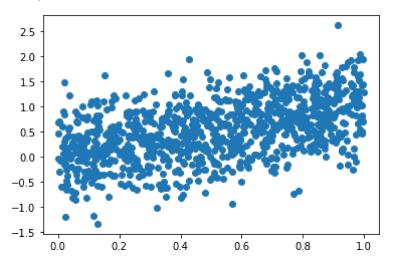
```
import numpy as np
from numpy.random import rand, randn
import matplotlib.pyplot as mpl
```

```
In [7]:
    n = 1000
    x = rand(n)
    eps = 0.5 * randn(n)
    y = x + eps
    p = np.argsort(x)
    x, y = x[p], y[p]
    fig, ax = mpl.subplots()
    ax.scatter(x, y)
```

Out[7]: <matplotlib.collections.PathCollection at 0x236c1161fd0>



```
In [8]:
    def f(a): return sum((x*a - y)**2)
    def Df(a): return sum(2*x * (x*a - y))
    DDf = sum(2 * x**2)

a, h = 0.5, 0.0
    f_new, f_old = f(a), np.inf
    while f_old - f_new > np.sqrt(np.finfo(float).eps):
        h = - Df(a)/DDf
        a += h
        f_old = f_new
        f_new = f(a)

y = a * x
ax.plot(x, y, color='r')
fig
```

Out[8]:

```
2.5 - 2.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.5 - 1.0 - 1.0 - 1.5 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 - 1.0 -
```

```
In [9]:
    from numpy.linalg import solve
    d = 4
    eps = 0.1 * randn(n)
    y = 30 * (x - .25)**2 * (x - .75)**2 + eps
    fig, ax = mpl.subplots()
    ax.scatter(x, y)

X = np.array([x**m for m in range(d+1)]).T
    a = solve(X.T @ X, X.T @ y)
    def interpolate(x): return sum(a[i] * x**i for i in range(d+1))
    interpolate = np.vectorize(interpolate)

y = interpolate(x)
    ax.plot(x, y, color='r')
```

Out[9]: [<matplotlib.lines.Line2D at 0x236c1a30d60>]

