# Analysis of Randomness of GAEN keys

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...ABSTRACT...

# Acknowledgments

Thank you Mum & Dad.

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APRIL SHEERAN

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Introduction

### 2.1.1 What are GAEN Keys

Introduction

**Contact Tracing Apps**

Google and Apple developed the Google/Apple Exposure Notification (GAEN) system to facilitate contact tracing in response to the Covid-19 Pandemic. (Mention conventional contact tracing). Nations across the world used this technology to create contract tracing apps, for example Covid Tracker in Ireland and SwissCovid in Switzerland (Leith and Farrell (2021)).

The way the contact tracing works, if a user enables it, is as follows:

- Every 10-20 minutes the user's device will generate a random 128-bit key, referred to as a Temporary Exposure Key (TEK).

- The user's device will broadcast these keys using Bluetooth Low Energy (BLE).

- The user's device will listen and store the TEKs being broadcasted from other devices within a certain radius. These TEKs are stored locally on the device.

- If a user tests positive for Covid, they can log this into the app. The app will send the user's recent TEKs (around the 14 days) to a central server managed by the local health authority.

- Every approx. 2 hours, the user's device will download the TEKs from the central server.

- The app compares these downloaded TEKs to the TEKs stored locally on the device.

- If there is a match, this means that the user has potentially been exposed to Covid and the app will notify them.

- CITES

**Studies on GAEN-based Apps**

There have been numerous studies done on contact tracing apps that use GAEN technology. Google and Apple acknowledge that keeping users' information private and secure is essential to the success of the contact tracing app and claim to have designed their system with this central to the design (Google (2020)). Why is privacy so important for this app? Don't want to share covid status etc.

The major privacy concerns of GAEN apps according to (Nguyen et al. (2022)) are the identification of users, tracking users or extracting the social graph of users. Contact tracing should aim to identify encounters rather than actual users, by doing so they

should not leak any information that could be used to identify the user. Similarly, the data collected should not be able to be used to create the social graph of the user, the social connections and relationships of a user. Having this information could potentially result in the user being identified.

Security is also a requirement for a contact tracing app according to (Nguyen et al. (2022)). The system should be resilient to large scale data pollution attacks. These could be fake exposure claims, where users may falsely claim they have been exposed in order to get out of work or another obligation or in an attempt to damage the reputation and credibility of the contact tracing apps. Fake exposure injection, a relay attack, may send users false notifications of potential exposures. The attacker could do this by capturing the TEKs of some user and broadcasting them in another location, leading to people being falsely notified of exposure. This could result in panic among users and the population, putting further strain on the healthcare system by creating a demand for unnecessary tests. It could also damage the trust in the contact tracing apps as their accuracy would be no longer trusted.

(Nguyen et al. (2022)) assess the GAEN apps on a variety of requirements. In terms of effectiveness, GAEN has been found to be imprecise at determining the distances between user devices (do i need to reference an internal reference?). Its use of BLE means scanning of the user's surroundings for other devices can only happen with frequent pauses to save battery life of the device. Many factors like positioning of the device's antenna, obstacles in the way and orientation of the device affect the computation of the distance between devices and the errors are significant. GAEN fails to account for 'superspreaders' of the virus, an individual who is very contagious and infects a number of other people. GAEN also does not have any mechanism for dealing with asymptomatic individuals, people that are infected with the virus and are contagious but do not show symptoms. Unknowingly, these people spread the virus. These individuals are unlikely to get tested and therefore

won't log their infection in the app, meaning those that come in contact with them will not be notified of a potential exposure. This significantly impacts the effectiveness of the contact tracing apps.

