# [S&B Book] Chapter 2: Multi-armed Bandits

Tags

> The most important feature distinguishing reinforcement learning from other types of learning is that it uses training information that *evaluates* the action taken rather than *instructs* by giving correct actions

- **Evaluative feedback**

  Indicates how good the action taken was, but not whether it was the best or the worst action possible;

- **Instructive feedback**

  Indicates the correction to take, independently of the action actually taken.

In this section we study the evaluative aspect of reinforcement learning in a simplified setting, *k*-armed bandit problem.

## ▼ 2.1 A *k*-armed Bandit Problem

- *The k-armed bandit problem*

  > You are faced repeatedly with a choice among *k* different options, or actions. After each choice you receive a numerical reward chosen from a stationary

> probability distribution that depends on the action you selected. Your objective is to maximize the expected total reward over some time period.

- The value of an arbitrary action: $q_*(a) \doteq \mathbb{E}[R_t | A_t = a]$
- **greedy actions**:

  When maintaining estimates of the action values, then at any time step there is at least one action whose estimated value is the greatest.
- *Exploration* and *exploitation:*
  - Exploitation - selecting one of the greedy actions; exploiting the current knowledge of the values of the actions;
  - Exploration - selecting one of the non-greedy actions; exploring enables you to improve your estimate of the non-greedy action's value;

  Reward is lower in the short run, during exploration, but higher in the long run because after you have discovered the better actions, you can exploit them many times.

## 2.2 Action-value Method

- The way of estimating action-value is by averaging the rewards actually received:

$$Q_t(a) \doteq \frac{\text{sum of rewards when } a \text{ taken prior to } t}{\text{number of times } a \text{ taken prior to } t} = \frac{\sum_{i=1}^{t-1} R_i \cdot 1_{A_i=a}}{\sum_{i=1}^{t-1} 1_{A_t=a}}$$

- *greedy* action selection method: $A_t \doteq \arg\max_a Q_t(a)$, with tie broken randomly; greedy action selection always exploits current knowledge to maximize immediate reward.
- $\varepsilon - greedy$ **method:** with a small probability $\varepsilon$, select randomly from among all the actions with equal probability; with probability $1 - \varepsilon$, select action greedily.

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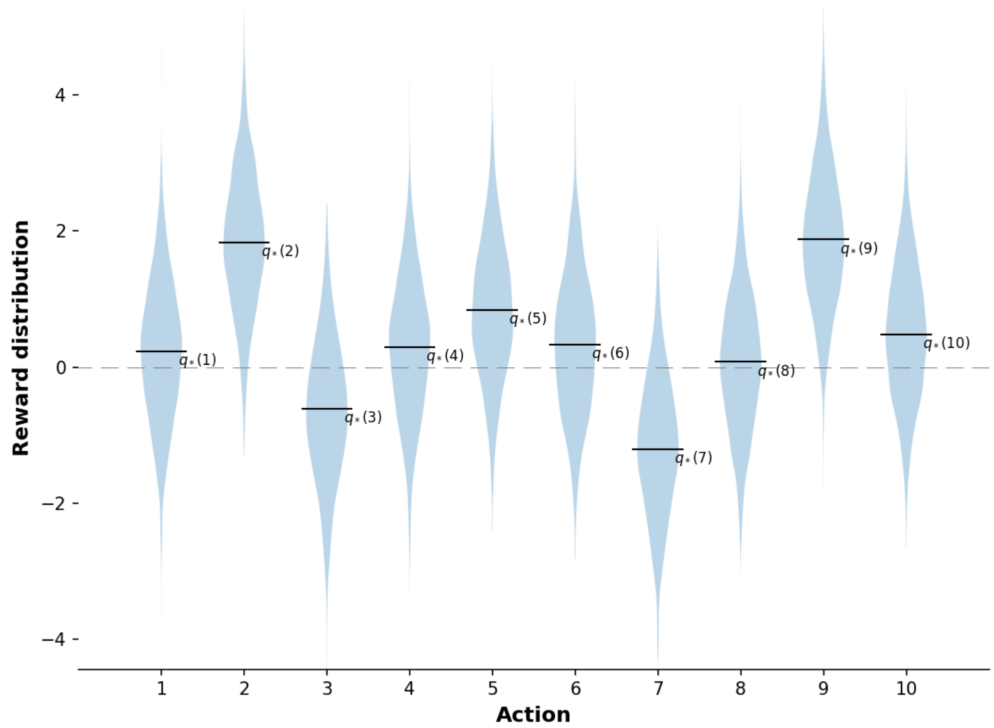
**Example:**

A set of 2000 randomly generated k-armed bandits problem with k = 10.




*Implementation:*

```
import numpy as np
from matplotlib import pyplot as plt


if __name__ == "__main__":

    # Randomly sample mean reward for each action
    means = np.random.normal(size=(10, ))

    # Generate sample data based on normal distribution
    data = [np.random.normal(mean, 1, 2000) for mean in means]

    # Create violin plot
    plt.figure(figsize=(8, 6), dpi=150)
    plt.violinplot(dataset=data,
                   showextrema=False,
                   showmeans=False,
                   points=2000)

    # Draw mean marks
    for i, mean in enumerate(means):
        idx = i + 1
        plt.plot([idx - 0.3, idx + 0.3], [mean, mean],
                 c='black',
                 linewidth=1)
        plt.text(idx + 0.2, mean - 0.2,
                 s=f"$q_*({idx})$",
                 fontsize=8)

    # Draw 0 dashed line
    plt.plot(np.arange(0, 12), np.zeros(12),
                c='gray',
```

```
             linewidth=0.5,
             linestyle=(5, (20, 10)))

plt.tick_params(axis='both', labelsize=10)
plt.xticks(np.arange(1, 11))

# get rid of the frame
for i, spine in enumerate(plt.gca().spines.values()):
    if i == 2: continue
    spine.set_visible(False)


# Draw labels
label_font = {
    'fontsize': 12,
    'fontweight': 'bold'
}

plt.xlabel('Action', fontdict=label_font)
plt.ylabel('Reward distribution', fontdict=label_font)
plt.margins(0)

plt.tight_layout()
# plt.show()
plt.savefig('./plots/example_2_1.png')
```

## 2.4 Incremental Implementation

*Here I want to first take notes on the 2.4 then back to the e-greedy examples in 2.3 because 2.4 introduced the algorithms to implement the 10-armed bandit problem;*

- **The action-value methods on a single action:**

  $R_i$ denotes the reward received after the $i\text{-th}$ selection of *this action*, and $Q_n$ denotes the estimate of its action value after it has been selected $n-1$ times:

$$Q_n \doteq \frac{R_1 + R_2 + \cdots + R_{n-1}}{n-1}$$

> This method maintain a record of all the rewards and then perform this computation whenever the estimated value was needed. But its disadvantage is that, if it is done, the memory and computational requirements would grow over time as more reward are seen. Each additional reward would require additional memory to store it and additional computation to compute the sum in the numerator.

