Chapter 16

Working with Lists

Lists are probably the most commonly used data structure in Scala programs. This chapter explains lists in detail. We will present many common operations that can be performed on lists. We'll also cover some important design principles for programs working on lists.

16.1 LIST LITERALS

You saw lists already in the preceding chapters, so you know that a list containing the elements 'a', 'b', and 'c' is written List('a', 'b', 'c'). Here are some other examples:

```
val fruit = List("apples", "oranges", "pears")
val nums = List(1, 2, 3, 4)
val diag3 =
  List(
    List(1, 0, 0),
    List(0, 1, 0),
    List(0, 0, 1)
  )
val empty = List()
```

Lists are quite similar to arrays, but there are two important differences. First, lists are immutable. That is, elements of a list cannot be changed by assignment. Second, lists have a recursive structure (*i.e.*, a *linked list*),[1] whereas arrays are flat.

16.2 THE LIST TYPE

Like arrays, lists are homogeneous: the elements of a list all have the same type. The type of a list that has elements of type T is written List[T]. For instance, here are the same four lists with explicit types added:

```
val fruit: List[String] = List("apples", "oranges", "pears")
val nums: List[Int] = List(1, 2, 3, 4)
val diag3: List[List[Int]] =
  List(
    List(1, 0, 0),
    List(0, 1, 0),
    List(0, 0, 1)
  )
val empty: List[Nothing] = List()
```

The list type in Scala is covariant. This means that for each pair of types S and T, if S is a subtype of T, then List[S] is a subtype of List[T]. For instance, List[String] is a subtype ofList[Object]. This is natural because every list of strings can also be seen as a list of objects.[2]

Note that the empty list has type List[Nothing]. You saw in Section 11.3 that Nothing is the bottom type in Scala's class hierarchy. It is a subtype of every other Scala type. Because lists are covariant, it follows that List[Nothing] is a subtype of List[T] for any type T. So the empty list object, which has type List[Nothing], can also be seen as an object of every other list type of the form List[T]. That's why it is permissible to write code like:

```
// List() is also of type List[String]!
val xs: List[String] = List()
```

16.3 CONSTRUCTING LISTS

All lists are built from two fundamental building blocks, Nil and :: (pronounced "cons"). Nilrepresents the empty list. The infix operator, ::, expresses list extension at the front. That is,x :: xs represents a list whose first element is x, followed by (the elements of) list xs. Hence, the previous list values could also have been defined as follows:

```
val fruit = "apples" :: ("oranges" :: ("pears" :: Nil))
val nums  = 1 :: (2 :: (3 :: (4 :: Nil)))
val diag3 = (1 :: (0 :: (0 :: Nil))) ::
            (0 :: (1 :: (0 :: Nil))) ::
            (0 :: (0 :: (1 :: Nil))) :: Nil
val empty = Nil
```

In fact the previous definitions of fruit, nums, diag3, and empty in terms of List(...) are just wrappers that expand to these definitions. For instance, List(1, 2, 3) creates the list1 :: (2 :: (3 :: Nil)).

Because it ends in a colon, the :: operation associates to the right: A :: B :: C is interpreted asA :: (B :: C). Therefore, you can drop the parentheses in the previous definitions. For instance:

```
val nums = 1 :: 2 :: 3 :: 4 :: Nil
```

is equivalent to the previous definition of nums.

16.4 BASIC OPERATIONS ON LISTS

All operations on lists can be expressed in terms of the following three:

head	returns the first element of a list
tail	returns a list consisting of all elements except the first
isEmpty	returns true if the list is empty

These operations are defined as methods of class List. Some examples are shown in Table 16.1.
The head and tail methods are defined only for non-empty lists. When selected from an empty list, they throw an exception:

```
scala> Nil.head
java.util.NoSuchElementException: head of empty list
```

As an example of how lists can be processed, consider sorting the elements of a list of numbers into ascending order. One simple way to do so is insertion sort, which works as follows: To sort a non-empty list x :: xs, sort the remainder xs and insert the first element x at the right position in the result. Sorting an empty list yields the empty list. Expressed as Scala code, the insertion sort algorithm looks like:

```
def isort(xs: List[Int]): List[Int] =
  if (xs.isEmpty) Nil
  else insert(xs.head, isort(xs.tail))

def insert(x: Int, xs: List[Int]): List[Int] =
  if (xs.isEmpty || x <= xs.head) x :: xs
  else xs.head :: insert(x, xs.tail)
```

Table 16.1 - Basic list operations

What it is	What it does
empty.isEmpty	returns true
fruit.isEmpty	returns false
fruit.head	returns "apples"
fruit.tail.head	returns "oranges"
diag3.head	returns List(1, 0, 0)

16.5 LIST PATTERNS

Lists can also be taken apart using pattern matching. List patterns correspond one-by-one to list expressions. You can either match on all elements of a list using a pattern of the formList(...), or you take lists apart bit by bit using patterns composed from the :: operator and the Nil constant.

Here's an example of the first kind of pattern:

```
scala> val List(a, b, c) = fruit
a: String = apples
b: String = oranges
c: String = pears
```

The pattern List(a, b, c) matches lists of length 3, and binds the three elements to the pattern variables a, b, and c. If you don't know the number of list elements beforehand, it's better to match with :: instead. For instance, the pattern a :: b :: rest matches lists of length 2 or greater:

ABOUT PATTERN MATCHING ON LISTS

If you review the possible forms of patterns explained in Chapter 15, you might find that neither List(...) nor :: look like it fits the kinds of patterns defined there. In fact,List(...) is an instance of a library-defined *extractor* pattern. Such patterns will be discussed in Chapter 26. The "cons" pattern x :: xs is a special case of an infix operation pattern. As an expression, an infix operation is equivalent to a method call. For patterns, the rules are different: As a pattern, an infix operation such as p op q is equivalent to op(p, q). That is, the infix operator op is treated as a constructor pattern. In particular, a cons pattern such as x :: xs is treated as ::(x, xs).