An investigation into the data shared by Europe's contact tracing apps that use GAEN (Leith and Farrell (2021)) discovered that a significant amount of data was being sent to Google servers. The android implementations of the GAEN systems use Google Play Services to facilitate GAEN-based contact tracing. The user must enable Google Play Services. It was found that Google Play Services connects to Google servers approximately every 20 minutes, sending requests that include the handset IP address, location data and persistent identifiers to link requests coming from the same device. The data sent to Google in other types of requests also include phone IMEI, device hardware serial number, SIM serial number and IMSI, phone number, WiFi MAC address, user email and Android ID. While sharing data to backend servers is not in itself an intrusion of privacy, the ability to link this data to a real-world user is problematic. Given that the user's IP address is being sent to Google very frequently, this could be used as location tracking. It is possible to de-anonymise this location data and potentially identify the user. Given that the user must enable Google Play Services, and therefore this data sharing, to do contact tracing, this does raise a concern to the privacy of the user.

(Avitabile et al. (2023)) examines several potential threats and privacy concerns of GAEN apps. They introduce a 'paparazzi attack' which involves using passive Bluetooth devices to capture the keys being broadcasted from a targeted user. If this user tests positive, the attacker can match their locally stored keys to those made publically available on the central server and learn that the user is positive for covid. This is a form of deanonymization. Similarly, an attacker could exploit the movements of a targeted user to gain money by linking the locations of the passive devices to the keys and sell this data to interested parties.

The effectiveness of these GAEN contact tracing apps is undetermined  (Leith and Farrell (2021)) (Nguyen et al. (2022))

Numerous alternative to the GAEN system have been proposed such as TraceCorona by  (Nguyen et al. (2022)) in an attempt to remedy the above privacy and security concerns.

### 2.1.2   What is Randomness

In order to test for randomness/non-randomness we must first define what randomness is. A random bit sequence could be explained as the result of flipping an unbiased coin, with two sides 1 and 0, which has an equal chance of 50 percent of landing on side 1 or side 0. Each flip of the coin does not affect any future coin flips which means the flips are independent of each other. This unbiased coin can therefore be considered a perfect random bit stream generator as the appearances of 1s and 0s will be randomly and uniformly distributed. All elements in the sequence are independent of each other and future elements in the sequence cannot be predicted using previous elements  (Dang (2012)). This simple example gives us an understanding of what it means for a set of keys to be random.

The keys must exhibit certain properties in order to be accepted as random. They should be independent meaning no previously generated keys affect a new key. Equally likely meaning that the probability of a 0 or 1 appearing at any point in the key is equal to 1/2. Scalable meaning that if the key is random, then any extracted subsequence is also random.  (Cortez et al. (2020))Any indication of a dependency or bias within the data would indicate nonrandomness.

## 2.2   Literature Review

Introduction

### 2.2.1   How to Test for Randomness

It is important to note that you can not say for certain whether something is random or not, you can only find evidence against non-randomness. It is not possible to give theoretical proof of randomness of a sequence. cITE 55555 Various statistical tests can be performed on the data in an attempt to compare and evaluate the data against a truly random sequence since the outcome when a statistical test is applied to a truly random sequence is known. (Bassham et al. (2010)).

A challenge when testing for randomness is that there is no agreed upon complete set of statistical tests to deem a sequence random (Bassham et al. (2010)). There is an infinite number of tests that you could run in order to find the presence or absence of a pattern or bias within the data. The existence of a pattern or bias within the data would indicate that it is non randomness.

**Hypothesis Testing**

Statistical testing is used to test against a defined null hypothesis (h0). The null hypothesis in this case is that the keys being tested are random. The alternative hypothesis (h1) is that the keys are not random. The challenge here is to determine which of these hypotheses can be accepted (Luengo and Villalba (2021)) . For each statistical test run on the data, the result accepts or rejects the null hypothesis.

The following table shows the possible results on a hypothesis test: ¡table¿ (Bassham et al. (2010))

The above situations are somewhat unknown but some control can be gained by knowing the probability of each of the error situations. The probability of Error Type 1 is defined as a, the level of significance (Luengo and Villalba (2021)). This value is typically 0.01, 0.05 or 0.10. The probability of Error Type 2 is defined as B, referred to as contrast power and is usually used as 1-B. (Luengo and Villalba (2021)). If the data is truly random, rejecting the null hypothesis, determining that the data is non-random, will occur a small percentage of the time. For example if a is 0.01, it would be expected that 1 sequence in 100 sequences is rejected (Bassham et al. (2010)).

