

# EWS Aufgabe 9.3 TeX Teil

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## 1 Introduction

$$E[T_1] = E\left[\frac{1}{n} \sum_{i=1}^n X_i\right] = \frac{1}{n} \sum_{i=1}^n E[X_i] = \frac{1}{n} n\theta$$

$$E[T_2] = E\left[\frac{1}{n+2} \left(\sum_{i=1}^n X_i + 1\right)\right] = \frac{1}{n+2} nE[X_i] + \frac{1}{n+2} = \frac{1}{n+2} (n\theta + 1)$$

$$\begin{aligned} E[T_1^2] &= \text{Var}(T_1) + E[T_1]^2 = \frac{1}{n^2} \sum_{i=1}^n \text{Var}(X_i) + \theta^2 \\ &= \frac{n}{n^2} \text{Var}(X_i) + \theta^2 = \theta(1 - \theta) + \theta^2 = \theta \end{aligned}$$

$$\begin{aligned} \Rightarrow MSI_\theta(T_1) &= E[T_1^2 - 2\theta T_1 + \theta] \\ &= E[T_1^2] - 2\theta E[T_1] + \theta^2 \\ &= \theta - \theta^2 \\ &= (\theta(1 - \theta)) \end{aligned}$$

$$\begin{aligned} E[T_2^2] &= \text{Var}(T_2) + E[T_2]^2 \\ &= \left(\frac{1}{n+2}\right)^2 \sum_{i=1}^n \text{Var}(X_i) + E[T_2]^2 \quad \leftarrow * \\ &= \frac{n}{(n+2)^2} (\theta(1 - \theta)) + \left(\frac{n\theta + 1}{n+2}\right)^2 \\ &= \frac{1 + 3n\theta}{(n+2)^2} \end{aligned}$$

$$\Rightarrow MSI_\theta(T_2) = E[T_2^2 - 2\theta T_2 + \theta^2] = \frac{1 + 3n\theta}{(n+2)^2} - 2\theta \frac{n\theta + 1}{n+2} + \theta^2$$

$$\begin{aligned}
MSE_{\theta}(T_1) &= Var(T_1) \quad \leftarrow \quad T_1 \quad \text{ist erwartungstreu} \\
&= Var\left(\frac{1}{n} \sum_{i=1}^n X_i\right) = \frac{1}{n^2} \sum_{i=1}^n Var(X_i) = \frac{1}{n} \theta(1 - \theta)
\end{aligned}$$

$$\begin{aligned}
MSE_{\theta}(T_2) &= Var(T_2) + B(T_2)^2 \\
&= Var\left(\frac{1}{n+2} \left(\sum_{i=1}^n X_i + 1\right)\right) + \left(\frac{n\theta + 1}{n+2} - \theta\right)^2 \\
* \quad \rightarrow \quad &= \frac{n}{(n+2)^2} (\theta - \theta^2) + \left(\frac{n\theta + 1}{n+2}\right)^2 - 2\theta \frac{n\theta + 1}{n+2} + \theta^2 \\
&= \frac{n-4}{(n+2)^2} \theta(1 - \theta)
\end{aligned}$$

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MSE_{\theta}(T_1) &= Var(T_1) \quad \leftarrow \quad T_1 \quad \text{ist erwartungstreu} \\
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\end{aligned}$$

$$\begin{aligned}
MSE_{\theta}(T_2) &= Var(T_2) + B(T_2)^2 \\
&= Var\left(\frac{1}{n+2} \left(\sum_{i=1}^n X_i + 1\right)\right) + \left(\frac{n\theta + 1}{n+2} - \theta\right)^2 \\
* \quad \rightarrow \quad &= \frac{n}{(n+2)^2} (\theta - \theta^2) + \left(\frac{n\theta + 1}{n+2}\right)^2 - 2\theta \frac{n\theta + 1}{n+2} + \theta^2 \\
&= \frac{n-4}{(n+2)^2} \theta(1 - \theta)
\end{aligned}$$