aligned} \hat{\theta} &= \text{argmin}_\theta\ R(\theta) \\ &= \text{argmin}_\theta\ ||\tilde{y} - \tilde{X}\theta||^2 + \lambda||\tilde{I}\,\theta||^2 \\ &= \langle \tilde{y} - \tilde{X}\theta,\ \tilde{y} - \tilde{X}\theta\rangle + \lambda\langle \tilde{I}\,\theta,\ \tilde{I}\,\theta\rangle \\ &= \tilde{y}^T\tilde{y} + \theta^T\tilde{X}^T\tilde{X}\theta - 2\theta^T\tilde{X}^T\tilde{y} + \lambda\theta^T\tilde{I}^T\tilde{I}\,\theta \end{aligned}$$

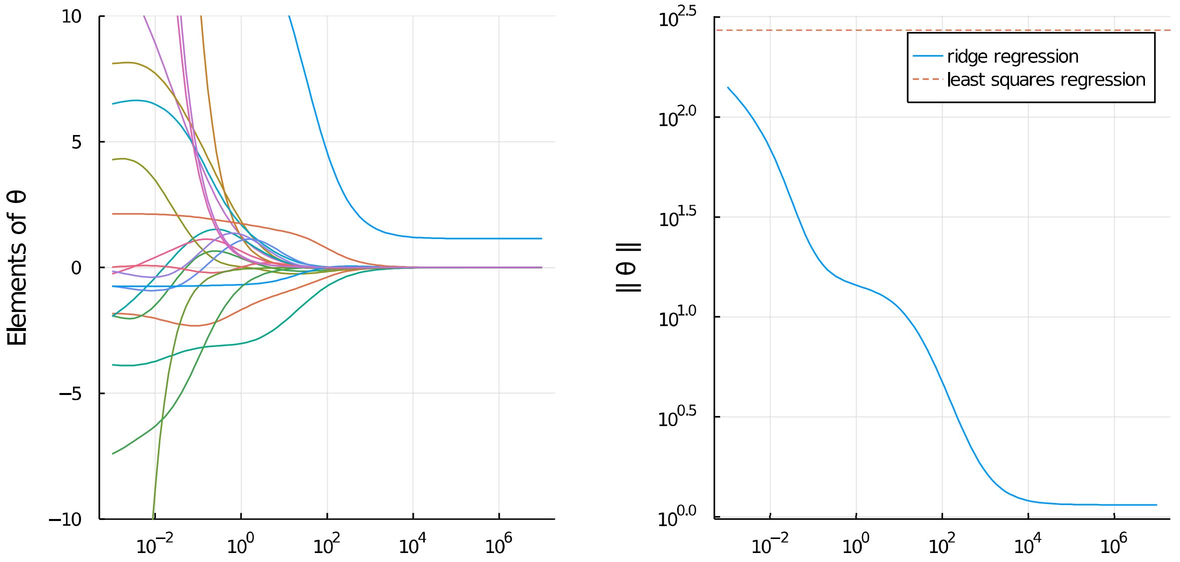
which is solved by $\left(\tilde{X}^T\tilde{X} + \lambda\tilde{I}\right)\hat{\theta} = \tilde{X}^T\tilde{y}$, after setting the gradient to 0. This can be solved via Cholesky decomposition since $\tilde{X}^T\tilde{X}$ and $\tilde{I}^T\tilde{I} = \tilde{I}$ are both symmetric positive definite matrices.

In [3]:
```
# 2.
X, d = hcat(ones(n), X), d+1
I˜= Diagonal([ 0; ones(d-1) ]);
```

In [4]:

```julia
# 3. and 4.
λ = 10 .^ range(-3, 7, length=100)
θ = [(X'X + λᵢ*I) \ X'y for λᵢ in λ]

fig1 = plot(λ, collect(eachrow(hcat(θ...))),
            xaxis=:log10, yaxis=((-10, 10), "Elements of θ"), lab=false)

θ = norm.(θ)
fig2 = plot(λ, θ, xaxis=:log10, yaxis=(:log10, "|| θ ||"), lab="ridge regression")
fig2 = hline!([norm( X'X \ X'y )], linestyle=:dash, lab="least squares regression")

fig = plot(fig1, fig2, layout=2, size=(800, 400), margin=5Plots.mm)
```

