

EWS Blatt 10

April Herwig

May 13, 2021

$$\mathbb{E} \left[\left(\hat{h}(x) - \mathbb{E} [\hat{h}(x)] \right)^2 \right] \quad \tilde{X} = [1 \quad X], \quad \tilde{y}^T = [1 \quad y^T] \quad \theta^* = \operatorname{argmin}_{\theta} \|\tilde{y} - \tilde{X}\theta\| + \lambda \left(\|\tilde{I}\theta\|^2 \right)$$

$$\tilde{I} = \begin{bmatrix} 0 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix} = I - \begin{bmatrix} 1 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}$$

$$\begin{aligned} \hat{\theta} &= \operatorname{argmin}_{\theta} R(\theta) \\ &= \operatorname{argmin}_{\theta} \|\tilde{y} - \tilde{X}\theta\|^2 + \lambda \|\tilde{I}\theta\|^2 \\ &= \langle \tilde{y} - \tilde{X}\theta, \tilde{y} - \tilde{X}\theta \rangle + \lambda \langle \tilde{I}\theta, \tilde{I}\theta \rangle \\ &= \tilde{y}^T \tilde{y} + \theta^T \tilde{X}^T \tilde{X} \theta - 2\theta^T \tilde{X}^T \tilde{y} + \lambda \theta^T \tilde{I}^T \tilde{I} \theta \end{aligned}$$

$$\left(\tilde{X}^T \tilde{X} + \tilde{I}^T \tilde{I} \right) \hat{\theta} = \tilde{X}^T \tilde{y} \quad \tilde{X}^T \tilde{X} \stackrel{(4.2)}{(5.6)} \nabla f(x) = \frac{x-b}{\|x-b\|} x - \nabla f(x) \quad \sum_{k=0}^{\infty} (5/6)^k$$