Problem 1

In [13]:
```
using DelimitedFiles, LinearAlgebra, Statistics, Plots
cd("C:\\Users\\april\\Documents\\schoolwork\\Numerik")
flatten(x) = [x[i] for i in eachindex(x)]

X, headers = readdlm("hitters.x.csv", ',', header=true)
y = readdlm("hitters.y.csv", ',', skipstart=1)
n, d = size(X)
```

Out[13]:
```
(263, 19)
```

1. Scaling the features to have variance 1 drastically decreases the norm of each feature, which in turn decreases the variance $\mathbb{E}\left[\left(\hat{h}(x) - \mathbb{E}\left[\hat{h}(x)\right]\right)^2\right]$. This allows the ridge regression penalty to be more finely tuned, since the normalizing factor can be larger without seeing as much effect. It also allows comparison between elements of θ.

In [14]:
```
# 1.
σX, σy = sqrt.(var(X, dims=2)), sqrt.(var(y))
y = y ./ σy
X = X ./ σX;                                                    # rescale
```

2. We can write $\tilde{X} = [1\ X]$, $\tilde{y}^T = [1\ y^T]$ and for theta we write $\hat{\theta} = \text{argmin}_\theta\ ||\tilde{y} - \tilde{X}\theta||^2 + \lambda||\tilde{I}\,\theta||^2$, with

$$\tilde{I} = \begin{bmatrix} 0 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix} = I - \begin{bmatrix} 1 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}.$$

This is solved almost identically to the least squares regression: we have

$$\begin{aligned} \hat{\theta} &= \text{argmin}_\theta\ R(\theta) \\ &= \text{argmin}_\theta\ ||\tilde{y} - \tilde{X}\theta||^2 + \lambda||\tilde{I}\,\theta||^2 \\ &= \langle \tilde{y} - \tilde{X}\theta,\ \tilde{y} - \tilde{X}\theta\rangle + \lambda\langle \tilde{I}\,\theta,\ \tilde{I}\,\theta\rangle \\ &= \tilde{y}^T\tilde{y} + \theta^T\tilde{X}^T\tilde{X}\theta - 2\theta^T\tilde{X}^T\tilde{y} + \lambda\theta^T\tilde{I}^T\tilde{I}\,\theta \end{aligned}$$

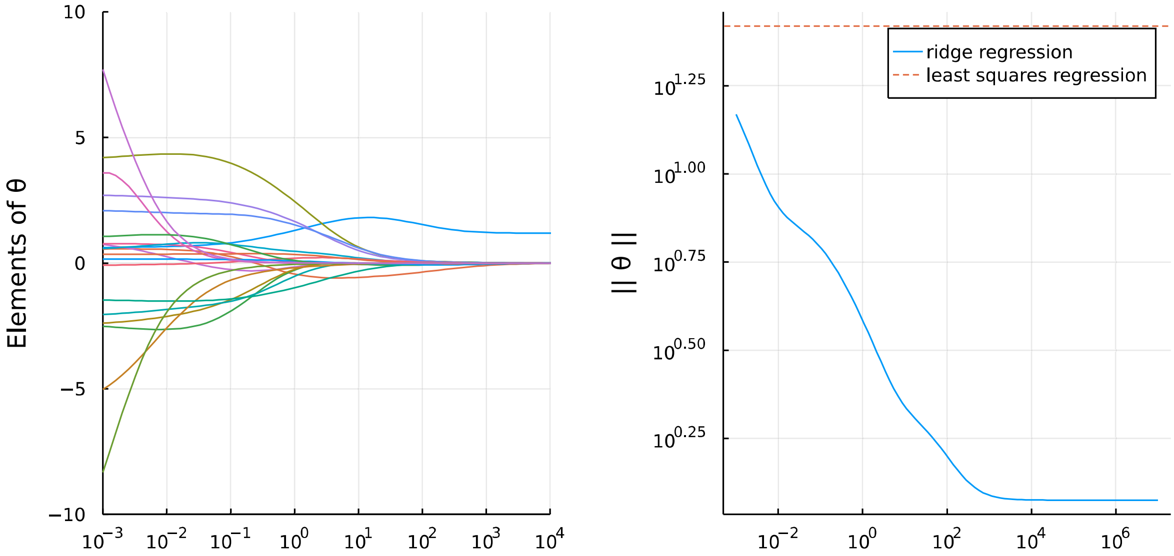
which is solved by $\left(\tilde{X}^T\tilde{X} + \lambda\tilde{I}\right)\hat{\theta} = \tilde{X}^T\tilde{y}$, after setting the gradient to 0. This can be solved via Cholesky decomposition since $\tilde{X}^T\tilde{X}$ and $\tilde{I}^T\tilde{I} = \tilde{I}$ are both symmetric positive definite matrices.