- Device incremental formulas for updating averages:

$$
\begin{aligned}
Q_{n+1} &= \frac{1}{n}\sum_{i=1}^{n} R_i \\
&= \frac{1}{n}\left(R_n + \sum_{i=1}^{n-1} R_i\right) \\
&= \frac{1}{n}\left(R_n + (n-1)\frac{1}{n-1}\sum_{i=1}^{n-1} R_i\right) \\
&= \frac{1}{n}\left(R_n + (n-1)Q_n\right) \\
&= \frac{1}{n}(R_n + nQ_n - Q_n) \\
&= Q_n + \frac{1}{n}\left[R_n - Q_n\right]
\end{aligned}
$$

Thus, a general form of this method is:

$$NewEstimate \leftarrow OldEstimate + StepSize\,[Target - OldEstimate]$$

- Put all together: Pseudocode for **A Simple Bandit Algorithm**

**A simple bandit algorithm**

Initialize, for $a = 1$ to $k$:
  $Q(a) \leftarrow 0$
  $N(a) \leftarrow 0$

Loop forever:

$$A \leftarrow \begin{cases} \arg\max_a Q(a) & \text{with probability } 1-\varepsilon \quad \text{(breaking ties randomly)} \\ \text{a random action} & \text{with probability } \varepsilon \end{cases}$$

  $R \leftarrow bandit(A)$
  $N(A) \leftarrow N(A) + 1$
  $Q(A) \leftarrow Q(A) + \frac{1}{N(A)}\left[R - Q(A)\right]$

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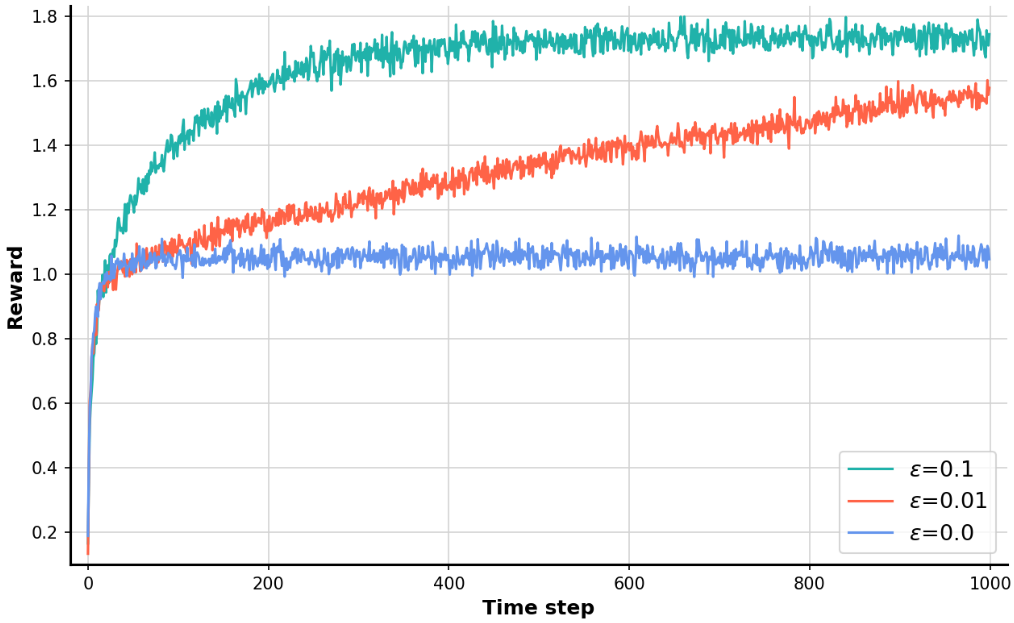
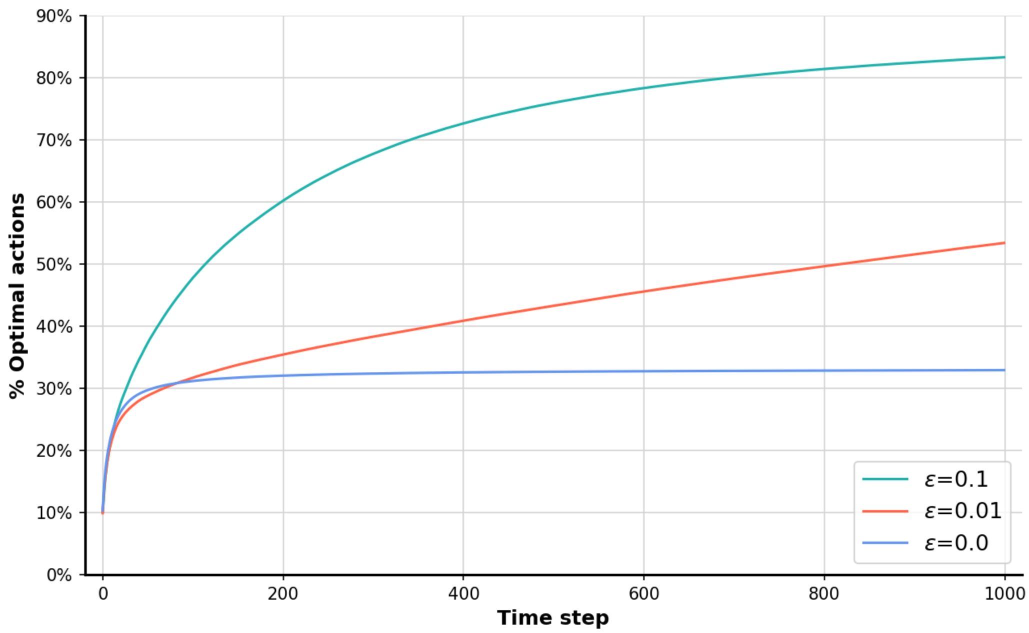
### Example:

Run the 10-armed bandit algorithm on the testbed and compare the effect of using different $\varepsilon$ value.