In practice, p-values are used to reject or accept the null hypothesis. In the context of this project, a p-value can be defined as the probability that a key produced is less random than the keys previously tested, given the kind of non-randomness the test is assessing (Bassham et al. (2010)). NEED TO FIX. A p-value equal to 1 indicates that the data is perfectly random while a p-value equal to 0 indicates that the data is completely non random. If the p-value is greater than or equal to a, the null hypothesis is accepted and the data appears to be random. If the p-value is less than a, the null hypothesis is rejected and the data is deemed non random.

Maybe K-S stuff here or later on TALK ABOUT test suites and what they are, not specific to one

## 2.2.2 Studies on Randomness Testing

Intro something like many examples of randomness testing being used on cryptographic techniques/applications. Multiple applications

Given that encryption is essential for maintaining data security in cloud computing, (Mohamed et al. (2012)) performed randomness testing on eight modern encryption tech-

niques, including AES, MARS and DES. They tested on two different platforms, desktop computer and Amazon EC2 Micro Instance. They evaluated the encryption techniques implemented as Pseudo Random Number Generators (PRNGs). They used the NIST Test Suite to perform the randomness testing. With a significance level of 0.01, any p-value less than 0.01 meant that sequence was rejected. They found no strong evidence of any statistical non randomness across the 8 encryption algorithms however some differences were found between them on the two different platforms.

Statistical analysis has been run on an enhanced SDEx encryption method based on the SHA-512 hash function  (Hłobaż (2020)). Using various tests like frequency, cumulative sums and runs, with a significance level of 0.01, it was found that this encryption algorithm was sufficiently random and passed the tests. They concluded that this SDEx method based on the SHA-512 hash function was quicker and equally or more secure than AES with a 256-bit key. They hope to use this method to secure end-to-end encryption for data transfer.

Similarly, statistical tests for randomness have been run on new algorithms, like a proposed stream cipher cryptographic algorithm based on the popular Vernam Cipher (Brosas et al. (2020)). The algorithm had a success rate of 99.5 percent across the statistical tests performed on it, which included frequency and longest runs of one's tests. Due to this success, the paper deemed the proposed algorithm effective in producing a random ciphertext sequence and detailed further work of implementing it to help secure medical records.

SHA256 is vulnerable to length extension attacks which involve misusing particular hashes as authentication codes and using them to include extra information. CITE 33333 introduces a new and improved padding scheme and hashing process for SHA256 to deal with this issue. To verify that the solution is cryptographically secure, statistical tests for

randomness are performed on the output of the Message Digest. Tests such as monobit frequency, frequency within a block and runs were carried out on the data. The results validate that the number of ones and zeros are randomly distributed in the final hash value.

Statistical tests were also used to identify encrypted and unencrypted bit sequences (Wu et al. (2015)). Unencrypted bit sequences are less random than encrypted ones. From the SP800-22 rev1a standard, five tests were selected and a significance level of 0.01 was chosen. If the sequence passes more than 3 of the tests, it was concluded that that sequence was encrypted. Otherwise the sequence was concluded as unencrypted. The results of the experiment were that 89 percent of the time, unencrypted sequences were identified correctly and 99 percent of the time encrypted sequences were identified correctly.

### 2.2.3   Examples of Randomness Failures

Randomness failures pose a serious threat to cryptographic security (Schuldt and Shinagawa (2017)). The consequences can be severe and there are many examples of real-world incidents.