This hints that there should be a class named :: that corresponds to the pattern constructor. Indeed, there is such a class—it is named scala.:: and is exactly the class that builds non-empty lists. So :: exists twice in Scala, once as a name of a class in package scala and again as a method in class List. The effect of the method :: is to produce an instance of the class scala.::. You'll find out more details about how the List class is implemented in Chapter 22.

```
scala> val a :: b :: rest = fruit
a: String = apples
b: String = oranges
rest: List[String] = List(pears)
```

Taking lists apart with patterns is an alternative to taking them apart with the basic methodshead, tail, and isEmpty. For instance, here's insertion sort again, this time written with pattern matching:

```
def isort(xs: List[Int]): List[Int] = xs match {
  case List()   => List()
  case x :: xs1 => insert(x, isort(xs1))
}

def insert(x: Int, xs: List[Int]): List[Int] = xs match {
  case List()  => List(x)
  case y :: ys => if (x <= y) x :: xs
                  else y :: insert(x, ys)
}
```

Often, pattern matching over lists is clearer than decomposing them with methods, so pattern matching should be a part of your list processing toolbox.

This is all you need to know about lists in Scala to use them correctly. However, there are also a large number of methods that capture common patterns of operations over lists. These methods make list processing programs more concise and often clearer. The next two sections present the most important methods defined in the List class.

16.6 FIRST-ORDER METHODS ON CLASS LIST

This section explains most first-order methods defined in the List class. A method is *first-order* if it does not take any functions as arguments. We will also introduce some recommended techniques to structure programs that operate on lists by using two examples.

Concatenating two lists

An operation similar to :: is list concatenation, written `:::'. Unlike ::, ::: takes two lists as operands. The result of xs ::: ys is a new list that contains all the elements of xs, followed by all the elements of ys.

Here are some examples:

```
scala> List(1, 2) ::: List(3, 4, 5)
res0: List[Int] = List(1, 2, 3, 4, 5)

scala> List() ::: List(1, 2, 3)
res1: List[Int] = List(1, 2, 3)

scala> List(1, 2, 3) ::: List(4)
res2: List[Int] = List(1, 2, 3, 4)
```

Like cons, list concatenation associates to the right. An expression like this:

```
xs ::: ys ::: zs
```

is interpreted like this:

```
xs ::: (ys ::: zs)
```

The Divide and Conquer principle

Concatenation (:::) is implemented as a method in class List. It would also be possible to implement concatenation "by hand," using pattern matching on lists. It's instructive to try to do that yourself, because it shows a common way to implement algorithms using lists. First, we'll settle on a signature for the concatenation method, which we'll call append. In order not to mix things up too much, assume that append is defined outside the List class, so it will take the two lists to be concatenated as parameters. These two lists must agree on their element type, but that element type can be arbitrary. This can be expressed by giving append a type parameter[3] that represents the element type of the two input lists:

```
def append[T](xs: List[T], ys: List[T]): List[T]
```

To design the implementation of append, it pays to remember the "divide and conquer" design principle for programs over recursive data structures such as lists. Many algorithms over lists first split an input list into simpler cases using a pattern match. That's the divide part of the principle. They then construct a result for each case. If the result is a non-empty list, some of its parts may be constructed by recursive invocations of the same algorithm. That's theconquer part of the principle.

To apply this principle to the implementation of the append method, the first question to ask is on which list to match. This is less trivial in the case of append than for many other methods because there are two choices. However, the subsequent "conquer" phase tells you that you need to construct a list consisting of all elements of both input lists. Since lists are constructed from the back towards the front, ys can remain intact, whereas xs will need to be taken apart and prepended to ys. Thus, it makes sense to concentrate on xs as a source for a pattern match. The most common pattern match over lists simply distinguishes an empty from a non-empty list. So this gives the following outline of an append method:

```
def append[T](xs: List[T], ys: List[T]): List[T] =
  xs match {
    case List() => ???
    case x :: xs1 => ???
  }
```

All that remains is to fill in the two places marked with ???.[4] The first such place is the alternative where the input list xs is empty. In this case concatenation yields the second list:

```
case List() => ys
```

The second place left open is the alternative where the input list xs consists of some head xfollowed by a tail xs1. In this case the result is also a non-empty list. To construct a non-empty list you need to know what the head and the tail of that list should be. You know that the first element of the result list is x. As for the remaining elements, these can be computed by appending the second list, ys, to the rest of the first list, xs1.

This completes the design and gives:

```
def append[T](xs: List[T], ys: List[T]): List[T] =
  xs match {
    case List() => ys
    case x :: xs1 => x :: append(xs1, ys)
  }
```

The computation of the second alternative illustrated the "conquer" part of the divide and conquer principle: Think first what the shape of the desired output should be, then compute the individual parts of that shape, using recursive invocations of the algorithm where appropriate. Finally, construct the output from these parts.

Taking the length of a list: length

The length method computes the length of a list.

```
scala> List(1, 2, 3).length
res3: Int = 3
```

On lists, unlike arrays, length is a relatively expensive operation. It needs to traverse the whole list to find its end, and therefore takes time proportional to the number of elements in the list. That's why it's not a good idea to replace a test such as xs.isEmpty by xs.length == 0. The result of the two tests is equivalent, but the second one is slower, in particular if the list xs is long.

Accessing the end of a list: init and last

You know already the basic operations head and tail, which respectively take the first element of a list, and the rest of the list except the first element. They each have a dual operation: lastreturns the last element of a (non-empty) list, whereas init returns a list consisting of all elements except the last one:

```
scala> val abcde = List('a', 'b', 'c', 'd', 'e')
abcde: List[Char] = List(a, b, c, d, e)

scala> abcde.last
res4: Char = e

scala> abcde.init
res5: List[Char] = List(a, b, c, d)
```

Like head and tail, these methods throw an exception when applied to an empty list:

```
scala> List().init
java.lang.UnsupportedOperationException: Nil.init
      at scala.List.init(List.scala:544)
      at ...

scala> List().last
java.util.NoSuchElementException: Nil.last
      at scala.List.last(List.scala:563)
```

```
        at ...
```

Unlike head and tail, which both run in constant time, init and last need to traverse the whole list to compute their result. As a result, they take time proportional to the length of the list.