*Implementation:*

```
import numpy as np
import matplotlib.pyplot as plt
import pickle
```

```python
SEED = 200
np.random.seed(SEED)

# Get the action with the max Q value
def get_argmax(G:np.array) -> int:
    candidates = np.argwhere(G == G.max()).flatten()
    # return the only index if there's only one max
    if len(candidates) == 1:
        return candidates[0]
    else:
        # instead break the tie randomly
        return np.random.choice(candidates)


# select arm and get the reward
def bandit(q_star:np.array,
           act:int) -> tuple:
    real_rewards = np.random.normal(q_star, 1.0)
    optim_choice = int(q_star[act] == q_star.max())
    return real_rewards[act], optim_choice


# running the k-armed bandit algorithm
def run_bandit(K:int,
            q_star:np.array,
            rewards:np.array,
            optim_acts_ratio: np.array,
            epsilon: float,
            num_steps:int=1000) -> None:

    Q = np.zeros(K) # The average action-value for each actions
    N = np.zeros(K) # The number of times each action been selected
    ttl_optim_acts = 0

    for i in range(num_steps):
        # print(q_star)
        # get action
        A = None
        if np.random.random() > epsilon:
            A = get_argmax(Q)
        else:
            A = np.random.randint(0, K)

        R, is_optim = bandit(q_star, A)
        N[A] += 1
        Q[A] += (R - Q[A]) / N[A]

        ttl_optim_acts += is_optim
        rewards[i] = R
        optim_acts_ratio[i] = ttl_optim_acts / (i + 1)


if __name__ == "__main__":

    # Initializing the hyper-parameters
    K = 10 # Number of arms
    epsilons = [0.0, 0.01, 0.1]
    num_steps = 1000
    total_rounds = 2000

    # Initialize the environment
    q_star = np.random.normal(loc=0, scale=1.0, size=K)
    rewards = np.zeros(shape=(len(epsilons), total_rounds, num_steps))
    optim_acts_ratio = np.zeros(shape=(len(epsilons), total_rounds, num_steps))
```

```
    # Run the k-armed bandits alg.
    for i, epsilon in enumerate(epsilons):
        for curr_round in range(total_rounds):
            run_bandit(K, q_star, rewards[i, curr_round], optim_acts_ratio[i, curr_round], epsilon, num_steps)

    rewards = rewards.mean(axis=1)
    optim_acts_ratio = optim_acts_ratio.mean(axis=1)

    record = {
        'epsilons': epsilons,
        'rewards': rewards,
        'optim_ratio': optim_acts_ratio
    }

    # for ratio in optim_acts_ratio:
    #     plt.plot(ratio)
    # plt.show()
    with open('./history/record.pkl', 'wb') as f:
        pickle.dump(record, f)
```

## ▼ 2.5 Tracking a Nonstationary Problem

- All of the discussions so far are based on the assumption that the bandit problem is stationary. But in most of the time, the reinforcement problem we're facing is nonstationary.
- In the nonstationary case, **it makes sense to give more weights to recent reward than to long-past rewards.** One of the most popular ways of doing this is to use a constant step-size parameter:

$$Q_{n+1} \doteq Q_n + \alpha\left[R_n - Q_n\right],\ \alpha \in (0, 1]$$

Proof: $Q_{n+1}$ is a weighted average of past rewards and the initial estimate $Q_1$:

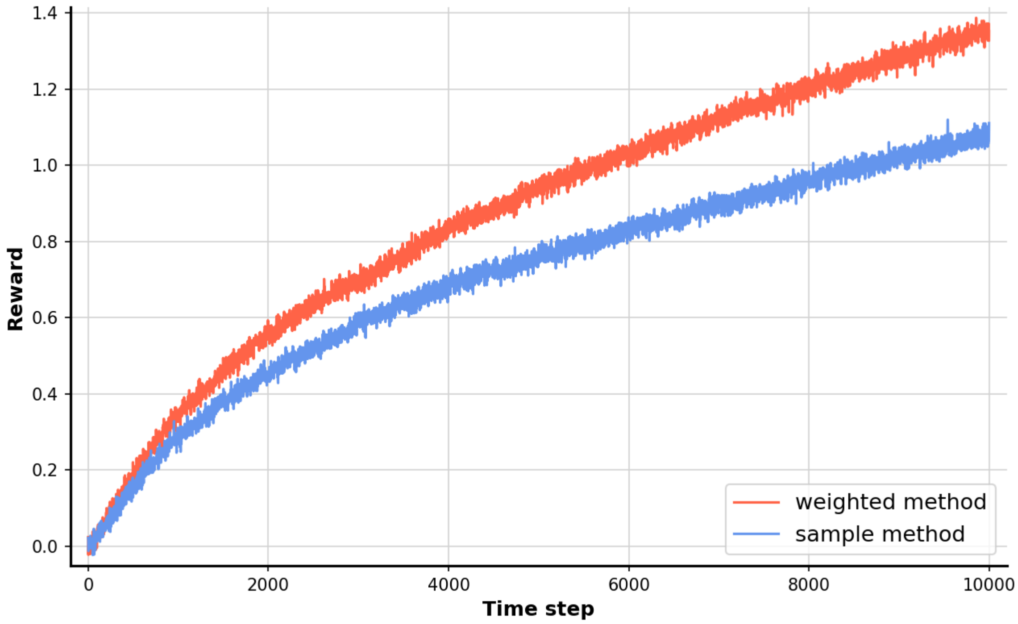
$$\begin{aligned} Q_{n+1} &= Q_n + \alpha\left[R_n - Q_n\right] \\ &= \alpha R_n + (1-\alpha)Q_n \\ &= \alpha R_n + (1-\alpha)\left[\alpha R_{n-1} + (1-\alpha)Q_{n-1}\right] \\ &= \alpha R_n + (1-\alpha)\alpha R_{n-1} + (1-\alpha)^2 Q_{n-1} \\ &= \alpha R_n + (1-\alpha)\alpha R_{n-1} + (1-\alpha)^2\alpha R_{n-2} + \cdots + (1-\alpha)^{n-1}\alpha R_1 + (1-\alpha)^n Q_1 \\ &= (1-\alpha)^n Q_1 + \sum_{i=1}^{n} \alpha(1-\alpha)^{n-i} R_i \end{aligned}$$