There are many examples of pseudo random number generators (PRNGs) failing and being guessable. Notably, the Debian Linux vulnerability in 2008 that left cryptographic keys to be guessable. It was caused by the code used to gather entropy, used to seed the PRNG used to create private keys, were removed. This resulted in only 32,768 possible keys meaning the connections made with these keys were insecure. CITE 11111

In 2015, Juniper Networks announced that there were multiple security vulnerabilities due to unauthorised code in their operating system, for their NetScreen VPN routers, called ScreenOS. CITE 22222 These vulnerabilities were due to Juniper's use of Dual

EC (Elliptical Curve) as a PRNG. Dual EC had a weakness that was exploited in the Juniper incident. It was possible for an attacker, who knew the discrete logarithm of an input parameter Q with respect to a generator point, to see a number of consecutive bytes from the output and hence calculate the internal state of the generator. This allowed the attacker to predict all the future output of the generator. They were able to exploit this lack of randomness and passively decrypt VPN traffic.

Bitcoin thefts in 2013 were due to a compromised PRNG used in Android wallets CITE 9999. Applications on Android using Java Cryptography Architecture (JCA) for key generation, signing and generating random numbers were not receiving cryptographically strong values because of an improper initialization of the underlying PRNG SecureRandom on Android devices CITE 991. The predictability of the values being generated by SecureRandom was exploited and attackers were able to guess the private keys used in Bitcoin Wallets and steal the Bitcoins the wallet contained. Again, attackers were able to exploit the lack of randomness.

## 2.3   Summary

# Chapter 3

# Design

## 3.1 Challenges

### 3.1.1 Review of Test Suites

There are a variety of statistical test suites that have been developed to test for randomness. In this section I will review these test suites in order to choose the appropriate one for this project's purposes. The notable test suites are Dieharder and NIST STS, being the most widely used and therefore most tested.

Dieharder is a random number test suite which was designed to test RNGs used in a variety of applications such as cryptography and computer simulation. CITE rbgwebsite. It consists of 26 tests, extending the original Diehard battery which was created by George Marsaglia in 1995 (Luengo and Villalba (2021)). This testing suite is widely used (Luengo (2022)) (Hurley-Smith and Hernandez-Castro (2020)) (Lu et al. (2023)) and is well tested. It is designed to be able to change the parameters in order to make failure unambiguous. It incorporates many of the tests in NIST STS. Because Dieharder is open source and encourages users to give feedback, contributions are continuously being made to improve and bug fix the tests. This results in a more reliable and stronger test suite than one that is closed or lacks a good feedback system like NIST STS CITE rgbwebsite.

NIST STS stands for the National Institute of Standards and Technology (NIST) Statistical Test Suite. It is used to test RNGs used in applications such as cryptography, modelling and simulation CITE NISTdoc. It contains 15 tests and is widely used (Mohamed et al. (2012)) (Hłobaż (2020)). These are standardised with its test parameters fixed, reducing its flexibility.

TestU01 CITE source is a test suite that was created by L'Ecuyer and Simard and implemented in C. It is an extensive battery that incorporates some of the tests in Dieharder and NIST STS (Luengo and Villalba (2021)). It consists of six test batteries each focusing on testing a different aspect of randomness. It only accepts 32-bit input and interprets it as values with the range of 0 and 1 CITE source. This can lead to inaccuracies in the most-significant bits.

Ent Cite where from is a lesser used test suite created by Walker (Luengo and Villalba (2021)). Its purpose is testing for simulation and cryptographic applications. It has two modes, binary and byte, with different statistics being calculated depending on the mode. Although it is fast and simple, the Ent battery has some issues with dependencies between tests, for example the Entropy test and the Chi-squared test (Luengo and Villalba (2021)).

## 3.2 Methodology

### 3.2.1 Data Preparation

The data used in this project was sourced from the Testing Apps for COVID-19 Tracing (TACT) project by Farrell and Leith CITE 2. The TACT project was a study on whether the BLE used in GAEN-based contact tracing applications was effective at identifying users who were in proximity for long enough to be deemed as exposed to covid, if one of the users was later positive for the virus. The project ran from April 2020 until September

2023 and a number of reports were written on the findings CITE some examples.

While the project was ongoing, the TEKs being published in 33 regions, including Ireland, Germany and Brazil, were downloaded hourly. This resulted in a huge amount of data and allowed for insight into the functioning of these apps. The TEKs downloaded were in a large number of zip files.