It's a good idea to organize your data so that most accesses are at the head of a list, rather than the last element.

Reversing lists: reverse

If at some point in the computation an algorithm demands frequent accesses to the end of a list, it's sometimes better to reverse the list first and work with the result instead. Here's how to do the reversal:

```
  scala> abcde.reverse
  res6: List[Char] = List(e, d, c, b, a)
```

Like all other list operations, reverse creates a new list rather than changing the one it operates on. Since lists are immutable, such a change would not be possible anyway. To verify this, check that the original value of abcde is unchanged after the reverse operation:

```
  scala> abcde
  res7: List[Char] = List(a, b, c, d, e)
```

The reverse, init, and last operations satisfy some laws that can be used for reasoning about computations and for simplifying programs.

1. reverse is its own inverse:

   ```
   xs.reverse.reverse  equals  xs
   ```

2. reverse turns init to tail and last to head, except that the elements are reversed:

   ```
   xs.reverse.init  equals  xs.tail.reverse
   xs.reverse.tail  equals  xs.init.reverse
   xs.reverse.head  equals  xs.last
   xs.reverse.last  equals  xs.head
   ```

Reverse could be implemented using concatenation (:::), like in the following method, rev:

```
  def rev[T](xs: List[T]): List[T] = xs match {
    case List() => xs
    case x :: xs1 => rev(xs1) ::: List(x)
  }
```

However, this method is less efficient than one would hope for. To study the complexity ofrev, assume that the list xs has length n. Notice that there are n recursive calls to rev. Each call except the last involves a list concatenation. List concatenation xs ::: ys takes time proportional to the length of its first argument xs. Hence, the total complexity of rev is:

$n + (n - 1) + ... + 1 = (1 + n) * n / 2$

In other words, rev's complexity is quadratic in the length of its input argument. This is disappointing when compared to the standard reversal of a mutable, linked list, which has linear complexity. However, the current implementation of rev is not the best implementation possible. In the example starting here, you will see how to speed it up.

Prefixes and suffixes: drop, take, and splitAt

The drop and take operations generalize tail and init in that they return arbitrary prefixes or suffixes of a list. The expression "xs take n" returns the first n elements of the list xs. If n is greater than xs.length, the whole list xs is returned. The operation "xs drop n" returns all elements of the list xs, except for the first n ones. If n is greater than xs.length, the empty list is returned.

The splitAt operation splits the list at a given index, returning a pair of two lists.[5] It is defined by the equality:

xs splitAt n   *equals*   (xs take n, xs drop n)

However, splitAt avoids traversing the list xs twice. Here are some examples of these three methods:

```
scala> abcde take 2
res8: List[Char] = List(a, b)

scala> abcde drop 2
res9: List[Char] = List(c, d, e)

scala> abcde splitAt 2
res10: (List[Char], List[Char]) = (List(a, b),List(c, d, e))
```

Element selection: apply and indices

Random element selection is supported through the apply method; however it is a less common operation for lists than it is for arrays.

```
scala> abcde apply 2 // rare in Scala
res11: Char = c
```

As for all other types, apply is implicitly inserted when an object appears in the function position in a method call. So the line above can be shortened to:

```
scala> abcde(2)      // rare in Scala
res12: Char = c
```

One reason why random element selection is less popular for lists than for arrays is that xs(n)takes time proportional to the index n. In fact, apply is simply defined by a combination of dropand head:

xs apply n   *equals*   (xs drop n).head

This definition also makes clear that list indices range from 0 up to the length of the list minus one, the same as for arrays. The indices method returns a list consisting of all valid indices of a given list:

```
scala> abcde.indices
res13: scala.collection.immutable.Range
```

```
    = Range(0, 1, 2, 3, 4)
```

Flattening a list of lists: flatten

The flatten method takes a list of lists and flattens it out to a single list:

```
  scala> List(List(1, 2), List(3), List(), List(4, 5)).flatten
  res14: List[Int] = List(1, 2, 3, 4, 5)
  scala> fruit.map(_.toCharArray).flatten
  res15: List[Char] = List(a, p, p, l, e, s, o, r, a, n, g, e,
  s, p, e, a, r, s)
```

It can only be applied to lists whose elements are all lists. Trying to flatten any other list will give a compilation error:

```
  scala> List(1, 2, 3).flatten
  <console>:8: error: No implicit view available from Int =>
  scala.collection.GenTraversableOnce[B].
                List(1, 2, 3).flatten
                              ^
```

Zipping lists: zip and unzip

The zip operation takes two lists and forms a list of pairs:

```
  scala> abcde.indices zip abcde
  res17: scala.collection.immutable.IndexedSeq[(Int, Char)] =
  Vector((0,a), (1,b), (2,c), (3,d), (4,e))
```

If the two lists are of different length, any unmatched elements are dropped:

```
  scala> val zipped = abcde zip List(1, 2, 3)
  zipped: List[(Char, Int)] = List((a,1), (b,2), (c,3))
```

A useful special case is to zip a list with its index. This is done most efficiently with thezipWithIndex method, which pairs every element of a list with the position where it appears in the list.

```
  scala> abcde.zipWithIndex
  res18: List[(Char, Int)] = List((a,0), (b,1), (c,2), (d,3),
      (e,4))
```

Any list of tuples can also be changed back to a tuple of lists by using the unzip method:

```
  scala> zipped.unzip
  res19: (List[Char], List[Int])
    = (List(a, b, c),List(1, 2, 3))
```

The zip and unzip methods provide one way to operate on multiple lists together. See Section 16.9 for a more concise way to do this.

Displaying lists: toString and mkString

The toString operation returns the canonical string representation of a list:

```
scala> abcde.toString
res20: String = List(a, b, c, d, e)
```

If you want a different representation you can use the mkString method. The operationxs mkString (pre, sep, post) involves four operands: the list xs to be displayed, a prefix string preto be displayed in front of all elements, a separator string sep to be displayed between successive elements, and a postfix string post to be displayed at the end.