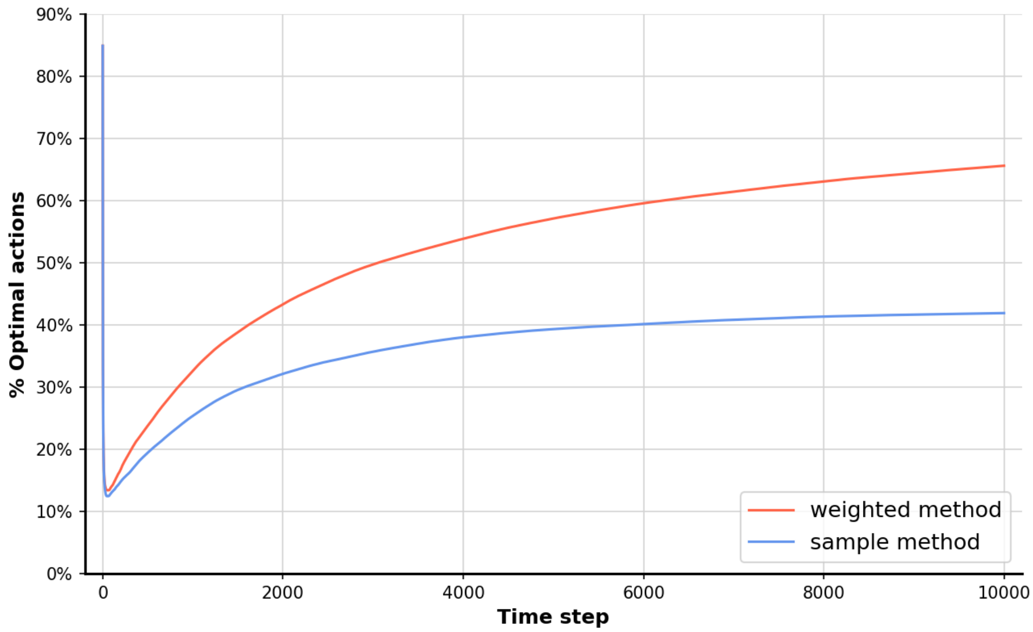
because $1-\alpha$ is less than 1, thus weight given to $R_i$ decreases as the number of intervening rewards increases.

**Exercise:**

Design and conduct an experiment to demonstrate the difficulties that sample-average methods have for non-stationary problems. Use a modified version of the 10-armed testbed in which all the $q_*(a)$ start out equal and then take independent random walks (say by adding a normally distributed increment with mean zero and standard deviation 0.01 to all the $q_*(a)$ on each step). Prepare plots like Figure 2.2 for an action-value method using sample averages, incrementally computed, and another action-value method using a constant step-size parameter, $\alpha = 0.1$ Use $\varepsilon = 0.1$ and longer runs, say of 10,000 steps.

*Implementation:*



```
# running the k-armed bandit algorithm on non-stationary environment
def run_bandit_non_stationary(K:int,
            q_star:np.array,
            rewards:np.array,
            optim_acts_ratio: np.array,
            epsilon: float,
            method: str, # method should be in ["sample", "weighted"]
            alpha: float=None,
            num_steps: int=1000) -> None:

    Q = np.zeros(K) # The action-value for each actions
    N = np.zeros(K) # The number of times each action been selected
    ttl_optim_acts = 0

    q_star_temp = np.copy(q_star)

    assert method in ['sample', 'weighted'], "The method should be 'sample' or 'weighted'"

    for i in range(num_steps):
        # print(q_star)
        # get action
        A = None
        if np.random.random() > epsilon:
            A = get_argmax(Q)
        else:
            A = np.random.randint(0, K)

        R, is_optim = bandit(q_star_temp, A)

        if method == 'sample':
            N[A] += 1
            Q[A] += (R - Q[A]) / N[A]
        else:
            Q[A] += alpha * (R - Q[A])

        ttl_optim_acts += is_optim
        rewards[i] = R
        optim_acts_ratio[i] = ttl_optim_acts / (i + 1)

        # Updating q_star values
        q_step_scale = 0.01
        q_steps = np.random.normal(loc=0, scale=q_step_scale, size=q_star_temp.shape)
        q_star_temp += q_steps
```

## 2.6 Optimistic Initial Values

- **Bias:**

  The initial action-value estimates $Q_1(a)$ , are **bias** in terms of statics.

  For the sample-average method, the bias disappears once all actions have been selected at lease once, but for the weighted-average method, the bias is permanent, though decreasing over time.

- **Optimistic initial values:**

  The assumption of this method: initial action values can also be used as a simple way to encourage exploration.

  In the k-armed bandit testbed, $q_*(a)$ are sampled from a normal distribution with mean 0 and variance 1. An initial estimate of +5 is thus wildly optimistic; Whichever actions are initially selected, the reward is less than the starting estimates; the learner switches to other actions, being "disappointed" with the rewards it is receiving.

- **Limitation:**

  This method is not well suited to nonstationary problems because its drive for exploration is inherently temporary. If the task changes, creating a renewed need for exploration, this method cannot help.