In [15]:
```
# 2.
X, d = hcat(ones(n), X), d+1
Ĩ = Diagonal([ 0; ones(d-1) ]);
```

In [16]:

```
# 3. and 4.
λ = 10 .^ range(-3, 7, length=100)                          # log range
θ = [(X'X + λᵢ*I) \ X'y for λᵢ in λ]                       # solve

fig1 = plot(λ, collect(eachrow(hcat(θ...))),
            xaxis=(:log10, (1e-3, 1e4)),
            yaxis=((-10, 10), "Elements of θ"), lab=false)

θ = norm.(θ)
fig2 = plot(λ, θ, xaxis=:log10, yaxis=(:log10, "|| θ ||"), lab="ridge regression")
fig2 = hline!([norm( X'X \ X'y )], linestyle=:dash, lab="least squares regression")

fig = plot(fig1, fig2, layout=2, size=(800, 400), margin=5Plots.mm)
```

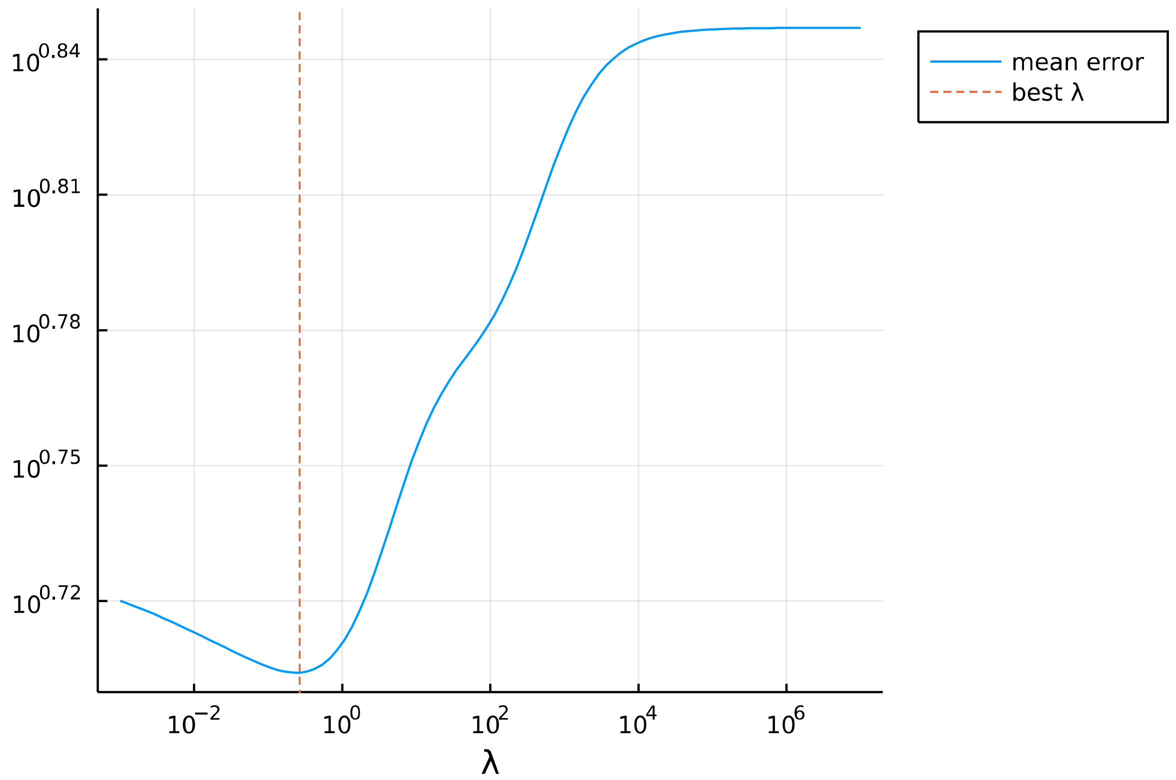
Out[16]:



In [17]:

```
# 5.
Xy = sortslices(hcat(X, y), dims=1, by=x->rand())              # shuffle
X, y = Xy[:, 1:end-1], Xy[:, end]

k = 5                                                           # initialize
m = round(Int, n / k, RoundDown)
err = zeros(100, k)
θ_cross = zeros(d, k)

for i in 1:k                                                    # split dataset
    val = m*(i-1) + 1 : m*i                                     # into k parts
    train = filter(j -> !(j in val), 1:n)
    X_train, y_train = X[train, :], y[train]

    θ = [ (X_train'X_train + λᵢ*I) \ X_train'y_train for λᵢ in λ ] # train model
    err[:, i] = [ norm( y[val] - X[val, :]*θᵢ ) for θᵢ in θ ]
    θ_cross[:, i] = θ[argmin( err[:, i] )]                      # save best resul
end

err = mean(err, dims=2)                                         # average error
fig = plot(λ, err, xaxis=(:log10, "λ"), yaxis=:log10,
           leg=:outertopright, lab="mean error")
fig = vline!([λ[argmin(err)]], linestyle=:dash, lab="best λ")
```

Out[17]:



In [18]:

```
# 6.
θ_cross = mean(θ_cross, dims=2)                                    # average best ϑ
p = filter(x -> x !== d, sortperm(flatten(θ_cross), by=abs, rev=true))
permutedims(headers[p])
# from most important to least important:
```

Out[18]:

```
1×19 Matrix{AbstractString}:
 "CHmRun"  "CRBI"  "CWalks"  "CHits"  …  "Years"  "Assists"  "NewLeagueN"
```