The result of the operation is the string:

pre + xs(0) + sep + ...+ sep + xs(xs.length - 1) + post

The mkString method has two overloaded variants that let you drop some or all of its arguments. The first variant only takes a separator string:

xs mkString sep *equals* xs mkString ("", sep, "")

The second variant lets you omit all arguments:

xs.mkString *equals* xs mkString ""

Here are some examples:

```
scala> abcde mkString ("[", ",", "]")
res21: String = [a,b,c,d,e]

scala> abcde mkString ""
res22: String = abcde

scala> abcde.mkString
res23: String = abcde

scala> abcde mkString ("List(", ", ", ")")
res24: String = List(a, b, c, d, e)
```

There are also variants of the mkString methods called addString which append the constructed string to a StringBuilder object,[6] rather than returning them as a result:

```
scala> val buf = new StringBuilder
buf: StringBuilder =

scala> abcde addString (buf, "(", ";", ")")
res25: StringBuilder = (a;b;c;d;e)
```

The mkString and addString methods are inherited from List's super trait Traversable, so they are applicable to all other collections as well.

Converting lists: iterator, toArray, copyToArray

To convert data between the flat world of arrays and the recursive world of lists, you can use method toArray in class List and toList in class Array:

```
scala> val arr = abcde.toArray
arr: Array[Char] = Array(a, b, c, d, e)
```

```
scala> arr.toList
res26: List[Char] = List(a, b, c, d, e)
```

There's also a method copyToArray, which copies list elements to successive array positions within some destination array. The operation:

```
xs copyToArray (arr, start)
```

copies all elements of the list xs to the array arr, beginning with position start. You must ensure that the destination array arr is large enough to hold the list in full. Here's an example:

```
scala> val arr2 = new Array[Int](10)
arr2: Array[Int] = Array(0, 0, 0, 0, 0, 0, 0, 0, 0, 0)

scala> List(1, 2, 3) copyToArray (arr2, 3)

scala> arr2
res28: Array[Int] = Array(0, 0, 0, 1, 2, 3, 0, 0, 0, 0)
```

Finally, if you need to access list elements via an iterator, you can use the iterator method:

```
scala> val it = abcde.iterator
it: Iterator[Char] = non-empty iterator

scala> it.next
res29: Char = a

scala> it.next
res30: Char = b
```

Example: Merge sort

The insertion sort presented earlier is concise to write, but it is not very efficient. Its average complexity is proportional to the square of the length of the input list. A more efficient algorithm is merge sort.

THE FAST TRACK

This example provides another illustration of the divide and conquer principle and currying, as well as a useful discussion of algorithmic complexity. If you prefer to move a bit faster on your first pass through this book, however, you can safely skip to Section 16.7.

Merge sort works as follows: First, if the list has zero or one elements, it is already sorted, so the list can be returned unchanged. Longer lists are split into two sub-lists, each containing about half the elements of the original list. Each sub-list is sorted by a recursive call to the sort function, and the resulting two sorted lists are then combined in a merge operation.

For a general implementation of merge sort, you want to leave open the type of list elements to be sorted and the function to be used for the comparison of elements. You obtain a function of maximal generality by passing these two items as parameters. This leads to the implementation shown in Listing 16.1.

```
def msort[T](less: (T, T) => Boolean)
    (xs: List[T]): List[T] = {

  def merge(xs: List[T], ys: List[T]): List[T] =
    (xs, ys) match {
      case (Nil, _) => ys
      case (_, Nil) => xs
      case (x :: xs1, y :: ys1) =>
        if (less(x, y)) x :: merge(xs1, ys)
        else y :: merge(xs, ys1)
    }

  val n = xs.length / 2
  if (n == 0) xs
  else {
    val (ys, zs) = xs splitAt n
    merge(msort(less)(ys), msort(less)(zs))
  }
}
```

Listing 16.1 - A merge sort function for Lists.

The complexity of msort is order (*n log*(*n*)), where *n* is the length of the input list. To see why, note that splitting a list in two and merging two sorted lists each take time proportional to the length of the argument list(s). Each recursive call of msort halves the number of elements in its input, so there are about *log*(*n*) consecutive recursive calls until the base case of lists of length 1 is reached. However, for longer lists each call spawns off two further calls. Adding everything up, we obtain at each of the *log*(*n*) call levels, every element of the original lists takes part in one split operation and one merge operation.

Hence, every call level has a total cost proportional to *n*. Since there are *log*(*n*) call levels, we obtain an overall cost proportional to *n log*(*n*). That cost does not depend on the initial distribution of elements in the list, so the worst case cost is the same as the average case cost. This property makes merge sort an attractive algorithm for sorting lists.

Here is an example of how msort is used:

```
scala> msort((x: Int, y: Int) => x < y)(List(5, 7, 1, 3))
res31: List[Int] = List(1, 3, 5, 7)
```

The msort function is a classical example of the currying concept discussed in Section 9.3. Currying makes it easy to specialize the function for particular comparison functions. Here's an example:

```
scala> val intSort = msort((x: Int, y: Int) => x < y) _
intSort: List[Int] => List[Int] = <function1>
```

The intSort variable refers to a function that takes a list of integers and sorts them in numerical order. As described in Section 8.6, an underscore stands for a missing argument list. In this case, the missing argument is the list that should be sorted. As another example, here's how you could define a function that sorts a list of integers in reverse numerical order:

```
scala> val reverseIntSort = msort((x: Int, y: Int) => x > y) _
reverseIntSort: (List[Int]) => List[Int] = <function>
```

Because you provided the comparison function already via currying, you now need only provide the list to sort when you invoke the intSort or reverseIntSort functions. Here are some examples:

```
scala> val mixedInts = List(4, 1, 9, 0, 5, 8, 3, 6, 2, 7)
mixedInts: List[Int] = List(4, 1, 9, 0, 5, 8, 3, 6, 2, 7)

scala> intSort(mixedInts)
res0: List[Int] = List(0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

scala> reverseIntSort(mixedInts)
res1: List[Int] = List(9, 8, 7, 6, 5, 4, 3, 2, 1, 0)
```

16.7 HIGHER-ORDER METHODS ON CLASS LIST

Many operations over lists have a similar structure. Several patterns appear time and time again. Some examples are: transforming every element of a list in some way, verifying whether a property holds for all elements of a list, extracting from a list elements satisfying a certain criterion, or combining the elements of a list using some operator. In Java, such patterns would usually be expressed by idiomatic combinations of for or while loops. In Scala, they can be expressed more concisely and directly using higher-order operators,[7] which are implemented as methods in class List. These higher-order operators are discussed in this section.