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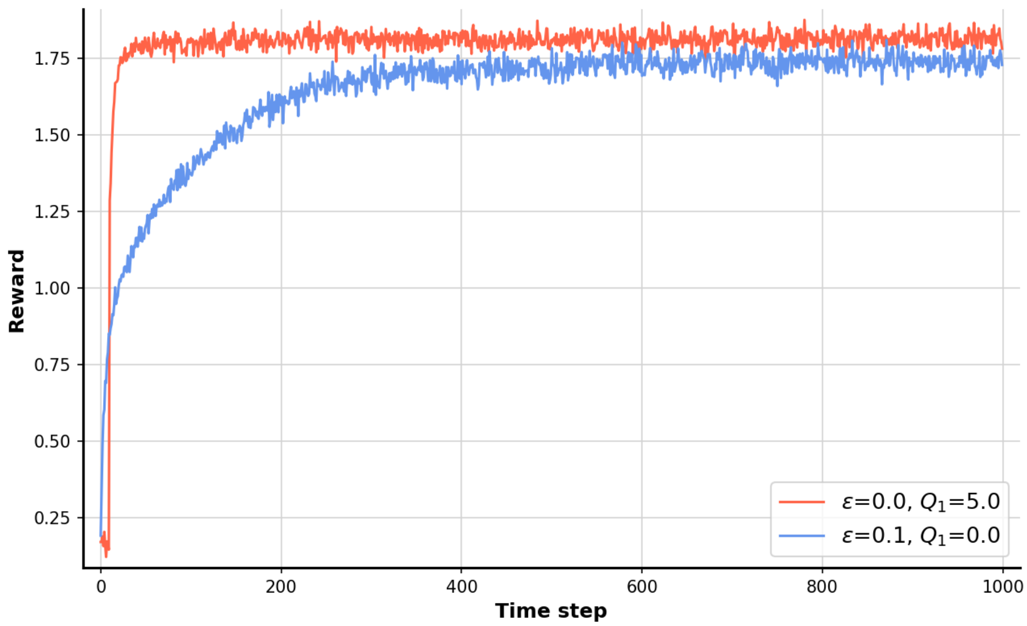
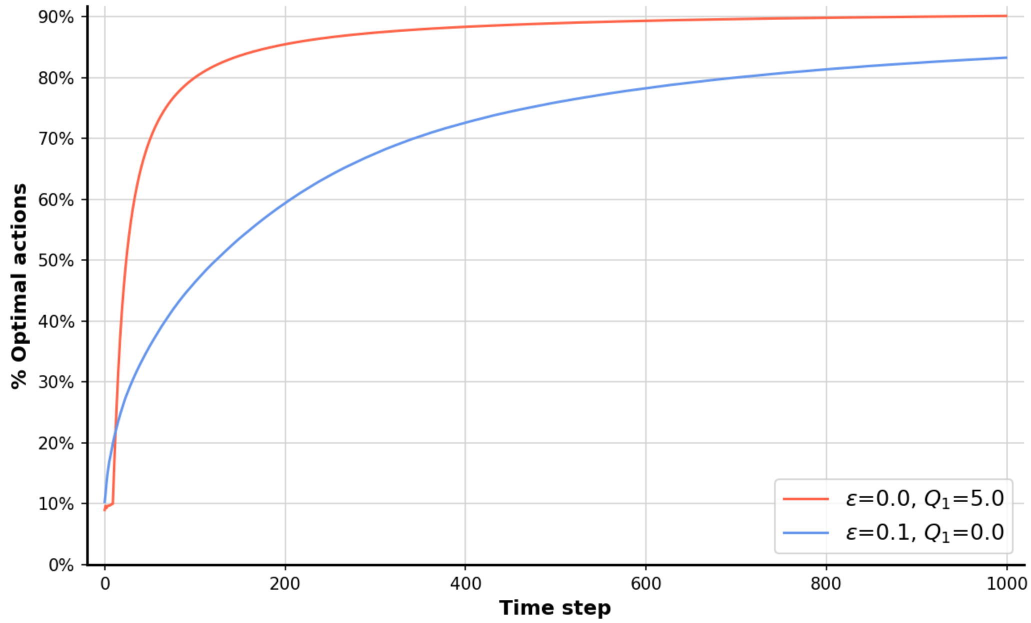
**Example:**

The effect of optimistic initial action-value estimates on the 10-armed testbed. Both methods used a constant step-size parameter, $\alpha = 0.1$






*Implementation:*

```
# running the k-armed bandit algorithm
def run_bandit(K:int,
            q_star:np.array,
            rewards:np.array,
            optim_acts_ratio: np.array,
            epsilon: float,
            num_steps:int=1000,
            init_val: int=0) -> None:

    Q = np.ones(K) * init_val
    N = np.zeros(K) # The number of times each action been selected
    ttl_optim_acts = 0

    for i in range(num_steps):
        # get action
        A = None
        if np.random.random() > epsilon:
            A = get_argmax(Q)
        else:
            A = np.random.randint(0, K)

        R, is_optim = bandit(q_star, A)
        N[A] += 1
        Q[A] += (R - Q[A]) / N[A]

        ttl_optim_acts += is_optim
        rewards[i] = R
        optim_acts_ratio[i] = ttl_optim_acts / (i + 1)
```

## ▼ 2.7 Upper-Confidence-Bound Action Selection

- The limitation of $\varepsilon\text{-greedy}$ method:

  The greedy actions look best at present, but some of other actions may be better;

  $\varepsilon\text{-greedy}$ action selection forces the non-greedy actions to be tried, but indiscriminately, with no preference for those that are nearly greedy or particularly uncertain.

- The idea of UCB helps select among the non-greedy actions according to their *potential for actually being optimal*.