In order to get the keys in the correct format to test, first the keys were extracted from the zipped files. Following this, duplicate keys were identified and removed, resulting in a file composed of only the unique keys. This significantly reduced the data size, from an initial 56GB of all the keys in the zip files, down to 4GB, removing a substantial amount of duplicate keys. The final dataset consists of a total of 129 million unique TEKs in ascii format, allowing for efficient analysis of the keys. For Dieharder, the TEKs were converted in raw binary.

### 3.2.2 Chosen Test Suite

Dieharder was selected as the test suite for this project as it is widely used and well tested. It also accepts files of numbers as input, which is useful as our data is a file of TEKs. NIST STS was the other potential candidate however it only accepts streams of data being produced by an RNG, which is not applicable in this project. Therefore Dieharder was the appropriate choice to test the TEKs, given its input type, wide range of tests and good reputation.

**Description of Dieharder Tests**

- "Birthdays" test: this test selects m birthdays of a year of n days. It creates a list of the intervals between two birthdays (time between two consecutive events). It expects that the repeated intervals are distributed in a Possion distribution, if the data is random. (cite dieharder man, (Luengo and Villalba (2021))).

- Overlapping 5-Permutations Test (OPERM5): this test studies a sequence of one million 32-bit random integers. There are 120 possible permutations of each set of 5 consecutive integers. The number of appearances of each permutation is recorded. Each permutation is expected to appear with equal probability across the sequence.

- 32x32 Binary Rank Test: 32x32 matrices are randomly formed from the data. The rank of the matrix is determined. What is rank?? The rank can be a value from 0-32, with infrequent ranks below 29 being pooled with those of rank 29. A chi-squared test is performed on the counts of matrices for ranks 32, 31, 30 and ¡= 29. This test is performed on 400,000 matrices each time.

- 6x8 Binary Rank Test: six random 32-bit integers are taken from the data. A specified byte is chosen and the six bytes, one from each 32-bit integer, create a 6x8 matrix. The rank of the matrix is determined. What is rank?? The rank can be a value from 0-6, with infrequent ranks below 4being pooled with those of rank 4. A chi-squared test is performed on the counts of matrices for ranks 6, 5 and ¡= 4. This test is performed on 100,000 matrices each time.

- Bitstream Test: in this test, the data is viewed as a stream of bits, a = ai and an alphabet with two letters 0 and 1. The stream is considered as successive 20 letter words that overlap. For example, the first word would be bits a1-a20 and the second word would be bits a2-a21. The test counts the number of missing 20-letter words in a string of 2power21 overlapping 20-letter words. Given that there are 2power20 possible overlapping 20-letter words, the number of missing words j in a 2power21+19 letter (bit) string is expected to be (nearly) normally distributed with a mean of 141,909 and sigma of 428. The z-score ?? This test is repeated 20 times each time.

- Overlapping Pairs Sparse Occupance (OPSO): This test considers 2 letter words from an alphabet containing 1024 letters. Each letter is found by a specified 10

bits from a 32-bit integer in the data. 2power21 overlapping 2-letter words from 2power21+1 "keystrokes" are generated by the test and it counts the number of 2-letter words that do not appear in the data. These counts are expected to be (nearly) a normal distribution with mean of 141,900 and sigma 290. Therefore the number of missing words minus the mean divided by sigma should be a standard normal variable. This test extracts 32 bits at a time from the data file and uses a specific 10 bits, the file is then restarted and the next 10 bits are taken and so on.

- Overlapping Quadruples Sparse Occupancy (OQSO): This test is similar to OPSO but takes 4-letter words from an alphabet of 32 letters. Each letter is determined by a specific string of 5 bits from the data, which is assumed to contains 32-bit random numbers. The test calculates the average number of missing words in a sequence of 2power21 four-letter words, which is equivalent to 2power21+3 "keystrokes". The average number of missing words is 141909, with a standard deviation (sigma) of 295. (The average is calculated based on theory, while the standard deviation is determined through extensive simulation?).

