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A subdivision algorithm for the computation of unstable manifolds and global attractors

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Summary. Each invariant set of a given dynamical system is part of the global attractor. Therefore the global attractor contains all the potentially interesting dynamics, and, in particular, it contains every (global) unstable manifold. For this reason it is of interest to have an algorithm which allows to approximate the global attractor numerically. In this article we develop such an algorithm using a subdivision technique. We prove convergence of this method in a very general setting, and, moreover, we describe the qualitative convergence behavior in the presence of a hyperbolic structure. The algorithm can successfully be applied to dynamical systems of moderate dimension, and we illustrate this fact by several numerical examples.

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1. Introduction

It is well known that unstable manifolds of invariant sets have a crucial influence on the complexity of the dynamical behavior which is present in a given dynamical system. For instance,

- a transverse intersection of an unstable manifold with a stable manifold leads to the existence of complicated dynamical behavior close to this intersection;
- frequently attracting sets coincide with the closure of unstable manifolds of hyperbolic periodic points;
- another important example is provided by homoclinic tangencies of stable and unstable manifolds. These also lead to extremely rich dynamical behavior.

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