Mapping over lists: map, flatMap and foreach

The operation xs map f takes as operands a list xs of type List[T] and a function f of type T => U. It returns the list that results from applying the function f to each list element in xs. For instance:

```
scala> List(1, 2, 3) map (_ + 1)
res32: List[Int] = List(2, 3, 4)

scala> val words = List("the", "quick", "brown", "fox")
words: List[String] = List(the, quick, brown, fox)

scala> words map (_.length)
res33: List[Int] = List(3, 5, 5, 3)

scala> words map (_.toList.reverse.mkString)
res34: List[String] = List(eht, kciuq, nworb, xof)
```

The flatMap operator is similar to map, but it takes a function returning a list of elements as its right operand. It applies the function to each list element and returns the concatenation of all function results. The difference between map and flatMap is illustrated in the following example:

```
scala> words map (_.toList)
res35: List[List[Char]] = List(List(t, h, e), List(q, u, i,
    c, k), List(b, r, o, w, n), List(f, o, x))
```

```
scala> words flatMap (_.toList)
res36: List[Char] = List(t, h, e, q, u, i, c, k, b, r, o, w,
    n, f, o, x)
```

You see that where map returns a list of lists, flatMap returns a single list in which all element lists are concatenated.

The differences and interplay between map and flatMap are also demonstrated by the following expression, which constructs a list of all pairs (*i*, *j*) such that $1 \leq j < i < 5$:

```
scala> List.range(1, 5) flatMap (
         i => List.range(1, i) map (j => (i, j))
       )
res37: List[(Int, Int)] = List((2,1), (3,1), (3,2), (4,1),
    (4,2), (4,3))
```

List.range is a utility method that creates a list of all integers in some range. It is used twice in this example: once to generate a list of integers from 1 (including) until 5 (excluding), and a second time to generate a list of integers from 1 until *i*, for each value of *i* taken from the first list. The map in this expression generates a list of tuples (*i*, *j*) where $j < i$. The outer flatMap in this example generates this list for each i between 1 and 5, and then concatenates all the results. Alternatively, the same list can be constructed with a for expression:

```
for (i <- List.range(1, 5); j <- List.range(1, i)) yield (i, j)
```

You'll learn more about the interplay of for expressions and list operations in Chapter 23.

The third map-like operation is foreach. Unlike map and flatMap, however, foreach takes a procedure (a function with result type Unit) as right operand. It simply applies the procedure to each list element. The result of the operation itself is again Unit; no list of results is assembled. As an example, here is a concise way of summing up all numbers in a list:

```
scala> var sum = 0
sum: Int = 0

scala> List(1, 2, 3, 4, 5) foreach (sum += _)

scala> sum
res39: Int = 15
```

Filtering lists: filter, partition, find, takeWhile, dropWhile, and span

The operation "xs filter p" takes as operands a list xs of type List[T] and a predicate function pof type T => Boolean. It yields the list of all elements x in xs for which p(x) is true. For instance:

```
scala> List(1, 2, 3, 4, 5) filter (_ % 2 == 0)
res40: List[Int] = List(2, 4)

scala> words filter (_.length == 3)
res41: List[String] = List(the, fox)
```

The partition method is like filter but returns a pair of lists. One list contains all elements for which the predicate is true, while the other contains all elements for which the predicate is false. It is defined by the equality:

xs partition p   *equals*   (xs filter p, xs filter (!p(_)))

Here's an example:

```
scala> List(1, 2, 3, 4, 5) partition (_ % 2 == 0)
res42: (List[Int], List[Int]) = (List(2, 4),List(1, 3, 5))
```

The find method is also similar to filter, but it returns the first element satisfying a given predicate, rather than all such elements. The operation xs find p takes a list xs and a predicatep as operands. It returns an optional value. If there is an element x in xs for which p(x) is true,Some(x) is returned. Otherwise, p is false for all elements, and None is returned. Here are some examples:

```
scala>  List(1, 2, 3, 4, 5) find (_ % 2 == 0)
res43: Option[Int] = Some(2)

scala>  List(1, 2, 3, 4, 5) find (_ <= 0)
res44: Option[Int] = None
```

The takeWhile and dropWhile operators also take a predicate as their right operand. The operation xs takeWhile p takes the longest prefix of list xs such that every element in the prefix satisfies p. Analogously, the operation xs dropWhile p removes the longest prefix from list xssuch that every element in the prefix satisfies p. Here are some examples:

```
scala> List(1, 2, 3, -4, 5) takeWhile (_ > 0)
res45: List[Int] = List(1, 2, 3)

scala> words dropWhile (_ startsWith "t")
res46: List[String] = List(quick, brown, fox)
```

The span method combines takeWhile and dropWhile in one operation, just like splitAt combinestake and drop. It returns a pair of two lists, defined by the equality:

xs span p   *equals*   (xs takeWhile p, xs dropWhile p)

Like splitAt, span avoids traversing the list xs twice:

```
scala> List(1, 2, 3, -4, 5) span (_ > 0)
res47: (List[Int], List[Int]) = (List(1, 2, 3),List(-4, 5))
```