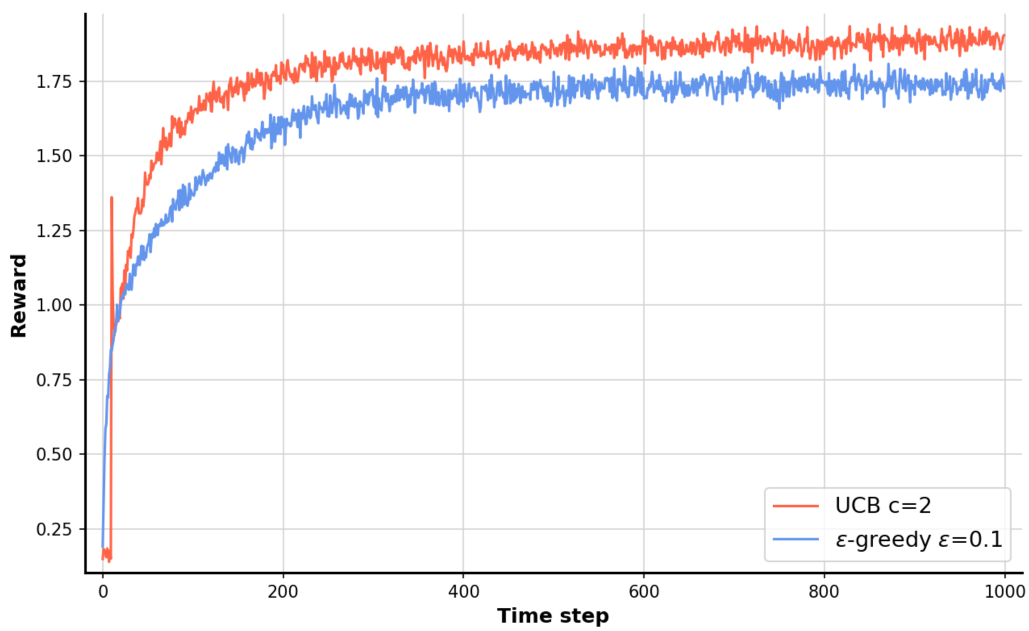
  $A_t \doteq \arg\max_a \left[Q_t(a) + c\sqrt{\frac{\ln t}{N_t(a)}}\right]$

  - The square-root term measures the uncertainty or variance in the estimate of a's value;
  - c determines the confidence level;
  - Each time a is selected the uncertainty is presumably reduced; each time an action other than a is selected, t increases but N_t(a) does not, the uncertainty estimate increases.

  The actions with lower value estimates, or that have already been selected frequently, will be selected with decreasing frequency over time.

---

**Example:**




*Implementation:*

```
# running the bandit algorithm with UCB
def run_bandit_UCB(K:int,
```

```
            q_star:np.array,
            rewards:np.array,
            optim_acts_ratio: np.array,
            c: float,
            num_steps:int=1000) -> None:

    Q = np.zeros(K)
    N = np.zeros(K) # The number of times each action been selected
    ttl_optim_acts = 0

    for i in range(num_steps):
        A = None

        # Avoid 0-division:
        # If there's 0 in N, then choose the action with N = 0
        if (0 in N):
            candidates = np.argwhere(N == 0).flatten()
            A = np.random.choice(candidates)
        else:
            confidence = c * np.sqrt(np.log(i) / N)
            freqs = Q + confidence
            A = np.argmax(freqs).flatten()

        R, is_optim = bandit(q_star, A)
        N[A] += 1
        Q[A] += (R - Q[A]) / N[A]

        ttl_optim_acts += is_optim
        rewards[i] = R
        optim_acts_ratio[i] = ttl_optim_acts / (i + 1)
```

- **The limitation of UCB**

  1. It is difficult in dealing with nonstationary problems;
  2. Another difficulty is dealing with large state space, particularly when using function approximation .

  In these settings, the idea of UCB action selection is usually not practical;

## 2.8 Gradient Bandit Algorithms

- In this section, we consider learning a numerical *preference* for each action $a$, which we denote $H_t(a) \in \mathbb{R}$; the larger the preference, the more often that action is taken.
- $\pi_t(a)$: a notation for the probability of taking action $a$ at time $t$;

$$\Pr\{A_t = a\} \doteq \frac{e^{H_t(a)}}{\sum_{b=1}^{k} e^{H_t(b)}} \doteq \pi_t(a)$$

  Initially, all action preferences are the same so that all actions have an equal probability of being selected.

- **Gradient ascent:**

On each step, after selecting action $A_t$ and receiving the reward $R_t$, the action preference are updated by:

$$
\begin{aligned}
&H_{t+1}(A_t) \doteq H_t(A_t) + \alpha(R_t - \bar{R}_t)(1 - \pi_t(A_t)), \quad \text{and} \\
&H_{t+1}(a) \doteq H_t(a) - \alpha(R_t - \bar{R}_t)\pi_t(a), \quad \text{for all } a \neq A_t
\end{aligned}
$$

- $\alpha > 0$ is a step size parameter;
- $\bar{R}_t \in \mathbb{R}$ is the average of the rewards up to but not including time $t$ (with $\bar{R}_1 \doteq R_1$). This term serves as a baseline with which reward is compared; if the reward is higher than baseline, then the probability of taking $A_t$ in the future is increased, and if the reward is below baseline, then the probability is decreased.

---

**proof of the gradient ascent equation:**

$$
H_{t+1}(a) \doteq H_t(a) + \alpha \frac{\partial \mathbb{E}[\mathbb{R}_t]}{\partial H_t(a)}, \text{where } \mathbb{E}[R_t] = \sum_x \pi_t(x) q_*(x)
$$

First we take a closer look at the exact performance gradient:

$$
\begin{aligned}
\frac{\partial \mathbb{E}[R_t]}{\partial H_t(a)} &= \frac{\partial}{\partial H_t(a)} \left[ \sum_x \pi_t(x) q_*(x) \right] \\
&= \sum_x q_*(x) \frac{\partial \pi_t(x)}{\partial H_t(a)} \\
&= \sum_x (q_*(x) - B_t) \frac{\partial \pi_t(x)}{\partial H_t(a)}
\end{aligned}
$$

Then we multiply each term for the sum by $\pi_t(x)/\pi_x(x)$:

$$
\begin{aligned}
\frac{\partial \mathbb{E}[R_t]}{\partial H_t(a)} &= \sum_x \pi_t(x) \left(q_*(x) - B_t\right) \frac{\partial \pi_t(x)}{\partial H_t(a)} / \pi_t(x) \\
&= \mathbb{E}\left[ (q_*(A_t) - B_t) \frac{\partial \pi_t(A_t)}{\partial H_t(a)} / \pi_t(A_t) \right] \\
&= \mathbb{E}\left[ (R_t - \bar{R}_t) \frac{\partial \pi_t(A_t)}{\partial H_t(a)} / \pi_t(A_t) \right]
\end{aligned}
$$

Get partial derivative $\frac{\partial \pi_t(x)}{\partial H_t(a)}$:

$$
\text{recall that: } \frac{\partial}{\partial x}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{\partial f(x)}{\partial x} g(x) - f(x) \frac{\partial g(x)}{\partial x}}{g(x)^2}
$$