- DNA Test: this test considers an alphabet made up of four letters: C, G, A, and T. These letters are determined by two designated bits in the sequence being tested. The test looks at words that are 10 letters long, like OPSO and OQSO tests, which means there are 2power20 possible words. For a string of 2power21 overlapping 10-letter words (which equals 2power21+9 "keystrokes"), the average number of missing words is 141909. The standard deviation, sigma, is 339, (which was calculated through simulation. However, for OPSO, the true standard deviation is 290, determined directly rather than through simulation.?)

- Count the 1s (stream): This test considers the data being tested as a stream of bytes, with four bytes making up each 32-bit integer. Each byte can contain anywhere from 0 to 8 occurrences of the number 1, with different probabilities for each count. This stream of bytes is considered as a series of overlapping 5-letter words. Each "letter"

in these words is determined by the number of 1s in a byte: 0, 1, or 2 gives A, 3 gives B, 4 gives C, 5 gives D, and 6, 7, or 8 gives E. (monkey at a typewriter hitting five keys, each with its own probability of being pressed?). There are 5power5 possible 5-letter words, the count of how often each word appears in a string of 256,000 overlapping 5-letter words is calculated. The quadratic form in the weak inverse of the covariance matrix of the cell counts provides a X2 test. The test returns two p-values for 5-letter and 4-letter cell counts.

- Count the 1s Test (byte): The test considers the data being tested as a series of 32-bit integers. From each integer, specific byte is selected, the leftmost byte (bits 1 to 8). This byte can have anywhere from 0 to 8 instances of the number 1, with probabilities of 1, 8, 28, 56, 70, 56, 28, 8, and 1 out of 256. These specified bytes are taken from consecutive integers and turned into a string of overlapping 5-letter words. In these words, each "letter" is determined by how many times the number 1 appears in that byte: 0, 1, or 2 gives A, 3 gives B, 4 gives C, 5 gives D, and 6, 7, or 8 gives E. (monkey at a typewriter hitting five keys, each with its own chances of being pressed: 37, 56, 70, 56, and 37 out of 256?). There are 5power5 possible 5-letter words, and from a set of 256,000 overlapping 5-letter words, how often each word appears is counted. (The quadratic form in the weak inverse of the covariance matrix of the cell counts provides a chisquare test:: Q5-Q4, the difference of the naive Pearson sums of (OBS-EXP)power2/EXP on counts for 5-and 4-letter cell counts?)

- Parking Lot Test: This test examines how attempts to randomly park a square car of length 1 on a 100x100 parking lot without crashing are distributed. The number of attempts (n) is plotted against the number of attempts that didn't "crash" because the car squares overlapped (k). This is compared to what would be expected from a perfectly random set of parking coordinates. The results are compared to when n=12,000, where k should average 3523 with a standard deviation of 21.9. This

average is very close to being normally distributed. The formula (k - 3523) / 21.9 is used to get a standard normal variable. Converting this to a uniform p-value, it is used as input for a KS test with a default of 100 samples.

- Minimum Distance (2d Circle) Test:

- 3d Sphere (Minimum Distance) Test:

- Squeeze Test:

- Sums Test (Broken)

- Runs Test:

- Craps Test:

- Greatest Common Divisor Marsaglia and Tsang Test:

- Monobit Test (STS):

- Runs Test (STS):

- Serial Test (STS):

- Bit Distribution Test:

- Generalised Minimum Distance Test:

- Permutations Test:

- Lagged Sums Test:

- Kolmongorov-Smirnov Test Test:

- Byte Distribution Test:

- Discrete Cosine Transform (DCT) (Frequency Analysis) Test:

- Fill Tree Test:

- Fill Tree 2 Test:

- Monobit 2 Test:

### 3.2.3   Other Tests

Other tests to supplement the Dieharder test suite were implemented, these include: chi-squared test, spectral test, lag plot and plot of counts.