Predicates over lists: forall and exists

The operation xs forall p takes as arguments a list xs and a predicate p. Its result is true if all elements in the list satisfy p. Conversely, the operation xs exists p returns true if there is an element in xs that satisfies the predicate p. For instance, to find out whether a matrix represented as a list of lists has a row with only zeroes as elements:

```
scala> def hasZeroRow(m: List[List[Int]]) =
         m exists (row => row forall (_ == 0))
```

```
hasZeroRow: (m: List[List[Int]])Boolean

scala> hasZeroRow(diag3)
res48: Boolean = false
```

Folding lists: /: and :

Another common kind of operation combines the elements of a list with some operator. For instance:

sum(List(a, b, c))   *equals*   0 + a + b + c

This is a special instance of a fold operation:

```
scala> def sum(xs: List[Int]): Int = (0 /: xs) (_ + _)
sum: (xs: List[Int])Int
```

Similarly:

product(List(a, b, c))   *equals*   1 * a * b * c

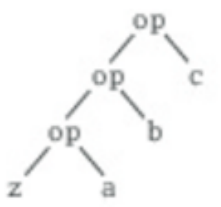
is a special instance of this fold operation:

```
scala> def product(xs: List[Int]): Int = (1 /: xs) (_ * _)
product: (xs: List[Int])Int
```

A fold left operation "(z /: xs) (op)" involves three objects: a start value z, a list xs, and a binary operation op. The result of the fold is op applied between successive elements of the list prefixed by z. For instance:

(z /: List(a, b, c)) (op)   *equals*   op(op(op(z, a), b), c)

Or, graphically:



Here's another example that illustrates how /: is used. To concatenate all words in a list of strings with spaces between them and in front, you can write this:

```
scala>  ("" /: words) (_ + " " + _)
res49: String = " the quick brown fox"
```

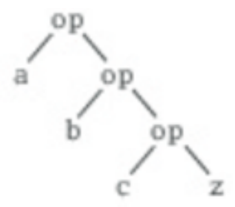
This gives an extra space at the beginning. To remove the space, you can use this slight variation:

```
scala> (words.head /: words.tail)  (_ + " " + _)
res50: String = the quick brown fox
```

The /: operator produces left-leaning operation trees (its syntax with the slash rising forward is intended to be a reflection of that). The operator has :\ as an analog that produces right-leaning trees. For instance:

(List(a, b, c) :\ z) (op)   *equals*   op(a, op(b, op(c, z)))

Or, graphically:



The :\ operator is pronounced fold right. It involves the same three operands as fold left, but the first two appear in reversed order: The first operand is the list to fold, the second is the start value.

For associative operations, fold left and fold right are equivalent, but there might be a difference in efficiency. Consider for instance an operation corresponding to the flattenmethod, which concatenates all elements in a list of lists. This could be implemented with either fold left or fold right:

```
def flattenLeft[T](xss: List[List[T]]) =
    (List[T]() /: xss) (_ ::: _)

def flattenRight[T](xss: List[List[T]]) =
    (xss :~List[T]()) (_ ::: _)
```

Because list concatenation, xs ::: ys, takes time proportional to its first argument xs, the implementation in terms of fold right in flattenRight is more efficient than the fold left implementation in flattenLeft. The problem is that flattenLeft(xss) copies the first element listxss.head *n*-1 times, where *n* is the length of the list xss.

Note that both versions of flatten require a type annotation on the empty list that is the start value of the fold. This is due to a limitation in Scala's type inferencer, which fails to infer the correct type of the list automatically. If you try to leave out the annotation, you get the following:

```
scala> def flattenRight[T](xss: List[List[T]]) =
           (xss :~List()) (_ ::: _)
<console>:8: error: type mismatch;
 found   : List[T]
 required: List[Nothing]
           (xss :~List()) (_ ::: _)
                               ^
```

To find out why the type inferencer goes wrong, you'll need to know about the types of the fold methods and how they are implemented. More on this in Section 16.10. Lastly, although the /: and :\ operators have the advantage that the direction of the slash resembles the graphical depiction of their respective left or right-leaning trees, and the associativity of the colon character places the start value in the same position in the expression as it is in the tree, some may find the

resulting code less than intuitive. If you prefer, you can use the methods
named foldLeft and foldRight instead, which are also defined on class List.

Example: List reversal using fold

Earlier in the chapter you saw an implementation of method reverse, named rev, whose running time was quadratic in the length of the list to be reversed. Here is a different implementation of reverse that has linear cost. The idea is to use a fold left operation based on the following scheme:

```
def reverseLeft[T](xs: List[T]) = (startvalue /: xs)(operation)
```

What remains is to fill in the *startvalue* and *operation* parts. In fact, you can try to deduce these parts from some simple examples. To deduce the correct value of *startvalue*, you can start with the smallest possible list, List(), and calculate as follows:

```
List()
  equals (by the properties of reverseLeft)

reverseLeft(List())
  equals (by the template for reverseLeft)

(startvalue /: List())(operation)
  equals (by the definition of /:)

startvalue
```

Hence, *startvalue* must be List(). To deduce the second operand, you can pick the next smallest list as an example case. You know already that *startvalue* is List(), so you can calculate as follows:

```
List(x)
  equals (by the properties of reverseLeft)

reverseLeft(List(x))
  equals (by the template for reverseLeft, with startvalue = List())

(List() /: List(x)) (operation)
  equals (by the definition of /:)

operation(List(), x)
```

Hence, *operation*(List(), x) equals List(x), which can also be written as x :: List(). This suggests taking as *operation* the :: operator with its operands exchanged. (This operation is sometimes called "snoc," in reference to ::, which is called cons.) We arrive then at the following implementation for reverseLeft:

```
def reverseLeft[T](xs: List[T]) =
  (List[T]() /: xs) {(ys, y) => y :: ys}
```

Again, the type annotation in List[T]() is needed to make the type inferencer work. If you analyze the complexity of reverseLeft, you'll find that it applies a constant-time operation ("snoc") *n* times, where *n* is the length of the argument list. Thus, the complexity ofreverseLeft is linear.