We can write:

$$
\begin{aligned}
\frac{\partial \pi_t(x)}{\partial H_t(a)} &= \frac{\partial}{\partial H_t(a)} \left[ \frac{e^{H_t(x)}}{\sum_{y=1}^{k} e^{H_t(y)}} \right] \\
&= \frac{\frac{\partial e^{H_t(x)}}{\partial H_t(a)} \sum_{y=1}^{k} e^{H_t(y)} - e^{H_t(x)} \frac{\partial \sum_{y=1}^{k} e^{H_t(y)}}{\partial H_t(a)}}{\left(\sum_{y=1}^{k} e^{H_t(y)}\right)^2} \\
&= \frac{1_{a=x} e^{H_t(x)} \sum_{y=1}^{k} e^{H_t(y)} - e^{H_t(x)} e^{H_t(a)}}{\left(\sum_{y=1}^{k} e^{H_t(y)}\right)^2} \\
&= \frac{1_{a=x} e^{H_t(x)}}{\sum_{y=1}^{k} e^{H_t(y)}} - \frac{e^{H_t(x)} e^{H_t(a)}}{\left(\sum_{y=1}^{k} e^{H_t(y)}\right)^2} \\
&= 1_{a=x} \pi_t(x) - \pi_t(x) \pi_t(a) \\
&= \pi_t(x)(1_{a=x} - \pi_t(a))
\end{aligned}
$$

Now we put this result back into the equation of $\frac{\partial \mathbb{E}[R_t]}{\partial H_t(a)}$:

$$
\begin{aligned}
\frac{\partial \mathbb{E}[R_t]}{\partial H_t(a)} &= \mathbb{E}\left[(R_t - \bar{R}_t) \pi_t(A_t)(1_{a=A_t} - \pi_t(a)) / \pi_t(A_t)\right] \\
&= \mathbb{E}\left[(R_t - \bar{R}_t)(1_{a=A_t} - \pi_t(a))\right]
\end{aligned}
$$

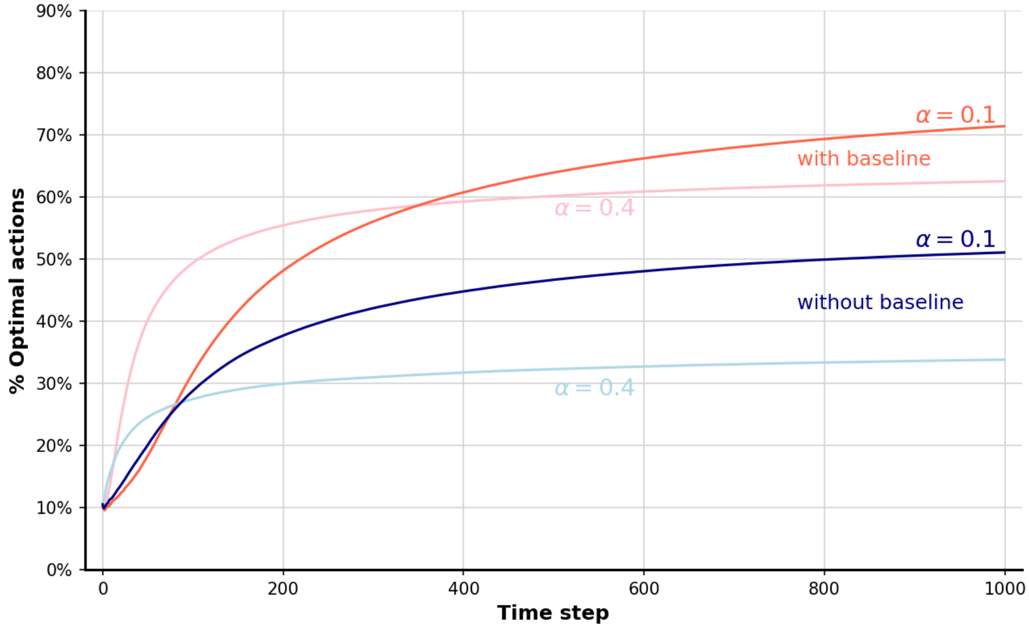
Thus, we get the gradient ascent equation:

$$
H_{t+1}(a) = H_t(a) + \alpha (R_t - \bar{R}_t)(1_{a=A_t} - \pi_t(a)), \quad \text{for all } a
$$

---

**Example:**

Average performance of the gradient bandit algorithm with and without reward baseline.  The reward are chosen from a normal distribution with mean=+4 and unit variance:




*Implementation:*

```
def update_policy(H:np.array) -> np.array:
    return np.exp(H) / np.exp(H).sum()

def update_H(H:np.array,
                policy:np.array,
                alpha:float,
                A:int,
                curr_reward:float,
                avg_reward:float) -> np.array:
    selec = np.zeros(len(H), dtype=np.float32)
    selec[A] = 1.0
    H = H + alpha * (curr_reward - avg_reward) * (selec - policy)
    return H

# running the k-armed bandit algorithm
def run_bandit(K:int,
            q_star:np.array,
            rewards:np.array,
            optim_acts_ratio: np.array,
            alpha: float,
            baseline:bool,
            num_steps:int=1000) -> None:

    H = np.zeros(K, dtype=np.float32) # initialize preference
    policy = np.ones(K, dtype=np.float32) / K
    ttl_reward = 0
    ttl_optim_acts = 0

    for i in range(num_steps):

        A = np.random.choice(np.arange(K), p=policy)
        reward, is_optim = bandit(q_star, A)
        avg_reward = 0
```