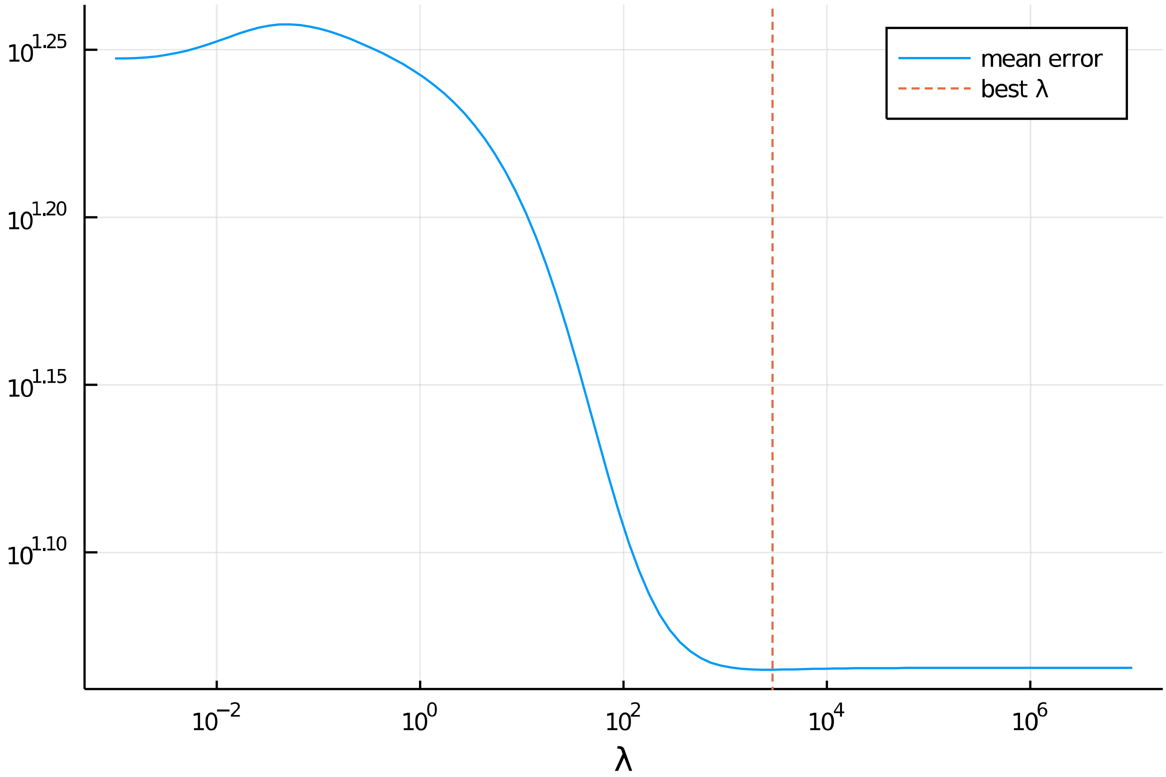
Out[4]:



In [5]:

```julia
# 5.
Xy = sortslices(hcat(X, y), by=x -> rand(), dims=1)                 # shuffle
X, y = Xy[:, 1:end-1], Xy[:, end]

k = 5                                                               # initialize
m = round(Int, n / k, RoundDown)
err = zeros(100, k)
θ_cross = zeros(d, k)

for i in 1:k                                                        # split dataset
    val = m*(i-1) + 1 : m*i                                         # into k parts
    train = filter(j -> !(j in val), 1:n)
    X_train, y_train = X[train, :], y[train]

    θ = [ (X_train'X_train + λᵢ*I) \ X_train'y_train for λᵢ in λ ] # train model
    err[:, i] = [ norm( y[val] - X[val, :]*θᵢ ) for θᵢ in θ ]
    θ_cross[:, i] = θ[argmin( err[:, i] )]                          # save best resul
end

err = mean(err, dims=2)                                             # average error
fig = plot(λ, err, xaxis=(:log10, "λ"), yaxis=:log10, lab="mean error")
fig = vline!([λ[argmin(err)]], linestyle=:dash, lab="best λ")
```

Out[5]:



In [7]:

```
# 6.
θ_cross = mean(θ_cross, dims=2)                                          # average best
p = filter(x -> x !== d, sortperm(flatten(θ_cross), by=abs, rev=true))
permutedims(headers[p])
# from most important to least important:
```

Out[7]:

```
1×19 Array{AbstractString,2}:
 "DivisionW"  "CAtBat"  "AtBat"  "NewLeagueN"  …  "Runs"  "Assists"  "CWalks"
```