Sorting lists: sortWith

The operation xs sortWith before, where "xs" is a list and "before" is a function that can be used to compare two elements, sorts the elements of list xs. The expression x before y should returntrue if x should come before y in the intended ordering for the sort. For instance:

```
scala> List(1, -3, 4, 2, 6) sortWith (_ < _)
res51: List[Int] = List(-3, 1, 2, 4, 6)

scala> words sortWith (_.length > _.length)
res52: List[String] = List(quick, brown, the, fox)
```

Note that sortWith performs a merge sort similar to the msort algorithm shown in the last section. But sortWith is a method of class List, whereas msort is defined outside lists.

16.8 METHODS OF THE LIST OBJECT

So far, all operations you have seen in this chapter are implemented as methods of class List, so you invoke them on individual list objects. There are also a number of methods in the globally accessible object scala.List, which is the companion object of class List. Some of these operations are factory methods that create lists. Others are operations that work on lists of some specific shape. Both kinds of methods will be presented in this section.

Creating lists from their elements: List.apply

You've already seen on several occasions list literals such as List(1, 2, 3). There's nothing special about their syntax. A literal like List(1, 2, 3) is simply the application of the object Listto the elements 1, 2, 3. That is, it is equivalent to List.apply(1, 2, 3):

```
scala> List.apply(1, 2, 3)
res53: List[Int] = List(1, 2, 3)
```

Creating a range of numbers: List.range

The range method, which you saw briefly earlier in the discussion of map and flatmap, creates a list consisting of a range of numbers. Its simplest form is List.range(from, until), which creates a list of all numbers starting at from and going up to until minus one. So the end value, until, does not form part of the range.

There's also a version of range that takes a step value as third parameter. This operation will yield list elements that are step values apart, starting at from. The step can be positive or negative:

```
scala> List.range(1, 5)
res54: List[Int] = List(1, 2, 3, 4)

scala> List.range(1, 9, 2)
res55: List[Int] = List(1, 3, 5, 7)

scala> List.range(9, 1, -3)
res56: List[Int] = List(9, 6, 3)
```

Creating uniform lists: List.fill

The fill method creates a list consisting of zero or more copies of the same element. It takes two parameters: the length of the list to be created, and the element to be repeated. Each parameter is given in a separate list:

```
scala> List.fill(5)('a')
res57: List[Char] = List(a, a, a, a, a)

scala> List.fill(3)("hello")
res58: List[String] = List(hello, hello, hello)
```

If fill is given more than two arguments, then it will make multi-dimensional lists. That is, it will make lists of lists, lists of lists of lists, *etc.* The additional arguments go in the first argument list.

```
scala> List.fill(2, 3)('b')
res59: List[List[Char]] = List(List(b, b, b), List(b, b, b))
```

Tabulating a function: List.tabulate

The tabulate method creates a list whose elements are computed according to a supplied function. Its arguments are just like those of List.fill: the first argument list gives the dimensions of the list to create, and the second describes the elements of the list. The only difference is that instead of the elements being fixed, they are computed from a function:

```
scala> val squares = List.tabulate(5)(n => n * n)
squares: List[Int] = List(0, 1, 4, 9, 16)
scala> val multiplication = List.tabulate(5,5)(_ * _)
multiplication: List[List[Int]] = List(List(0, 0, 0, 0, 0),
    List(0, 1, 2, 3, 4), List(0, 2, 4, 6, 8),
    List(0, 3, 6, 9, 12), List(0, 4, 8, 12, 16))
```

Concatenating multiple lists: List.concat

The concat method concatenates a number of element lists. The lists to be concatenated are supplied as direct arguments to concat:

```
scala> List.concat(List('a', 'b'), List('c'))
res60: List[Char] = List(a, b, c)

scala> List.concat(List(), List('b'), List('c'))
res61: List[Char] = List(b, c)

scala> List.concat()
res62: List[Nothing] = List()
```

16.9 PROCESSING MULTIPLE LISTS TOGETHER

The zipped method on tuples generalizes several common operations to work on multiple lists instead of just one. One such operation is map. The map method for two zipped lists maps pairs of elements rather than individual elements. One pair is for the first element of each list, another pair is for the second element of each list, and so on—as many pairs as the lists are long. Here is an example of its use:

```
scala> (List(10, 20), List(3, 4, 5)).zipped.map(_ * _)
res63: List[Int] = List(30, 80)
```

Notice that the third element of the second list is discarded. The zipped method zips up only as many elements as appear in all the lists together. Any extra elements on the end are discarded.

There are also zipped analogs to exists and forall. They are just like the single-list versions of those methods except they operate on elements from multiple lists instead of just one:

```
scala> (List("abc", "de"), List(3, 2)).zipped.
         forall(_.length == _)
res64: Boolean = true
scala> (List("abc", "de"), List(3, 2)).zipped.
         exists(_.length != _)
res65: Boolean = false
```

THE FAST TRACK

In the next (and final) section of this chapter, we provide insight into Scala's type inference algorithm. If you're not interested in such details right now, you can skip the entire section and go straight to the conclusion here.

16.10 UNDERSTANDING SCALA'S TYPE INFERENCE ALGORITHM

One difference between the previous uses of sortWith and msort concerns the admissible syntactic forms of the comparison function.

Compare:

```
scala> msort((x: Char, y: Char) => x > y)(abcde)
res66: List[Char] = List(e, d, c, b, a)
```

with:

```
scala> abcde sortWith (_ > _)
res67: List[Char] = List(e, d, c, b, a)
```

The two expressions are equivalent, but the first uses a longer form of comparison function with named parameters and explicit types. The second uses the concise form, (_ > _), where named parameters are replaced by underscores. Of course, you could also use the first, longer form of comparison with sortWith.