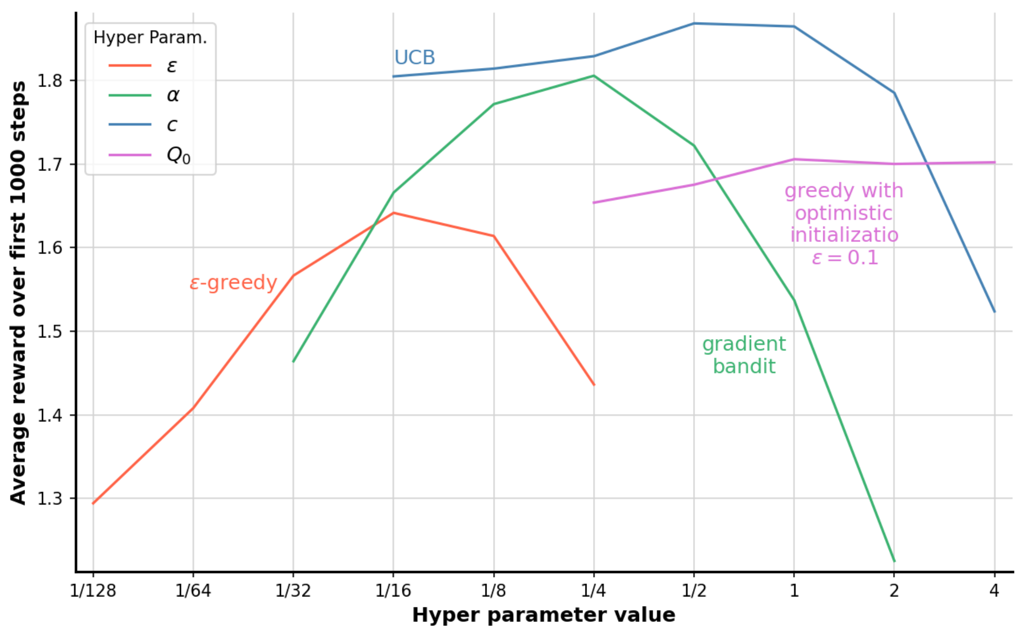
```
        if baseline:
            # Get average reward unitl timestep=i
            avg_reward = ttl_reward / i if i > 0 else reward

        # Update preference and policy
        H = update_H(H, policy, alpha, A, reward, avg_reward)
        policy = update_policy(H)

        ttl_reward += reward
        ttl_optim_acts += is_optim
        rewards[i] = reward
        optim_acts_ratio[i] = ttl_optim_acts / (i + 1)
```

## 2.9 Summary

It is natural to ask which of the method in this chapter is the best; it is a difficult question to answer, but we can run them all on the 10-armed bandit testbed and compare their performance

**Example:**

A *parameter study* of the various bandit algorithm in this chapter, each as a function of its own parameter shown on a single scale on the x-axis.




*Implementation:*

```
import numpy as np
from matplotlib import pyplot as plt
from collections import namedtuple
import pickle
```

```
from example_2_2_bandits_algo import run_bandit as e_greedy
from example_2_3_OIV import run_bandit as OIV
from example_2_4_UCB import run_bandit_UCB as UCB
from example_2_5_gradient import run_bandit as gradient


SEED = 200
np.random.seed(SEED)

# A wraper function for running differen algorithms
def run_algorithm(fn_name:str,
                    fn:'function',
                    params:np.array,
                    args:dict,
                    total_rounds:int) -> np.array:

    if fn_name == 'e_greedy':
        hyper_param = 'epsilon'
    elif fn_name == 'ucb':
        hyper_param = 'c'
    elif fn_name == 'gradient':
        hyper_param = 'alpha'
    elif fn_name == 'oiv':
        hyper_param = 'init_val'

    args[hyper_param] = None

    rewards_hist = np.zeros(shape=(len(params), total_rounds, args['num_steps']))
    optm_acts_hist = np.zeros_like(rewards_hist)
    for i, param in enumerate(params):
        args[hyper_param] = param
        for curr_round in range(total_rounds):
            fn(**args,
               rewards=rewards_hist[i, curr_round],
               optim_acts_ratio=optm_acts_hist[i, curr_round])

    return rewards_hist.mean(axis=1).mean(axis=1)


if __name__ == "__main__":
    K = 10
    num_steps = 1000
    total_rounds = 2000
    q_star = np.random.normal(loc=0, scale=1.0, size=K)

    # Creating parameter array: [1/128, 1/64, 1/32, 1/16, ...]
    multiplier = np.exp2(np.arange(10))
    params = np.ones(10) * (1 / 128)
    params *= multiplier
    x_labels = ['1/128', '1/64', '1/32', '1/16', '1/8', '1/4', '1/2', '1', '2', '4']

    # Creating a dict to record running histories
    records = {'params': params,
               'x_labels': x_labels}
    history = namedtuple('history', ['bounds', 'data'])

    base_args = {
        'K': K,
        'q_star': q_star,
        'num_steps': num_steps
    }

    # ======== e_greedy ========
    eps_bounds = [0, 6]
    fn_params = params[eps_bounds[0]: eps_bounds[1]]
```

```
eps_rewards = run_algorithm('e_greedy', e_greedy, fn_params, base_args.copy(), total_rounds)
records['e_greedy'] = history(eps_bounds, eps_rewards)


# ======== UCB ========
ucb_bounds = [3, 10]
fn_params = params[ucb_bounds[0]: ucb_bounds[1]]

ucb_rewards = run_algorithm('ucb', UCB, fn_params, base_args.copy(), total_rounds)
records['ucb'] = history(ucb_bounds, ucb_rewards)


# ======== Gradient ========
gd_bounds = [2, 9]
fn_params = params[gd_bounds[0]:gd_bounds[1]]
gd_args = base_args.copy()
gd_args['baseline'] = True

gd_rewards = run_algorithm('gradient', gradient, fn_params, gd_args, total_rounds)
records['gradient'] = history(gd_bounds, gd_rewards)


# ======== OIV ========
oiv_bounds = [5, 10]
fn_params = params[oiv_bounds[0]:oiv_bounds[1]]
oiv_args = base_args.copy()
oiv_args['epsilon'] = 0.1
oiv_rewards = run_algorithm('oiv', OIV, fn_params, oiv_args, total_rounds)
records['oiv'] = history(oiv_bounds, oiv_rewards)


with open('./history/summary.pkl', 'wb') as f:
    pickle.dump(records, f)
```

> Despite the simplicity of these algorithms, the methods presented in this chapter can fairly be considered the state of art. There are more sophisticated methods, but their complexity and assumptions make them impractical for the full reinforcement learning problem that is our real focus.

**Exercise:**

Make a figure analogous to the previous figure for the non-stationary case in Exercise 2.5. Include the constant-step-size $\varepsilon$-greedy algorithm with $\alpha$=0.1. Use runs of 200,000 steps and, as a performance measure for each algorithm and parameter setting, use the average reward over the last 100,000 steps.