**Lag Plot**

A lag plot is a graphical test for randomness, it displays any patterns or relationships in the data. Random data should not have any identifiable structure in the lag plot, a structure in the lag plot indicates that the data is not random CITE 1.3.3.15. Lag Plot (nist.gov). A lag plot involves plotting a set of the data against another set of the data that occurs later. For example, given a data set Y1, Y2 ..., Yn, Y2 and Y7 have lag 5 since 7 - 2 = 5. Lag plots can be generated for any arbitrary lag CITE 1.3.3.15. Lag Plot (nist.gov) . A plot of lag 1 would be a plot of Yi against Yi-1. PICTURES of examples from NIST of random v non random

**Chi-Squared Test**

The chi-squared test for randomness iterates through the list of TEKs, counting the occurrences of '0' and '1' in each key. The observed frequencies are calculated based on these counts. Assuming equal probability for '0' and '1', the expected frequencies are calculated. The chi-squared statistic is computed and the corresponding p-value for the observed and expected frequencies. The chi-squared statistic, p-value, and expected frequencies are returned. This test helps to quantify the degree of randomness in the TEK dataset, providing valuable insights into its distribution and potential biases. get cites

**Spectral Test**

**Plot of Counts**

**Hilbert Curve**

REWRITE and cite: the Hilbert curve can be used to visually represent randomness or non-randomness in data. The Hilbert curve is a space-filling curve that can map one-dimensional data onto a two-dimensional space in a way that preserves locality: nearby points in the original data remain close to each other in the curve's representation.

When data points are randomly distributed, the resulting Hilbert curve will exhibit a relatively uniform distribution throughout its space-filling path. This means that nearby points in the original dataset will likely remain close to each other in the curve, creating a visually uniform pattern.

Conversely, if the data exhibits some form of non-random structure or clustering, the resulting Hilbert curve will reflect this by exhibiting areas of higher density or clustering along certain portions of the curve. In other words, points that are close to each other in the original dataset may end up far apart or clustered together in the Hilbert curve representation.

By visualizing data using the Hilbert curve, one can quickly discern patterns or lack thereof, providing insights into the randomness or non-randomness of the dataset. However, it's important to note that the Hilbert curve visualization is not a formal statistical test for randomness but rather a tool for qualitative assessment. Formal statistical tests, such as the chi-squared test or others, are typically used for quantitative assessments of randomness.

### 3.2.4   Random Dataset

A dataset containing random numbers was sourced from CITE Random.org. This website generates true random numbers from atmospheric noise. These numbers are better than pseudo random numbers as they do not require a seed and are truly random. This

service was created in 1998 by Dr Mads Haahr in Trinity College Dublin and is now run by Randomness and Integrity Services Ltd.

The same randomness testing is applied to this random dataset as the TEKs. This allows for a baseline to compare the results to as well as validating the tests. The dataset contains 129 million 128-bit numbers, equal to the set of TEKs.

# Chapter 4

# Evaluation

## 4.1 Dieharder Results

Dieharder is designed to push the tests to unambiguous failure CITE Robert G. Brown's General Tools Page (duke.edu) . It contains a number of flag options to alter the parameters of the tests and change their acceptance criteria. The command used to run the Dieharder test suite for this project was:

Dieharder -a -k 2 -Y 1 -f ¡filename¿

The -a flag runs all the tests in the dieharder test suite, as described in section BLANK. The -k flag is the ks-flag K sminorv thing. The -Y flag is the Xtrategy flag which is used to control the 'test to failure' modes. CITE rgb. This flag is set to 1 to use the 'resolve ambiguity' mode. Dieharder can return 'weak' as a test result which can be difficult to interpret. Even perfect random numbers will return some 'weak' results at some point because the p-values are uniformly distributed and will have a result in the tails of the distribution from time to time. Even if a test returns more than one weak result, this is not conclusive evidence that the data is non-random. The 'resolve ambiguity' mode resolves this issue by adding p-samples (in blocks of 100) until the test results in a definitive pass

or it proceeds to failure.

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