However, the short form cannot be used with msort.

```
scala> msort(_ > _)(abcde)
<console>:12: error: missing parameter type for expanded
function ((x$1, x$2) => x$1.$greater(x$2))
       msort(_ > _)(abcde)
             ^
```

To understand why, you need to know some details of Scala's type inference algorithm. Type inference in Scala is flow based. In a method application m(args), the inferencer first checks whether the method m has a known type. If it does, that type is used to infer the expected type of the arguments. For instance, in abcde.sortWith(_ > _), the type of abcde is List[Char]. Hence,sortWith is known to be a method that takes an argument of type (Char, Char) => Boolean and produces a result of type List[Char]. Since the parameter types of the function arguments are known, they need not be written explicitly. With what it knows about sortWith, the inferencer can deduce that (_ > _) should expand to ((x: Char, y: Char) => x > y) where x and y are some arbitrary fresh names.

Now consider the second case, msort(_ > _)(abcde). The type of msort is a curried, polymorphic method type that takes an argument of type (T, T) => Boolean to a function from List[T] toList[T] where T is some as-yet unknown type. The msort method needs to be instantiated with a type parameter before it can be applied to its arguments.

Because the precise instance type of msort in the application is not yet known, it cannot be used to infer the type of its first argument. The type inferencer changes its strategy in this case; it first type checks method arguments to determine the proper instance type of the method. However, when tasked to type check the short-hand function literal, (_ > _), it fails because it has no information about the types of the implicit function parameters that are indicated by underscores.

One way to resolve the problem is to pass an explicit type parameter to msort, as in:

```
scala> msort[Char](_ > _)(abcde)
res68: List[Char] = List(e, d, c, b, a)
```

Because the correct instance type of msort is now known, it can be used to infer the type of the arguments. Another possible solution is to rewrite the msort method so that its parameters are swapped:

```
def msortSwapped[T](xs: List[T])(less:
    (T, T) => Boolean): List[T] = {

  // same implementation as msort,
  // but with arguments swapped
}
```

Now type inference would succeed:

```
scala> msortSwapped(abcde)(_ > _)
res69: List[Char] = List(e, d, c, b, a)
```

What has happened is that the inferencer used the known type of the first parameter abcde to determine the type parameter of msortSwapped. Once the precise type of msortSwapped was known, it could be used in turn to infer the type of the second parameter, (_ > _).

Generally, when tasked to infer the type parameters of a polymorphic method, the type inferencer consults the types of all value arguments in the first parameter list but no arguments beyond that. Since msortSwapped is a curried method with two parameter lists, the second argument (*i.e.*, the function value) did not need to be consulted to determine the type parameter of the method.

This inference scheme suggests the following library design principle: When designing a polymorphic method that takes some non-function arguments and a function argument, place the function argument last in a curried parameter list on its own. That way, the method's correct instance type can be inferred from the non-function arguments, and that type can in turn be used to type check the function argument. The net effect is that users of the method will be able to give less type information and write function literals in more compact ways.

Now to the more complicated case of a fold operation. Why is there the need for an explicit type parameter in an expression like the body of the flattenRight method shown here?

```
(xss :~List[T]()) (_ ::: _)
```

The type of the fold-right operation is polymorphic in two type variables. Given an expression:

```
(xs :~z) (op)
```

The type of xs must be a list of some arbitrary type A, say xs: List[A]. The start value z can be of some other type B. The operation op must then take two arguments of type A and B, and return a result of type B, *i.e.*, op: (A, B) => B. Because the type of z is not related to the type of the listxs, type inference has no context information for z.

Now consider the expression in the erroneous version of flattenRight, also shown here:

```
(xss :~List()) (_ ::: _)  // this won't compile
```

The start value z in this fold is an empty list, List(), so without additional type information its type is inferred to be a List[Nothing]. Hence, the inferencer will infer that the B type of the fold is List[Nothing]. Therefore, the operation (_ ::: _) of the fold is expected to be of the following type:

```
(List[T], List[Nothing]) => List[Nothing]
```

This is indeed a possible type for the operation in that fold but it is not a very useful one! It says that the operation always takes an empty list as second argument and always produces an empty list as result.

In other words, the type inference settled too early on a type for List(); it should have waited until it had seen the type of the operation op. So the (otherwise very useful) rule to only consider the first argument section in a curried method application for determining the method's type is at the root of the problem here. On the other hand, even if that rule were relaxed, the inferencer still could not come up with a type for op because its parameter types are not given. Hence, there is a Catch-22 situation that can only be resolved by an explicit type annotation from the programmer.

This example highlights some limitations of the local, flow-based type inference scheme of Scala. It is not present in the more global Hindley-Milner style of type inference used in functional languages, such as ML or Haskell. However, Scala's local type inference deals much more gracefully with object-oriented subtyping than the Hindley-Milner style does. Fortunately, the limitations show up only in some corner cases, and are usually easily fixed by adding an explicit type annotation.

Adding type annotations is also a useful debugging technique when you get confused by type error messages related to polymorphic methods. If you are unsure what caused a particular type error, just add some type arguments or other type annotations, which you think are correct. Then you should be able to quickly see where the real problem is.

16.11 CONCLUSION

Now you have seen many ways to work with lists. You have seen the basic operations like headand tail, the first-order operations like reverse, the higher-order operations like map, and the utility methods in the List object. Along the way, you learned a bit about how Scala's type inference works.

Lists are a real work horse in Scala, so you will benefit from knowing how to use them. For that reason, this chapter has delved deeply into how to use lists. Lists are just one kind of collection that Scala supports, however. The next chapter is broad, rather than deep, and shows you how to use a variety of Scala's collection types.

Footnotes for Chapter 16:

[1] For a graphical depiction of the structure of a List, see Figure 22.2 here.

[2] Chapter 19 gives more details on covariance and other kinds of variance.

[3] Type parameters will be explained in more detail in Chapter 19.

[4] The ??? method, which throws scala.NotImplementedError and has result type Nothing, can be used as a temporary implementation during development.

[5] As mentioned in Section 10.12, the term *pair* is an informal name for Tuple2.

[6] This is class scala.StringBuilder, not java.lang.StringBuilder.

[7] By *higher-order operators*, we mean higher-order functions used in operator notation. As mentioned in Section 9.1, a function is "higher-order" if it takes one or more other functions as a parameters.