Problem 5

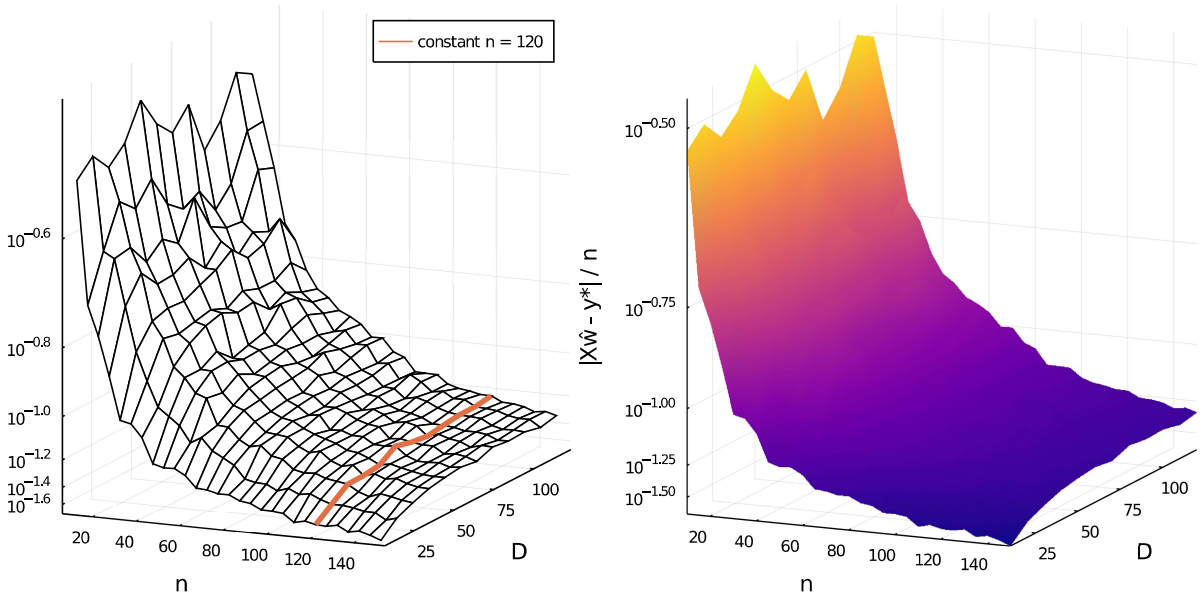
In [60]:
```
# Julia only because numpy.polyfit produced unstable results
using Plots, LinearAlgebra, Statistics
```

In [61]:
```
w0, w1 = 1, 1

function interpolate(n, D)                                  # I suppose it's
    α = sort!( 2 .* rand(n) .- 1 )                          # actually regression
    ỹ = w0 .+ w1 .* α                                       # undisturbed y
    y = ỹ + randn(n)                                        # disturbed y
    X = hcat( [α.^i for i in 0:D]... )
    Q, R = qr(X)                                            # qr factorization
    n > D+1 && ( R = vcat( R, zeros( n-(D+1), D+1 ) ) )     # for backward stability

    ŵ = R \ Q'y
    err = norm( X * ŵ - ỹ ) / n
    return err
end
interpolate(n, D, trials) = mean( interpolate(n, D) for _ in 1:trials )

x, y = 10:5:150, 10:10:120
xy = [(n, D) for D in y, n in x];
```

In [62]:
```
z = map(xy) do nD
    interpolate(nD..., 10)
end;
```

In [63]:
```
fig1 = wireframe(x, y, z,
                 xaxis="n", yaxis="D", zaxis=:log10)
fig1 = plot!(fig1,
             120 .* ones(length(y)), y, z[:, 23],
             linewidth=4, lab="constant n = 120")
fig2 = surface(x, y, z,
               colorbar=false, c=:plasma,
               xaxis="n", yaxis="D", zaxis=("|Xŵ - y*| / n", :log10))
fig = plot(fig1, fig2, layout=2, size=(900, 450))
```

Out[63]:



Problem 7

In [64]:
```
function interpolate(n, D)                                  # redefine interpolate
    α = sort!( 7 .* rand(n) .- 4 )                          # function for exp(α)
    ỹ = exp.(α)                                             # undisturbed y
    y = ỹ + randn(n)                                        # disturbed y
    X = hcat( [α.^i for i in 0:D]... )
    Q, R = qr(X)                                            # qr factorization
    n > D+1 && ( R = vcat( R, zeros( n-(D+1), D+1 ) ) )
    ŵ = R \ Q'y
    err = norm( X * ŵ - ỹ ) / n
    return err
end;
```

In [65]:
```
z = map(xy) do nD
    interpolate(nD..., 10)
end;
```

In [66]:
```
fig1 = wireframe(x, y, z,
                 xaxis="n", yaxis="D", zaxis=:log10)
fig1 = plot!(fig1,
             120 .* ones(length(y)), y, z[:, 23],
             linewidth=4, lab="constant n = 120")
fig2 = surface(x, y, z,
               colorbar=false, c=:plasma,
               xaxis="n", yaxis="D", zaxis=("|Xŵ - y*| / n", :log10))

fig = plot(fig1, fig2, layout=2, size=(900, 450))
```

Out[66]: