GAIO

25 years later

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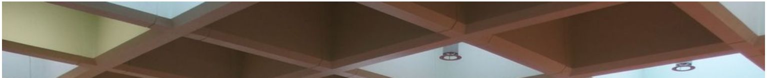
Where it all started



Who it all started

With what it all started

Numer. Math. 75: 293–317 (1997)

A subdivision algorithm for the computation of unstable manifolds and global attractors

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Received May 11, 1995 / Revised version received December 6, 1995

Summary. Each invariant set of a given dynamical system is part of the global attractor. Therefore the global attractor contains all the potentially interesting dynamics, and, in particular, it contains every (global) unstable manifold. For this reason it is of interest to have an algorithm which allows to approximate the global attractor numerically. In this article we develop such an algorithm using a subdivision technique. We prove convergence of this method in a very general setting, and, moreover, we describe the qualitative convergence behavior in the presence of a hyperbolic structure. The algorithm can successfully be applied to dynamical systems of moderate dimension, and we illustrate this fact by several numerical examples.

Mathematics Subject Classification (1991): 65L05, 65L70, 58F15, 58F12

1. Introduction

It is well known that unstable manifolds of invariant sets have a crucial influence on the complexity of the dynamical behavior which is present in a given dynamical system. For instance,

– a transverse intersection of an unstable manifold with a stable manifold leads

Dynamical systems

Consider a continuous map

$$f : \mathbb{R}^d \to \mathbb{R}^d.$$

The map defines a discrete dynamical system by iteration:

$$x_{k+1} = f(x_k), \qquad k = 0, 1, 2, \ldots$$

Basic question: *What is the fate of some x_0 as $k \to \infty$?*

Attractors

Definition

A set $A \subset \mathbb{R}^d$ is invariant, if

$$f(A) = A.$$

Example: $A = \{\bar{x}\}$ with $\bar{x} = f(\bar{x})$ a fixed point.

Definition

An invariant set A is attracting if there is a neighborhood U of A such that for every open set $V \supset A$ there is $K \in \mathbb{N}$ such that

$$f^k(U) \subset V \quad \text{for all } k \geq K.$$

Relative attractors

Proposition

If A is a closed invariant set then

$$A = \bigcap_{k\in\mathbb{N}} f^k(U).$$

Definition

For some compact set $Q \subset \mathbb{R}^d$, the attractor relative to Q is

$$A_Q \overset{def}{=} \bigcap_{k\in\mathbb{N}} f^k(Q).$$

We have

- A_Q is not necessarily invariant,
- but any invariant subset of Q is contained in A_Q.

Computing A_Q

Generate a sequence $\mathcal{B}_0, \mathcal{B}_1, \mathcal{B}_2, \ldots$ of finite families of compact sets as follows:

Let $\mathcal{B}_0 = \{Q\}$, $\theta \in (0,1)$. For $k = 1, 2, \ldots$ do

- construct $\hat{\mathcal{B}}_k$ such that

$$|\hat{\mathcal{B}}_k| = |\mathcal{B}_{k-1}| \quad \text{and} \quad \operatorname{diam} \hat{\mathcal{B}}_k \leq \theta \operatorname{diam} \mathcal{B}_{k-1}.$$

- set

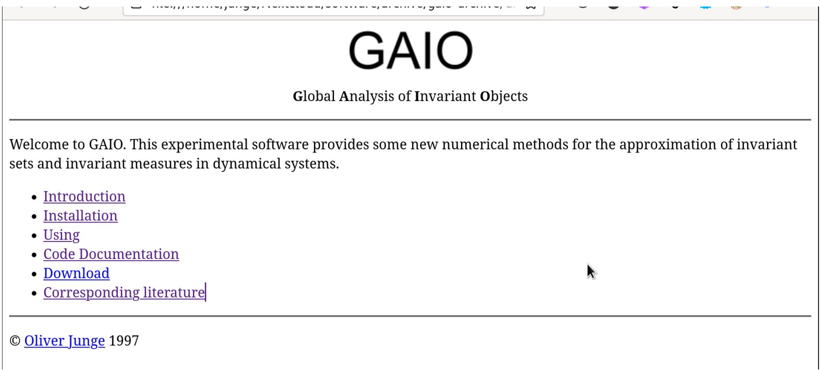
$$\mathcal{B}_k = \{B \in \hat{\mathcal{B}}_k \mid \exists B' \in \hat{\mathcal{B}}_k : f^{-1}(B) \cap B' \neq \varnothing\}.$$

Theorem
$|\mathcal{B}_k| \to A_Q$ as $k \to \infty$ in the Hausdorff metric.

Implementation

Storage of the $\mathcal{B}_k$ in a binary tree, subdivision by bisection:

GAIO v0.2



GAIO

Global **A**nalysis of **I**nvariant **O**bjects

Welcome to GAIO. This experimental software provides some new numerical methods for the approximation of invariant sets and invariant measures in dynamical systems.

- Introduction
- Installation
- Using
- Code Documentation
- Download
- Corresponding literature

Architecture

Architecture

[fragile]

def incmatrix(genl1,genl2): m = len(genl1) n = len(genl2) M = None to become the incidence matrix VT = np.zeros((n*m,1), int) dummy variable

return M