[S&B Book] Chapter 4: Dynamic Programming

Tags

- The key idea of **Dynamic Programming** in reinforcement learning:

  The use of value functions to organize and structure the search for good policies.

▼ 4.1 Policy Evaluation (Prediction)

- ***Policy evaluation:*** How to compute the state-value function v_π for an arbitrary policy π. It is also been referred as the *prediction problem.*
- ***Iterative policy evaluation:***

  The initial approximation, v_0 is chosen arbitrarily (except that the terminal state, if any, must be given value 0), and each successive approximation is obtained by using Bellman equation for v_π as an update rule:

$$
\begin{aligned}
v_{k+1}(s) &\doteq \mathbb{E}[R_{t+1} + \gamma v_k(S_{t+1}) | S_t = s] \\
&= \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a)[r + \gamma v_k(s')]
\end{aligned}
$$

The sequence $\{v_k\}$ can be shown in general to **converge to** v_π as $k \to \infty$ under the same conditions that guarantee the existence of v_π.

- ***Expected update:***

  The iterative policy evaluation applies the same operation to each state s: it replaces the old value of s with a new value obtained from the old values of the successor states of s, and the expected immediate rewards, along all the one-step transitions possible under the policy being evaluated.

  The algorithms are called *expected* updates because they are based on **an expectation over all possible next states** rather than on a sample next state.

- Two methods of implementing policy evaluation:

  1. Two-array:

     Using two arrays, one for the old values $v_k(s)$, and one for the new values, $v_{k+1}(s)$. With two arrays, the new value can be computed one by one from the old values without the old values being changed.

  2. In-place:

     With each new value immediately overwriting the old one. Then, depending on the order in which the states are updated, sometimes new values are used instead of old ones.

  The “in-place” method usually converges faster than the two-array version, because it uses new data as soon as they are available.

- **In-place iterative policy evaluation algorithm:**

Iterative Policy Evaluation, for estimating $V \approx v_\pi$

Input π, the policy to be evaluated
Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(terminal) = 0$

Loop:
  $\Delta \leftarrow 0$
  Loop for each $s \in \mathcal{S}$:
    $v \leftarrow V(s)$
    $V(s) \leftarrow \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a)\big[r + \gamma V(s')\big]$
    $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
until $\Delta < \theta$

▼ 4.2 Policy Improvement

- The reason for computing the value function for a policy is to help find better policies.
- *Policy improvement theorem:*

  Let π and π' be any pair of deterministic policies such that, for all $s \in \mathcal{S}$, $q_\pi(s, \pi'(s)) \geq v_\pi(s)$. Then the policy π' must be as good as, or better than, π. That is, it must obtain greater or equal expected return for all states $s \in \mathcal{S}$: $v_{\pi'}(s) \geq v_\pi(s)$.
- ***Policy improvement:***

  The process of making a new policy that improves on an original policy, by making it greedy with respect to the value function of the original policy.

  The greedy policy takes action that looks best in the short term:

$$
\begin{aligned}
\pi'(s) &\doteq \arg\max_a q_\pi(s, a) \\
&= \arg\max_a \mathbb{E}[R_{t+1} + \gamma v_\pi(S_{t+1}) | S_t = s, A_t = a] \\
&= \arg\max_a \sum_{s',r} p(s', r|s, a)[r + \gamma v(s')]
\end{aligned}
$$

Suppose the new greedy policy π' is as good as, but not better than, the old policy π, then $v_\pi = v_{\pi'}$, for all $s \in \mathcal{S}$:

$$\begin{aligned} v_{\pi'} &= \max_a \mathbb{E}[R_{t+1} + \gamma v_{\pi'} | S_t = s, A_t = a] \\ &= \max_a \sum_{s', a} p(s', r | s, a)[r + v_{\pi'}(s')] \end{aligned}$$

This is the same as the Bellman optimality equation, and therefore $v_{\pi'}$ must be v_*, and both π and π' must be optimal policies. Policy improvement must give us a strictly better policy except when the original policy is already optimal.

Example

Implementation
